Algorithms - CheatSheet

IN BA4 - Ola Nils Anders Svensson Notes by Ali EL AZDI

This is a cheat sheet for the Algorithms midterm exam. For suggestions, contact me on $Telegram\ (elazdi_al)$ or via $EPFL\ email\ (ali.elazdi@epfl.ch)$.

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\label{eq:master Theorem} \begin{array}{l} \textbf{Master Theorem} \\ \textbf{If } T(n) = aT \binom{n}{\delta} + f(n), a \geq 1, b > 1, \text{ and } f(n) \text{ asymptotically positive.} \\ \textbf{Case 1: } \textbf{If } f(n) = O(n^{\log_b a - \varepsilon}) \text{ for some } \varepsilon > 0, \text{ then } T(n) = \Theta(n^{\log_b a}). \\ \textbf{Case 2: } \textbf{If } f(n) = \Theta(n^{\log_b a}), \text{ then } T(n) = \Theta(n^{\log_b a}) \text{ some } \varepsilon > 0, \text{ and } \text{ if } af \left(\frac{n}{\delta}\right) \leq c f(n) \text{ for some } \varepsilon < 1 \\ \textbf{Case 3: } \textbf{If } f(n) = \Omega(n^{\log_b a + \varepsilon}) \text{ for some } \varepsilon > 0, \text{ and } \text{ if } af \left(\frac{n}{\delta}\right) \leq c f(n) \text{ for some } c < 1 \\ \textbf{Case 3: } \textbf{If } f(n) = \Omega(n^{\log_b a + \varepsilon}) \text{ for some } \varepsilon > 0, \text{ and } \text{ if } af \left(\frac{n}{\delta}\right) \leq c f(n) \text{ for some } c < 1 \\ \textbf{Case 3: } \textbf{If } f(n) = \Omega(n^{\log_b a + \varepsilon}) \text{ for some } \varepsilon > 0, \text{ and } \text{ if } af \left(\frac{n}{\delta}\right) \leq c f(n) \text{ for some } c < 1 \\ \textbf{Case 3: } \textbf{If } f(n) = \Omega(n^{\log_b a - \varepsilon}) \text{ for some } \epsilon > 0, \text{ and } \text{ if } af \left(\frac{n}{\delta}\right) \leq c f(n) \text{ for some } c < 1 \\ \textbf{Case 3: } \textbf{If } f(n) = \Omega(n^{\log_b a - \varepsilon}) \text{ for some } c < 1 \\ \textbf{Case 3: } \textbf{If } f(n) = \Omega(n^{\log_b a - \varepsilon}) \text{ for some } c < 1 \\ \textbf{Case 3: } \textbf{If } f(n) = \Omega(n^{\log_b a - \varepsilon}) \text{ for some } c < 1 \\ \textbf{Case 3: } \textbf{If } f(n) = \Omega(n^{\log_b a - \varepsilon}) \text{ for some } c < 1 \\ \textbf{Case 3: } \textbf{If } f(n) = \Omega(n^{\log_b a - \varepsilon}) \text{ for some } c < 1 \\ \textbf{Case 3: } \textbf{If } f(n) = \Omega(n^{\log_b a - \varepsilon}) \text{ for some } c < 1 \\ \textbf{Case 3: } \textbf{If } f(n) = \Omega(n^{\log_b a - \varepsilon}) \text{ for some } c < 1 \\ \textbf{Case 3: } \textbf{If } f(n) = \Omega(n^{\log_b a - \varepsilon}) \text{ for some } c < 1 \\ \textbf{Case 3: } \textbf{If } f(n) = \Omega(n^{\log_b a - \varepsilon}) \text{ for some } c < 1 \\ \textbf{Case 3: } \textbf{If } f(n) = \Omega(n^{\log_b a - \varepsilon}) \text{ for some } c < 1 \\ \textbf{Case 3: } \textbf{If } f(n) = \Omega(n^{\log_b a - \varepsilon}) \text{ for some } c < 1 \\ \textbf{Case 3: } \textbf{If } f(n) = \Omega(n^{\log_b a - \varepsilon}) \text{ for some } c < 1 \\ \textbf{Case 3: } \textbf{If } f(n) = \Omega(n^{\log_b a - \varepsilon}) \text{ for some } c < 1 \\ \textbf{Case 3: } \textbf{If } f(n) = \Omega(n^{\log_b a - \varepsilon}) \text{ for some } c < 1 \\ \textbf{Case 3: } \textbf{If } f(n) = \Omega(n^{\log_b a - \varepsilon}) \text{ for some } c < 1 \\ \textbf{Case 3: } \textbf{If } f(n) = \Omega(n^{\log_b a - \varepsilon}) \text{ for some } c < 1 \\ \textbf{Case 3: } \textbf{If } f(n) = \Omega(n^{\log_b a - \varepsilon}) \text{ for some } c < 1 \\ \textbf{Case 4: } \textbf{Case 4: } \textbf{Case 4: } \textbf{Cas
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  If d > 0 and a n_0 > 0, 0 \le f \le n, 0 \le f \le n. Big-Onega If d > 0 and d n_0 > 0, 0 \le c \cdot g(n) \le f(n) \ \forall n \ge n_0, f(n) = \Omega(g(n)). Big-Theta
                                                                                                                                                                                                                                                                                                Then:
                                                                                                                                                                                                                                                                                                                                                                             p > d \Rightarrow T(n) = \Theta(n^p),
                                                                                                                                                                                                                                                                                                                                                                         p = d \Rightarrow T(n) = \Theta(n^d \log n),
  Case 5: If J(n) = s_2(n^{-d-1}) to some \varepsilon > 0, and if a_J(\frac{\pi}{b}) \le \varepsilon f(n) I and all sufficiently large n, then T(n) = \Theta(f(n)).

