

# High-level shape models



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# Core problem: correspondence

'what is here comes from here'

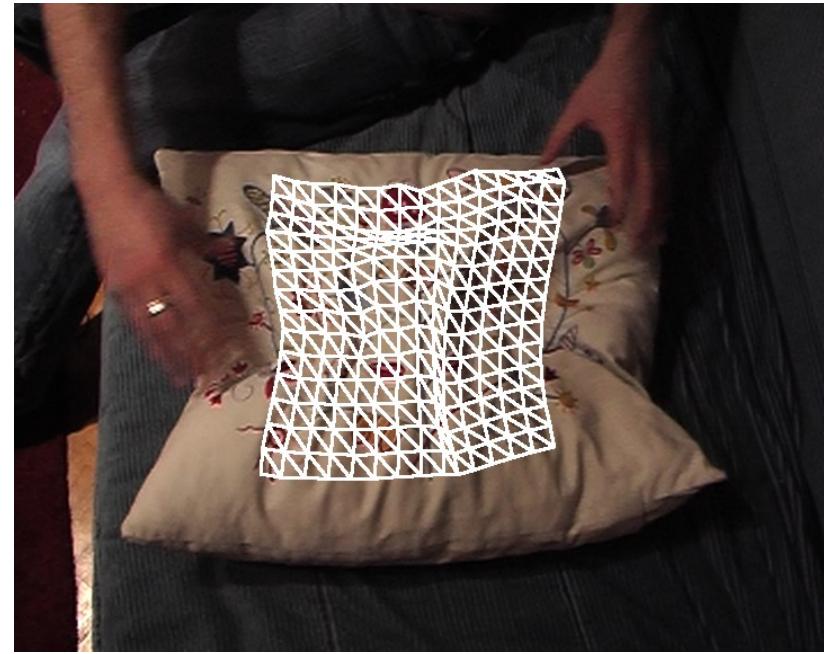
Simple case: global rigid transformation (4-9 DoF)



# Core problem: correspondence

'what is here comes from here'

Complications: non-rigid transformation



# Core problem: correspondence

‘what is here comes from here’

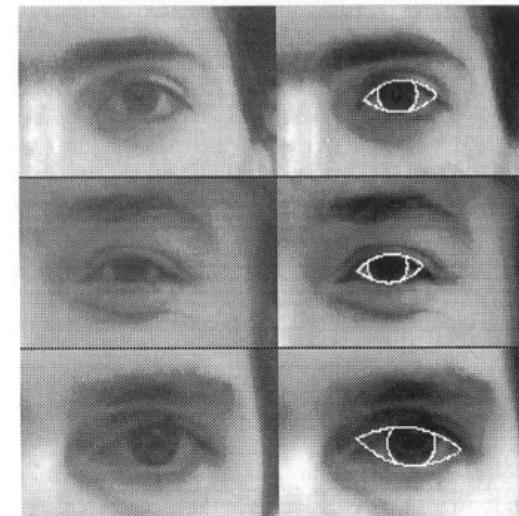
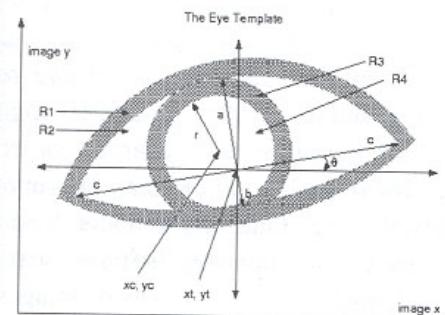
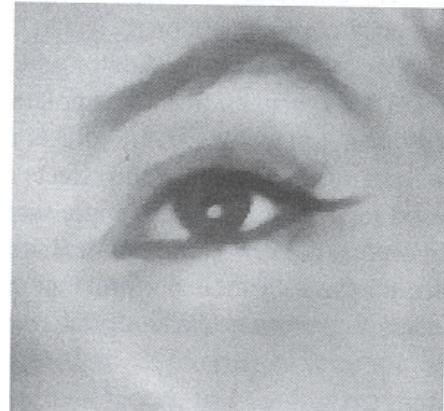
Complications: background changes



# Core problem: correspondence

'what is here comes from there'

Complications: intra-category variation



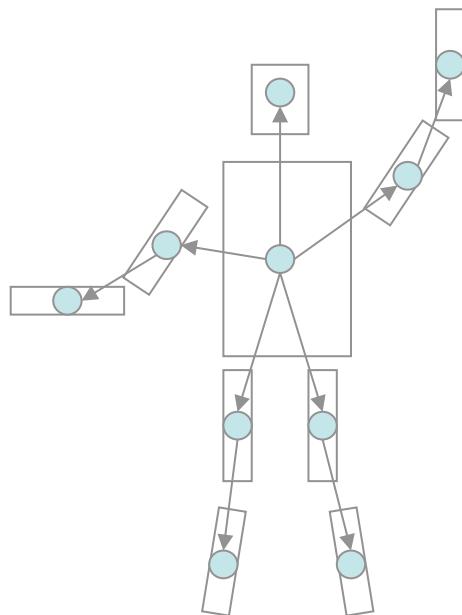
'One shape fits all'

A.L. Yuille, D.S. Cohen and P.W. Hallinan. Feature extraction from faces using deformable templates. CVPR 1989.

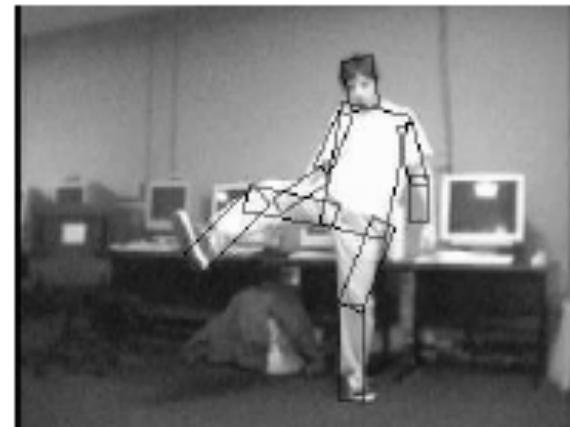
# Core problem: correspondence

'what is here comes from there'

Complications: computation



Is there a shape? Where?



# Core problem: correspondence

‘what is here comes from there’

Complications: 3D, pose variability



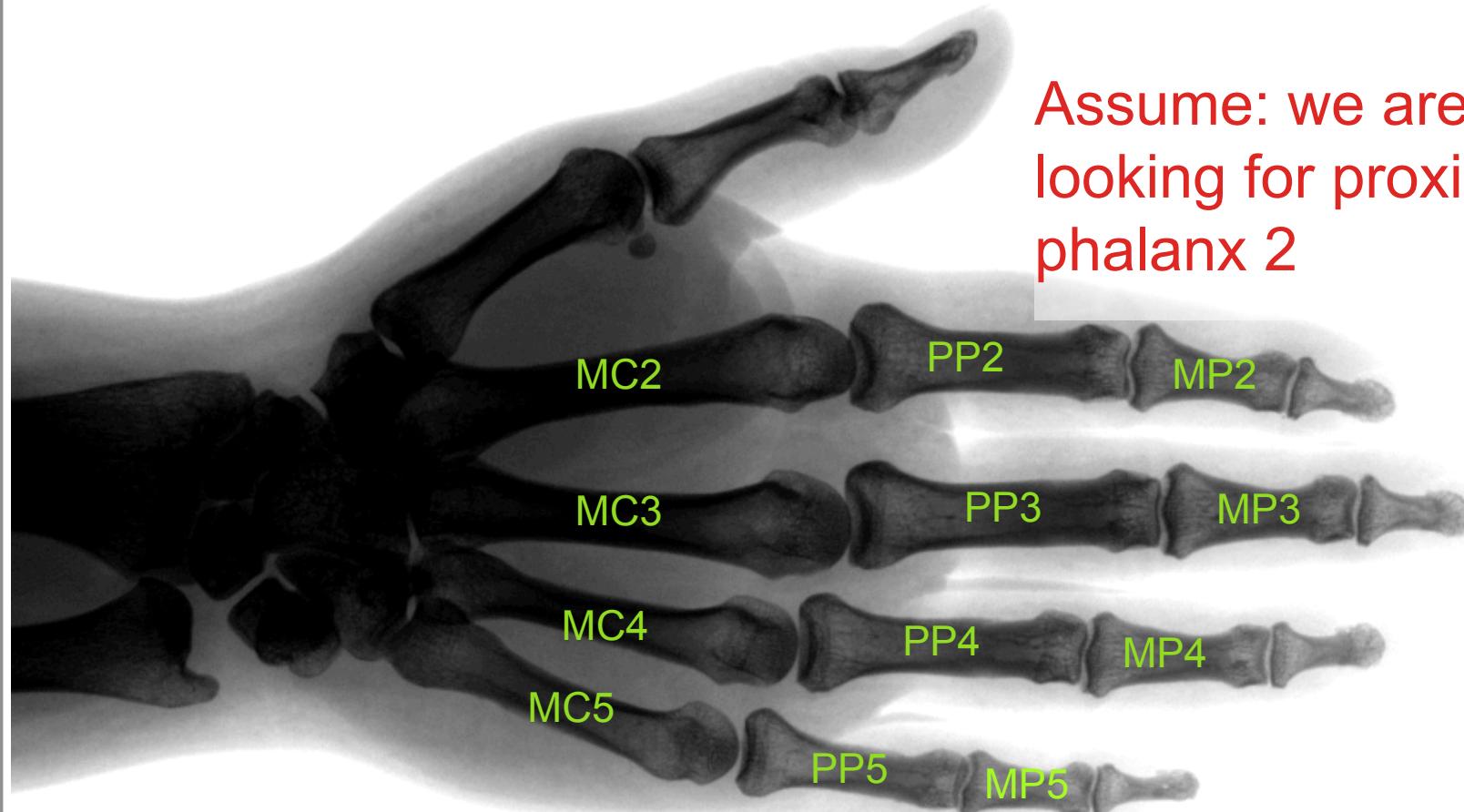
T. J. Cashman, A. W. Fitzgibbon: What Shape Are Dolphins? Building 3D Morphable Models from 2D Images, 2013  
<http://research.microsoft.com/en-us/um/people/awf/dolphins/>

# Motivation: Analyze a hand radiograph



Slide credit: G. Langs

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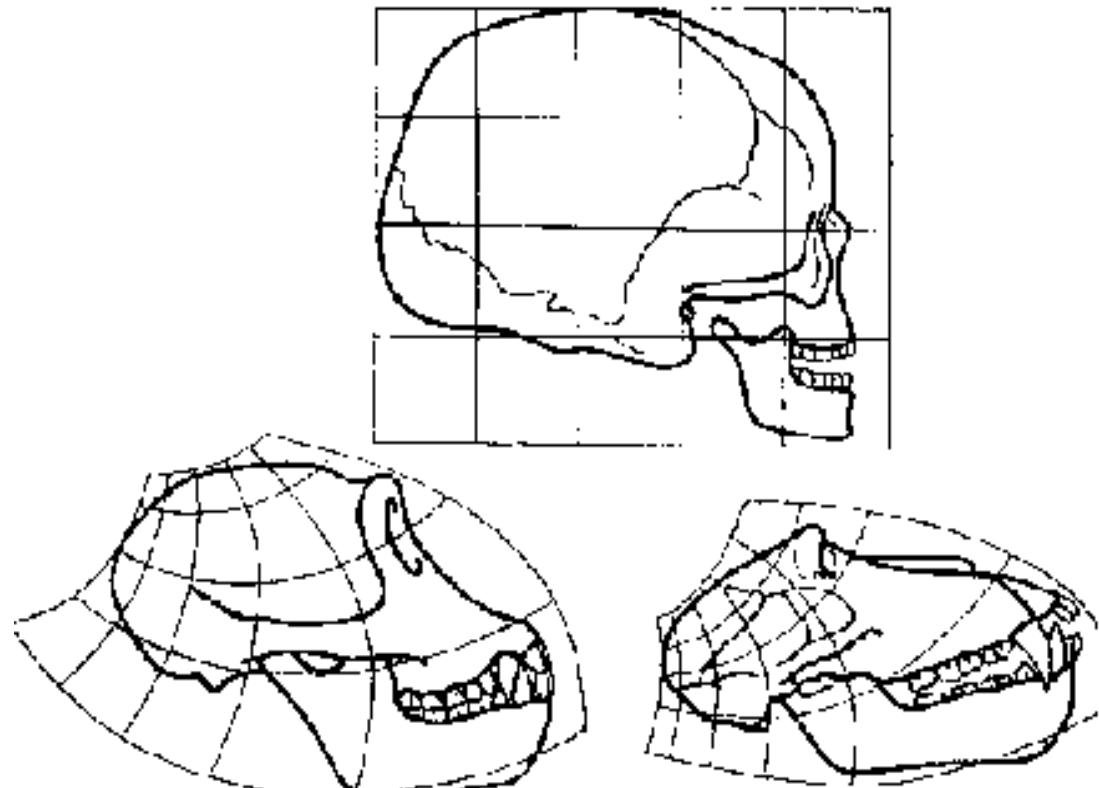


How can we represent this knowledge?

How can we exploit it?

Slide credit: G. Langs

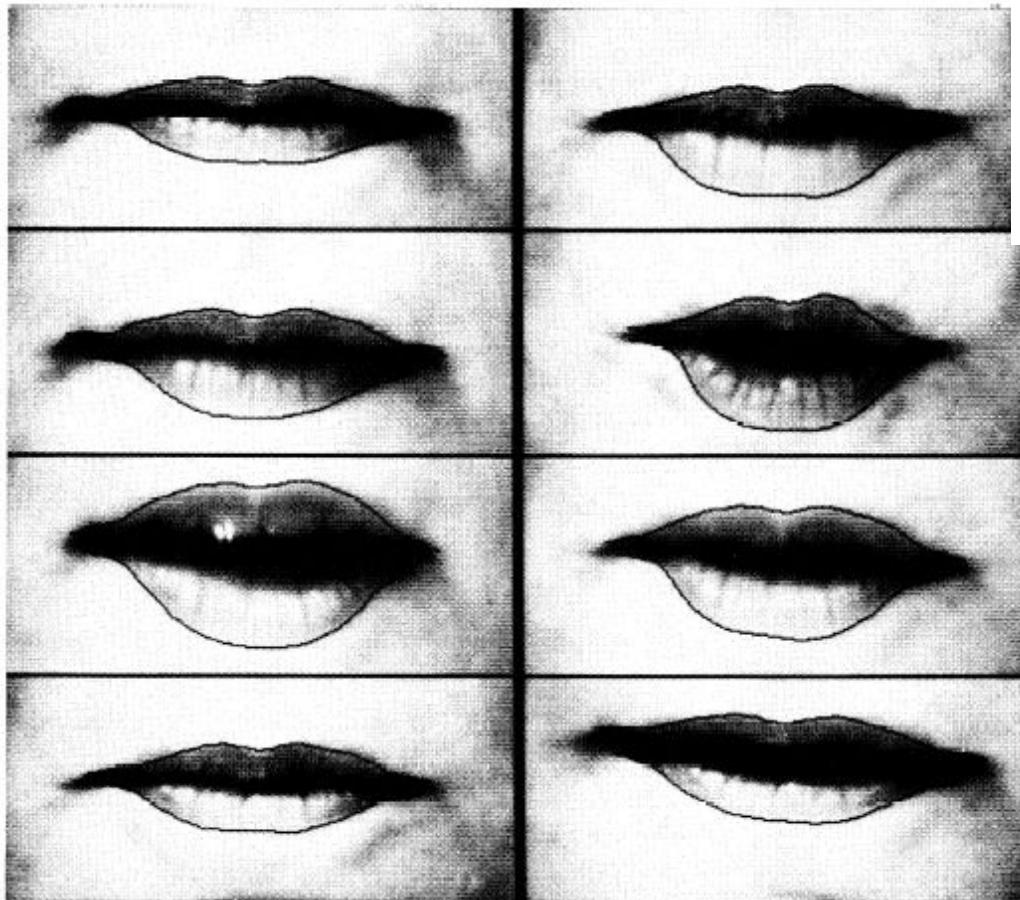
# On growth and form (1917)



Skulls of a human, a chimpanzee and a baboon  
and transformations between them

D'Arcy Thompson, "On Growth and Form" (1917)

## Snakes-Active Contour Models (1987)



$$\begin{aligned} E_{\text{snake}}^* &= \int_0^1 E_{\text{snake}}(\mathbf{v}(s)) \, ds \\ &= \int_0^1 E_{\text{int}}(\mathbf{v}(s)) + E_{\text{image}}(\mathbf{v}(s)) \\ &\quad + E_{\text{con}}(\mathbf{v}(s)) \, ds \end{aligned}$$

M. Kass, A. Witkin and D. Terzopoulos, 'Snakes', 1987

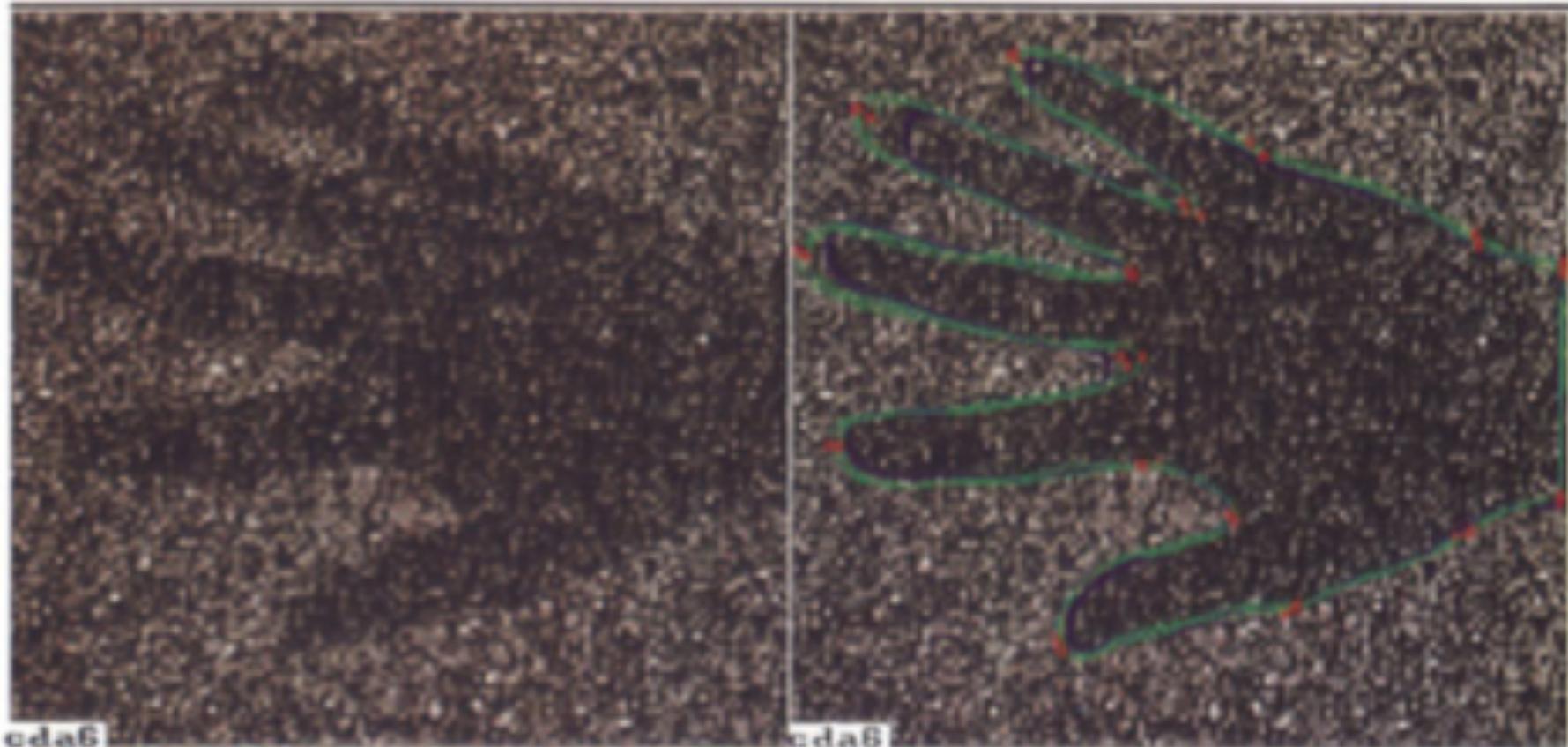
## Pattern theory - Hands (1990)



