Smoothed Local Histogram Filters

Sparsity and Compressed Sensing

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Overview

- Motivation
- Smooth Local Histograms
- Morphological filters
- Mode filters
- Applications

Motivation

- An image histogram specifies the intensities distribution within the whole image: **It does not contain spatial information.**
- Local histograms summarize the tonal distribution within an image neighborhood.
- Many applications in computer vision make use of the smooth local histograms (SLH).
- Caveat: Very expensive computation over large neighborhoods. $\mathcal{O}(n^2 \log n)$: binned histogram of neighborhood size $(n \times n)$

Definition

Density estimation: The SLH can be seen as a kernel density estimator:

$$\hat{f}_p(s) = \sum_i K(I_{q_i} - s)W(p - q_i)$$

Where (values space parameters):

- K is the smoothing kernel, generally a gaussian kernel of wifth σ_v denoted by G_{σ_v}.
- I_{q_i} the intensity of the pixel q_i in p's neighborhood/
- s the shift
- we sample the shift values $(s_i)_{1 \leq i \leq m}$ at a frequency larger than the Nyquist frequency.

$$\boxed{f_p(s) = G_{\sigma_v}(I_p - s) \otimes W}$$

The kernel shouldn't introduce additional extrema

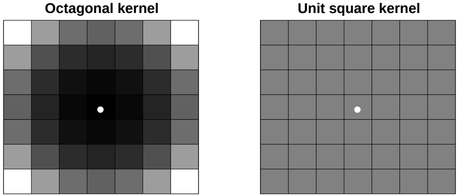
Definition

Density estimation: The SLH can be seen as a kernel density estimator:

$$\boxed{f_p(s) = G_{\sigma_v}(I_p - s) \otimes W}$$

Where (spatial space parameters):

- W is a weighting function positive, has unit sum and ideally with valued diminishing with distance.
- Generally we consider W a gaussian kernel window with width $= \sigma_x$



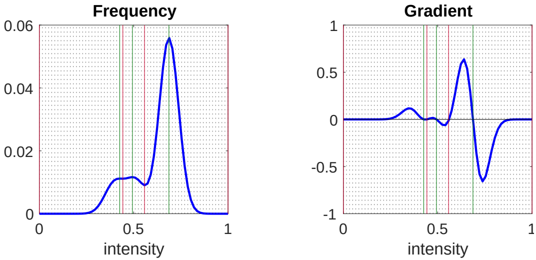
Integration and derivation

Derivative

$$\forall i\{1, .., m\},\ D_i(p) = \frac{1}{\sigma_v^2}(I_p - s_i)K(I_p - s_i) \otimes W$$

We interpolate the signal $(D_i(p))_i$ to approximate the value of $D(I_p)$ and locate:

- The modes (local maxima : negative going zero crossings of D_p).
- The antimodes (local minima: positive going zero crossings of D_p)



Integration and derivation

Integral

$$R_i(p) = \phi(I_p - s_i) \otimes W$$

Where for the Gaussian kernel G_{σ_v}:

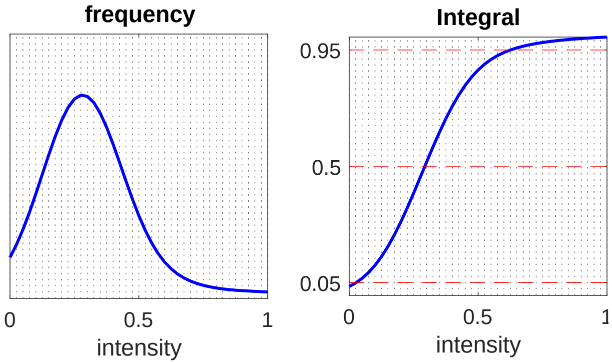
$$\phi(z) = \sigma_v \sqrt{\frac{\pi}{2}} \left(1 + \operatorname{erf}\left(\frac{z}{\sigma_v \sqrt{2}}\right)\right)$$

Whenever we need to evaluate $R(I_p)$ or $D(I_p)$ we linearly interpolate between two consecutive shifts such that $I_p \in [s_i, s_{i+1}]$.
The sampling frequency should be large enough to ensure sufficient approximations.