Common case - if f(n) = \Theta(n^d) for some exponent d:

- If \frac{a}{b^d} < 1 (or d > \log_b a), then T(n) = \Theta(n^d).
                                                                                                                                                                                                                                                                                                                                                                               p < d \Rightarrow T(n) = \Theta(n^d)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  Little-o If \forall c > 0 \ \exists n_0 > 0, \ 0 \le f(n) < c \cdot g(n) \ \forall n \ge n_0, \ f(n) = o(g(n)).
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                The second of t
                                                                                                                                                                                                                                                                                                If p is not easily found, compare \alpha a^d + \beta b^d with 1:
                                                                                                                                                                                                                                                                                             with p determined by \alpha a^p + \beta b^p with 1: \alpha a^d + \beta b^d < 1 \Rightarrow T(n) = \Theta(n^d), \alpha a^d + \beta b^d = 1 \Rightarrow T(n) = \Theta(n^d \log n), \alpha a^d + \beta b^d > 1 \Rightarrow T(n) = \Theta(n^p), with p determined by \alpha a^p + \beta b^p = 1.
  - If \frac{a}{b^d} = 1 (or d = \log_b a), then T(n) = \Theta(n^d \log n).
   If \frac{a}{b^d} > 1 (or d < \log_b a), then T(n) = \Theta(n^{\log_b a}).
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    \ll O(c^n) \ll O(n!) \ll O(n^n)
 Queue Operations
Queue-Empty(Q): Time: O(1), Auxiliary Space: O(1)

1. Returns TRUE if the queue is empty (Q.head = Q.tail).

2. Returns FALSE otherwise.

Enqueue(Q, x): Time: O(1), Auxiliary Space: O(1)

1. Adds element x to the rear of queue Q.

2. Q[Q,tail] = x

3. Q,tail = Q,tail + 1 (or wrap around if using circular array)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           MAX-HEAPIFY(A, i, n)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  Heap Root is A[1] Left(i) = 2i Right(i) = 2i + 1 Parent(i) = |i/2|
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          MAX-HEAPIF

l \leftarrow \text{LEFT}(i)

r \leftarrow \text{RIGHT}(i)

if l \le n \land A[l] > largest \leftarrow l

else

largest \leftarrow i

if r \le n \land A[l] > largest
                                                                                                                                                                                                                                                                                          Insertion Sort.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     Max-Heapify (heapify subtree rooted at i)
                                                                                                                                                                                                                                                                                                 - Select the key
                                                                                                                                                                                                                                                                                                                                                                                                                       INSERTION-SORT(A, n)

for j = 2 to n do

key = A[j] // Insert A[j] in the sorted :

A[1..j-1].

Starting at the root

Compare A[i], A[Left(i)], A[Right(i)]

Max-Heapify the swapped child

Max-Heapify the swapped child

Continue comparing and swapping down the heap until subtree rooted at is max-heap

Time Complexity: O(log(n))
                                                                                                                                                                                                                                                                                           Begin with the
                                                                                                                                                                                                                                                                                                                                                econd element (at in-
                                                                                                                                                                                                                                                                                           dex 1) as the key
                                                                                                                                                                                                                                                                                        dex 1) as the key.

2 - Compare and Shift
Compare the key with elements in the sorted section (to its left).

3 - Shift Elements
If an element is greater than the key, shift that element one position to the right.
                                                                                                                                                                                                                                                                                                                                                                                                                            a_i \cdot a_j \cdot f:

i = j \cdot 1

while i > 0 and A[i] > \text{key do}

A[i + 1] = A[i]

i = i \cdot f
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          largest \leftarrow i

largest \leftarrow r

largest \leftarrow r

if largest \neq i

exchange A[i] with A[largest]

MAX-HEAPIFV(A, largest, n)

MAX-HEAPINSERT(A, key, n)
 3. Q.tan = Q.tan + 1 (or wrap around it using circular arr Dequeue(Q): Time: O(I), Auxiliary Space: O(I)
1. If Queue-Empty(Q), return error "underflow".
2. Otherwise, remove and return the element at the front.
3. x = Q[Q.head]
4. Q.head = Q.head + 1 (or wrap around)
                                                                                                                                                                                                                                                                                                                                                                                                                                i = i - 1