Samples from the  
prior distribution

U. Grenander, Hands: a pattern theoretic study of biological shapes, 1990

## Pattern theory - Hands (1990)

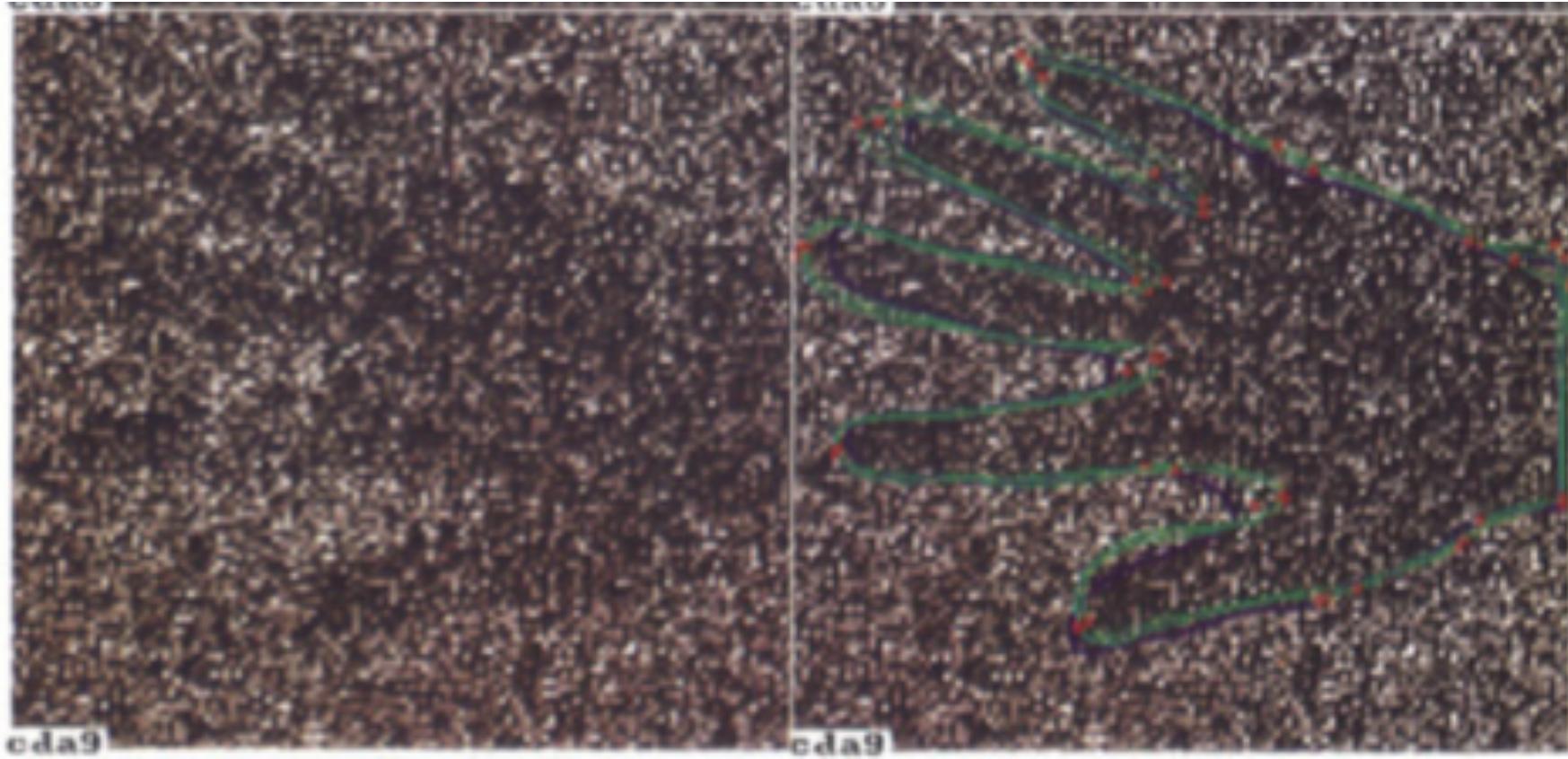


Input

Samples from posterior

U. Grenander, Hands: a pattern theoretic study of biological shapes, 1990

## Pattern theory - Hands (1990)

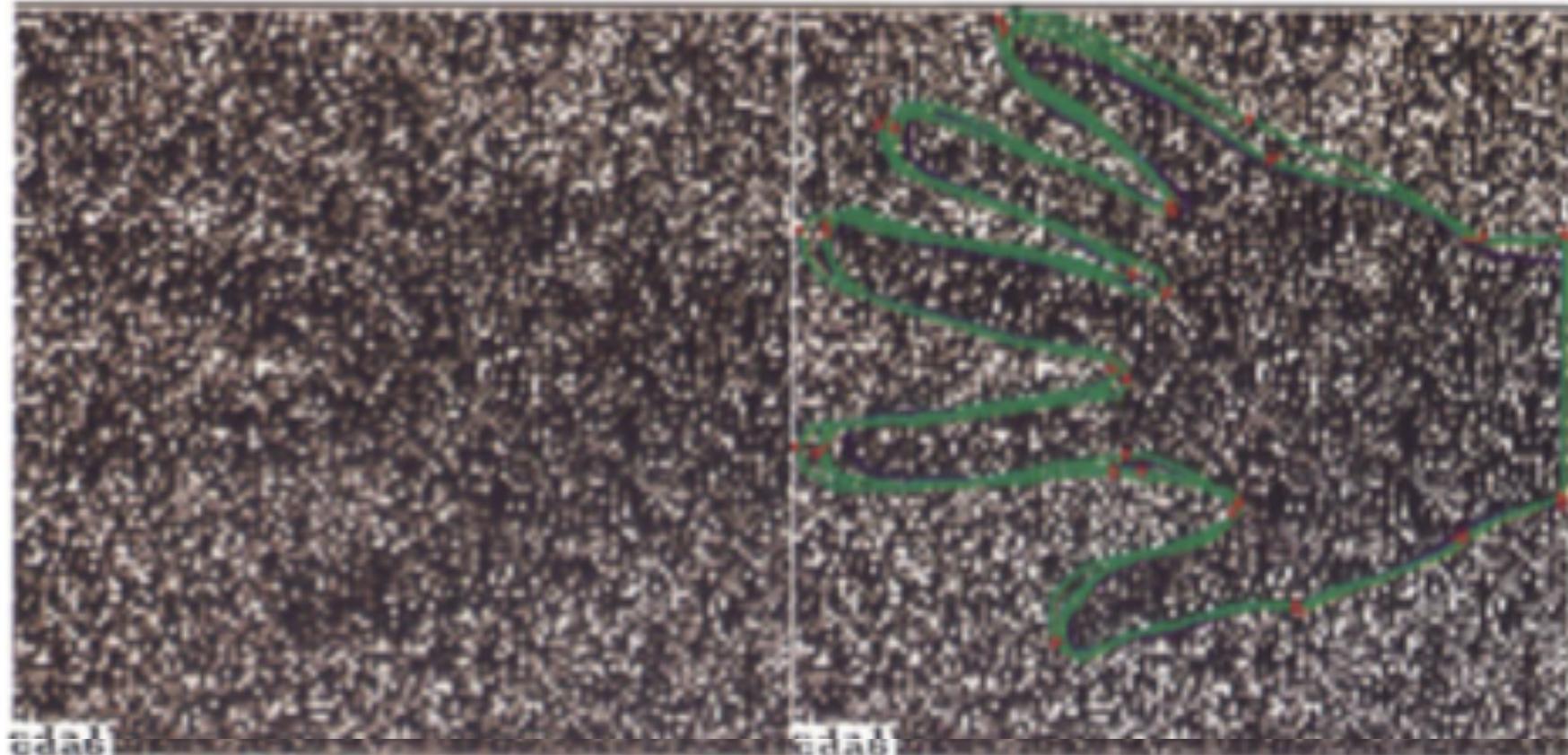


Input

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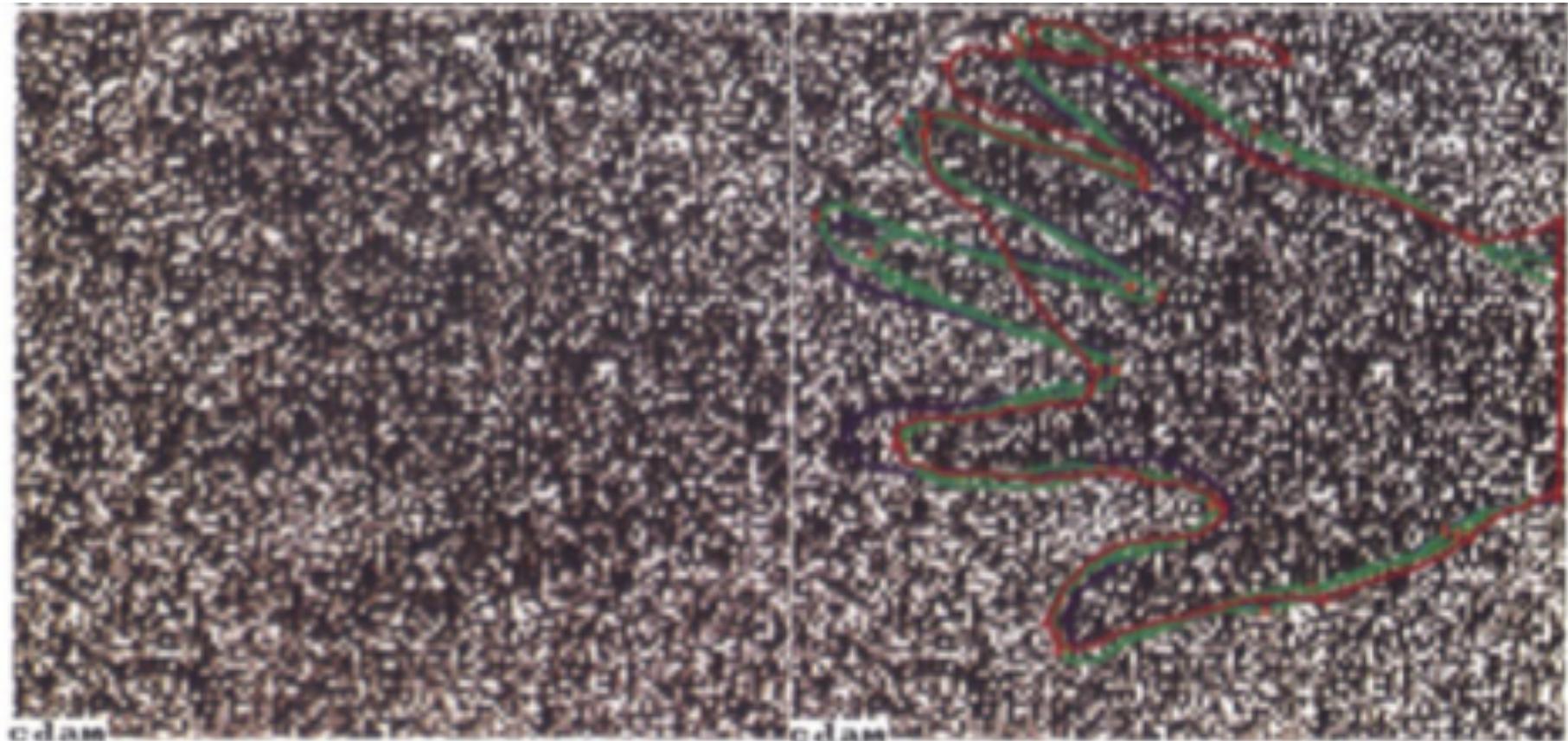


Input

Samples from posterior

U. Grenander, Hands: a pattern theoretic study of biological shapes, 1990

## Pattern theory - Hands (1990)

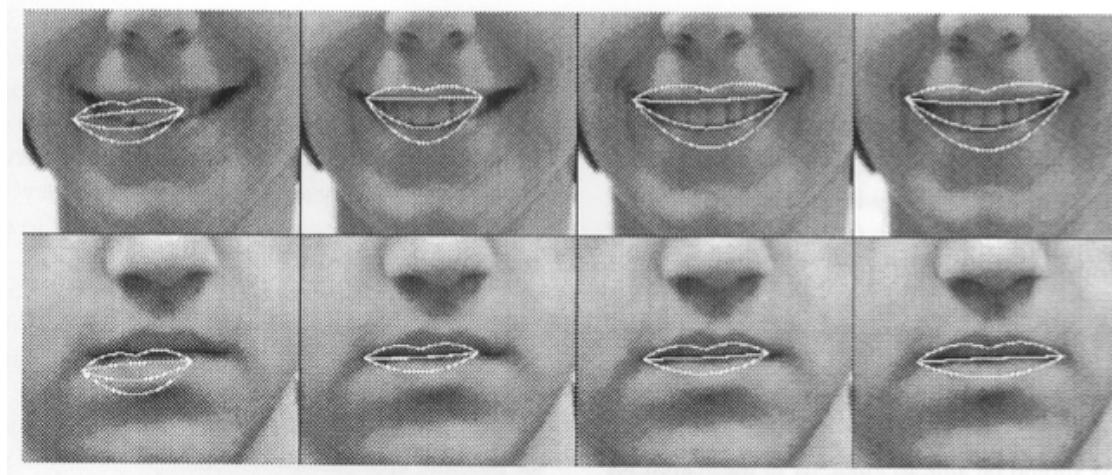
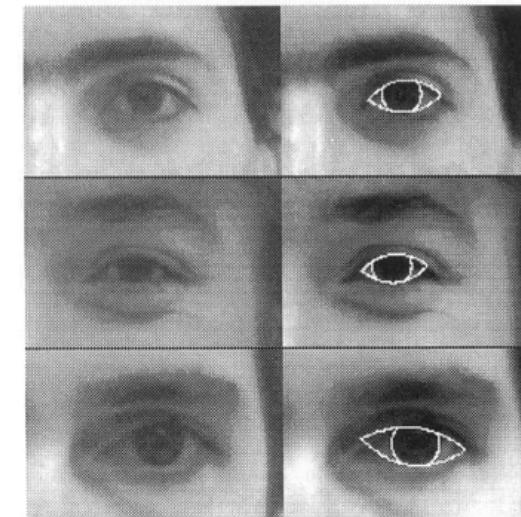
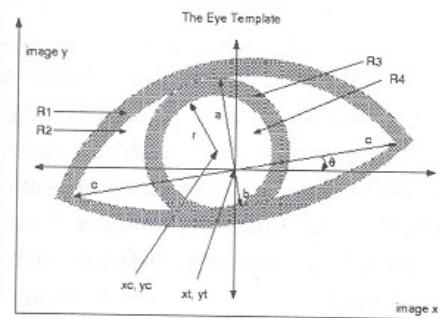
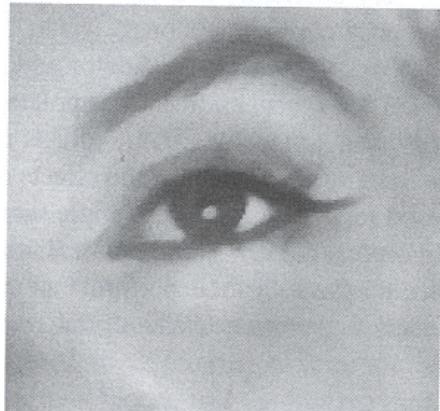


Input

Samples from posterior

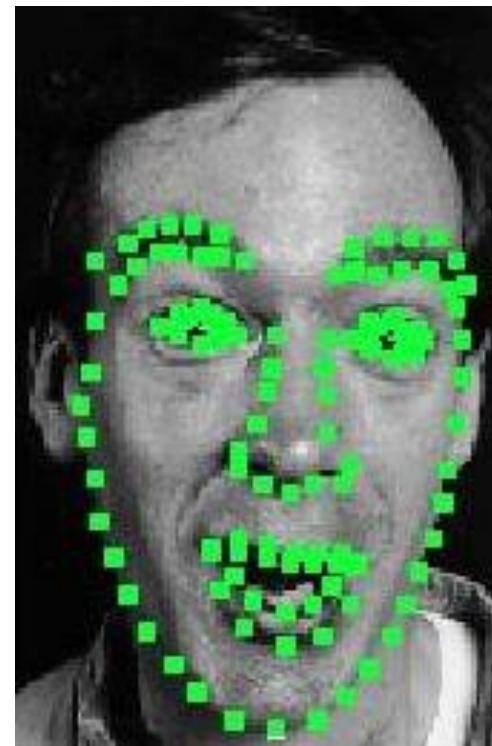
U. Grenander, Hands: a pattern theoretic study of biological shapes, 1990

# Deformable templates - 1989



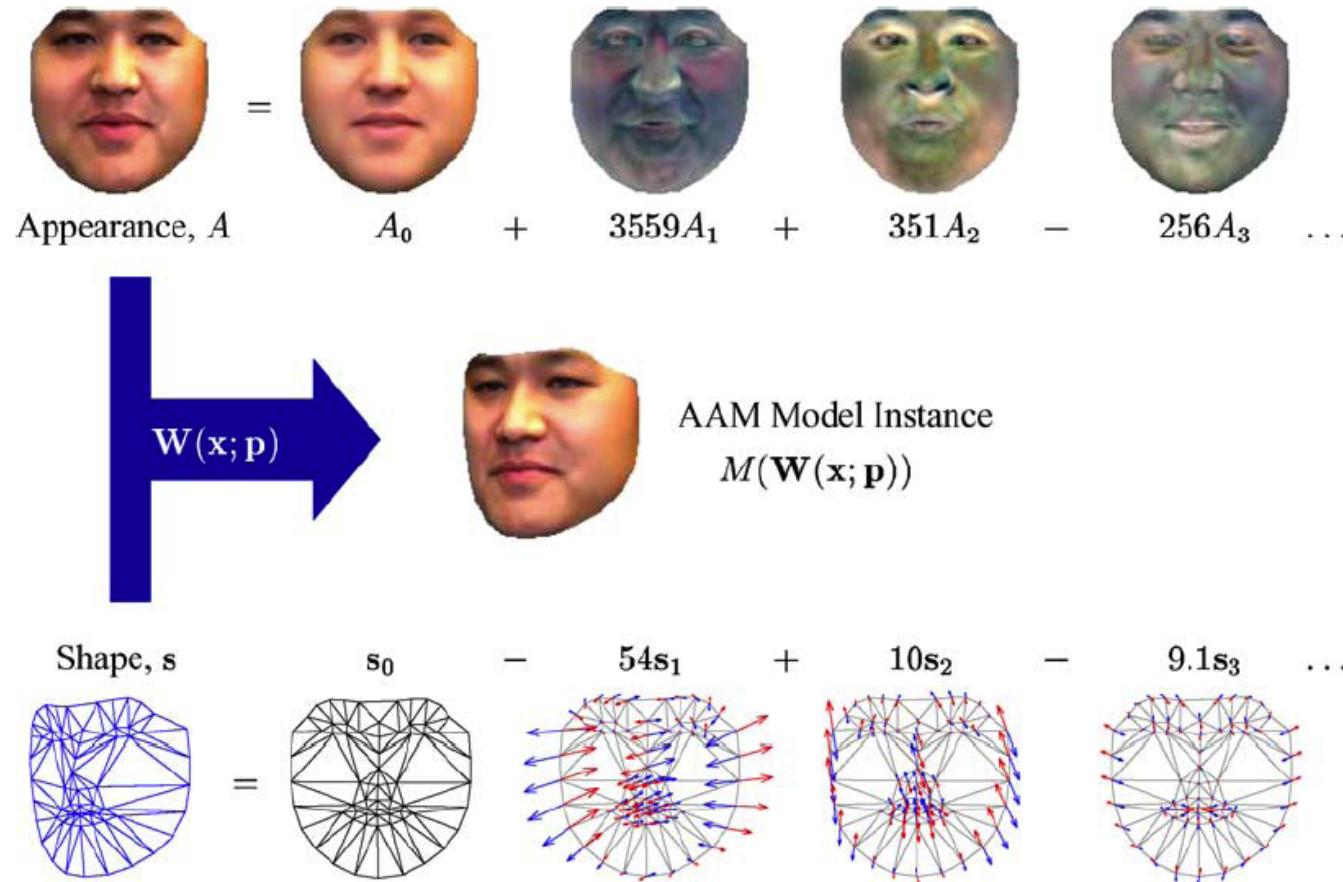
A.L. Yuille, D.S. Cohen and P.W. Hallinan. Feature extraction from faces using deformable templates. CVPR 1989.

## Active Shape Models - 1992



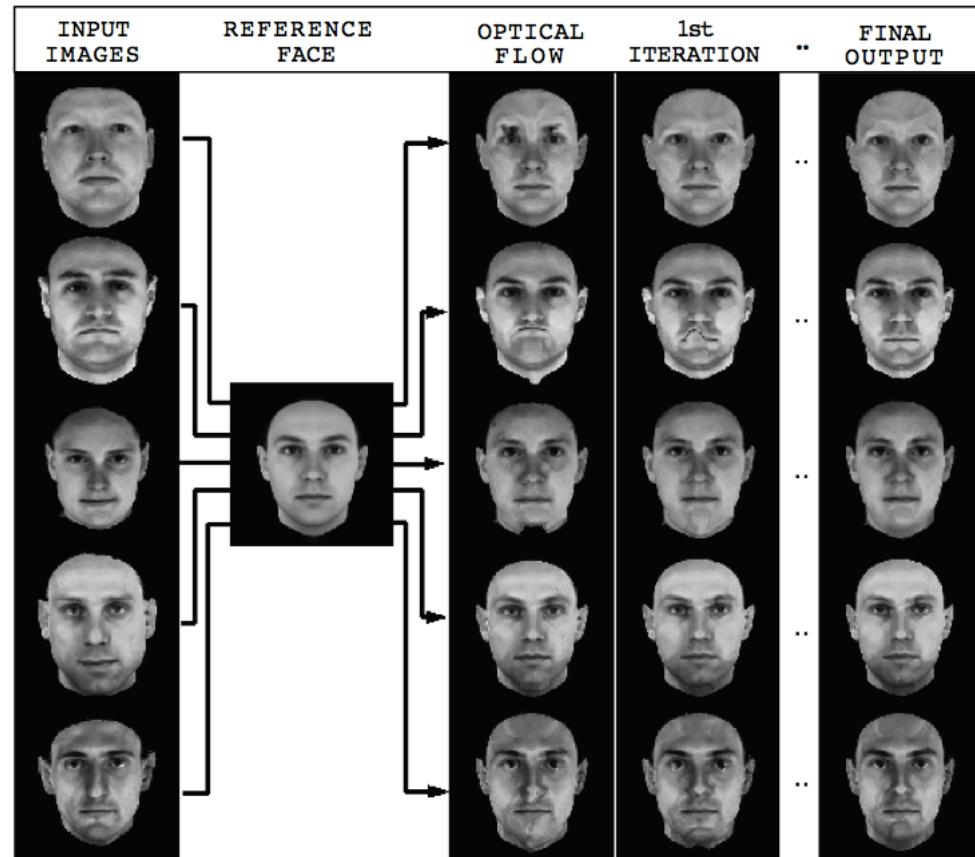
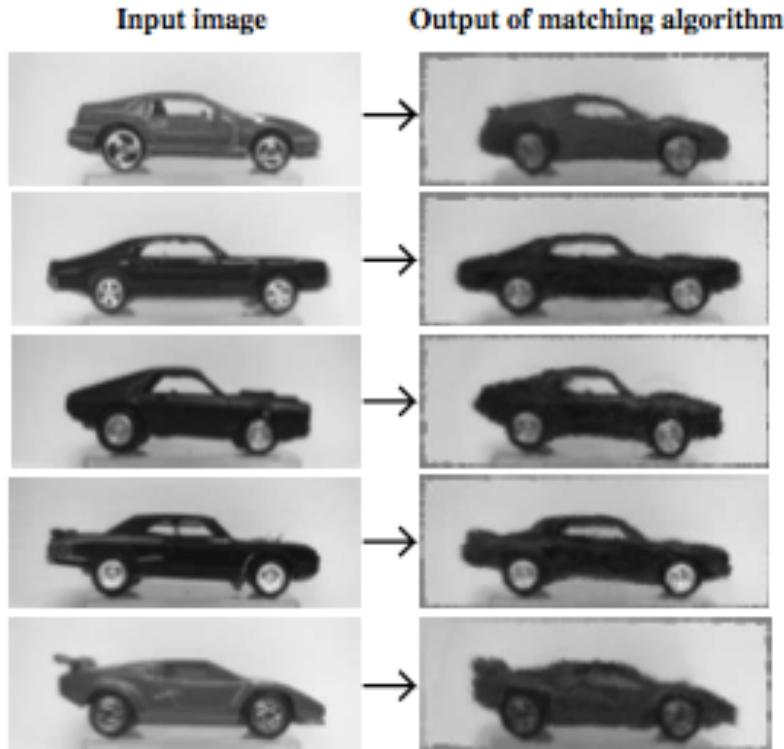
T. F. Cootes, C. J. Taylor, D. H. Cooper, and J. Graham. Training models of shape **from sets of examples**. BMVC 1992

# Active Appearance Models - 1998



T.F. Cootes, G. J. Edwards, and C. J. Taylor. Active appearance models, 1998  
 T. F. Cootes, G. V. Wheeler, K. N. Walker, C. J. Taylor: View-based active appearance models.2002,  
 I. Matthews and S. Baker, "Active Appearance Models Revisited," 2004.

# Morphable Models - 1996

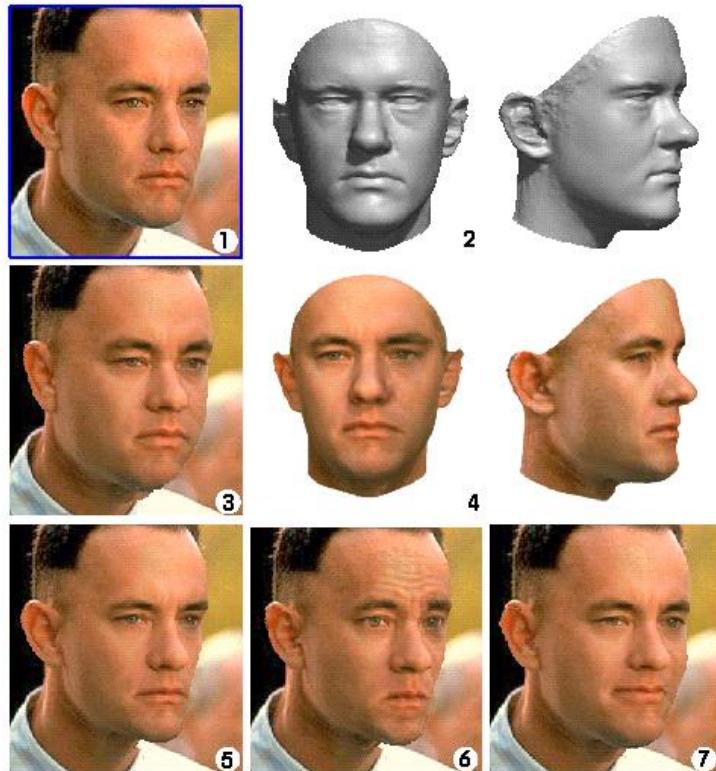


T. Vetter, T. Poggio: Linear Object Classes and Image Synthesis From a Single Example Image., 1997.

T. Vetter, M. J. Jones, T. Poggio: A bootstrapping algorithm for learning linear models of object classes. CVPR 1997

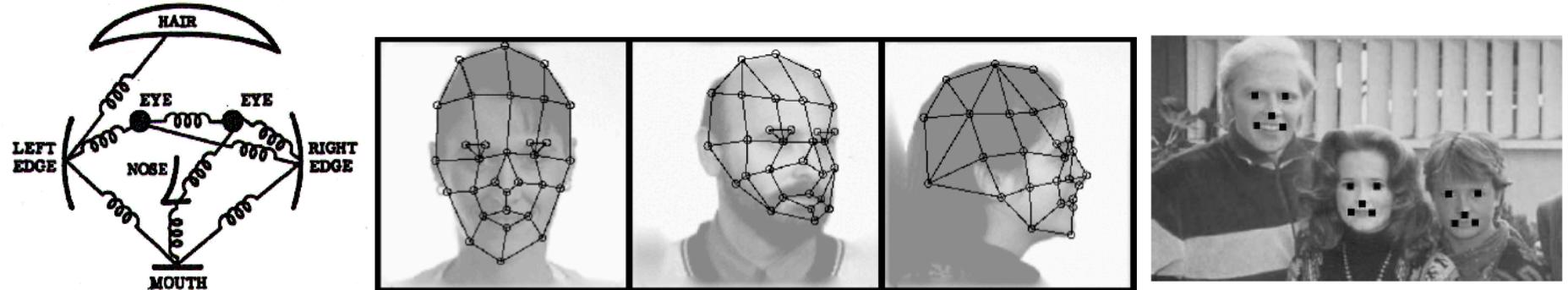
M. J. Jones, Y. Poggio: Multidimensional Morphable Models: A Framework for Representing and Matching Object Classes. IJCV 1998

## 3D Morphable Models - 1999



- V. Blanz, T. Vetter: A Morphable Model for the Synthesis of 3D Faces. SIGGRAPH 1999  
V. Blanz, T. Vetter: Face Recognition Based on Fitting a 3D Morphable Model. IEEE PAMI, 2003  
T. J. Cashman, A. W. Fitzgibbon: What Shape Are Dolphins? Building 3D Morphable Models from 2D Images, 2013