Morphological filters

The morphological filter of order β outputs the $\beta-$percentile of the intensities within the neighborhood. i.e.

$$\mathbb{P}(I \leq x) = \beta$$



Morphological filters : Particular cases

Median: $\beta = \mathbf{50\%}$
Erosion: $\beta = \mathbf{5\%}$

The default erosion filter consider the minimum intensity in the neighborhood in case W is the unit area, to integrate the spatial variation we set β for the erosion at 5%.

Dilation: $\beta = \mathbf{95\%}$

The default erosion filter consider the maximum intensity (we move β from 100% to 95%).

Closing: Erosion followed by dilation
Opening: Dilation followed by erosion.

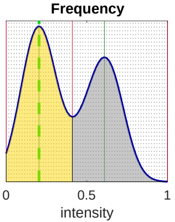
Mode filters

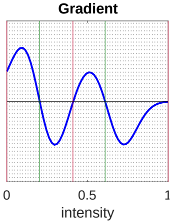
Closest mode:
It's the mode one would reach by steepest ascent in the smoothed local histogram.

- Estimate $D(I_p)$
- if $D(I_p) \geq 0$, select the first mode greater than I_p.
  - otherwise, select the largest mode smaller than I_p.

Relies heavily on the input value.

Dominant mode:
Choose the mode with the largest population:
population $\approx$ area between the two antimodes surrounding the considered mode





Original image

(Closest mode) $\sigma_x = 3 \mid \sigma_v = 0.05 \mid m = 100$

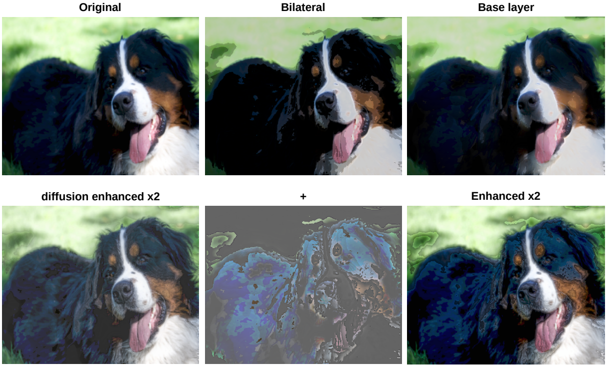
(Dominant mode) $\sigma_x = 3 \mid \sigma_v = 0.05 \mid m = 100$

(Median) $\beta = 50\% \mid \sigma_x = 3 \mid \sigma_v = 0.05 \mid m = 100$

Detail enhancement

Selective Diffusion:

- Decompose an image into a base layer and an additional detail layer.
- Find the base layer with an edge-preserving filter
- Diffuse the base layer with gaussian blurring.

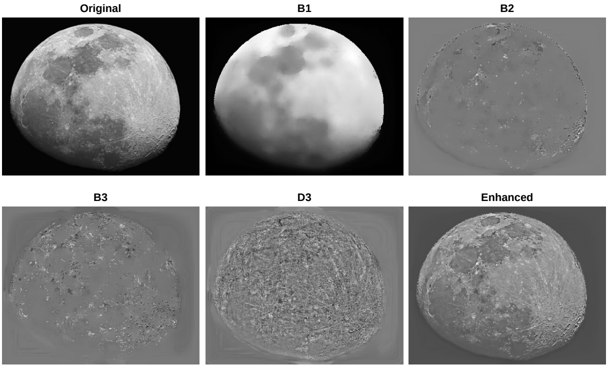


Detail enhancement

Iterative selective Diffusion:

- Reiterate the selective diffusion over the detail layer.

$$\boxed{I = \left(\sum_{i=1}^{n} \mathcal{B}_i\right) + \mathcal{D}_n}$$



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