A[i + 1] = key
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               Time Complexity: O(\log(n)) Space Complexity: O(n) including heap array, O(1) auxiliary Max-Heap-Insert (insert new key into heap)

1. Increase heap size: A.heap-size = A.heap-size + 1

2. Set the last element to negative infinity: A[A.heap-size] = -\infty

3. Call Max-Heap-Increase-Key to update to the correct value Time Complexity: O(\log n)

Space Complexity: O(\log n)

Space Complexity: O(n) including heap array, O(1) auxiliary Max-Heap-Increase-Key (increase key at position i)

1. Ensure new key is larger than current: f(xe) < A[i] then error

2. Set A[i] = key

3. Compare with parent and swap if necessary: while i_i 1 and A[Parent(i)] 2. Exchange A[i] with A[Parent(i)]

5. Set i = Parent(i) and continue upward Time Complexity: O(\log n)

Space Complexity: O(\log n)
 4. Q.nead = Q.nead + 1 (or wrap around)
5. Return x
Queue Implementation:
1. Q.head: Index of the front element
2. Q.tail: Index where next element will be inserted
3. In a circular array, indices wrap around
4. Leave one slot empty to distinguish full/empty states
Overall Space Complexity: O(n) for a queue of capacity n
                                                                                                                                                                                                                                                                                           A- Insert the Key
Once an element less than or equal to the key is found (or you reach the start), insert the key immediately after that element.

5 - Repeat
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            HEAP-INCREASE-KEY(A, n, key)
                                                                                                                                                                                                                                                                                            Move forward to the next element, treating it as the new key, and repeat until the
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           HEAP-INCREASE-KEY(A, i, key)
                                                                                                                                                                                                                                                                                            array is sorted.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          HEAP-INCKEASE-KEY(A, i, key) if key < A[i] error "new key is smaller than curre key" A[i] \leftarrow key while i > 1 and A[Parent(i)] < A[i] do exchange A[i] with A[Parent(i)] i \leftarrow Parent(i)
                                                                                                                                                                                                                                                                                            Time Complexity: Worst-case O(n^2), Best-case O(n).
  Merge Sort

1. Divide: Split the array evenly into two smaller
                                                                                                                                                                                                                                                                                            Space Complexity: O(1).
                                                                                                                                                                                                                                                                                             Strassen's Matrix Multiplication
Divide: Partition each of A,B,C into four \frac{n}{2} \times \frac{n}{2} submatrices:
  subarrays, and continue dividing recursively.

2. Sort (Recursively): Apply merge sort recursively on each subarray until each has only one element
                                                                                                                                                                             MERGE(A, p, q, r)
                                                                                                                                                                                                                                                                                               \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} \, = \, \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \cdot \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}.
                                                                                                                                                                              MERGER, p, q, r, n_1 \leftarrow q - p + 1

n_2 \leftarrow r - q

let L[1...n_1 + 1] and R[1...n_2 + 1]

be new arrays

for i \leftarrow 1 to n_1 do

L[i] \leftarrow A[p + i - 1]
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 Time Complexity: O(\log n)
Space Complexity: O(n) including heap array, O(1) auxiliary
Build-Max-Heap (build a max-heap from an array)
1. Start from the last non-leaf node at index \frac{n}{2}-1
 (base case). 

MERGE-SORT(A, p, r) 

if p < r 

q \leftarrow \lfloor (p+r)/2 \rfloor 

MERGE-SORT(A, p, q) 

MERGE-SORT(A, q + 1, r) 

MERGE-R, p, q + 1, r) 

3. Merge: Combine the two sorted subarrays into a
                                                                                                                                                                                                                                                                                              \begin{array}{ll} \textbf{Conquer:} \ \textbf{Compute 7} \ \textbf{products} \ (\textbf{re-cursively on} \ \frac{\alpha}{2} \times \frac{g}{2} \ \textbf{matrices}) : \\ \textbf{M}_1 := (A_{11} + A_{22})(B_{11} + B_{22}), \\ \end{array} \qquad \begin{array}{ll} \textbf{Combine:} \ \textbf{Assemble the resulting submatrices to form} \ C: \\ \textbf{C}_{11} &= M_1 + M_4 - M_5 + M_7, \end{array} 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            BUILD-MAX-HEAP(A, n)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              for i \leftarrow \lfloor n/2 \rfloor to 1 do

Max-Heapify(A, i, n)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 2. Move upwards to the root (index 0) and:
                                                                                                                                                                                                                                                                                                                                                                                                                       C_{21} = M_2 + M_4
                                                                                                                                                                                                                                                                                               M_2 := (A_{21} + A_{22})B_{11}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  a. Max-Heapify the current node
                                                                                                                                                                                for j \leftarrow 1 to n_2 do

R[j] \leftarrow A[q+j]
                                                                                                                                                                                                                                                                                                                                                                                                                       C_{12} = M_3 + M_5
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 b. Ensure the subtree rooted here satisfies max-heap property
                                                                                                                                                                                                                                                                                             M_3 := A_{11} (B_{12} - B_{22}),
 3. Merge: Combine the two sorted subarrays into a single sorted array;

a) Initializing pointers at the start of each subarray.
b) Comparing the elements pointed to, and appending the smaller one into a new array.
c) Advancing the pointer in the subarray from which the element was chosen.
d) Repeating this process until all elements in both subarrays are merged into the sorted array.

