

# Computer Security - CheatSheet

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CompSec Properties

- **Confidentiality.** prevention of unauthorized disclosure of information.
- **Integrity.** prevention of unauthorized modification of information.
- **Availability.** prevention of unauthorized denial of service or access to information and resources.
- **Authenticity.** assurance that entities (users, systems, or data) are genuine and can be verified as such.
- **Anonymity.** protection of an individual’s identity from being disclosed or linked to specific actions or data.
- **Non-repudiation.** assurance that a party in a communication cannot deny the authenticity of their signature or the sending of a message.

**The Adversary.** malicious entity aiming at breaching the security policy and **will** choose the optimal way to use her ressources to mount an attack that violates the security properties.

**Threat Model.** describes the ressources available to the adversary and their capabilities (*has access to internet, but doesn’t have access to the internal network of the company.*)

**Threat.** Who might attack which assets, using what resources, with what goal, how, and with what probability

**Vulnerability.** Specific weakness that could be exploited by adversaries with interest in a lot of different assets (*API is not protected, password appears in plain text...*)

**Harm.** The bad thing that could happen when the **threat** materializes. (*adversary steals the money, learns my password...*)

Security Policy

A high level description of the security properties that must hold in the system in relation to assets and principals

- **Assets (objects).** anything with value (data, files , memory) that needs to be protected
- **Principals (subjects).** people, computer programs, services
- Confidentiality.* authorized users may read a file
- Integrity.* authorized programs may write a file
- Availability.* authorized services can access a file

**Security Mechanism.** Technical mechanism used to ensure that the security policy is not violated by an adversary within the threat model, **we can only prepare for threats we’re aware of** (*Policy. ensure messages cannot be read by anyone but the sender and the receiver, Mechanism. encrypt the message before sending*)

Composition of Security Mechanisms

- **Defence in depth.** As long as one remains unbroken the Security Policy isn’t broken) (*two-factor auth*)
- **Weakest Link.** if anyone fails the Security Policy is broken security questions in case of a lost password, no need to know the password but just the answer

*Humans are also part of the mechanism and allow vulnerabilities like phishing attacks, bad use of passwords...*)

**To show a system is secure. (under a specific threat model)** Attacker - Just one way to violate **one** security property is enough (within the threat model).

Defender - No adversary strategy can violate the security policy.

Security Argument

Rigorous argument that the security mechanisms in place are indeed effective in maintaining the security policy subject to the assumptions of the Threat Model.

Principles of CompSec.

1. Economy of mechanism

- Keep the security mechanism/implementation design as simple and small as possible. Why ?
- a. Easier to audit and verify.
- b. Testing is not appropriate to evaluate security.

Trusted Computing Base (TCB).

Every component of the system on which the security policy relies upo hardware, firmware, software.  
The TCB is trusted to operate correctly for the security policy to hold. → If something goes wrong in it, the security policy may be violated  
It **must** be kept small to ease verification (economy of mechanism) and diminish the attack surface

2. Fail-safe defaults. Base access decisions on permission rather than exclusion. (*Whitelist, do not blacklist*)

- If something fails, be as secure as it does not fail errors / uncertainty should error on the side of the security policy Do **not** try to fix on error !
- Automated doors: if they cannot close, stay open
- Form input: if no permission to write in X, do not write anywhere

3. Complete mediation. Every access to every object must be checked for authority A Reference Monitor mediates all actions from subjects on objects and ensures they are according to the policy. Tradeoff time\_to\_check vs. time\_to\_use

4. Open design The design should not be secret

- Always design as if the enemy knows the system.
  - When you design...
    - \* Crypto. Only keep the key secret
    - \* Authentication. Only keep the password secret
    - \* Obfuscation. Only keep the used noise secret
- assuming the thread model can’t get a hold of the system is unrealistic (employee corruption, ...)

