Fundamentals of digital systems

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A description or preface here

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Chapter 1

Number Systems

1.1 Digital Representations

In digital representation, a number is represented by an ordered n-tuple (a Digit-vector string) :

In a digital representation, a number is represented by an ordered n-tuple:

The n-tuple is called a **digit vector**, each element is a **digit**

The number of digits n is called the precision of the representation. (careful! leftward indexing)

$$X = (X_{n-1}, X_{n-1}, \ldots, X_0)$$

Each digit is given **a set of values** D_i (eg. For base 10 representation of numbers, $D_i = \{0, 1, 2, \ldots, 9\}$)

the **Set size**, the maximum number of representable digit vectors is : $K = \prod_{i=0}^{n-1} |D_i|$

1.1.1 (Non)Redundant Number Systems

A number system is nonredundant if each digit-vector represents a different integer

1.1.2 Weighted Number Systems

The rule of representation if a Weighted (Positional) Number Systems is as follows : $\sum_{i=0}^{n-1} X_i W_i$ where $W = (W_{n-1}, W_{n-2}, \ldots, W_0)$

1.1.3 Radix Systems

When weights are in this format :

$$\begin{cases} W_0 = 1 \\ W_{i+1} = W_i R_i \text{ with } 1 \leq i \leq n-1 \end{cases}$$

Also written : $W_0 = 1,\ \prod_{j=0}^{i-1} R_j$

1.1.4 Fixed and Mixed-Radix Number Systems

In a **fixed-radix system**, all elements of the radix-vector have the same value r (*the radix*)

The weight vector in a fixed-radix system is given by:

$$W = \left(r^{n-1}, r^{n-2}, \ldots, r^2, r, 1\right)$$

and the integer x becomes:

$$x = \sum_{i=0}^{n-1} X_i \times r^i$$

In a **mixed-radix system**, the elements of the radix-vector differ

Example: Decimal Number System

The decimal number system has the following characteristics:

- Radix $r = 10$, indicating it is a fixed-radix system.

The weight vector W is defined as:

$$W = \left(10^{n-1}, 10^{n-2}, \ldots, 10^2, 10, 1\right)$$

An integer x in this system is represented by:

$$x = \sum_{i=0}^{n-1} X_i \times 10^i$$

For example:

$$854703 = 8 \times 10^5 + 5 \times 10^4 + 4 \times 10^3 + 7 \times 10^2 + 0 \times 10^1 + 3 \times 10^0$$

In contrast, the representation can be fixed or mixed:

Fixed: This category includes:

- Decimal – radix 10
- Binary – radix 2
- Octal – radix 8
- Hexadecimal – radix 16

Mixed: An example of mixed representation is the way time is represented in terms of hours, minutes, and seconds:

- Radix-vector $R = (24, 60, 60)$
- Weight-vector $W = (3600, 60, 1)$

1.1.5 Canonical Number Systems

In a **canonical number system**, the set of values for a digit D_i is with $|D_i| = R_i$, the corresponding element of the radix vector

$$D_i = \{0, 1, \ldots, R_i - 1\}$$

Canonical digit sets with fixed radix:

- Decimal: {0, 1, ..., 9}
- Binary: {0, 1}
- Hexadecimal: {0, 1, 2, ..., 15}

Range of values of x represented with n fixed-radix-r digits:

$$0 \leq x \leq r^n - 1$$

A system with fixed positive radix r and a canonical set of digit values is called a radix-r conventional number system.

1.2 Binary/Octal/Hexadecimal to/from Decimal

Conversion Table

The hexadecimal system supplements 0-9 digits with the letters A-F.

Remark. Programming languages often use the prefix 0x to denote a hexadecimal number.

Table 1.1: Conversion table up to 15.

Decimal	**Binary**	**Octal**	**Hexadecimal**
0	0000	00	0
1	0001	01	1
2	0010	02	2
3	0011	03	3
4	0100	04	4
5	0101	05	5
6	0110	06	6
7	0111	07	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

To convert a binary number to hexadecimal, group the binary digits in sets of four, starting from the right. Each group of four binary digits corresponds to one hexadecimal digit.

Example:

$$\begin{aligned} \text{Binary:}\quad & \underline{1010}\,\underline{1100} \\ \text{Hexadecimal:}\quad & A\,C \quad (\text{Since } \underline{1010}_2 = A_{16} \text{ and } \underline{1100}_2 = C_{16}) \end{aligned}$$

To convert a binary number to octal, group the binary digits in sets of three, also starting from the right. Each group of three binary digits corresponds to one octal digit.

Example:

$$\begin{aligned} \text{Binary:}\quad & \underline{100}\,\underline{101}\,\underline{110} \\ \text{Octal:}\quad & 4\,5\,6 \quad (\text{Since } \underline{100}_2 = 4_8, \underline{101}_2 = 5_8, \text{ and } \underline{110}_2 = 6_8) \end{aligned}$$

Chapter 2

Literature Review

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2.1 Previous Work

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