Merge Cost Complexity: O(n) per merge operation.
The Cost of the process of the p

    Repeat until the root node is processed
    After completion, array A represents a valid max heap

                                                                                                                                                                                L[n_1 + 1] \leftarrow \infty

R[n_2 + 1] \leftarrow \infty
                                                                                                                                                                                                                                                                                                                                                                                                                        C_{22} = M_1 + M_3 - M_2 + M_6.
                                                                                                                                                                                                                                                                                             M_4 := A_{22} (B_{21} - B_{11}),
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               4. After completion, array A represents a valid max heap Time Complexity: O(n) including heap array, O(1) auxiliary Heap Sort

    Build a Max Heap:
    Convert the given array into a max heap
    Start from the last non-leaf node and heapify upwards
    Ensure each parent node is greater than its children
    Extract Maximum Elements:
    Swap the root (maximum value) with the last element

                                                                                                                                                                                                                                                                                                                                                                                                                        Time Complexity: O(n^{\log_2 7}) \approx O(n^{2.81})
                                                                                                                                                                                                                                                                                             M_5 := (A_{11} + A_{12})B_{22},
                                                                                                                                                                                                                                                                                                                                                                                                                       Space Complexity: O(n^2)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             HEAPSORT(A, n)
BUILD-MAX-HEAP(A, n)
for i = n to 2 do
                                                                                                                                                                                   \begin{aligned} & j \leftarrow 1 \\ & \text{for } k \leftarrow p \text{ to } r \text{ do} \\ & \text{if } L[i] \leq R[j] \\ & A[k] \leftarrow L[i] \\ & i \leftarrow i+1 \\ & \text{else} \\ & A[k] \leftarrow R[j] \\ & j \leftarrow j+1 \end{aligned} 
                                                                                                                                                                                                                                                                                             M_6 := (A_{21} - A_{11})(B_{11} + B_{12}),
                                                                                                                                                                                                                                                                                             M_7 := (A_{12} - A_{22})(B_{21} + B_{22}).
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                exchange A[1] with A[i]

MAX-HEAPIFY(A, 1, i-1)
                                                                                                                                                                                                                                                                                           Binary Search Trees (BST)
BST-Search
                                                                                                                                                                                                                                                                                                                                                                                                                     BST-Postorder
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             HEAP-EXTRACT-MAX(A,n)
                                                                                                                                                                                                                                                                                                                                                                                                                   1. Recursively traverse left
2. Recursively traverse right
3. Visit current node
(Children first, then root)

    Start at root
    If NULL, return NULL

 Time Complexity: O(n \log n)
Space Complexity: O(n)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              if n < 1
error "heap underflow"

    Swap the root (maximum value) with the last element
    Reduce heap size by one to exclude the last element
    Heapify the root to maintain max heap property

                                                                                                                                                                                                                                                                                          2. If NOLL, return NOLL
3. If key = root's key, return root
4. If key < root's key, search left
5. If key > root's key, search right
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              max \leftarrow A[1]

A[1] \leftarrow A[n]
 Priority Queue
Maintains a dynamic set of elements with associated priority values (keys).

Maximum(S): Return element of S with highest priority (return A[1], complexity O(1))
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                c. Heaptly the root to maintain max heap property d. Repeat until heap size becomes 1

3. Final Sorted Array:
a. After extraction, the sorted array in ascending order is obtained b. Maximum elements are placed at the end Time Complexity: O(nlogn)

Space Complexity: O(n) including array, O(1) auxiliary
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              n \leftarrow n-1

Max-Heapify(A,1,n)
                                                                                                                                                                                                                                                                                                                                                                                                                            POSTORDER-TREE-WALK(x)
                                                                                                                                                                                                                                                                                                                                                                                                                  at x \neq \text{NIL}
Postorder-Tree-Walk(x.left)
Postorder-Tree-Walk(x.right)
Print key(x]
Time: O(n)
Space O(n)
                                                                                                                                                                                                                                                                                                  TREE-SEARCH(x,k)
                                                                                                                                                                                                                                                                                                   TREE-SEARCH(x, k)

if x = \text{NIL or } k = \text{key}[x]

return x

else if k < x.key

return TREE-SEARCH(x.left, k)