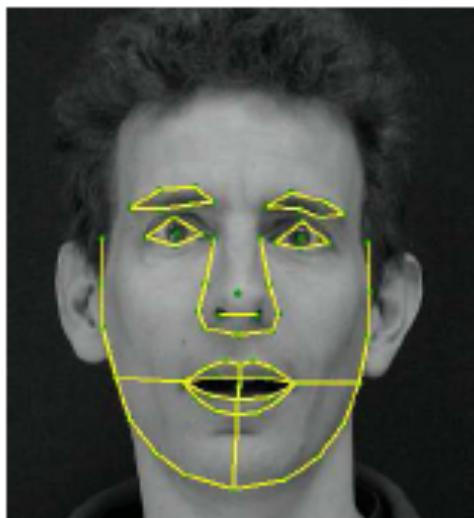
## Deformable Part Models (DPMs)



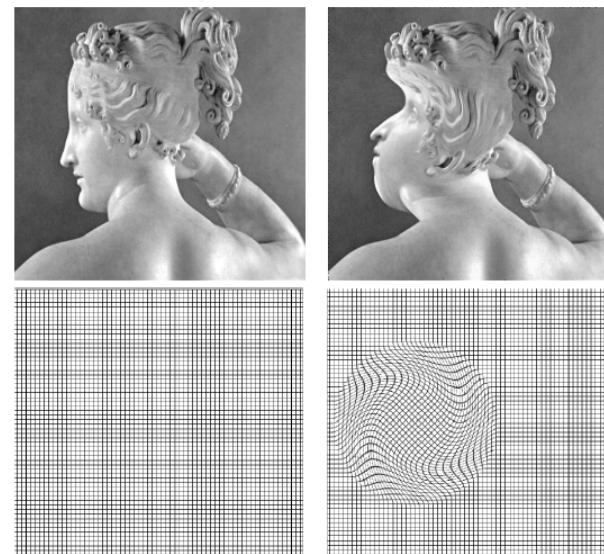
- M. Fischler and R. Erschlanger: The Representation and Matching of Pictorial Structures '73.*
- M. Lades, et al: Distortion Invariant Object Recognition in the Dynamic Link Architecture. '93*
- Y. Amit, A. Kong: Graphical Templates for Model Registration. '96*
- A. L. Yuille, J. M. Coughlan: An A\* perspective on deterministic optimization for deformable templates. '00*
- M. C. Burl, P. Perona: Recognition of Planar Object Classes. '96*
- M. C. Burl, M. Weber, P. Perona: A Probabilistic Approach to Object Recognition Using Local Photometry and Global Geometry. '98*
- M. Weber, M. Welling, P. Perona: Unsupervised Learning of Models for Recognition. '00*
- P. Felzenszwalb, and D. Huttenlocher, Pictorial Structures for Object Recognition, IJCV '05*
- P. Felzenszwalb, et. al., Object Detection with Discriminatively Trained DPMs, '10*

# Three classes of deformable models

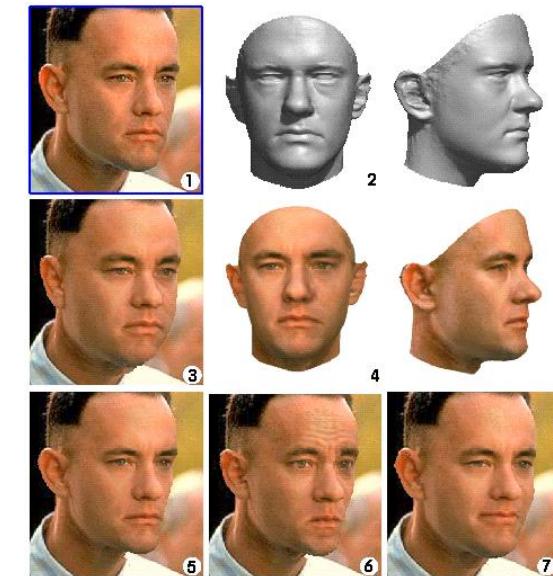
**ASM**



**AAM**



**3D Morphable**



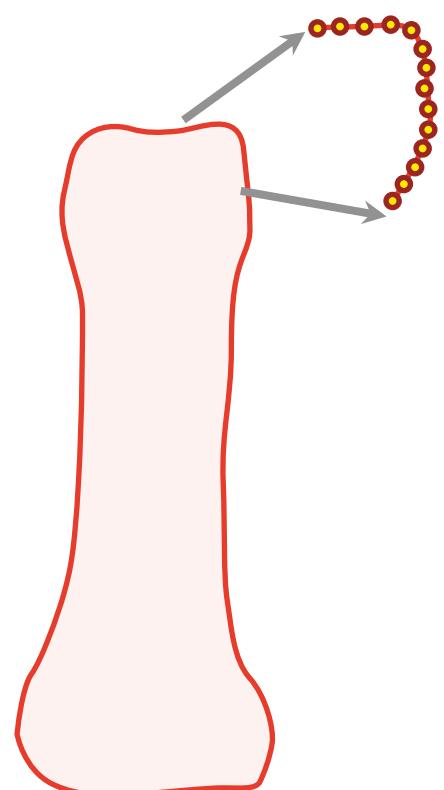
# Analyzing a hand radiograph



We have *a priori* knowledge  
about the typical appearance:  
e.g. bone shapes and texture

How can we represent this knowledge?  
How can we exploit it?

## How to capture *a priori knowledge*?



$$\mathbf{x}_i = \begin{pmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ \vdots \\ x_m \\ y_m \end{pmatrix}$$

Each example is represented by a vector containing the coordinates of the landmarks.

Learning: Model Acquisition  
Inference: Model Fitting

Goal:

- capture common properties of the bone
- find a representation that is restricted to plausible bones.

# Density Estimation

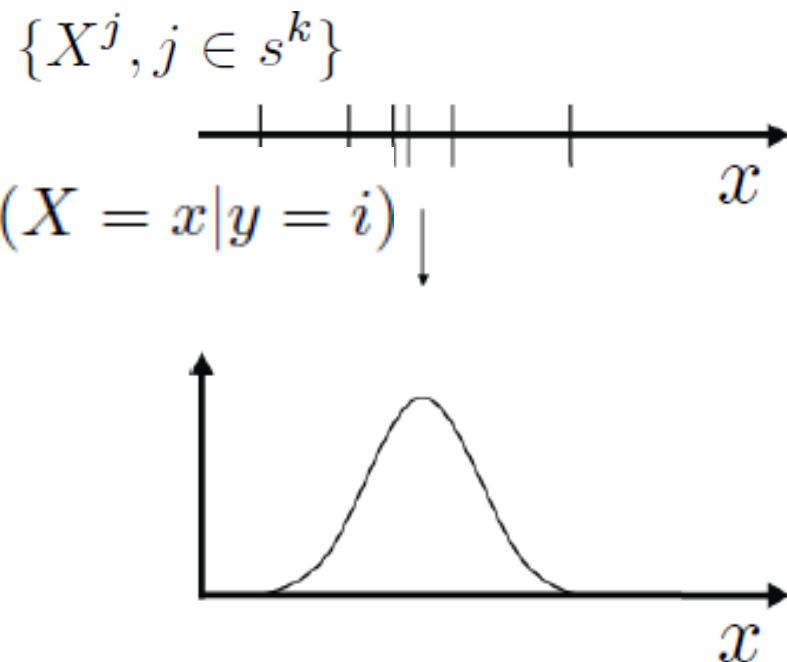
- Training set:  $\{(X^i, y^i)\}, \quad i = 1 \dots N$

- Examples corresponding to class k:  $s^k = \{i : y^i = k\}$

- Training data for class k:  $\{X^j, j \in s^k\}$

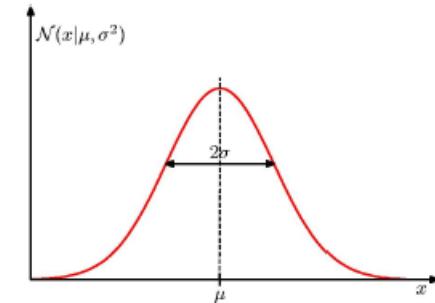
- One density estimate per class:  $P(X = x | y = i)$

for short:  $P(x)$



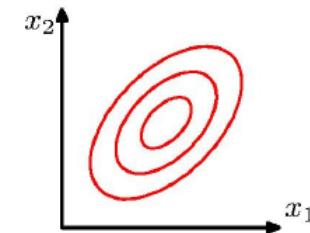
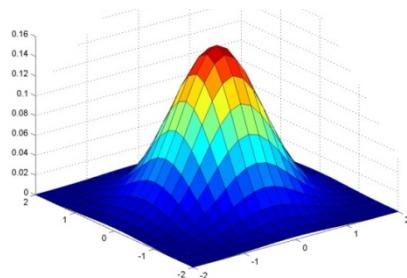
## Parametric Distributions: Gaussian

- 1D  $\mathcal{N}(x|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$



- ND  $\mathcal{N}(\mathbf{x}|\boldsymbol{\mu}, \boldsymbol{\Sigma}) = \frac{1}{(2\pi)^{D/2}|\boldsymbol{\Sigma}|^{1/2}} \exp\left\{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{x}-\boldsymbol{\mu})\right\}$

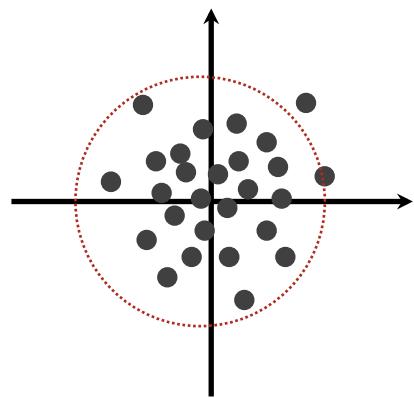
- e.g. 2D:



## Covariance matrix reminder

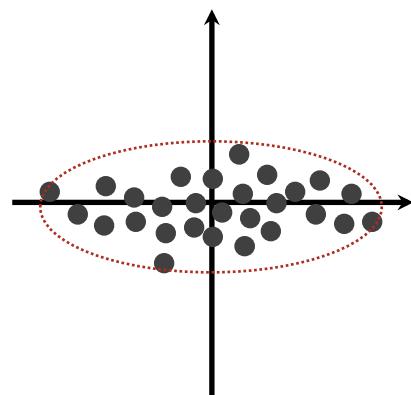
Covariance matrix:

$$\Sigma_{i,j} = E((x_i - E(x_i))(x_j - E(x_j)))$$

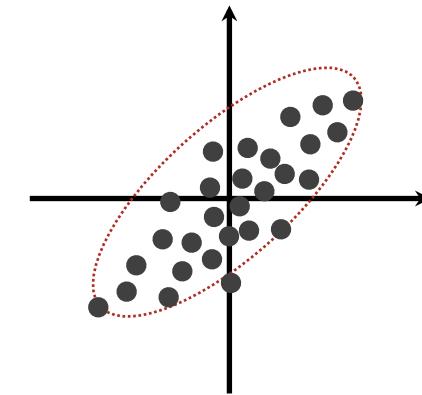


$$\Sigma = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Height, Income



$$\Sigma = \begin{pmatrix} 4 & 0 \\ 0 & 0.5 \end{pmatrix}$$



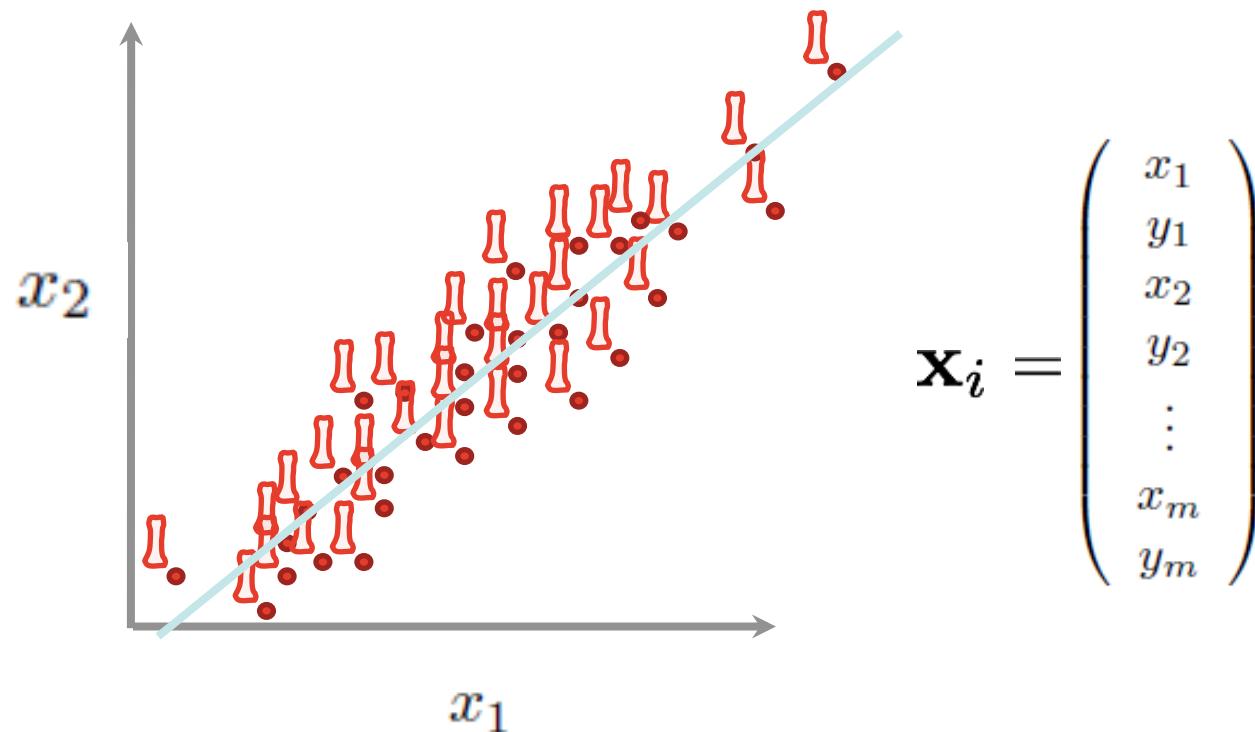
$$\Sigma = \begin{pmatrix} 2.45 & 1.2 \\ 1.2 & 2.1 \end{pmatrix}$$

Height, Weight

Uncorrelated coordinates: diagonal covariance

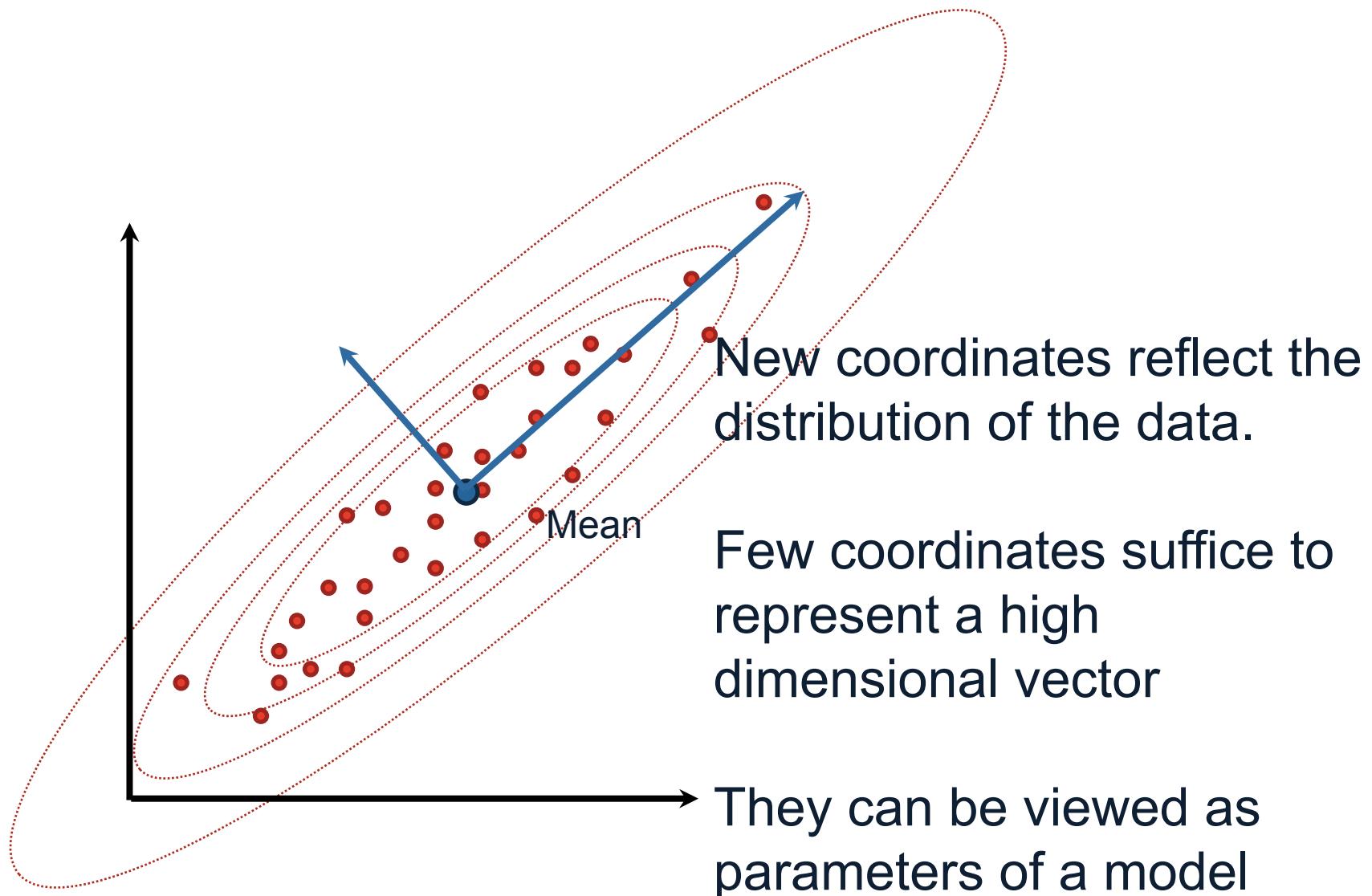
# The space of all bone shapes

- Bone shapes: vectors in  $R^{2m}$



- Goal: project data onto a low-dimensional linear subspace that best explains their variation.

# New subspace: 'better' coordinate system



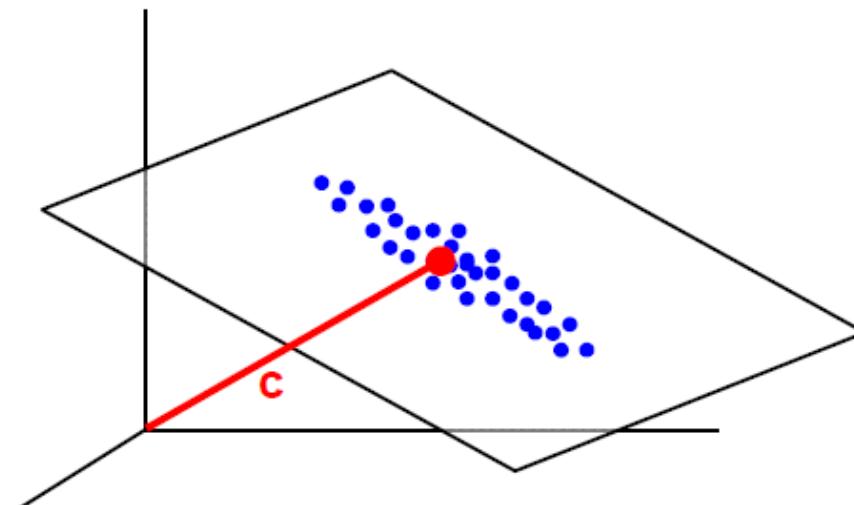
# Principal Component Analysis (PCA)

- Find a low-dimensional subspace to reconstruct high-dimensional data
- Reconstruction on orthogonal basis      Approximation with K terms

$$x^i = \sum_{j=1}^M h_j^i b_j$$

$$= \sum_{j=1}^M (h^{iT} b_j) b_j$$

$$\bar{x}^i = \sum_{j=1}^K (h^{iT} b_j) b_j$$



# Principal component analysis

- The  $k$  orthogonal directions that capture most of the data variance are the  $k$  leading (largest-eigenvalue) covariance eigenvectors

Factor Analysis

$\Lambda$  matrix

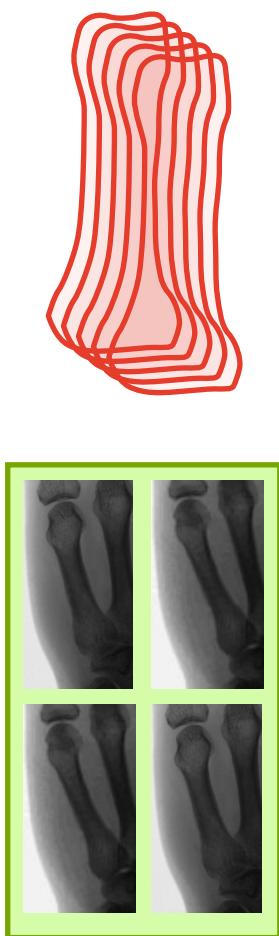
Hidden variables

PCA

Leading K eigenvectors of covariance

Inner product of data with eigenvectors

# Shape Eigenbasis



$$\text{shape} = \hat{\mathbf{m}} + b_1 \mathbf{e}_1 + b_2 \mathbf{e}_2 + b_3 \mathbf{e}_3$$

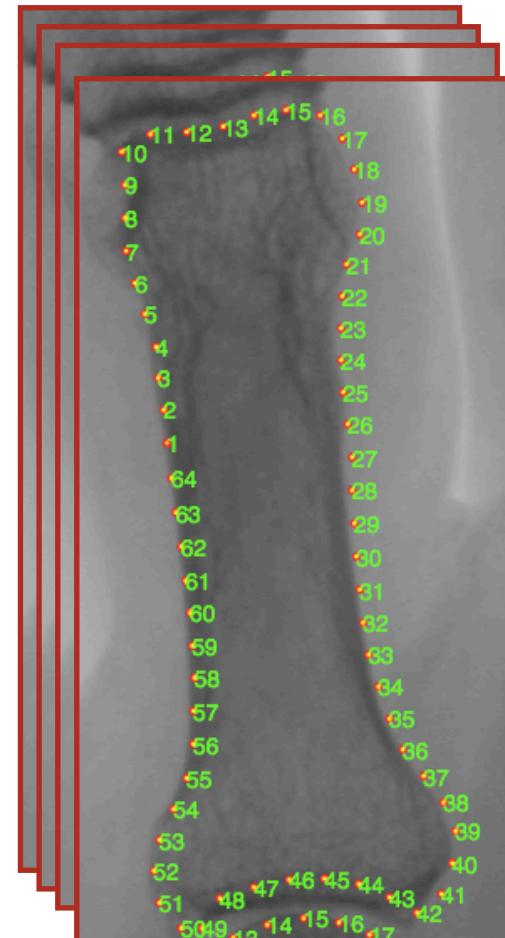
The diagram illustrates the decomposition of a target shape into a mean shape and three principal components. On the left, a red outline of a hand is shown. To its right is an equals sign (=). Following the equals sign are three red outlines of hands, each enclosed in a vertical teal line. After the first outline, there is a plus sign (+). After the second outline, there is another plus sign (+). After the third outline, there is a final plus sign (+).