5. Separation of privilege. No single accident, deception, or breach of trust is sufficient to compromise the protected information

- **Privilege.** A privilege allows a user to perform an action on a computer system that may have security consequences. (*create a file in a directory, access a device, write to a socket for communicating over the internet...*)

6. Least Privilege.

Every program and every user of the system should operate using the least set of privileges necessary to complete the job. *Rights are added as needed, discarded after use. Users should get to know about things if they have to.*

7. Least Common Mechanism

Minimize the amount of mechanism common to more than one user and depended on by all users. Every shared mechanism represents a potential information path between users.

8. Psychological acceptability It is essential that the human interface be designed for ease of use, so that users routinely and automatically apply the protection mechanisms correctly. (*hide complexity, keep ressources as accessible as before, mental model of users must match security policy/mechanisms, cultural acceptability...*)

9. Work Factor

Compare the cost of breaking the mechanism with the resources of a potential attacker. (*cost of compromising insiders, cost of finding a bug, monetization...*)

10. Compromise recording

Reliably record that a compromise of information has occurred [...] in place of more elaborate mechanisms that completely prevent loss. (*keep tamper-evidence logs, what if you cannot recover them? (confidentiality), how to keep integrity ? (Blockchain), logs may be a vulnerability ? (Privacy), logging the log ? (Availability)...*)

Piecewise Continuity & Differentiability

$f : [a, b] \rightarrow \mathbb{R}$  is *piecewise continuous* if there is a partition

$$a = a_0 < a_1 < \cdots < a_n = b$$

such that  $\lim_{x \rightarrow a_i^-} f(x)$  and  $\lim_{x \rightarrow a_i^+} f(x)$  exist (finite).

Similarly,  $f$  is *piecewise  $C^1$*  if it is continuously differentiable on each open subinterval and the one-sided derivatives at boundaries exist.

Euler’s Formulas

$$e^{x+iy} = e^x (\cos y + i \sin y), \quad \sin x = \frac{e^{ix} - e^{-ix}}{2i}, \quad \cos x = \frac{e^{ix} + e^{-ix}}{2}.$$

Orthogonality (Sine/Cosine Products)

For  $n, m \in \mathbb{N}_{\geq 1}$  and period  $T > 0$ :

$$\frac{2}{T} \int_0^T \cos\left(\frac{2\pi n}{T}x\right) \cos\left(\frac{2\pi m}{T}x\right) dx = \begin{cases} 1 & n = m, \\ 0 & n \neq m \end{cases}$$

(Same for  $\sin \sin$ , and  $\cos \sin$  integrates to 0.)

Integration Over One Period

If  $f$  is  $T$ -periodic and piecewise continuous, then for any  $a \in \mathbb{R}$ :

$$\int_a^{a+T} f(x) dx = \int_0^T f(x) dx.$$

Dirichlet’s Theorem (Pointwise Convergence)

Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be  $T$ -periodic and piecewise  $C^1$ . Then, for all  $x \in \mathbb{R}$ ,

$$Ff(x) = \lim_{t \rightarrow 0} \frac{f(x-t) + f(x+t)}{2}.$$

Real Fourier Series

For  $f : \mathbb{R} \rightarrow \mathbb{R}$ ,  $T$ -periodic, piecewise  $C^1$ , the real Fourier series is

$$Ff(x) = \frac{a_0}{2} + \sum_{n=1}^\infty \left[ a_n \cos\left(\frac{2\pi n}{T}x\right) + b_n \sin\left(\frac{2\pi n}{T}x\right) \right].$$

Fourier Coefficients:

$$a_n = \frac{2}{T} \int_0^T f(x) \cos\left(\frac{2\pi n}{T}x\right) dx, \quad b_n = \frac{2}{T} \int_0^T f(x) \sin\left(\frac{2\pi n}{T}x\right) dx,$$
$$a_0 = \frac{2}{T} \int_0^T f(x) dx.$$