else

return TREE-SEARCH(x.right, k)
Insert(S,x): Insert element x into set S

1. Increment the heap size

2. Insert a new node in the last position in the heap, with key -∞

3. Increase the -∞ value to key using Heap-Increase-Key

Extract-Max(S): Remove and return element of S with highest priority

1. Make sure heap is not empty

2. Make a copy of the maximum element (the root)

3. Make the last node in the tree the new root

4. Re-heapify the heap, with one fewer node

5. Return the copy of the maximum element

Increase-Key(S,x,k): Increase the value of element x's key to the new value k

1. Make sure key ≥ A[i]

2. Update A[i]'s value to key

3. Traverse the tree upward comparing new key to the parent and swapping if necessary

Time Complexity: Insert, Extract-Max, Increase-Key: O(logn) Maximum: O(1)
   Insert(S,x): Insert element x into set S
                                                                                                                                                                                                                                                                                                                                                                                                                     Space: O(n) for tree, O(h) auxiliary
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  FIND-MAX-SUBARRAY(A, low, high)
if high == low
return (low, high, A[low])
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              Maximum Subarray Problem
Problem: Find contiguous subarray with largest
                                                                                                                                                                                                                                                                                                                                                                                                                     BST-Inorder
                                                                                                                                                                                                                                                                                           return TREE-SEARCH(x.ngnt,k)

Time: O(\log n) avg, O(h) worst

Space: O(n) for tree, O(h) auxiliary

BST-Minimum

1. Recursively traverse left
2. Visit current node
3. Recursively traverse right
(Visits nodes in sorted order)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  sum
Divide and Conquer Approach:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              Divide and Conquer Approach:

1 - Divide.

Split array at midpoint mid = [(low + high)/2].

2 - Conquer.

Find maximum subarrays recursively Left maximum subarrays recursively.

    Start at root
    If NULL, return NULL
    Follow left pointers until no left

                                                                                                                                                                                                                                                                                                                                                                                                                           {\tt INORDER-TREE-WALK}(x)
                                                                                                                                                                                                                                                                                          child
4. Return leftmost node
                                                                                                                                                                                                                                                                                                                                                                                                                     if x ≠ NIL

INORDER-TREE-WALK(x.left)

print key[x]

INORDER-TREE-WALK(x.right)

Time: O(n)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     low, mid, high)

if left-sum ≥ right-sum and left-sum ≥ cross-sum

return (left-low, left-high, left-sum)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         m retr-sum ≥ right-sum and lett-sum ≥ cross-sum

return (left-low, left-high, left-sum)

else if right-sum ≥ left-sum and right-sum ≥ cross-sum

return (right-low, right-high, right-sum)

else
                                                                                                                                                                                                                                                                                                  TREE-MINIMUM(x)

while x.left \neq NIL do

x \leftarrow x.left
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               Left max: in A[low...mid]
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              Right max: in A[mid + 1...high]
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   rossing max: spans the midpoint

- Combine: Return the largest of the three hax(left_max,right_max,crossing_max)
  Time Complexity: Insert, Extract-Max, Increase-Key: O(log n)
                                                                                                                                                                                                                                                                                                                                                                                                                     Space: O(n) for tree, O(h) auxiliary
   Space Complexity: O(n)
                                                                                                                                                                                                                                                                                                    return x
ne: O(h)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             return (cross-low, cross-high, cross-sum)
FIND-MAX-CROSSING-SUBARRAY(A, low, mid, high)
                                                                                                                                                                                                                                                                                                                                                                                                                     BST-Preorder
                                                                                                                                                                                                                                                                                                                                                                                                                   1. Visit current node
2. Recursively traverse left
3. Recursively traverse right
(Root first, then children)
  Stack Operations
                                                                                                                                                                                                                                                                                             Space: O(n) for tree, O(1) auxiliary
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      Finding the Crossing Maximum:
   Stack-Empty(S): Time: O(1). Auxiliary Space: O(1)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               left-sum \leftarrow -\infty, sum \leftarrow 0

for i = mid to low do

sum \leftarrow sum + A[i]
 Stack-Empty(S): Time: O(1), Auxiliary Space:
1. Returns TRUE if the stack is empty.
2. Returns FALSE otherwise.
Push(S, x): Time: O(1), Auxiliary Space: O(1)
1. Adds element x to the top of stack S.
2. Increments the stack pointer.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   Find maximum suffix in left half (from mid down to low)
Find maximum prefix in right half (from mid+1 up to high)
Crossing max = max suffix + max prefix
                                                                                                                                                                                                                                                                                           BST-Maximum

    Start at root
    I. Start at root
    If NULL, return NULL
    Follow right pointers until no right

                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 if sum > left-sum

left-sum \leftarrow sum, max-left \leftarrow i
                                                                                                                                                                                                                                                                                                                                                                                                                           PREORDER-TREE-WALK(x)
if x ≠ NIL
print key[x]
PREORDER-TREE-WALK/>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               Crossing Subarray: The maximum subarray that crosses the 
midpoint combines the maximum suffix of the left half with the
                                                                                                                                                                                                                                                                                          child
4. Return rightmost node
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               right-sum \leftarrow -\infty, sum \leftarrow 0

for j = mid + 1 to high do
                                                                                                                                                                                                                                                                                                                                                                                                                   PREORDER-TREE-WALK(x.left)
PREORDER-TREE-WALK(x.right
Time: O(n)
  Pop(S): Time: O(1), Auxiliary Space: O(1)
                                                                                                                                                                                                                                                                                           TREE-MAXIMUM(x)

while xright ≠ NIL do

x ← xright
return x.