Slide credits: G. Langs

# Active shape models (ASM)

- A set of training examples (images)
- A set of landmarks, that are present on all images
- Build a statistical model of shape variation (PCA)
- Build a statistical model of the local texture (PCA)
- Use the model for the search in a new image

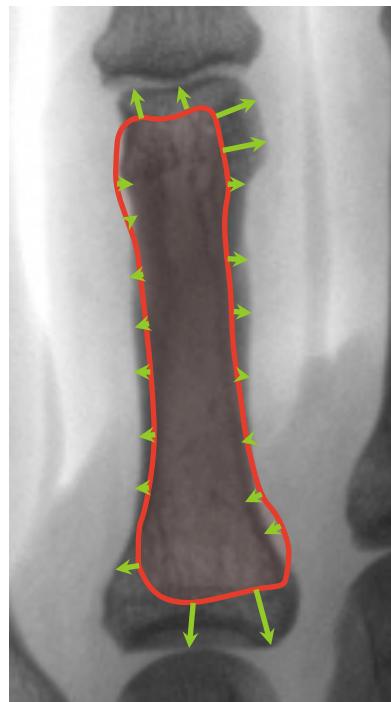
T. F. Cootes, C. J. Taylor, D. H. Cooper, and J. Graham. Training models of shape **from sets of examples**. BMVC 1992



## ASM search



Initialize



Adjust to texture



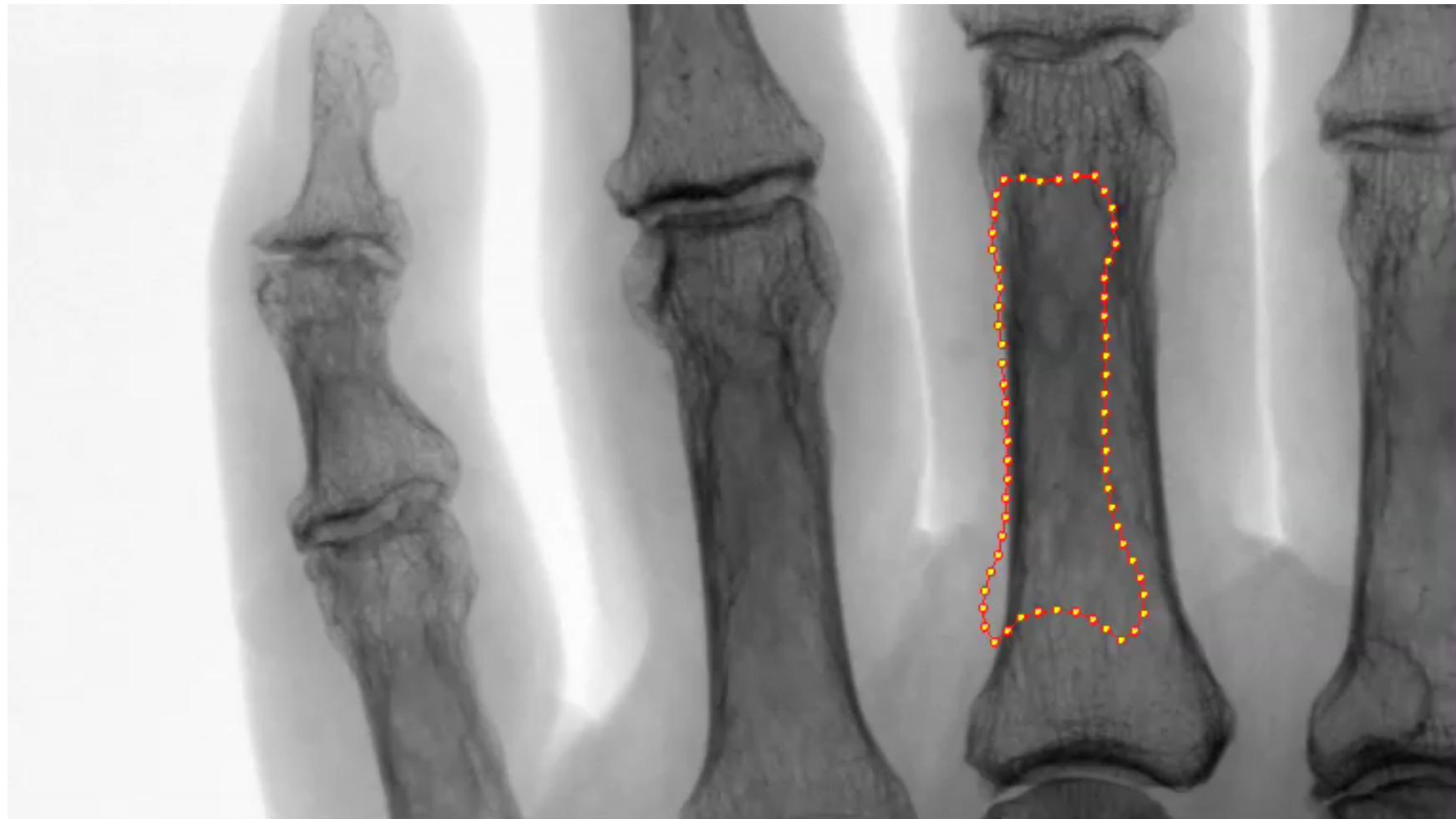
Fit to shape model



## ASM search

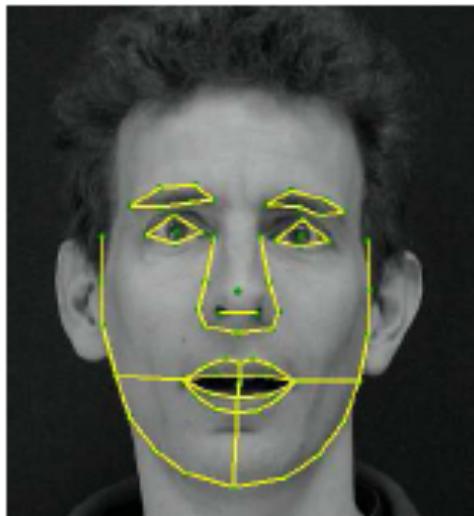


# ASM search

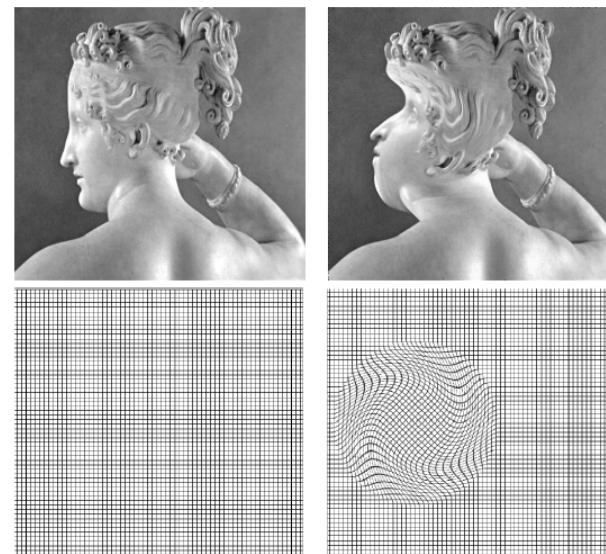


# Three classes of deformable models

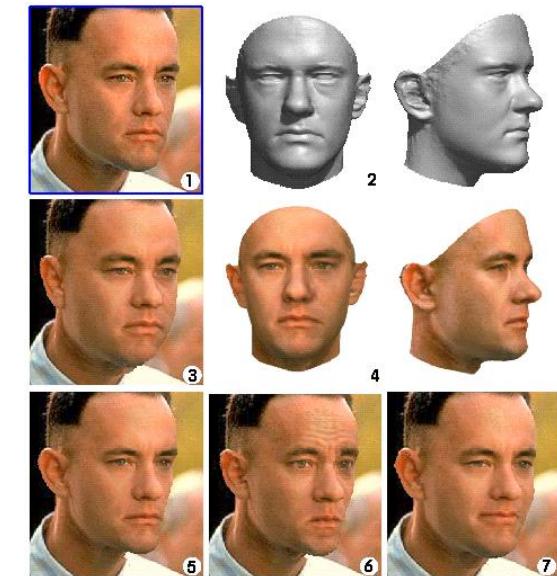
ASM



AAM



3D Morphable



# Eigenfaces (Sirovich & Kirby 87, Turk & Pentland 91)

- Very few 100x100 vectors correspond to valid face images



- model the subspace ('manifold') of face images

# Eigenfaces

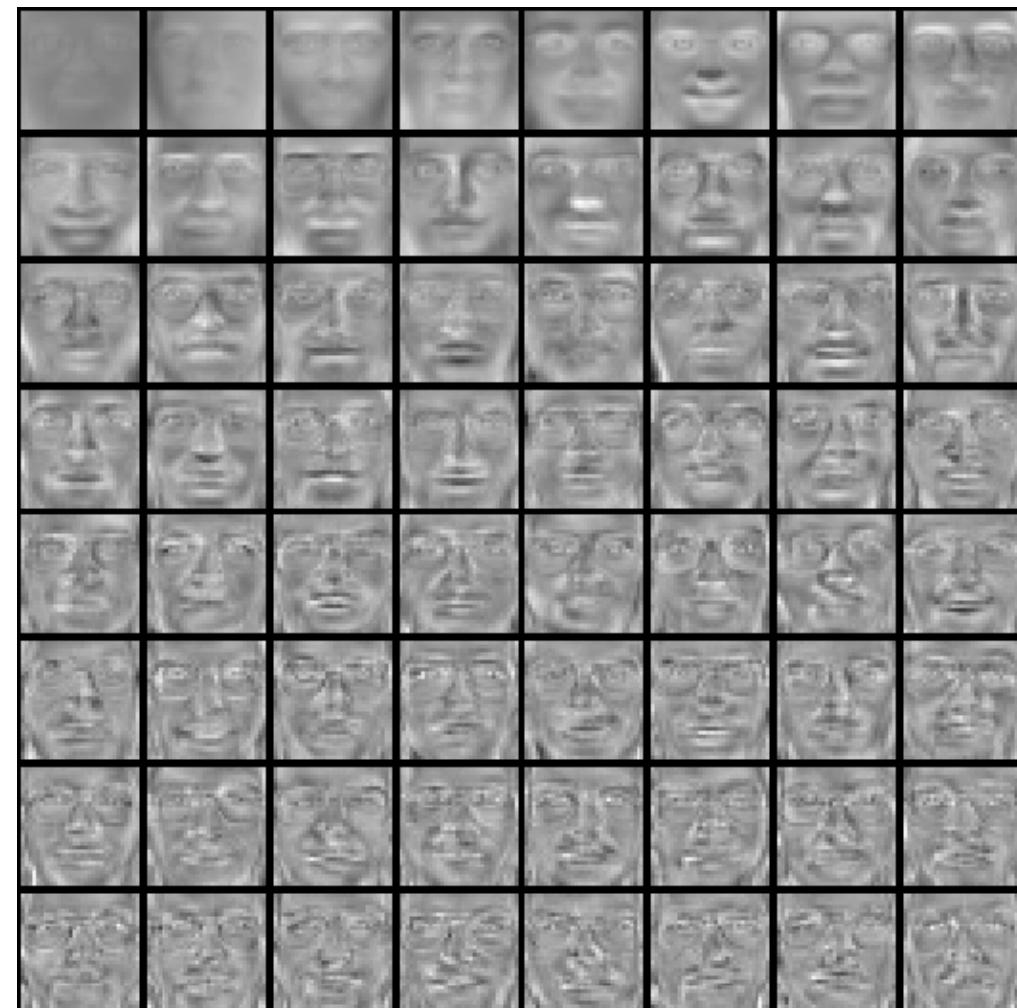
- Training images
- $x_1, \dots, x_N$



# Eigenfaces

Top eigenvectors:  $u_1, \dots, u_k$

Mean:  $\mu$



# Eigenfaces

Principal component (eigenvector)  $u_k$



$\mu + 3\sigma_k u_k$



$\mu - 3\sigma_k u_k$



# Eigenfaces example

- Face  $x$  in “face space” coordinates:



$$\mathbf{x} \rightarrow [\mathbf{u}_1^T(\mathbf{x} - \mu), \dots, \mathbf{u}_k^T(\mathbf{x} - \mu)]$$

$$= w_1, \dots, w_k$$

- Reconstruction:

$$\begin{aligned} \hat{\mathbf{x}} &= \mathbf{\mu} + \begin{matrix} \mathbf{u}_1 \\ \mathbf{u}_2 \\ \mathbf{u}_3 \\ \mathbf{u}_4 \\ \vdots \end{matrix} w_1 + w_2 + w_3 + w_4 + \dots \end{aligned}$$

# Limitations

- Global appearance method: not robust to misalignment, background variation

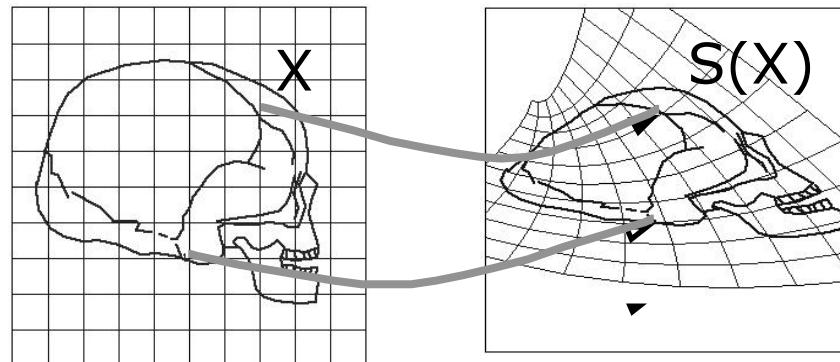


# Appearance and Shape interpolation



# Active Appearance Models (AAMs)

$$T(\mathbf{x}) = I(\mathcal{S}(\mathbf{x}))$$



Shape:

$$\mathcal{S}(\mathbf{x}; \mathbf{s}) = \sum_i \mathbf{s}_i S_i(\mathbf{x}),$$

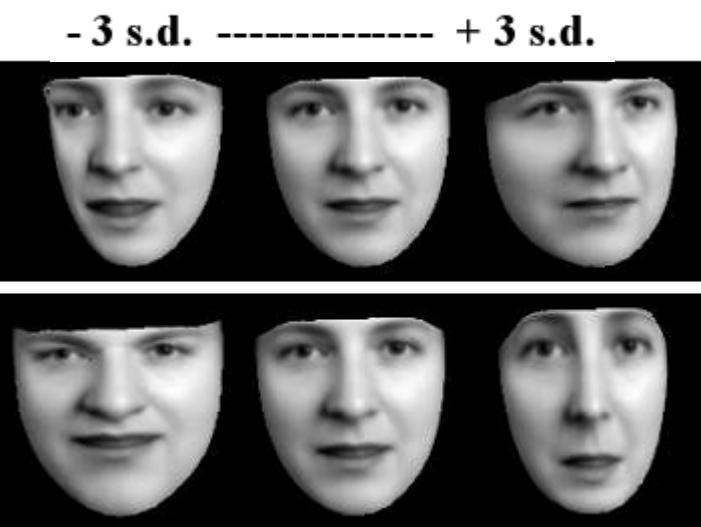
Appearance:

$$\mathcal{T}(\mathbf{x}; \mathbf{t}) = \sum_i \mathbf{t}_i T_i(\mathbf{x})$$

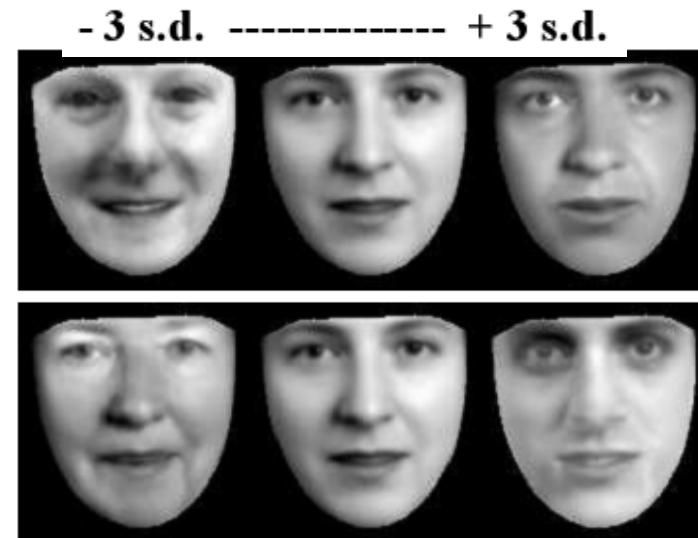
Synthesis:

$$I(\mathcal{S}(\mathbf{x}; \mathbf{s})) \simeq \mathcal{T}(\mathbf{x}; \mathbf{t})$$

# Active Appearance Models (AAMs)



First two modes of shape variation

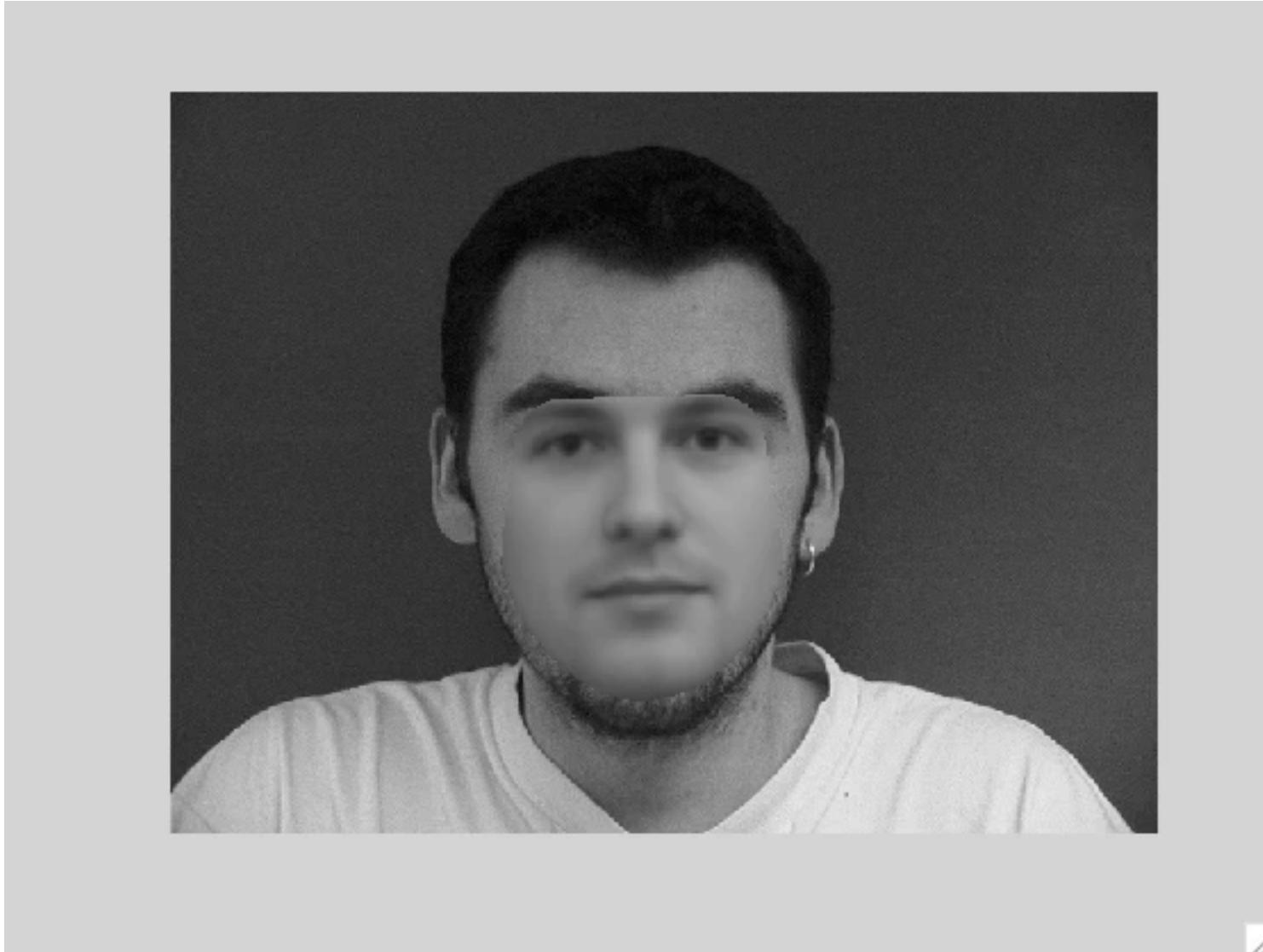


First two modes of gray-level variation



First four  
modes of  
appearance  
variation

## AAM Search



Slide credits: Tim Cootes

## Active Appearance Model Search (Results)



Initial

2 its

8 its

14 its

20 its

converged

Slide credits: Tim Cootes

# Active Appearance Model fitting

Given:

- 1) an appearance model,
- 2) a new image,
- 3) a starting approximation

Find:

the best matching  
synthetic image

- Minimize reconstruction error

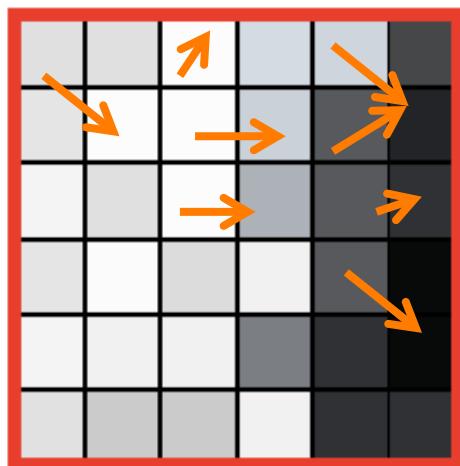
$$E(\mathbf{s}, \mathbf{t}) = \sum_{\mathbf{x}} [I(\mathcal{S}(\mathbf{x}; \mathbf{s})) - \mathcal{T}(\mathbf{x}; \mathbf{t})]^2$$

- Alternate between estimating  $\mathbf{s}$  and  $\mathbf{t}$
- For  $\mathbf{t}$ : projection of deformed image  $I(\mathcal{S}(\mathbf{x}; \mathbf{s}))$  onto PCA basis
- For  $\mathbf{s}$ ?

## Reminder: Lucas-Kanade method

Brightness constancy constraint

$$I(x_i + u_i, y + v_i, t) = I(x_i, y_i, t + 1)$$



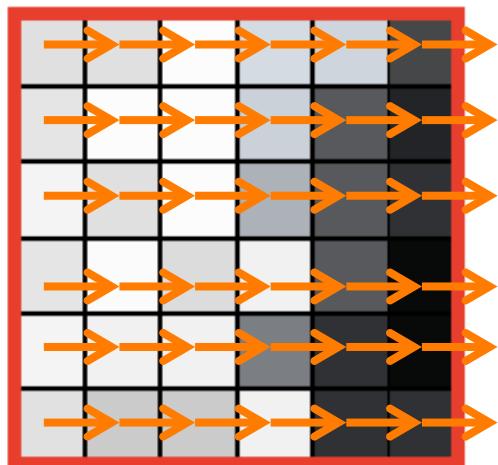
Linearization:

$$I_x(p_i)u_i + I_y(p_i)v_i + I_t(p_i) = 0$$

## Reminder: Lucas-Kanade method

Brightness constancy constraint

$$I(x_i + u, y + v, t) = I(x_i, y_i, t + 1)$$



Linearization:

$$I_x(p_i)u + I_y(p_i)v + I_t(p_i) = 0$$

## From Lucas-Kanade to AAMs

Brightness constancy constraint

$$I(x_i + u, y + v, t) = I(x_i, y_i, t + 1)$$

$$I(\mathbf{x}_i + u \cdot (1, 0) + v \cdot (0, 1), t) = I(\mathbf{x}_i, t + 1)$$

AAM synthesis equation:

$$I_1(\mathbf{x}_i + \sum_k a_k \mathbf{b}_k(\mathbf{x}_i)) = I_2(\mathbf{x}_i)$$

## AAM parameter estimation: shape

- Iterative scheme

$$E(\mathbf{s}, \mathbf{t}) = \sum_{\mathbf{x}} [I(\mathcal{S}(\mathbf{x}; \mathbf{s})) - T(\mathbf{x}; \mathbf{t})]^2$$

$$E(\mathbf{s} + \Delta \mathbf{s}, \mathbf{t}) = E(\mathbf{s}, \mathbf{t}) + \mathcal{J} \Delta \mathbf{s} + \frac{1}{2} \Delta \mathbf{s}^T \mathcal{H} \Delta \mathbf{s}$$

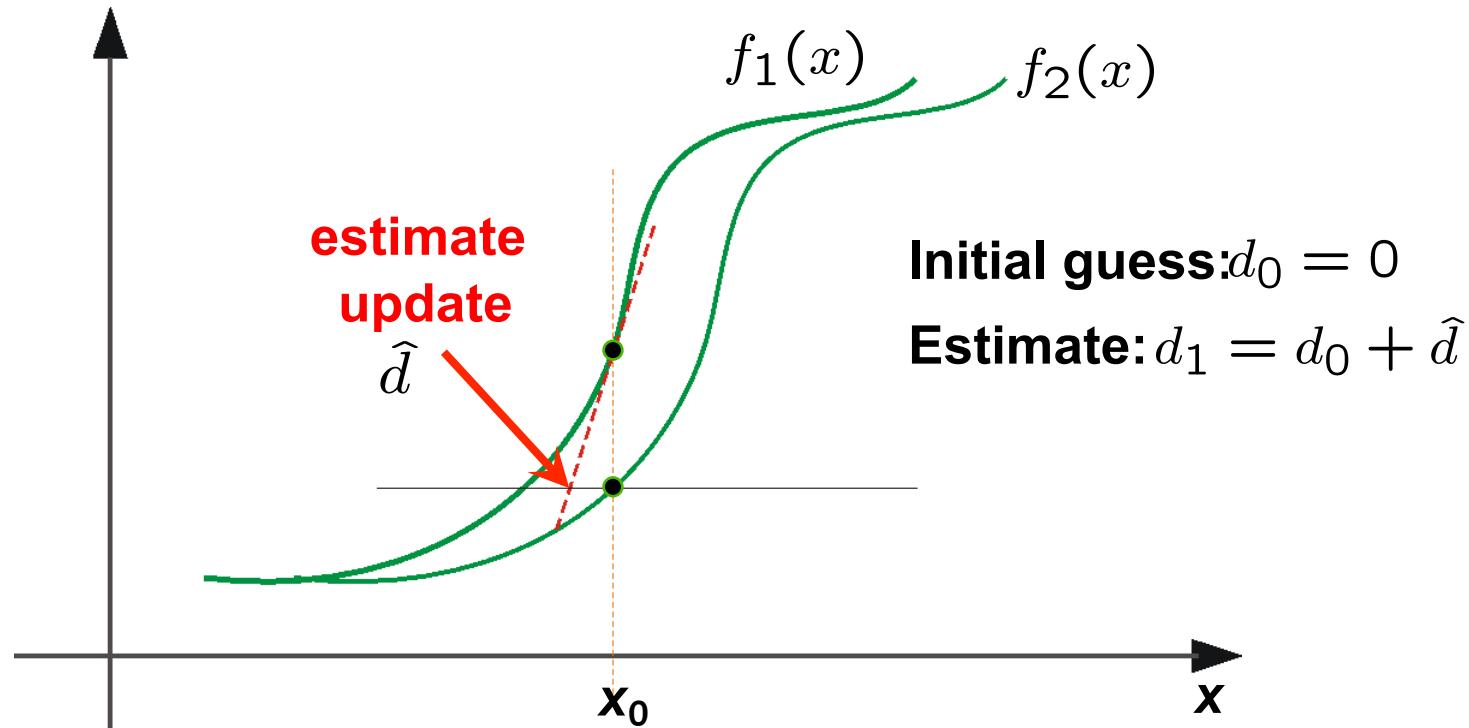
$$\Delta \mathbf{s}^* = -\mathcal{J} \mathcal{H}^{-1} \quad \mathbf{s}' = \mathbf{s} - \mathcal{J} \mathcal{H}^{-1}$$

$$I(S(\mathbf{x}; \mathbf{s} + \Delta \mathbf{s})) \simeq I(S(\mathbf{x}; \mathbf{s})) + \sum_{i=1}^{N_S} \frac{dI}{d\mathbf{s}_i}(\mathbf{x}; \mathbf{s}) \Delta \mathbf{s}_i$$

$$\frac{dI}{d\mathbf{s}_i}(\mathbf{x}; \mathbf{s}) = \frac{\partial I(S(\mathbf{x}; \mathbf{s}))}{\partial x} \frac{\partial S_x}{\partial \mathbf{s}_i} + \frac{\partial I(S(\mathbf{x}; \mathbf{s}))}{\partial y} \frac{\partial S_y}{\partial \mathbf{s}_i},$$