**Parity:** If  $f$  is even,  $b_n = 0$ ; if  $f$  is odd,  $a_n = 0$ .

Term-by-Term Differentiation

If  $f$  is  $T$ -periodic, continuous, and piecewise  $C^1$ , then

$$\frac{d}{dx} [Ff(x)] = \sum_{n=1}^\infty \frac{2\pi n}{T} \left[ -a_n \sin\left(\frac{2\pi n}{T}x\right) + b_n \cos\left(\frac{2\pi n}{T}x\right) \right]$$
$$= \lim_{t \rightarrow 0} \frac{f'(x-t) + f'(x+t)}{2}.$$

Term-by-Term Integration

If  $f$  is  $T$ -periodic, continuous, and piecewise  $C^1$ , then

$$\int Ff(x) dx = \sum_{n=1}^\infty \frac{T}{2n\pi} \left[ a_n \sin\left(\frac{2\pi n}{T}x\right) - b_n \cos\left(\frac{2\pi n}{T}x\right) \right] + C$$
$$= \lim_{h \rightarrow 0} \frac{1}{2h} \int_{x-h}^{x+h} Ff(t) dt,$$

where  $C$  is the constant of integration.

Poisson on  $[a, b]$

$$\begin{cases} -u''(x) = f(x), \\ u(a) = g_a, \quad u(b) = g_b, \end{cases} \quad L = b - a$$
$$u^g(x) = \frac{g_b - g_a}{b - a} x + \frac{b g_a - a g_b}{b - a}.$$
$$f(x) = \sum_{n=1}^\infty b_n \sin\left(\frac{n\pi x}{L}\right), \quad b_n = \frac{2}{L} \int_0^L f(t) \sin\left(\frac{n\pi t}{L}\right) dt,$$
$$u^f(x) = \sum_{n=1}^\infty b_n \frac{L^2}{\pi^2 n^2} \sin\left(\frac{n\pi x}{L}\right).$$

$$u(x) = u^g(x) + u^f(x) \quad (\text{superposition principle}).$$

Poisson with mass term on  $\mathbb{R}$

$$-u''(x) + k^2 u(x) = f(x), \quad \widehat{u}(\alpha) = \frac{\widehat{f}(\alpha)}{\alpha^2 + k^2}.$$
$$g(x) = \sqrt{\frac{\pi}{2}} \frac{1}{k} e^{-k|x|}, \quad \widehat{g}(\alpha) = \frac{1}{\alpha^2 + k^2},$$
$$u(x) = (g * f)(x) = \frac{1}{2k} \int_{-\infty}^\infty f(y) e^{-k|x-y|} dy.$$

Complex Fourier Coefficient

Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be  $T$ -periodic and piecewise continuous. The complex Fourier coefficients are:

$$c_n = \frac{1}{T} \int_0^T f(x) e^{-i\frac{2\pi}{T}nx} dx, \quad Ff(x) = \sum_{n=-\infty}^\infty c_n e^{i\frac{2\pi n}{T}x}.$$

For  $\phi : \mathbb{R} \rightarrow \mathbb{C}$ ,

$$\int_a^b \phi(x) dx = \int_a^b \operatorname{Re}(\phi(x)) dx + i \int_a^b \operatorname{Im}(\phi(x)) dx.$$

Relation to  $(a_n, b_n)$

$$c_n = \frac{1}{2}(a_n - i b_n), \quad c_{-n} = \frac{1}{2}(a_n + i b_n), \quad c_0 = \frac{a_0}{2}.$$
$$a_n = c_n + c_{-n} \quad a_0 = 2c_0 \quad b_n = \operatorname{Re}(c_{-n} - c_n)$$

Fourier Series on  $[0, L]$

For  $f : [0, L] \rightarrow \mathbb{R}$  (piecewise  $C^1$ ):

$$F_c f(x) = \frac{\tilde{a}_0}{2} + \sum_{n=1}^\infty \tilde{a}_n \cos\left(\frac{\pi n}{L}x\right), \quad \tilde{a}_n = \frac{2}{L} \int_0^L f(x) \cos\left(\frac{\pi n}{L}x\right) dx.$$

$$F_s f(x) = \sum_{n=1}^\infty \tilde{b}_n \sin\left(\frac{\pi n}{L}x\right), \quad \tilde{b}_n = \frac{2}{L} \int_0^L f(x) \sin\left(\frac{\pi n}{L}x\right) dx.$$