Time: O(h)

Space: O(n) for tree, O(1) auxiliary
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                mum prefix of the right half.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 sum \leftarrow sum + A[j]

if sum > right-sum
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               Time Complexity: \Theta(n \log n) due to T(n) = 2T(n/2) + \Theta(n)

    If Stack-Empty(S), return error "underflow

                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               \begin{aligned} \textit{right-sum} \leftarrow \textit{sum}, & \textit{max-right} \leftarrow j \\ \textbf{return} & (\textit{max-left}, \textit{max-right}, \textit{left-sum} + \textit{right-sum}) \end{aligned}

    Otherwise, remove and return the top element.
    Decrements the stack pointer.

                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               Space Complexity: O(\log n) for recursion stack
                                                                                                                                                                                                                                                                                                                                                                                                                     Space: O(n) for tree, O(h) auxiliary
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 Data Structure Operations Summary
Priority Queue Operations

Operation
Construction

Descript
Construction
Create pi

Stack Implementation:

1. Elements are stored in a simple array
2. S.top: Index of the topmost element
3. An empty stack has S.top = 0 or S.top = -1 (implementation dependent)

Overall Space Complexity: O(n) for a stack of size n
                                                                                                                                                                                                                                                                                                                                                                                                                     BST-Transplant
                                                                                                                                                                                                                                                                                                                                                                                                                           eplace subtree at u with v:

If u is root, set v as root

If u is left child, make v left child o
                                                                                                                                                                                                                                                                                           BST-Successor
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              Description
Create priority queue from array of n ele-
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       Time
O(n)

    If right subtree exists:

                                                                                                                                                                                                                                                                                            Return minimum in right subtree
2. Otherwise:
Find first ancestor where
node is in left subtree
                                                                                                                                                                                                                                                                                                                                                                                                                     u's parent
3. Else make v right child of u's paren
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               ments
Returns element with highest priority
Removes and returns element with highest
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         Maximum(S)
Extract-Max(S)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       O(1)
O(1)
 Dynamic Programming
Problem: Optimal solutions to overlap- Cut-Rod Problem:
                                                                                                                                                                                                                                                                                                                                                                                                                     4. Set v's parent to u's paren
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 Removes and returns element with
priority
Inserts element x into set S
Increases priority of element x to k
                                                                                                                                                                                                                                                                                                                                                                                                                              TRANSPLANT(T. n. v)
                                                                                                                                                                                                                                                                                                 TREE-SUCCESSOR(x)
if x.right ≠ NIL
return TREE-MINIMUM(x.right)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          Insert(S, x)
Increase-Key(S, x, k)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      O(\log n)
O(\log n)
                                                                                                                                            Find optimal way to cut rod to maximize
                                                                                                                                                                                                                                                                                                                                                                                                                            if u.p = \text{NIL}

T.\text{root} \leftarrow v

else if u = u.p.\text{left}

u.p.\text{left} \leftarrow v

else
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      O(1)
   ping subproblems
Fibonacci Sequence:
                                                                                                                                           revenue
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 Stack Operations
Operation
                                                                                                                                           Top-Down (Memoization):
  Top-Down (Memoization):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 Description
Create stack from array of n elements
Returns TRUE if stack is empty, FALSE
                                                                                                                                                                                                                                                                                                     y \leftarrow x.p

while y \neq NIL and x = y.right do
 1. Create memo array F[0...n] initialized to NIL 2. Base cases: F[0] = 0, F[1] = 1
                                                                                                                                                   Create memo array r[0...n] with r[0] = 0
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      Space
O(n)
                                                                                                                                          1. Create memo array r[0...n] with r[0 2. For uncalculated r[j], compute: r[j] = \max_i s_i c_j (p[i] + r[j-i]) 3. Return r[n] 3. Return r[n] with r[n] \geq 0 if r[n] \geq 0 return r[n] if n = 0 q = 0 q = 0
                                                                                                                                                                                                                                                                                                                                                                                                                              else

u.p.right \leftarrow v

if v \neq NIL
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           Stack-Empty(S)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       O(1)
                                                                                                                                                                                                                                                                                                      x \leftarrow y

y \leftarrow y.p
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       0(1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 Adds element x to top of stack S
Removes and returns top element from stack S
  3. Recursive with memo
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         Push(S, x)
Pop(S)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      O(1)
O(1)
                                                                                                                                                                                                                                                                                                                                                                                                                     v.p \leftarrow u.p

Time: O(1)
 S. Recursive with memo:

- Return F[n] if already computed

- Otherwise compute

F[n] = F[n-1] + F[n-2]

- Store result in F[n] and return
                                                                                                                                                                                                                                                                                                                                                                                                                   Space: O(n) for tree, O(1) auxiliary BST-Delete
1. If z has no left: transplant right
                                                                                                                                                                                                                                                                                           Space: O(n) for tree, O(1) auxiliary
                                                                                                                                                                                                                                                                                         Space: O(n) for tree, O(1) auxiliary

Sp. BST-Insert

1. Create new node z with key

2. Start at root, track parent y = NIL

2. Start at root, track parent y = NIL

key, right otherwise)