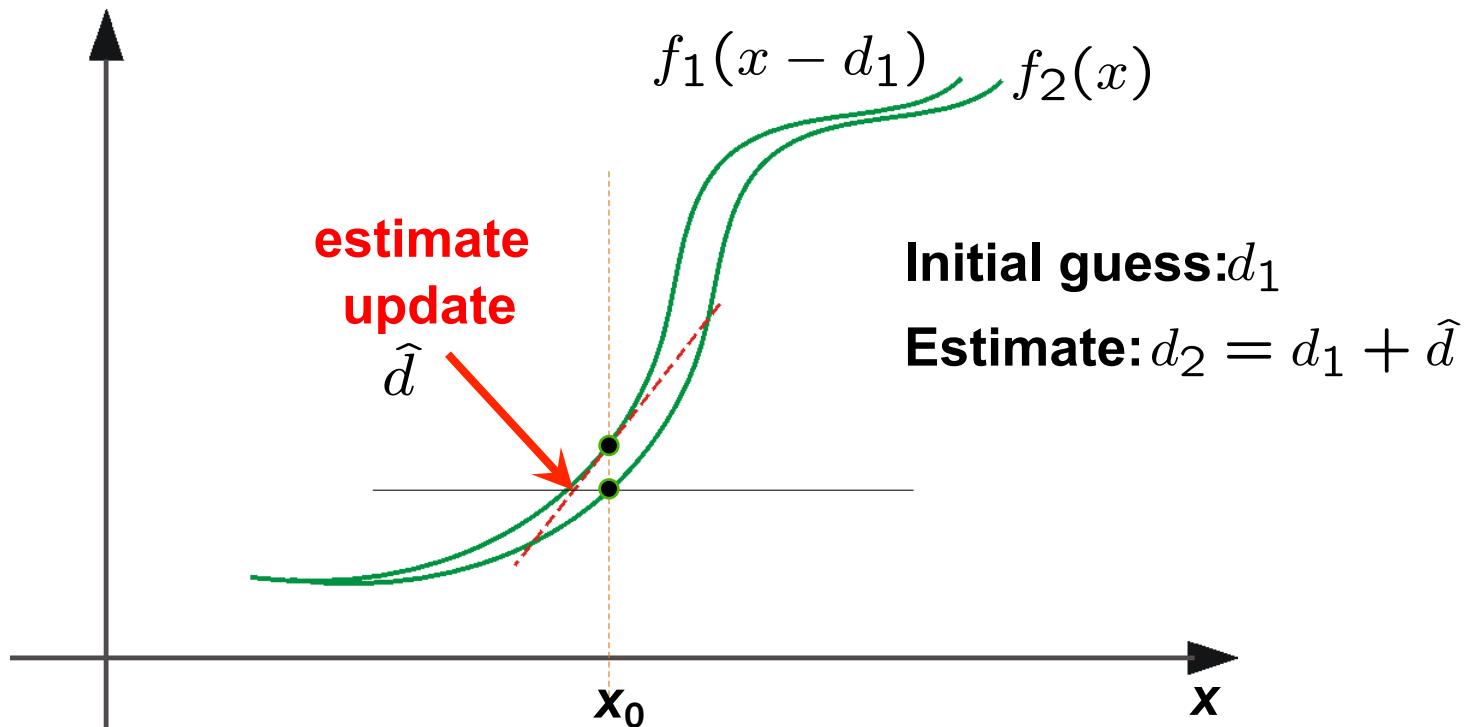
$$\mathcal{J}_i = \sum_x [I(S(\mathbf{x}, \mathbf{s})) - T(\mathbf{x}, \mathbf{t})] \frac{dI}{d\mathbf{s}_i}(x) \quad \mathcal{H}_{i,j} = \sum_x \frac{dI}{d\mathbf{s}_i}(x) \frac{dI}{d\mathbf{s}_j}(x)$$

# Optical Flow: Iterative Estimation

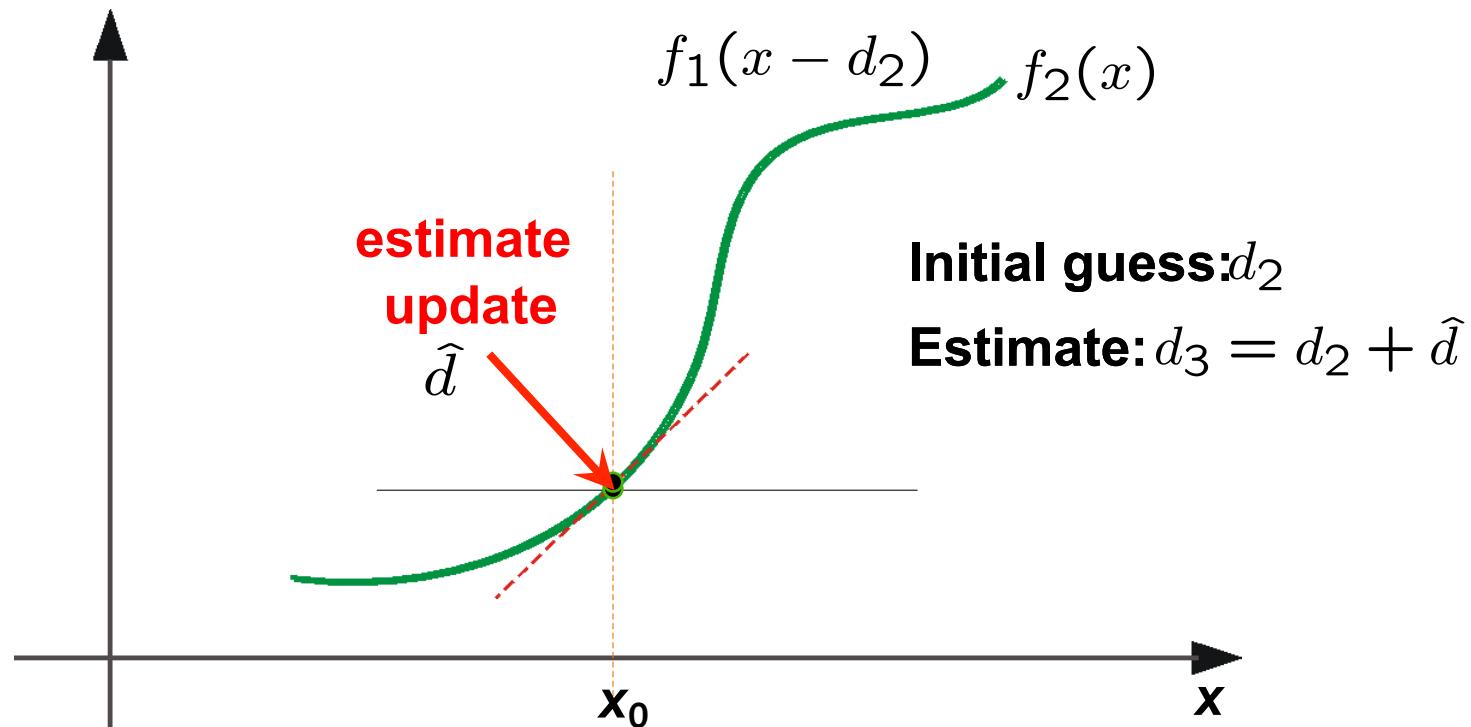


(using  $d$  for *displacement* here instead of  $u$ )