Parseval’s Identity (Periodic Case)

If  $f$  is  $T$ -periodic (piecewise  $C^1$ ),

$$\frac{2}{T} \int_0^T f^2(x) dx = \frac{a_0^2}{2} + \sum_{n=1}^\infty (a_n^2 + b_n^2) = 2 \sum_{n=-\infty}^\infty |c_n|^2.$$

Plancherel Theorem

Let  $f \in L^2(\mathbb{R})$ . Then its Fourier transform  $\hat{f}$  is also in  $L^2(\mathbb{R})$ , and:

$$\int_{-\infty}^\infty |f(x)|^2 dx = \int_{-\infty}^\infty |\hat{f}(\xi)|^2 d\xi.$$

The Fourier Transform

If  $f : \mathbb{R} \rightarrow \mathbb{R}$  with  $\int_{-\infty}^\infty |f(x)| dx < \infty$ , its (unitary) Fourier transform is

$$\hat{f}(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty f(x) e^{-i\alpha x} dx,$$

Inverse Transform

If  $\varphi(\alpha)$  is similarly integrable,

$$\mathcal{F}^{-1}(\varphi)(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty \varphi(\alpha) e^{i\alpha x} d\alpha.$$

Convolution Product

Let  $f, g : \mathbb{R} \rightarrow \mathbb{R}$  such that  $\int_{-\infty}^{+\infty} |f(x)| dx < +\infty$ ,  $\int_{-\infty}^{+\infty} |g(x)| dx < +\infty$ .

$$(f * g)(x) = \int_{-\infty}^{+\infty} f(x-t) g(t) dt = \int_{-\infty}^{+\infty} f(t) g(x-t) dt$$

Scaling

$$\mathcal{F}\{f(ax)\} = \frac{1}{|a|} \hat{f}\left(\frac{\alpha}{a}\right)$$

Shifting

$$\mathcal{F}\{f(x - x_0)\} = e^{-i\alpha x_0} \hat{f}(\alpha)$$

Modulation

$$\mathcal{F}\{e^{i\omega_0 x} f(x)\} = \hat{f}(\alpha - \omega_0)$$

Convolution

$$\mathcal{F}[f * g](\alpha) = \sqrt{2\pi} \hat{f}(\alpha) \hat{g}(\alpha)$$

Differentiation

$$\mathcal{F}\left\{\frac{d^n}{dx^n} f(x)\right\} = (i\alpha)^n \hat{f}(\alpha)$$

Integration

$$\mathcal{F}\left\{\int_{-\infty}^x f(\xi) d\xi\right\} = \frac{1}{i\alpha} \hat{f}(\alpha), \quad \alpha \neq 0$$

Differentiation of Transform

$$\mathcal{F}((-ix)^n f(x))(\alpha) = \frac{\partial^n}{\partial \alpha^n} \hat{f}(\alpha)$$

Multiplication

$$\mathcal{F}\{f(x) \cdot g(x)\} = \sqrt{2\pi} (\mathcal{F}\{f(x)\} * \mathcal{F}\{g(x)\})$$

Important Trigonometric Identities

$$\sin(2x) = 2 \sin x \cos x,$$
$$\cos(2x) = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x = \cos^2 x - \sin^2 x,$$
$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b,$$
$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b,$$
$$\cos a \cos b = \frac{1}{2} [\cos(a - b) + \cos(a + b)],$$
$$\sin a \sin b = \frac{1}{2} [\cos(a - b) - \cos(a + b)],$$
$$\sin a \cos b = \frac{1}{2} [\sin(a + b) + \sin(a - b)],$$
$$\cos a \sin b = \frac{1}{2} [\sin(a + b) - \sin(a - b)].$$
$$\cos(n\pi) = (-1)^n$$