4. Once NULL found, link z as child b.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 Heap Operations
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             Description
Create heap from array of n elements
Maintains max-heap property at node i
Converts array A of n elements into max
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         Operation
Construction
Max-Heapify(A, i)
Build-Max-
Heap(A,n)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     O(n)
O(1)
O(1)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       Time
                                                                                                                                                                                                                                                                                                                                                                                                                           If z has no right: transplant left
       MEMOIZED-FIB(n)

Let r = [0...n] be a new array

for i = 0 to n do
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      O(n)
O(\log n)
O(n)
                                                                                                                                                                                                                                                                                                                                                                                                                              With both children
                                                                                                                                                                                                                                                                                                                                                                                                                           Find successor y
Handle y's children
Replace z with y
                                                                                                                                                  q = max(q, p[i])

MEMOIZED-CUT-ROD-AUX(p, n-i, r)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  heap
Sorts array A of n elements using heap

    Once NULL found, link z
    of y
    If y is NIL, z becomes root
    Otherwise insert z as left.
           \begin{aligned} r[i] \leftarrow -\infty \\ \textbf{return} \ \text{MEMOIZED-FIB-AUX}(n, r) \end{aligned}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       O(n \log n
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       O(1)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         Heap-Sort(A, n)
                                                                                                                                                                                                                                                                                                                                                                                                                            TREE-DELETE(T, z)
          return MEMOIZED-FIB-AUX(n, r)

MEMOIZED-FIB-AUX(n, r)

if r[n] \ge 0

return r[n]

if n = 0 or n = 1

ans \leftarrow 1

else

ans \leftarrow 1

MEMOIZED-FIB-AUX(n - 1, r) +

MEMOIZED-FIB-AUX(n - 1, r) +
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 Inserts key k into heap A
                                                                                                                                                    \begin{array}{l} r[n] = q \\ \textbf{return} \ q \end{array}

    Otherwise, insert z as left or right child based on key comparison

                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          Max-Heap-Insert(A
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       O(\log n)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       O(1)
                                                                                                                                                                                                                                                                                                                                                                                                                                  TRANSPLANT(T. z. z.right) // z has n
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          k)
Heap-Extract
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  Returns and removes largest element from
                                                                                                                                                    MEMOIZED-CUT-ROD(p, n)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       O(\log n)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       O(1)
                                                                                                                                                                                                                                                                                                  TREE-INSERT(T. z)
                                                                                                                                                                                                                                                                                                                                                                                                                          left child

else if z.right == NIL

TRANSPLANT(T, z, z.left) // z has just
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        Max(A)
Heap-Increas
Key(A,i,k)
                                                                                                                                                     let r[0...n] be a new array

for i = 0 to n do

r[i] = -\infty
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  heap A
Increases key at index i to new value k
                                                                                                                                                                                                                                                                                                     y \leftarrow \text{NIL}

x \leftarrow T.\text{root}

while x \neq \text{NIL} do
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       O(\log n)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       O(1)
                                                                                                                                                                                                                                                                                                                                                                                                                           left child
                                                                                                                                                     return MEMOIZED-CUT-ROD-AUX(n. n.
                                                                                                                                                                                                                                                                                                                                                                                                                              else// z has two children

y \leftarrow TREE-MINIMUM(z.right) // y is z
       MEMOIZED-FIB-AUX(n-2.r
                                                                                                                                                                                                                                                                                                    \begin{aligned} & y \leftarrow x \\ & \textbf{if } z.\text{key} < x.\text{key} \\ & x \leftarrow x.\text{left} \\ & \textbf{else} \\ & x \leftarrow x.\text{right} \end{aligned}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  BST Operations
                                                                                                                                                    ottom-Up (Tabulation):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  Description
Create BST from array of n elements
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   Operation
Construction
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         Time
Best:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       Space O(n)
                                                                                                                                            1. Create array r[0...n] with r[0] = 0
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  O(n \log n)
                                                                                                                                                                                                                                                                                                                                                                                                                                if y.p \neq z

TRANSPLANT(T, y, y.right)
                                                                                                                                            2. For i = 1 to n:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       Worst: O(n^2)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 Finds node with key k in tree T
Returns node with smallest key in T
Returns node with largest key in T
Returns node with smallest key greater than
  1. Create array F[0...n]
2. Base cases: F[0] = 0, F[1] = 1
3. For i = 2 to n:
- Compute F[i] = F[i-1] + F[i-2]
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       O(h)
                                                                                                                                           - Compute r[j] = \max_{1 \le i \le j} (p[i] + r[j-i])
3. Return r[n]
                                                                                                                                                                                                                                                                                                                                                                                                                                  y.right \leftarrow z.right

y.right.p \leftarrow y

TRANSPLANT(T, z, y)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          BST-Search(T, k
                                                                                                                                                                                                                                                                                                    z.p \leftarrow y
if y = NIL
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       BST-Maximum(T
BST-Maximum(T
BST-Successor(x)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     O(1)
O(1)
O(1)
                                                                                                                                                                                                                                                                                                  If y = NIL