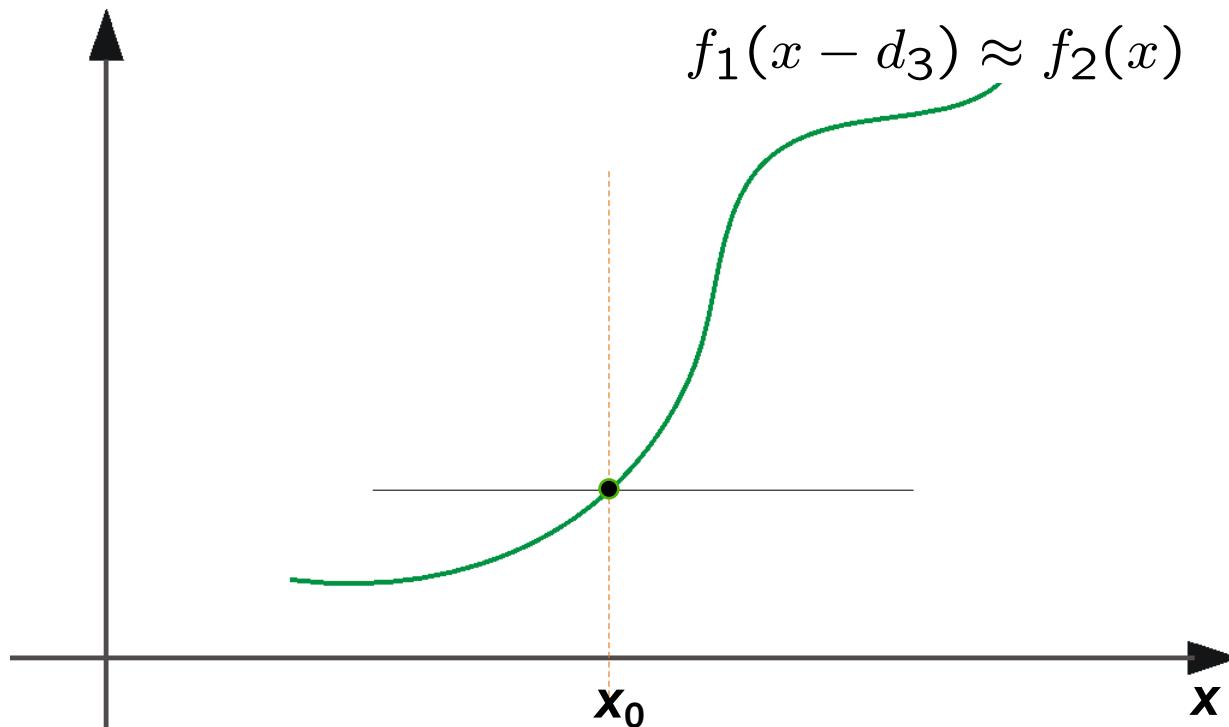
# Optical Flow: Iterative Estimation



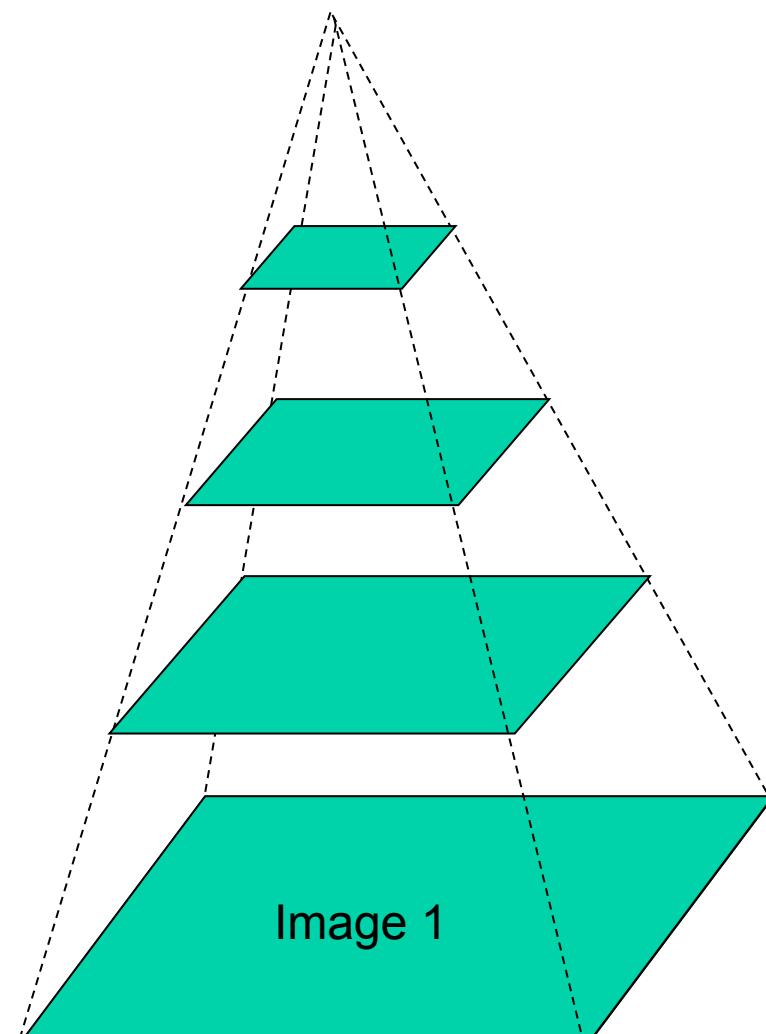
# Optical Flow: Iterative Estimation



# Optical Flow: Iterative Estimation



# Coarse-to-fine Optical Flow Estimation



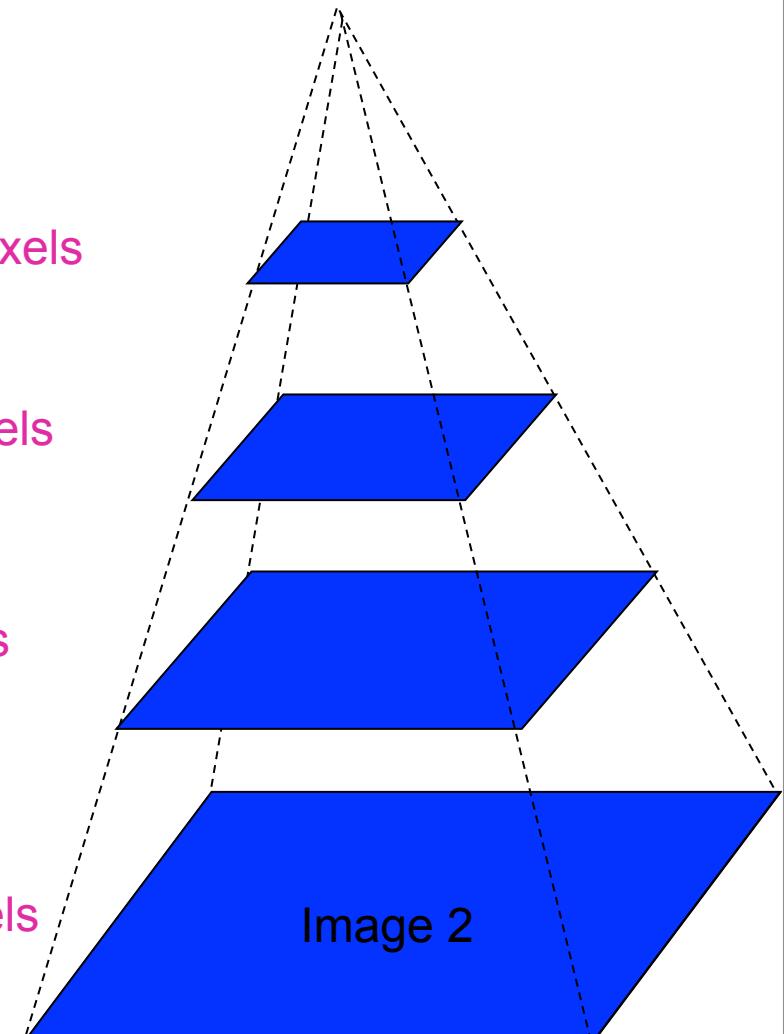
Gaussian pyramid of image 1

$u=1.25$  pixels

$u=2.5$  pixels

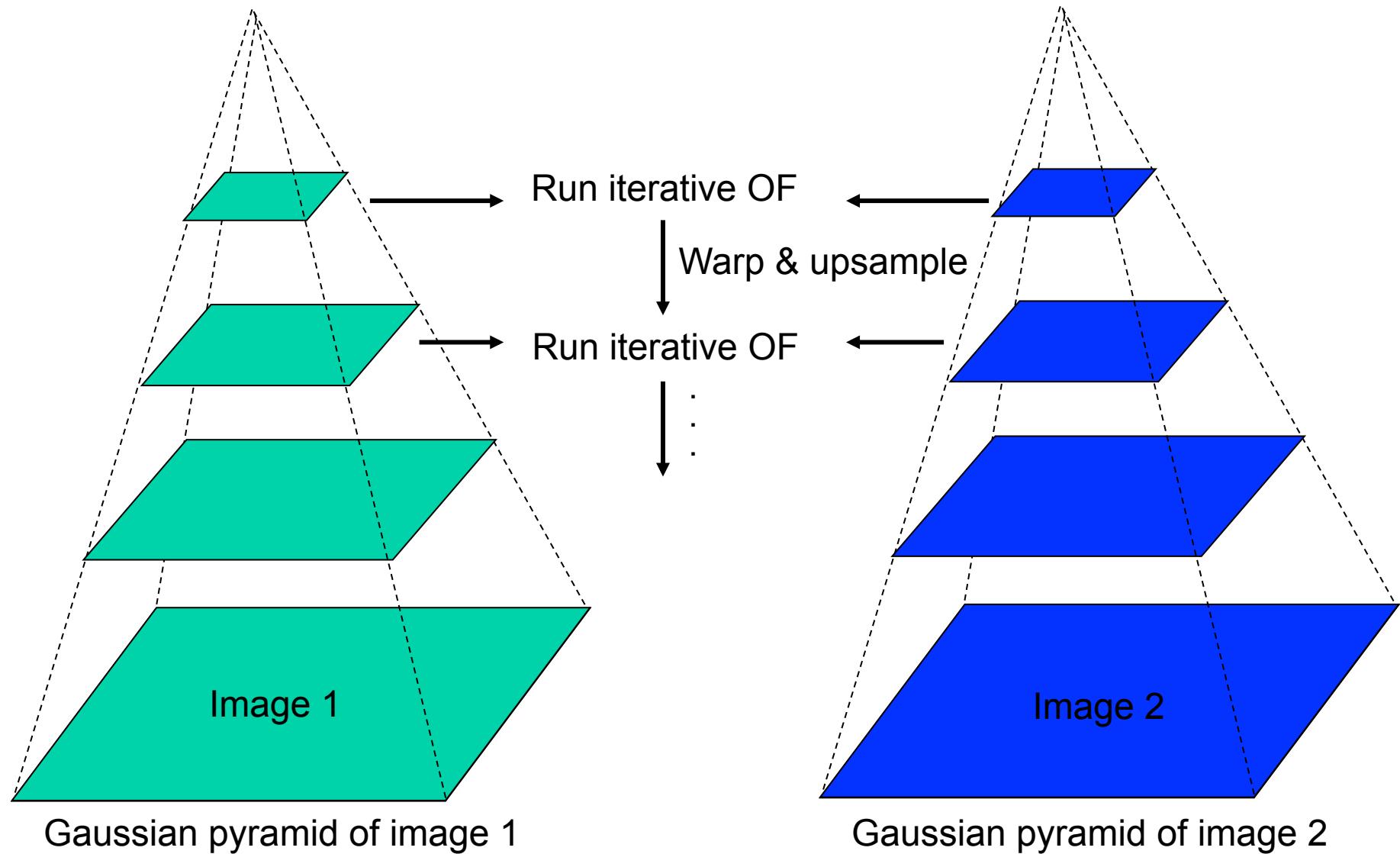
$u=5$  pixels

$u=10$  pixels

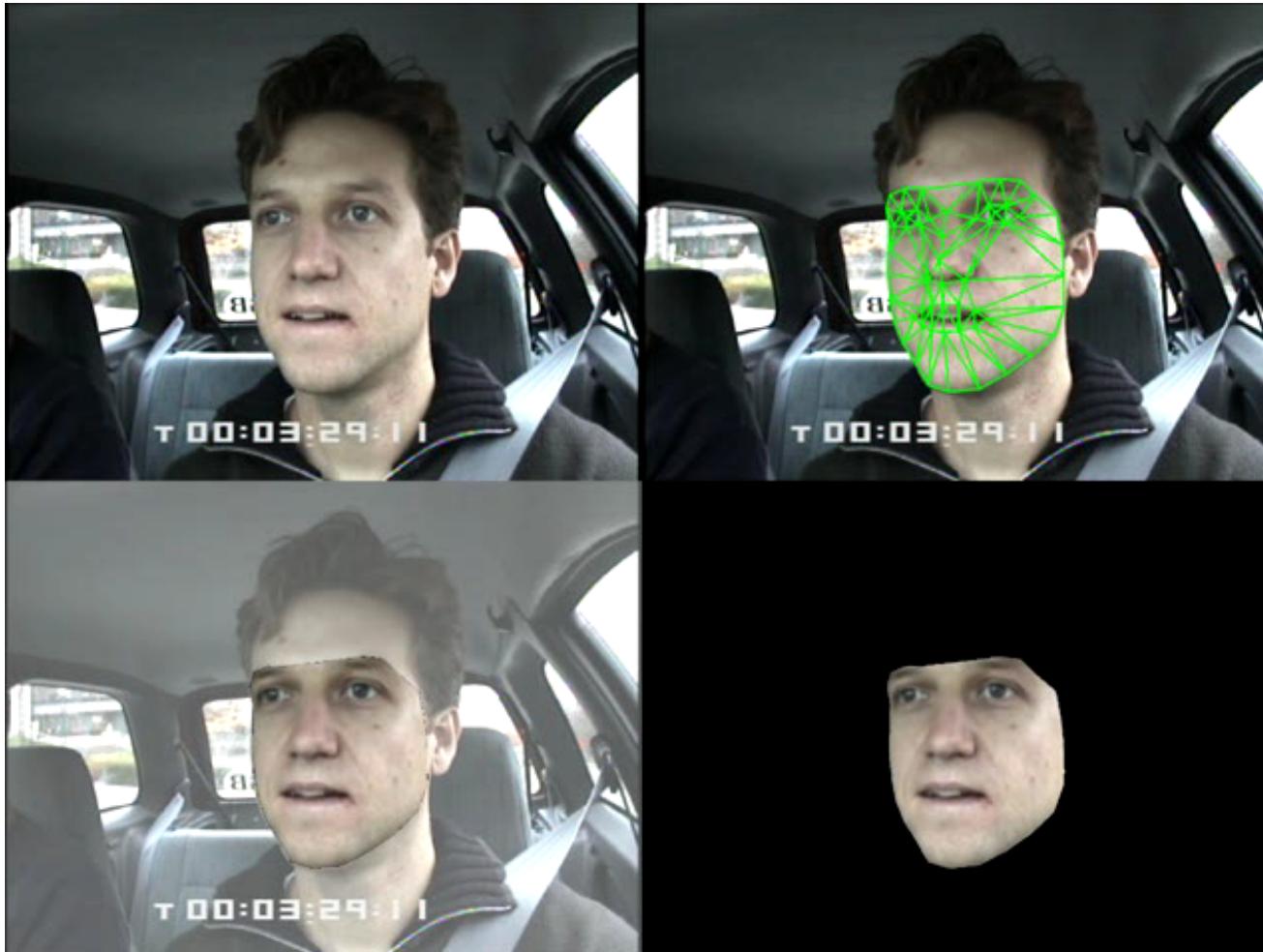


Gaussian pyramid of image 2

# Coarse-to-fine Optical Flow Estimation

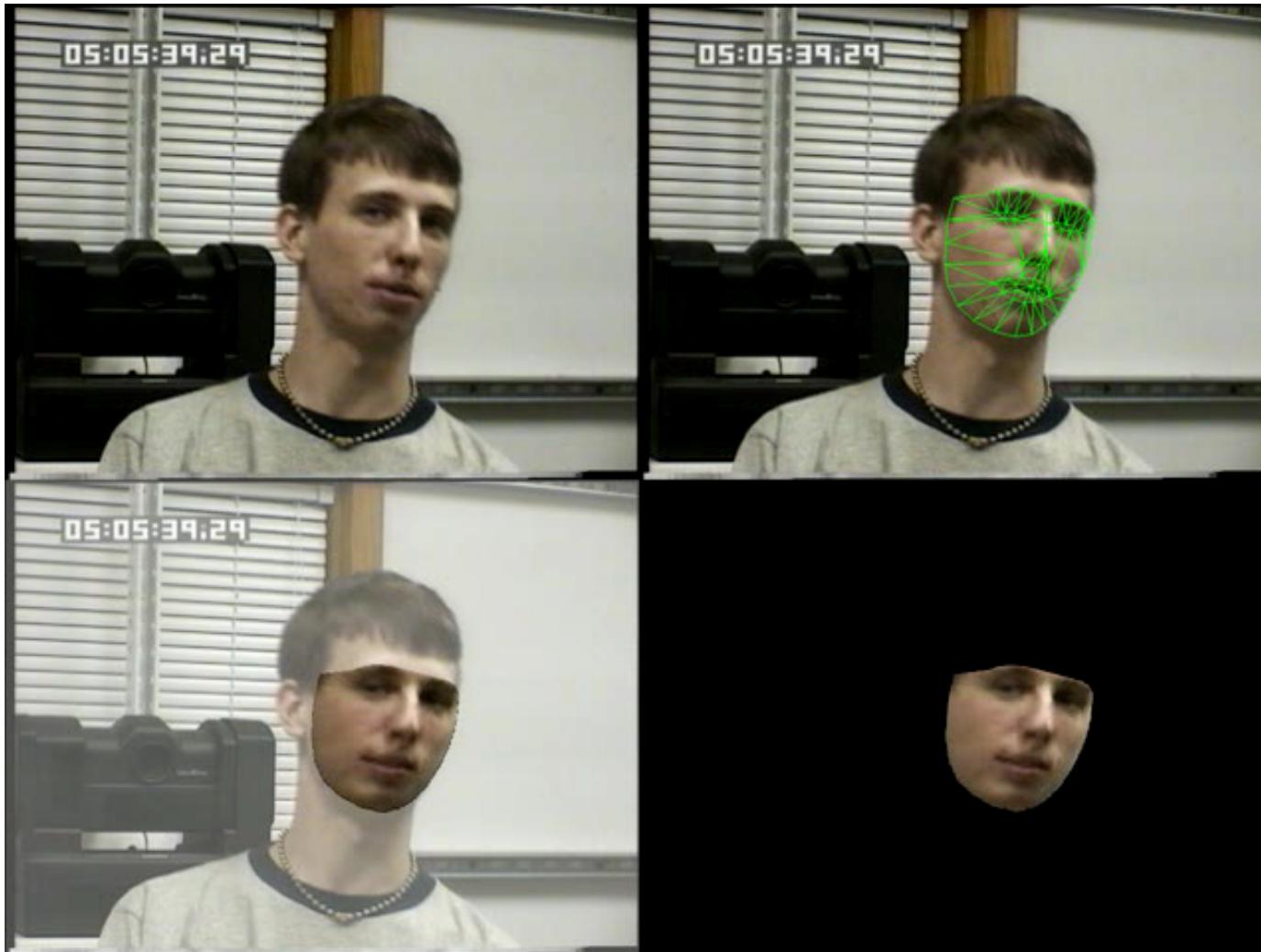


## AAM for face tracking



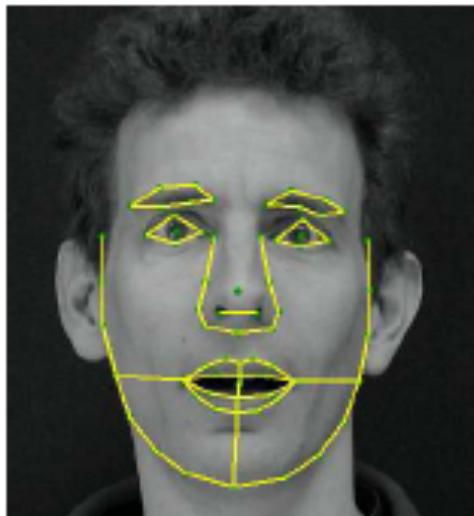
CMU group: I. Matthews, S. Baker, R. Gross  
(230 Frames per second, 2004)

## AAM for face tracking

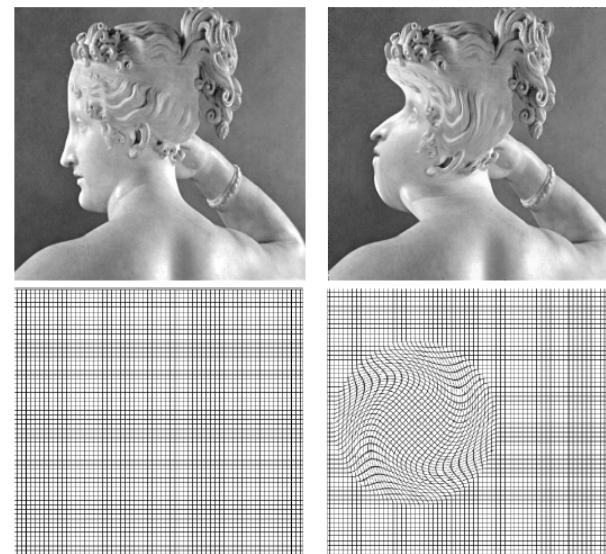


# Three classes of deformable models

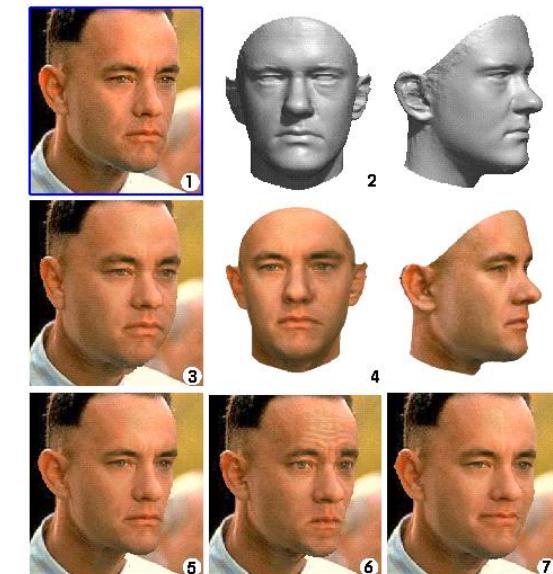
ASM



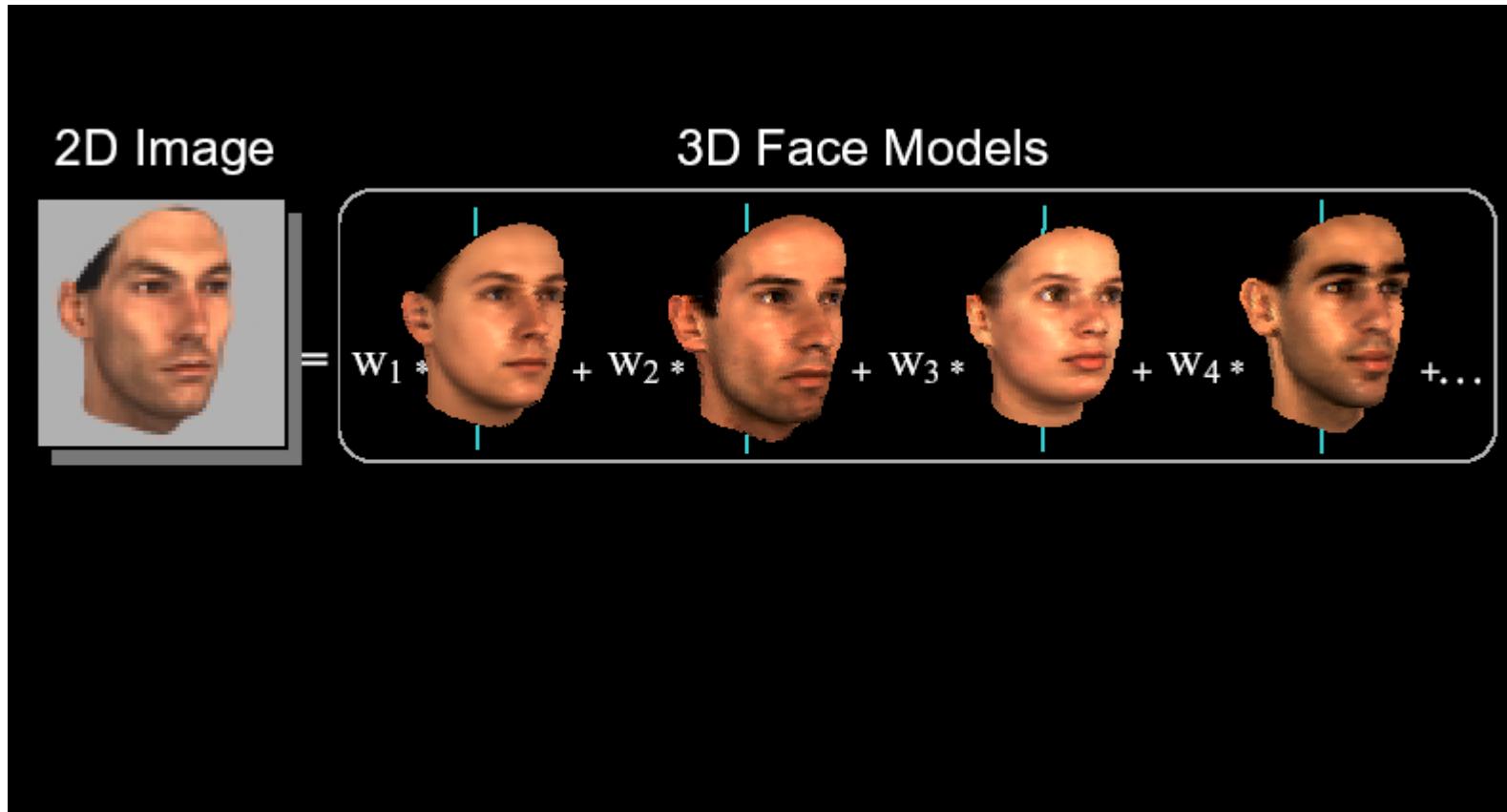
AAM



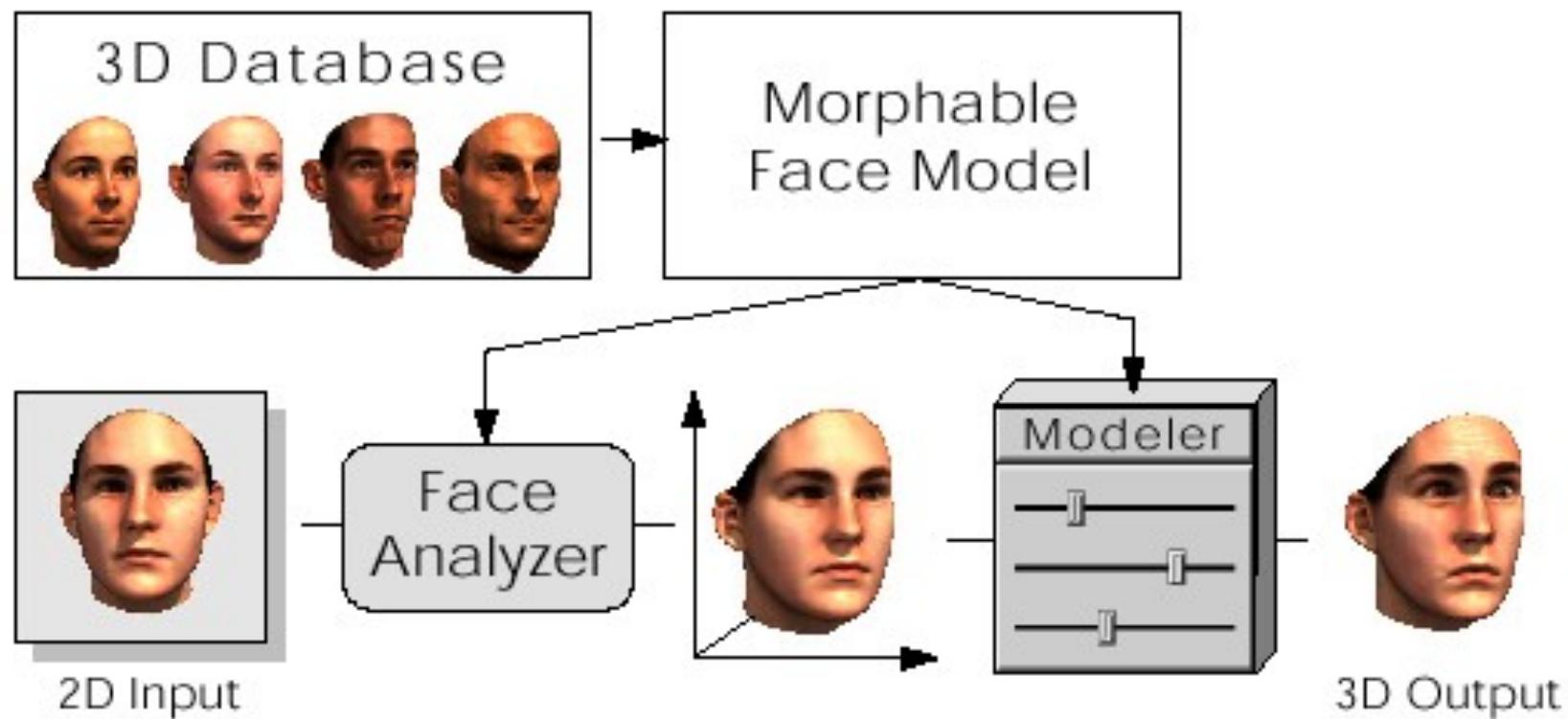
3D Morphable



# Morphable Models: Blanz and Vetter



# 3-D Morphable Models

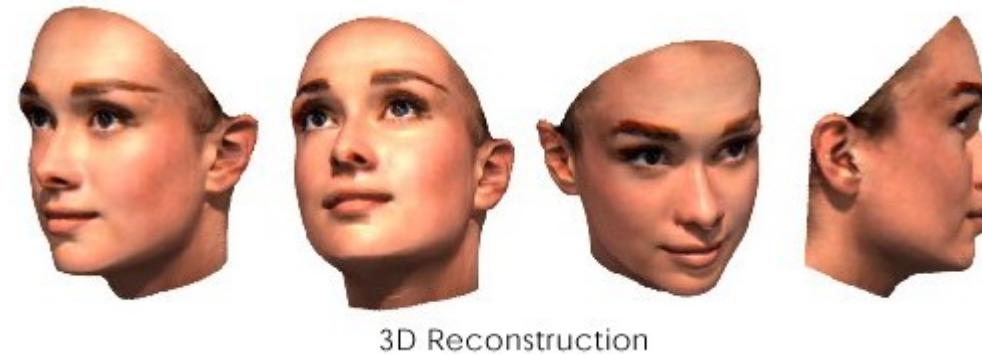


## 3-D Morphable Model fitting

- Rough manual initialization
- Gradient descent to minimize reconstruction error functional

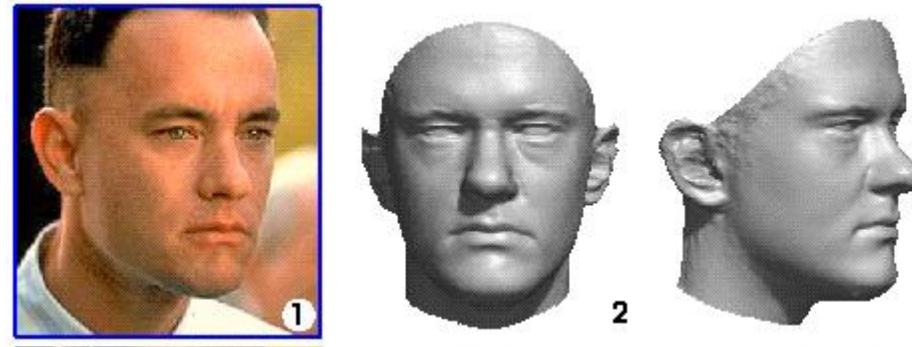


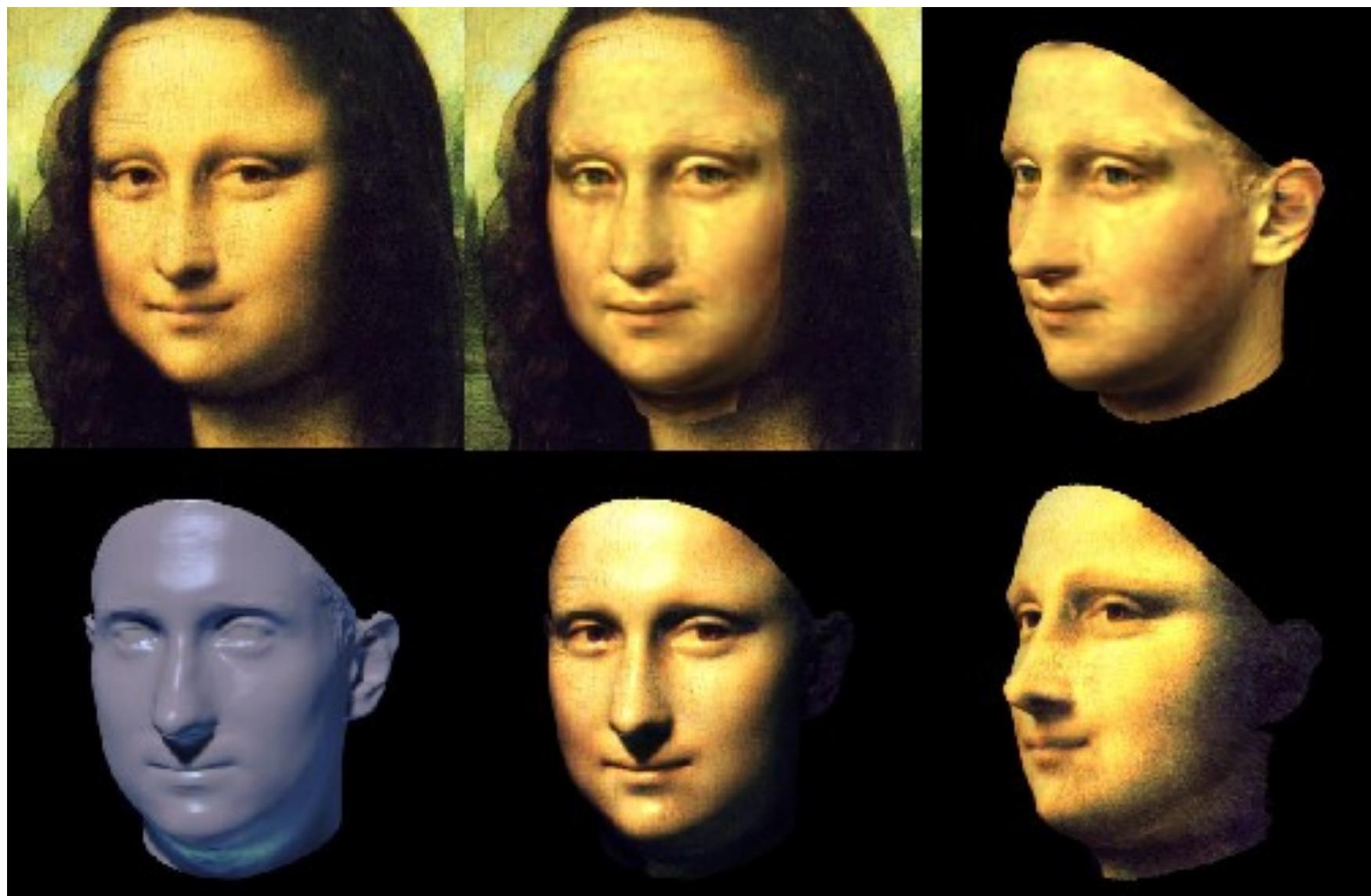
- And then



# 3D Morphable models

Recover Shape



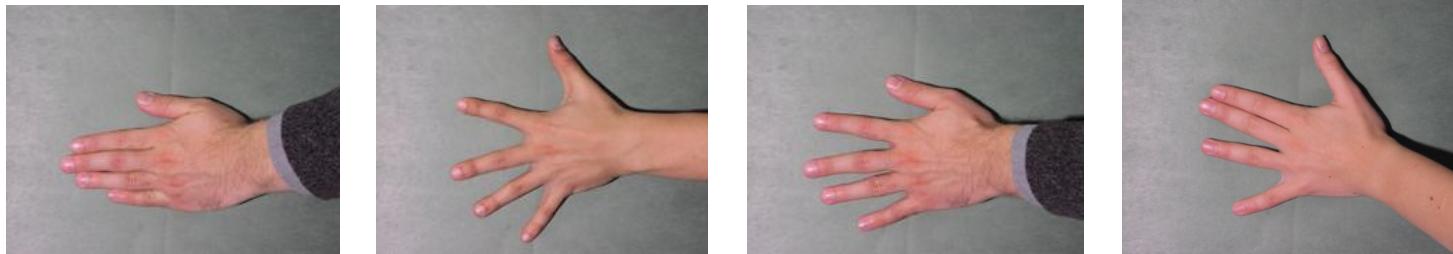


# A Morphable Model for the Synthesis of 3D Faces

Volker Blanz & Thomas Vetter

MPI for Biological Cybernetics  
Tübingen, Germany

# Unsupervised learning of deformable models

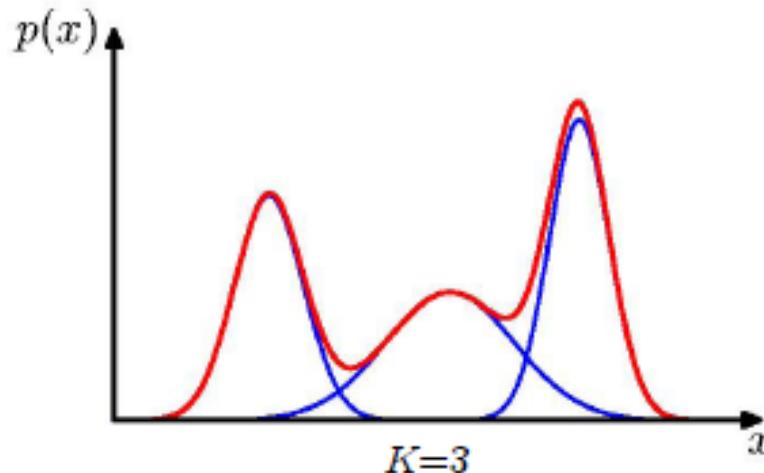


# Mixture of Gaussians model

Combine simple models  
into a complex model:

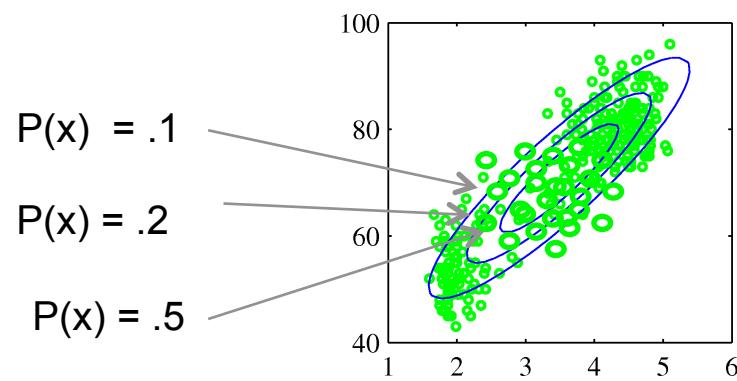
$$p(\mathbf{x}) = \sum_{k=1}^K \pi_k \mathcal{N}(\mathbf{x} | \boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)$$

↑ Component  
Mixing coefficient

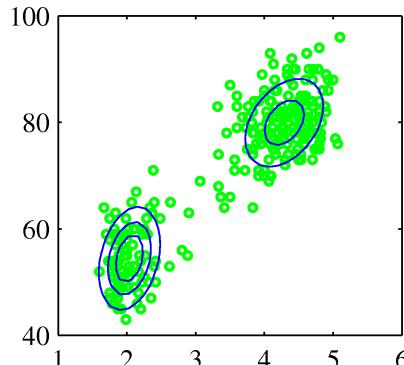


$$\forall k : \pi_k \geq 0 \quad \sum_{k=1}^K \pi_k = 1$$

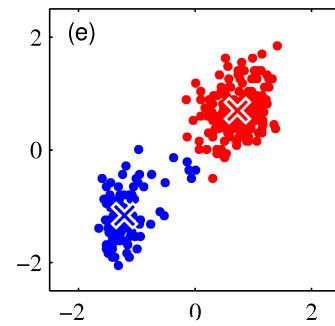
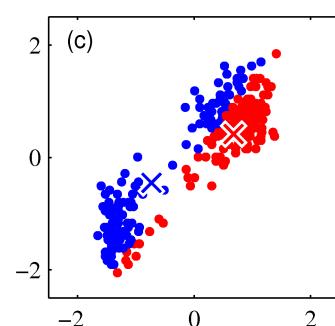
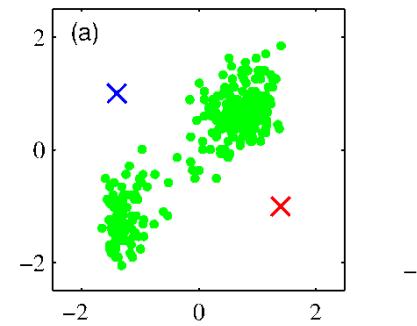
Main challenge: parameter estimation



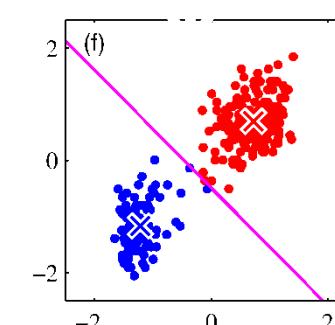
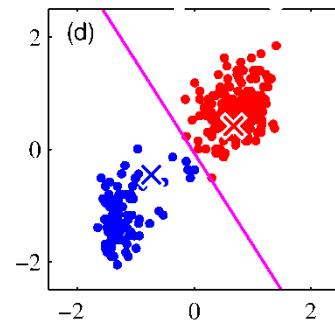
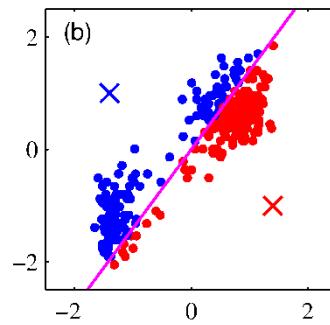
Which points go with which cluster?