T.root \leftarrow z // tree T was empty

else if z.key < y.key

y.left \leftarrow z

else
                                                                                                                                                  EXTENDED-BOTTOM-UP-CUT-ROD(p, n) let r[0..n] and s[0..n] be new arrays
                                                                                                                                                                                                                                                                                                                                                                                                                     Time: O(h)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 Inserts node z into BST T
Removes node z from BST T
Visits all nodes in sorted order

 Return F[n]

                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         BST-Insert(T, z)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     O(1)
                                                                                                                                                     r[0] \leftarrow 0
for j \leftarrow 1 to n do
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       O(h)
         Return F[n]

BOTTOM-UP-FIB(n)

Let r be a new array of size n+1

r[0] \leftarrow 1

r[i] \leftarrow 1

for i = 2 to n do

r[i] \leftarrow r[i-1] + r[i-2]

return r[n]
                                                                                                                                                                                                                                                                                                                                                                                                                     Space: O(n) for tree, O(1) auxiliary
                                                                                                                                                                                                                                                                                          y.right \leftarrow z

Time: O(h)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     O(1)
O(h)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         BST-Delete(T, z)
BST-Inorder(T)
BST-Preorder(T)
BST-Postorder(T)
                                                                                                                                                        \begin{array}{l} q \leftarrow -\infty \\ \textbf{for } i \leftarrow 1 \textbf{ to } j \textbf{ do} \\ \textbf{if } q < p[i] + r[j-i] \\ q \leftarrow p[i] + r[j-i] \\ s[j] \leftarrow i \end{array}
                                                                                                                                                                                                                                                                                            Space: O(n) for tree, O(1) auxiliary
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     O(n)
O(n)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     O(h)
O(h)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 Visits root before its childr
Visits children before root
                                                                                                                                                                                                                                                                                          Properties:
                                                                                                                                                                                                                                                                                           - Left subtree: all keys ¡ node's key

- Right subtree: all keys ¿ node's key

- Left and right subtrees are also BSTs
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 Queue Operation
Operation
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 Description
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      Space O(n) O(1)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 Create queue from array of n elements
Returns TRUE if queue is empty, FALSE
                                                                                                                                           Time Complexity: O(n^2)
  \begin{array}{ll} \textbf{Time Complexity: } O(n) \\ \textbf{Space Complexity: } O(n) \end{array} 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           Queue-Empty(Q)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       O(n)
                                                                                                                                                                                                                                                                                             A Node has: key (value), left & right (child pointers), parent (optional)
Tree height h: length of longest path from root to leaf
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 otherwise
Adds element x to rear of queue Q
Removes and returns front element from queue Q
                                                                                                                                            Space Complexity: O(n)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       O(1)
O(1)
```

Enqueue(Q, x Dequeue(Q)

List-Insert(L, x)

Operation Construction List-Search(L, k)

Linked List Operatio

O(1)

O(1) O(n) find, O(1) del

O(1)O(1)

Description
Create linked list from array of n elements
Returns pointer to first node with key k,

NULL if not found
Inserts node x at beginning of list L
Parecuse node x from list L

(includes input and memo array)

(includes input prices and array)

Akra-Bazzi, Ali Najib Variation:

For recurrence $T(n) = \alpha T(an) + \beta T(bn) + \Theta(n^d)$, where $\alpha, \beta > 0$, $a,b \in (0,1)$, $d \ge 0$, and unique p with $\alpha a^p + \beta b^p = 1$

> 0 and $\exists n_0 > 0, 0 \le f(n) \le c \cdot g(n) \ \forall n \ge n_0, f(n) = O(g(n)).$

Matrix Chain Multiplication:
Find optimal parenthesization to minimize multiplications **Bottom-Up** (Tabulation):

1. Create table m[1...n, 1...n] with m[i,i] = 0 m[i,j] stores minimal cost of multiplying matrices i through j2. For l = 2 to n (chain length):

3. For i = 1 to n - l + 1:
Set j = i + l - 1Compute $m[i,j] = \min_{i \leq k < j} \{m[i,k] + m[k+1,j] + p_{i-1}p_kp_j\}$ Store k in s[i,j] that achieved minimum cost

4. Return m[1,n]MATRIX-CHAIN-ORDER(p) $n \leftarrow p$, length -1 $t \in m[1,n]$, and s[1..n, 1..n] be new tables
for i = 1 to n do $m[i,i] \leftarrow 0$ for i = 1 to $n \leftarrow l + 1$ do $j \leftarrow i + l \leftarrow 1$ $j \leftarrow l \rightarrow l$ do $j \leftarrow i + l \leftarrow 1$ $j \leftarrow l \rightarrow l$ do $j \leftarrow m[i,j] \leftarrow m$ for $j \leftarrow 1$ to $n \leftarrow l \leftarrow l$ do $j \leftarrow m[i,j] \leftarrow m$ for $j \leftarrow m[i,j] \leftarrow m$ $j \leftarrow m[i,j] \leftarrow m$