## K-Means algorithm



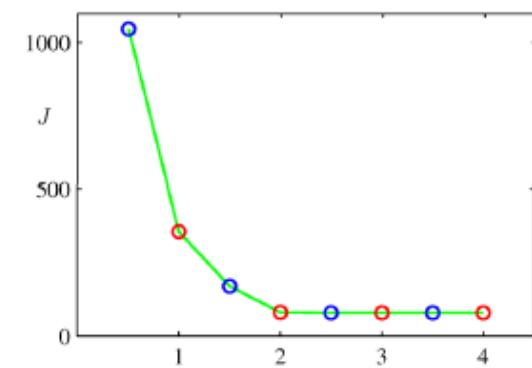
$$c^j = \frac{\sum_{i=1}^N (m(i) = j)x^i}{\sum_{i=1}^N (m(i) = j)}$$



$$m(i) = \operatorname{argmin}_j |x^i - c^j|$$

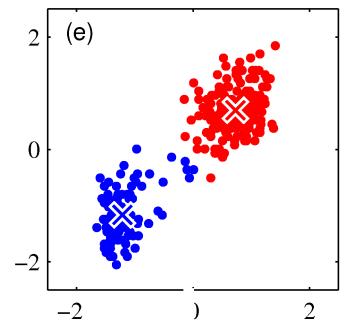
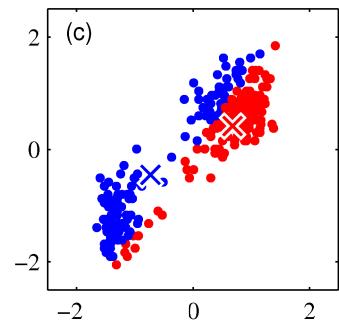
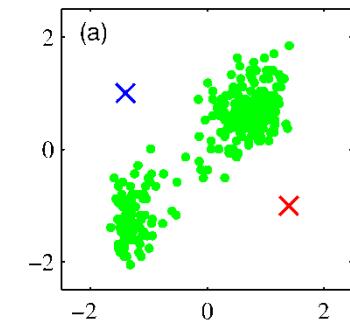
- Coordinate descent on distortion cost:

$$F(m, c) = \sum_{i=1}^N |x^i - c^{m(i)}|^2$$

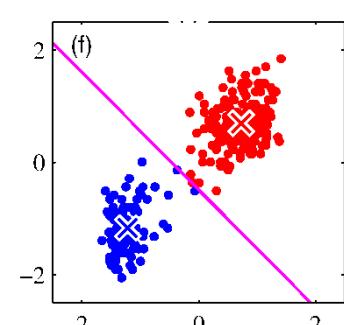
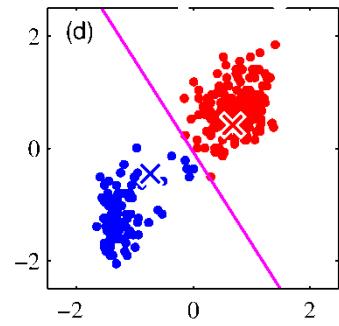
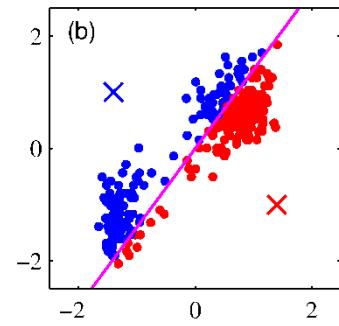


- Local minima (multiple initializations to find better solution)

## K-Means algorithm

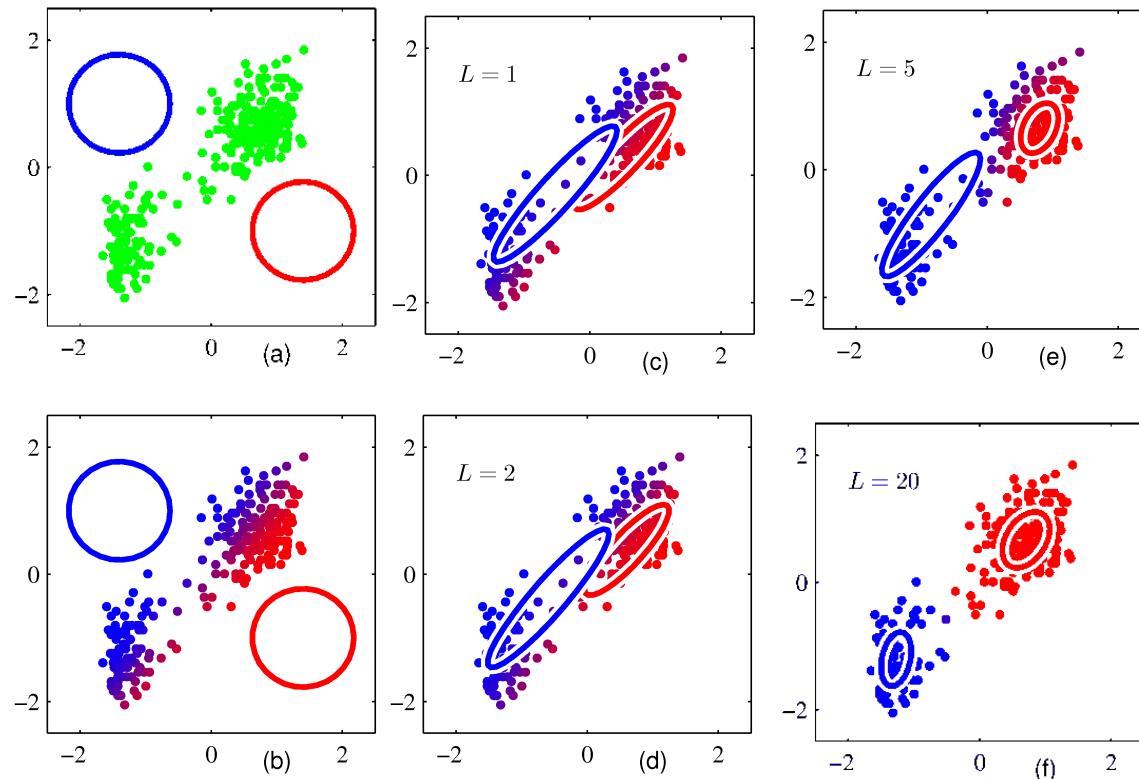


$$c^j = \frac{\sum_{i=1}^N (m(i) = j) x^i}{\sum_{i=1}^N (m(i) = j)}$$



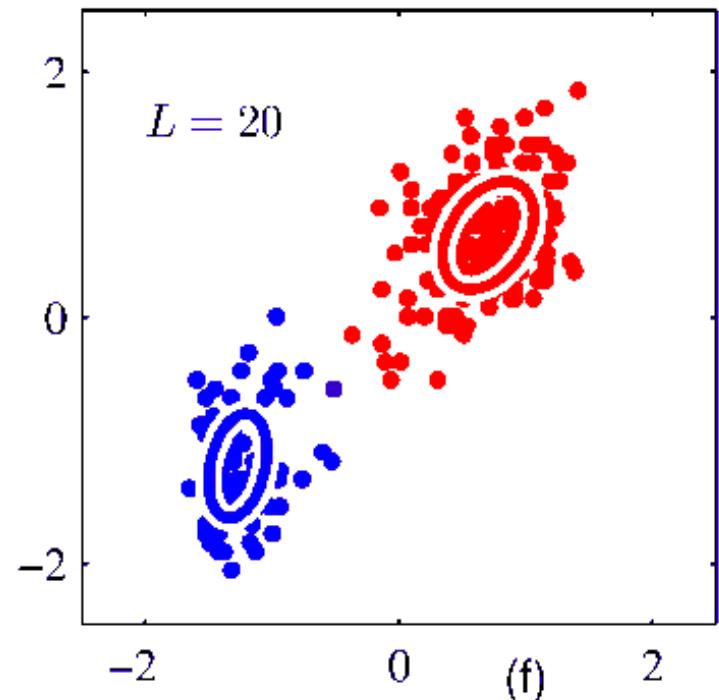
$$m(i) = \operatorname{argmin}_j |x^i - c^j|$$

# Adaptation for Gaussian distributions



$$\begin{aligned} \mu^j &= \frac{\sum_{i=1}^N R_{i,j} x^i}{\sum_{i=1}^N R_{i,j}} \\ \Sigma^j &= \frac{\sum_{i=1}^N R_{i,j} (x^i - \mu^j)^T (x^i - \mu^j)}{\sum_{i=1}^N R_{i,j}} \\ R_{i,j} &= P(z^i = j | \theta) \\ &= \frac{\pi_j P(x^i | \mu_j, \Sigma_j)}{\sum_{k=1}^K P(x^i | \mu_k, \Sigma_k) \pi_k} \end{aligned}$$

# Expectation Maximization algorithm



E-step

$$\begin{aligned} R_{i,j} &= P(z^i = j | \theta) \\ &= \frac{\pi_j P(x^i | \mu_j, \Sigma_j)}{\sum_{k=1}^K P(x^i | \mu_k, \Sigma_k) \pi_k} \end{aligned}$$

M-step

$$\mu^j = \frac{\sum_{i=1}^N R_{i,j} x^i}{\sum_{i=1}^N R_{i,j}}$$

$$\pi^j = \frac{\sum_{i=1}^N R_{i,j}}{N}$$

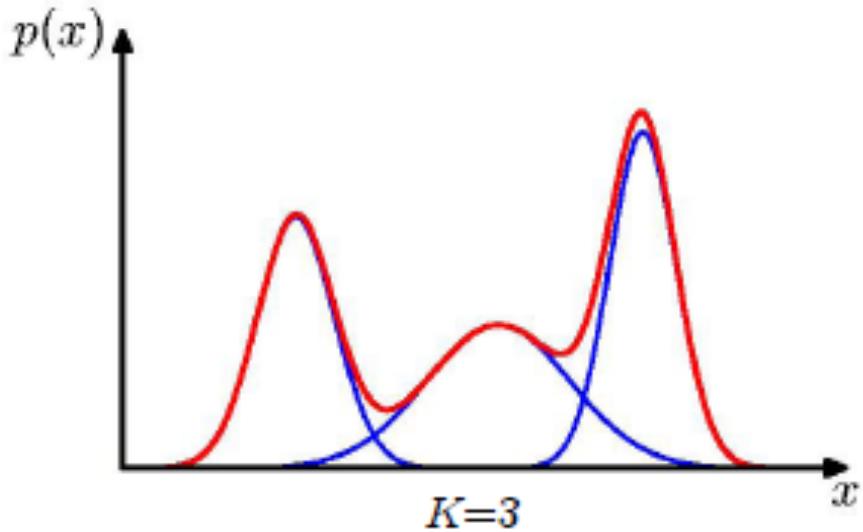
$$\Sigma^j = \frac{\sum_{i=1}^N R_{i,j} (x^i - \mu^j)^T (x^i - \mu^j)}{\sum_{i=1}^N R_{i,j}}$$

# Mixture of Gaussians

- Combine simple models into a complex model:

$$p(\mathbf{x}) = \sum_{k=1}^K \pi_k \mathcal{N}(\mathbf{x} | \boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)$$

↓ Component  
 Mixing coefficient



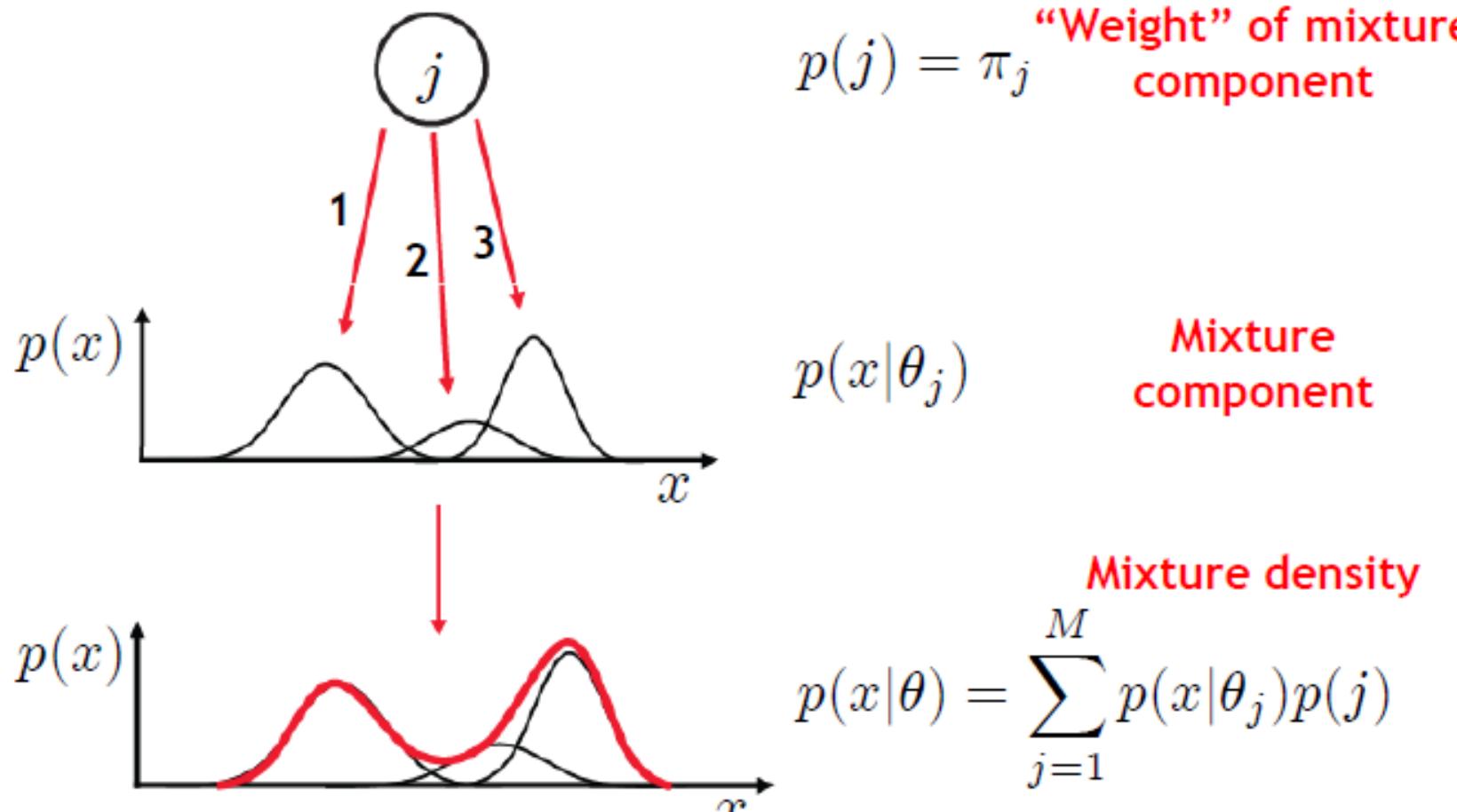
$$\forall k : \pi_k \geq 0 \quad \sum_{k=1}^K \pi_k = 1$$

- Maximum Likelihood Estimation:

$$P(x|\theta) = \prod_{i=1}^N P(x^i|\theta)$$

$$l(\theta; x) = \log P(x|\theta) = \sum_{i=1}^N \log \sum_{k=1}^K P(x^i | \boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k) \pi_k$$

- “Generative model”

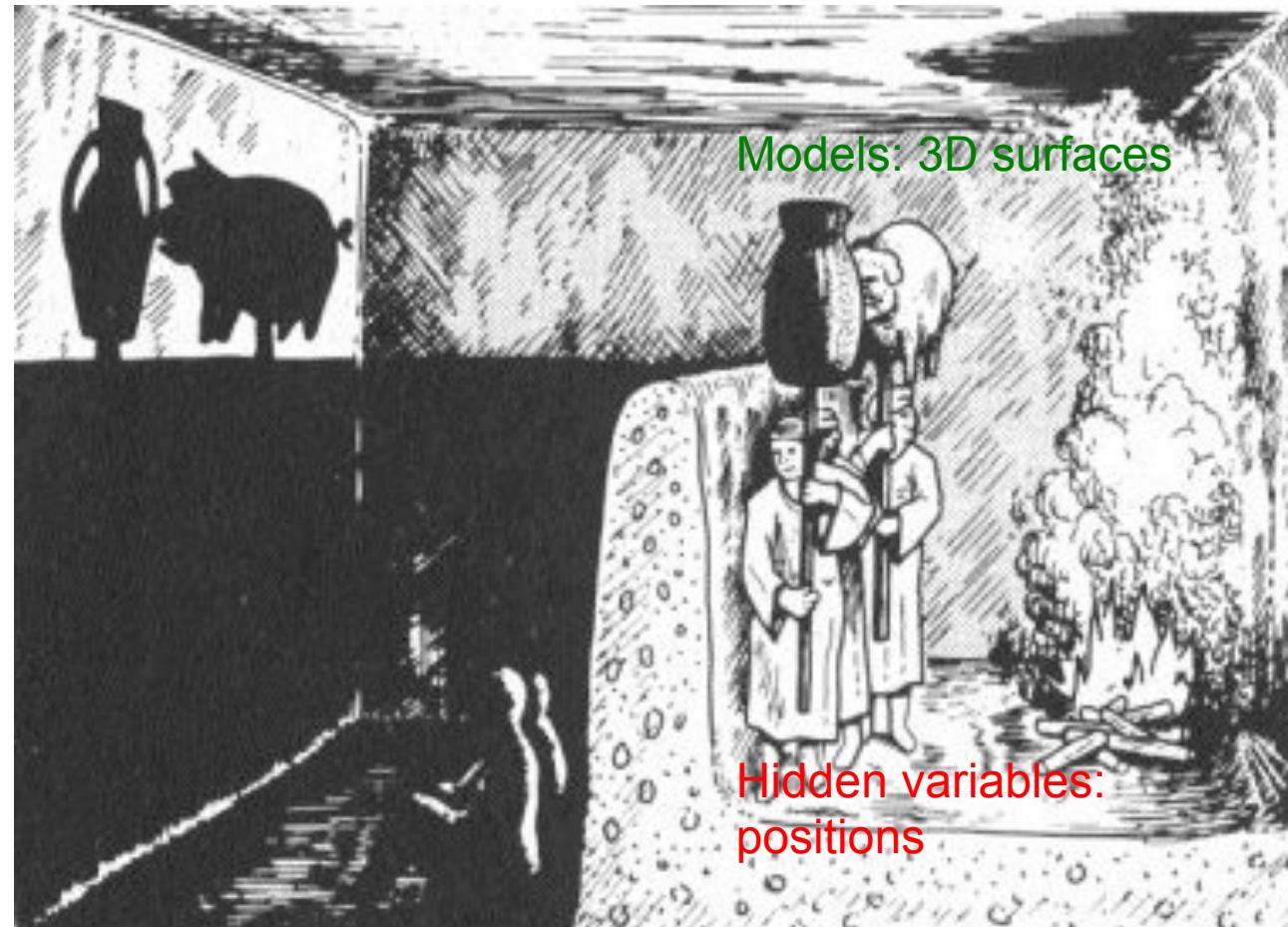


## Hidden Variables:

- - Criterion: 
$$l(\theta; x) = \log P(x|\theta) = \sum_{i=1}^N \log \sum_{k=1}^K P(x^i|\mu_k, \Sigma_k)\pi_k$$
  - Problem: Summation inside logarithm
  - We do not know which component generated each point
  - What if we knew?

# Plato's cave

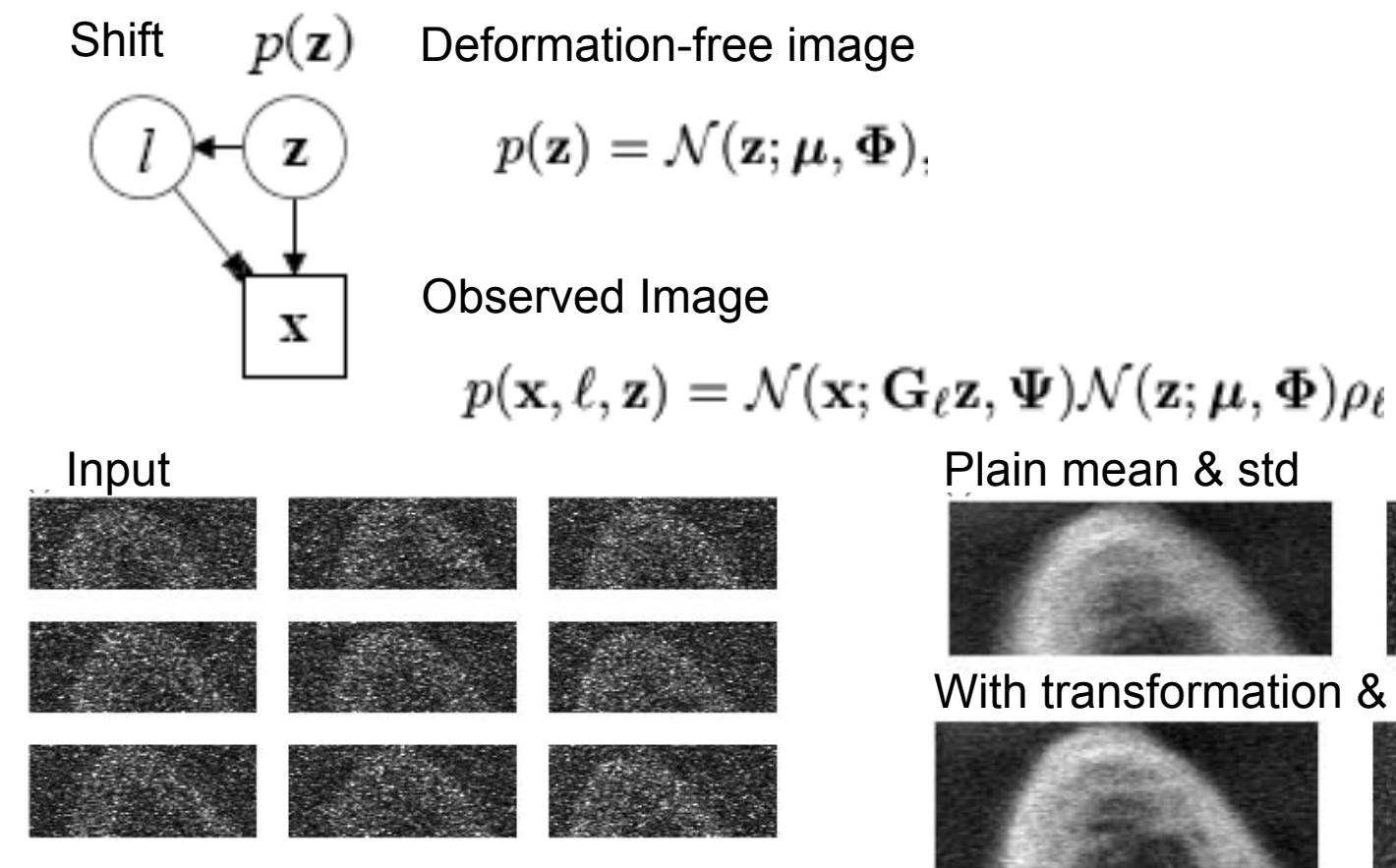
Observations:  
B&W Images



# Transformation-invariant image averaging

Consider shift as a hidden variable,  $\ell$

Estimate model with EM



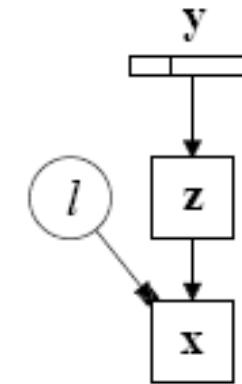
Transformation-Invariant Clustering Using the EM Algorithm, Frey & Jojic, 2003

## Transformed Components Analysis

Latent variables for synthesis (continuous)

Latent variables for shift (discrete)

Estimate mean basis using EM



Plain mean & PCA



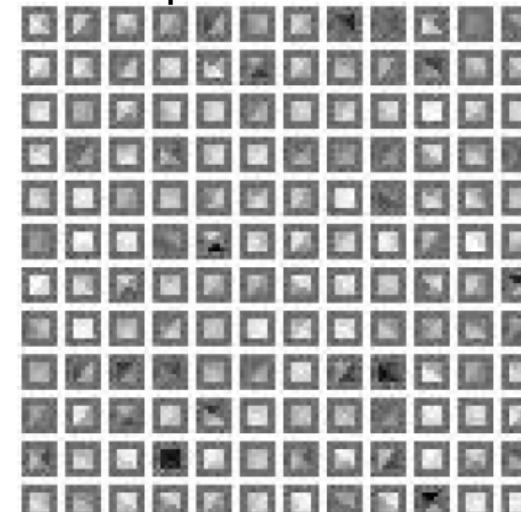
Input



With offset



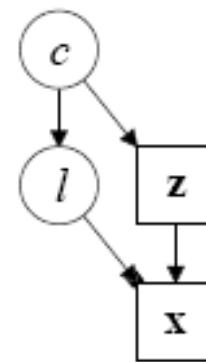
Samples of model



## Transformed Mixture of Gaussians

Latent variables for cluster (discrete)

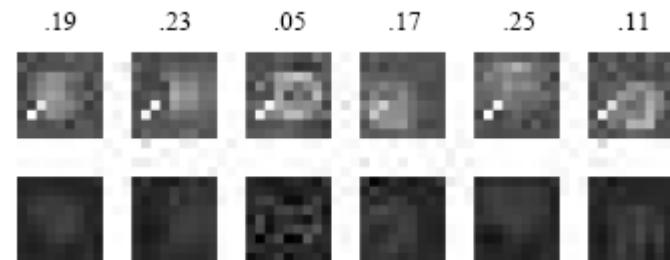
Latent variables for shift (discrete)



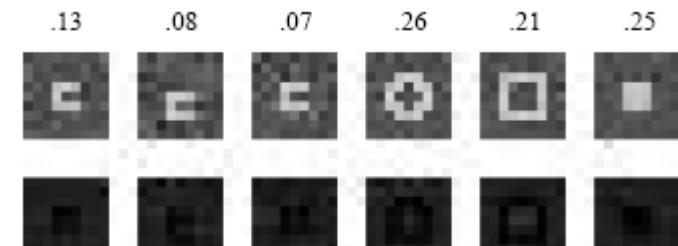
Input



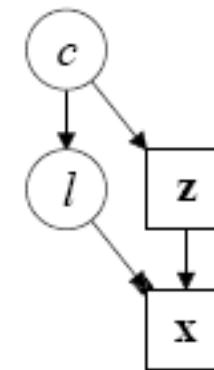
Plain Mixture-of-Gaussians



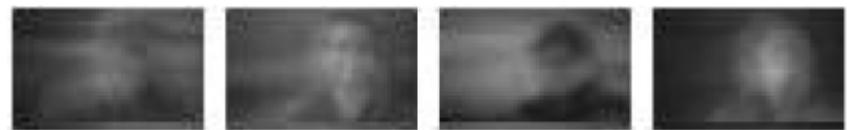
With offset



# Transformed Mixture of Gaussians



Plain Mixture-of-Gaussians



With offset

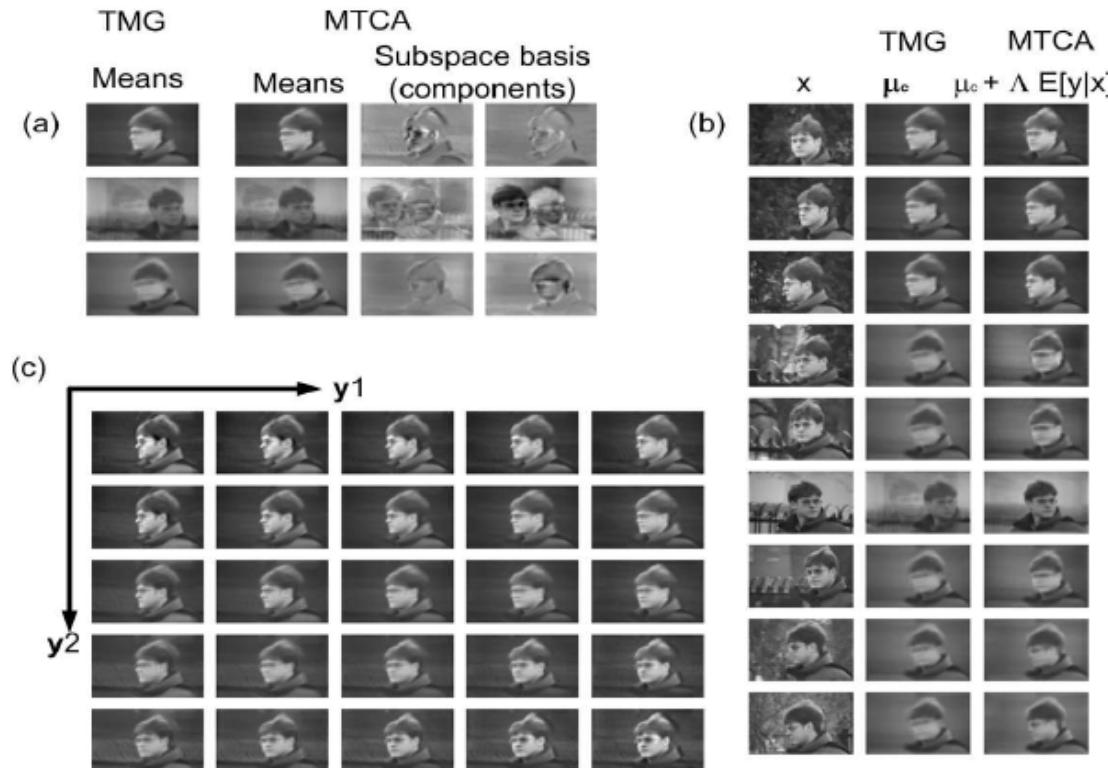
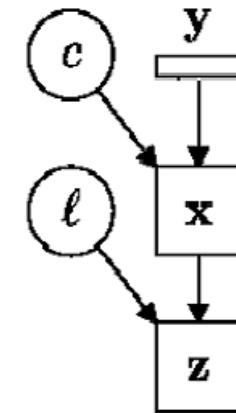


## Mixture of Transformed Components

Latent variables for cluster

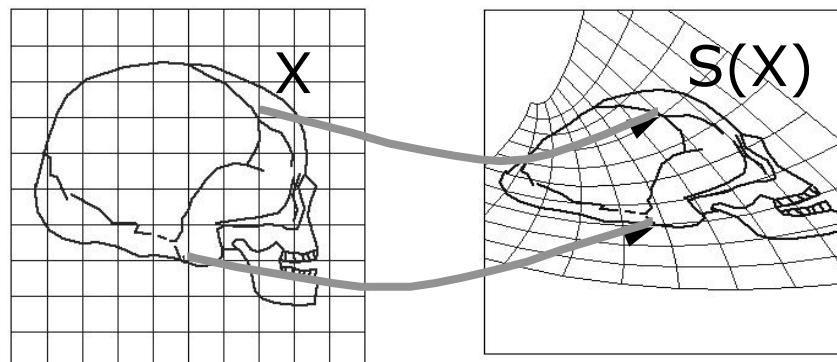
Latent variables for components

Latent variables for shift



## Nonrigid deformations: AAMs

$$T(\mathbf{x}) = I(\mathcal{S}(\mathbf{x}))$$



- Active Appearance Models

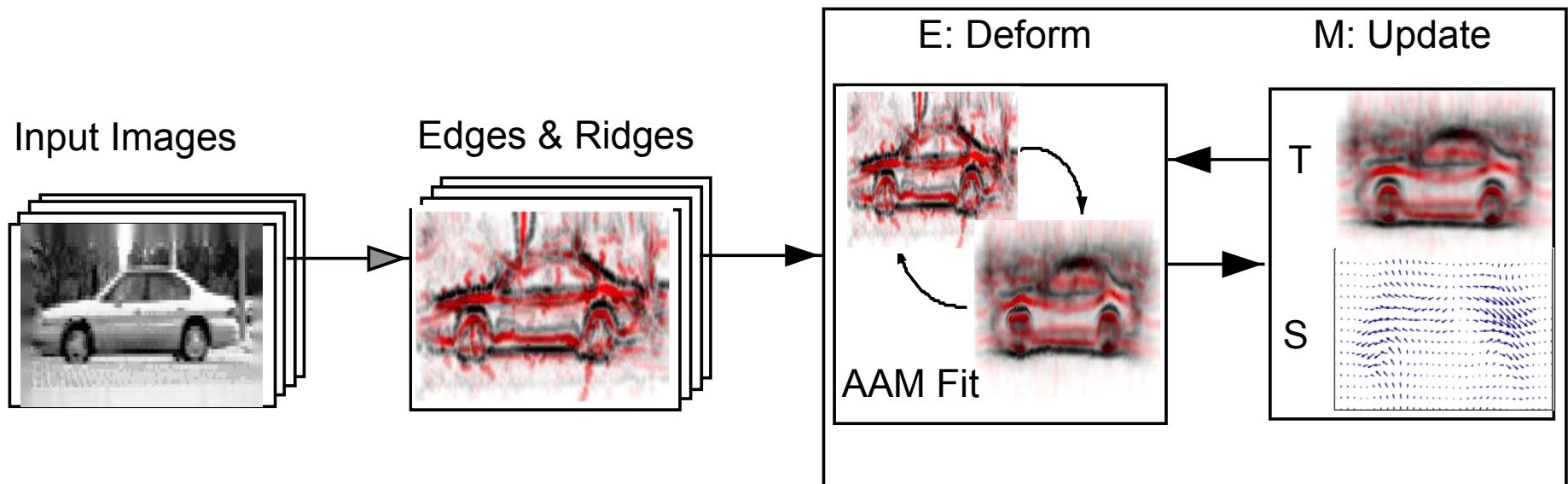
$$\mathcal{S}(\mathbf{x}; \mathbf{s}) = \sum_i \mathbf{s}_i S_i(\mathbf{x}) \quad \mathcal{T}(\mathbf{x}; \mathbf{t}) = \sum_i \mathbf{t}_i T_i(\mathbf{x})$$

## EM-based AAM learning

Training criterion:

$$\min_{\mathbf{s}_k, S_i, \mathcal{T}} \sum_{k=1}^K \sum_{\mathbf{x}} [I_k(\mathcal{S}(\mathbf{x}; \mathbf{s}_k)) - \mathcal{T}(\mathbf{x})]^2$$

$$\mathcal{S}(\mathbf{x}; \mathbf{s}_k) = \sum_i \mathbf{s}_{i,k} S_i(\mathbf{x})$$

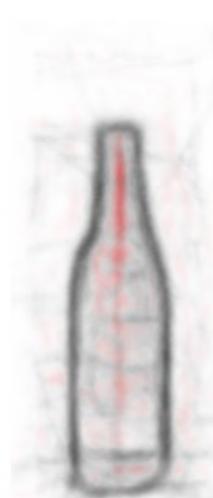


# Bottle models

## Observations



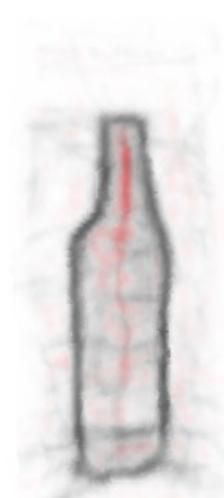
Template



1<sup>st</sup> basis element



2<sup>nd</sup> basis element

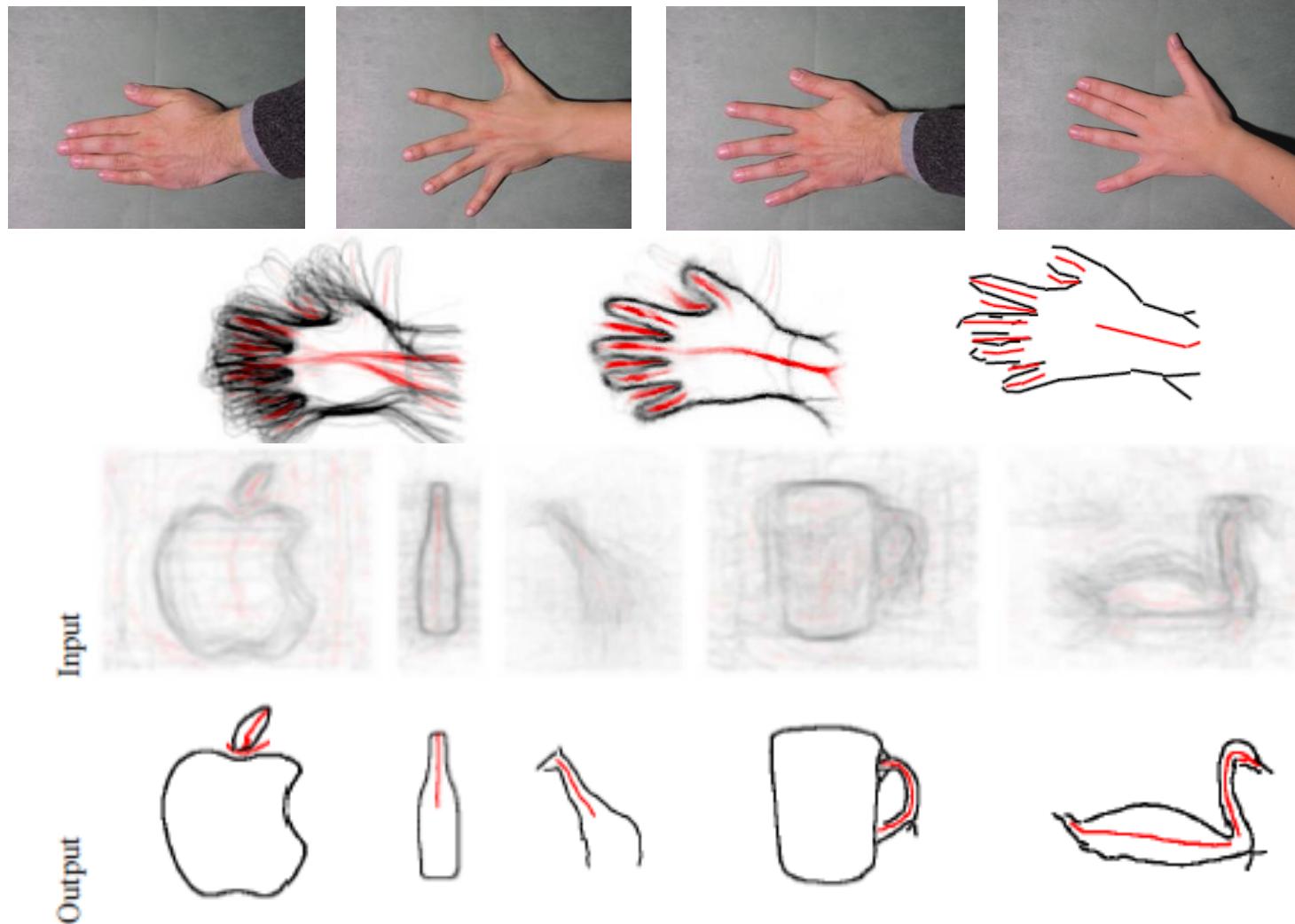


# Recovering Object Contours (2007)



I. Kokkinos and A. Yuille, Unsupervised learning of object deformation models, ICCV 2007

# Hand, apple, giraffe, mug, swan models (2011)



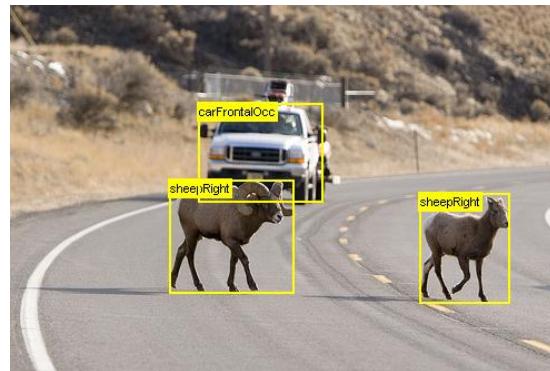
# Pascal Dataset

20 Categories, 25000 images

‘Bus’



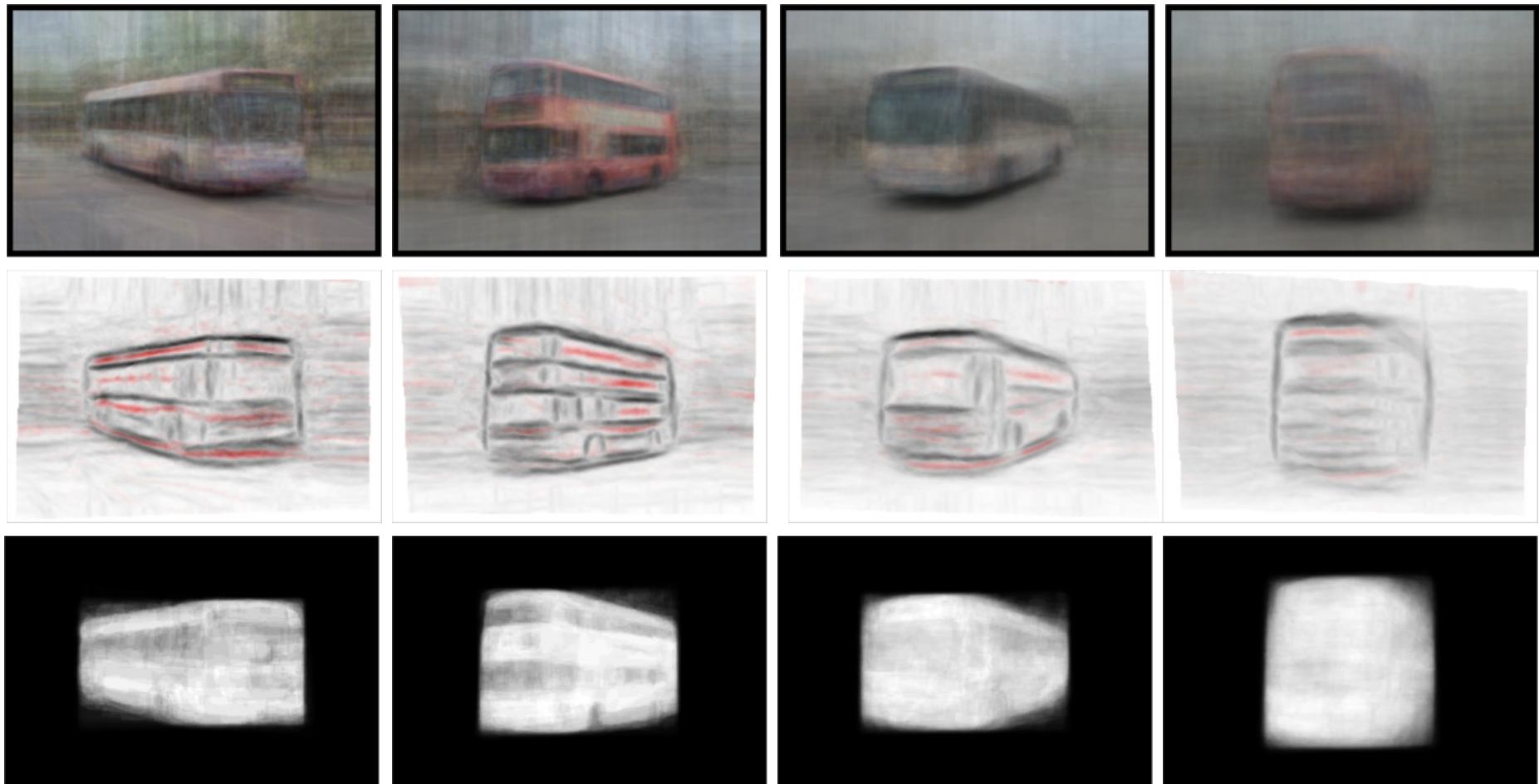
‘Car’



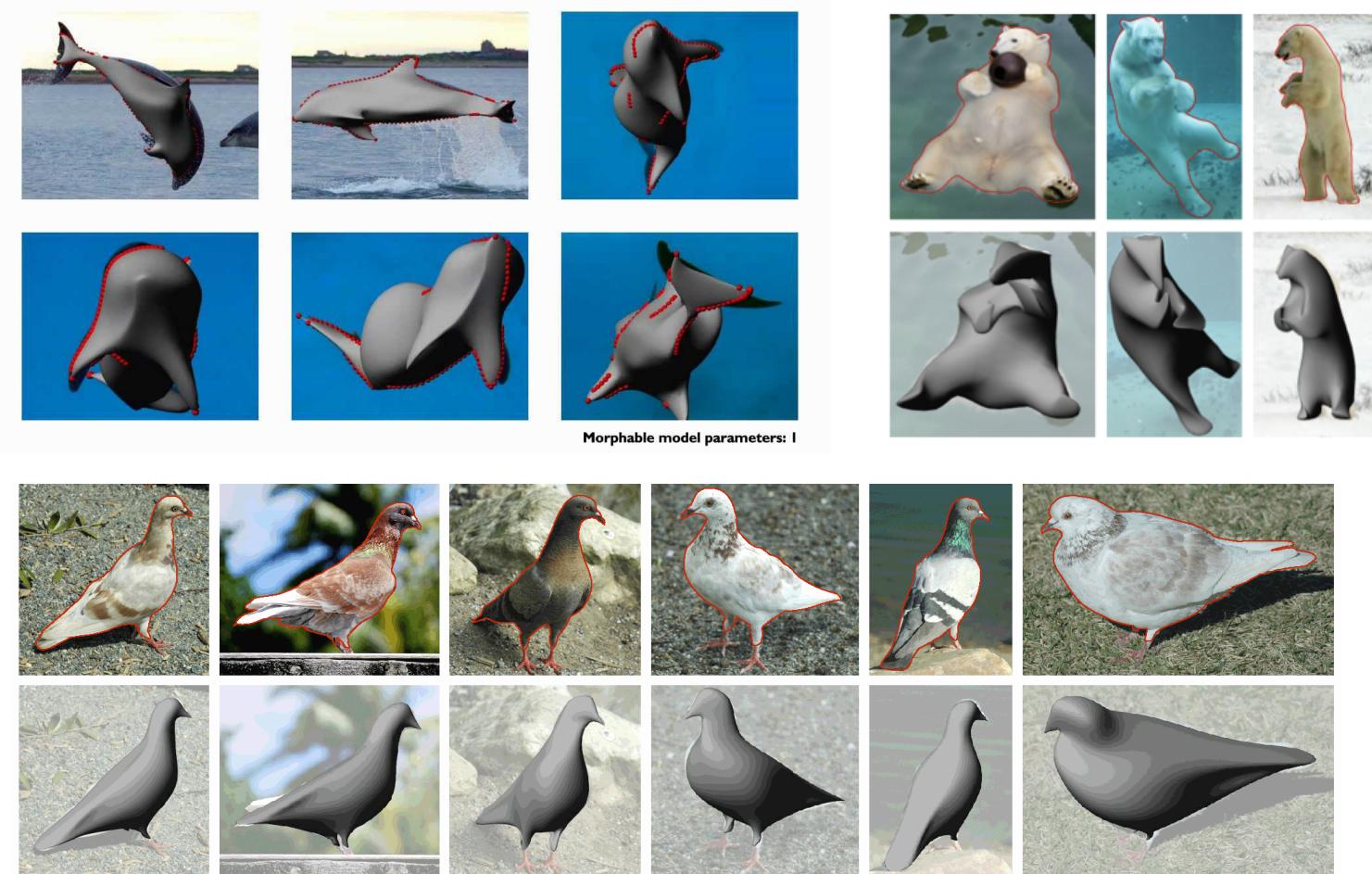
## Multi-view models (20??) – work in progress



## Multi-view models + segmentation



# Semi-automated learning of 3D morphable models



T. J. Cashman, A. W. Fitzgibbon: What Shape Are Dolphins? Building 3D Morphable Models from 2D Images, 2013  
<http://research.microsoft.com/en-us/um/people/awf/dolphins/>