Name: Amit Elbaz

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Homework 1

Problem 1

a.) First I will identify all the matrices to implement the kalman filter:

$A_t = 1, B_t = 0, C_t = 2, Q_t = 1, R_t = 2.$

After that, I will write the pseudo code with this information:

Algorithm 1 Kalman Filter question 1

Input $\mu_0 = 2$, $\Sigma_0 = 8$, $u_t = 0$, $z_1 = 4$, $z_2 = 2$
function KALMAN_FILTER($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$):
$\bar{\mu_t} = A_t \cdot \mu_{t-1} + B_t \cdot u_t = \mu_{t-1}$
$\bar{\Sigma}_t = A_t \cdot \Sigma_{t-1} \cdot A_t^T + R_t = \Sigma_{t-1} + 2$
$K_t = \bar{\Sigma}_t \cdot C_t^T \cdot (C_t \cdot \bar{\Sigma}_t \cdot C_t^T + Q_t)^{-1} = \frac{\bar{\Sigma}_t}{\bar{\Sigma}_t + 1} = \frac{\Sigma_{t-1}+2}{\Sigma_{t-1}+3}$
$\mu_t = \bar{\mu_t} + K_t \cdot (z_t - C_t \cdot \bar{\mu_t}) = \bar{\mu_t} + K_t \cdot (z_t - \bar{\mu_t})$
$\Sigma_t = (I - K_t \cdot C_t) \cdot \bar{\Sigma}_t = \frac{\bar{\Sigma}_t}{\bar{\Sigma}_t + 1} = \frac{\Sigma_{t-1}+2}{\Sigma_{t-1}+3}$
return μ_t, Σ_t
end function

After calculating with this algorithm, the table will be:

t	$\bar{\mu_t}$	$\bar{\Sigma}_t$	K_t	μ_t	Σ_t
1	2.000	10.000	0.909	3.8182	0.909
2	3.8182	2.909	0.744	2.465	0.744

b.) $\bar{\mu_t}$: The value of x_t with respect only to the motion equation, the prediction of the state vector.

$\bar{\Sigma}_t$: The covariance for the motion prediction. How much error our model has with only respecting the motion, and the prediction of the covariance for the state vector.

K_t: Represent the relative importance we give to the difference between the motion prediction and the measurement. The bigger it is, the more our measurements influence the state vector.

μ_t: The value of x_t with respect to the prediction and the measurement.

Σ_t: The covariance for the state vector. How much error we think our model has with depending on the measurements and the motion.

c.) By looking at Algorithm 1, we can see that Σ_t is a recursive sequence, to calculate its limit, we will guess that it has a finite one and its value for Σ_{t-1} is the same. In other words:

$$\lim_{t\to\infty} \Sigma_t = \lim_{t\to\infty} \Sigma_{t-1} = L \tag{1}$$

The equation we need to solve is:

$$L = \lim_{t\to\infty} \frac{\Sigma_{t-1}+2}{\Sigma_{t-1}+3} = \frac{\lim_{t\to\infty}(\Sigma_{t-1}+2)}{\lim_{t\to\infty}(\Sigma_{t-1}+3)} = \frac{L+2}{L+3} \tag{2}$$

The solution will be:

$$L = 0.732 \tag{3}$$

Problem 2

a.) The definition of the state vector will be: $X_t = \left\{x_t \quad y_t \quad \theta_t \quad v_t^x \quad v_t^y \quad \omega_t\right\}^T$. The definition of the control vector will be: $u_t = \left\{a_t^x \quad a_t^y \quad \alpha_t\right\}^T$. The motion model will be:

$$X_t = A_t \cdot X_{t-1} + B_t \cdot u_t + \epsilon_t \tag{4}$$

The measurement model will be:

$$z_t = C_t \cdot X_t + \delta_t \tag{5}$$

Where $\epsilon_t = N(0, R_t)$ and $\delta_t = N(0, Q_t)$ are the noises of the Model and the measurments. The following equations will describe all the matrices that are needed for the implantation of the Kalman filter. A_t will be:

$$A_t = \begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

B_t will be:

$$B_t = \begin{bmatrix} 0.5\Delta t^2 & 0 & 0 \\ 0 & 0.5\Delta t^2 & 0 \\ 0 & 0 & 0.5\Delta t^2 \\ \Delta t & 0 & 0 \\ 0 & \Delta t & 0 \\ 0 & 0 & \Delta t \end{bmatrix} \tag{7}$$

C_t will be:

$$C_t = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{8}$$

For R_t I will another use 2 elements, first $\eta = N(0,1)$.With this, we can identify ϵ_t to be:

$$\epsilon_t = B_t \cdot \begin{pmatrix} \sigma_{ax} \\ \sigma_{ay} \\ \sigma_{\alpha} \end{pmatrix} \cdot \eta = \begin{bmatrix} 0.5\Delta t^2 & 0 & 0 \\ 0 & 0.5\Delta t^2 & 0 \\ 0 & 0 & 0.5\Delta t^2 \\ \Delta t & 0 & 0 \\ 0 & \Delta t & 0 \\ 0 & 0 & \Delta t \end{bmatrix} \cdot \begin{pmatrix} \sigma_{ax} \\ \sigma_{ay} \\ \sigma_{\alpha} \end{pmatrix} \cdot \eta \tag{9}$$

And now, R_t will be:

$$R_t = E\left[\epsilon_t \cdot \epsilon_t^T\right] = \begin{bmatrix} 0.5\Delta t^2 \cdot \sigma_{ax} \\ 0.5\Delta t^2 \cdot \sigma_{ay} \\ 0.5\Delta t^2 \cdot \sigma_{\alpha} \\ \Delta t \cdot \sigma_{ax} \\ \Delta t \cdot \sigma_{ay} \\ \Delta t \cdot \sigma_{\alpha} \end{bmatrix} \cdot \begin{bmatrix} 0.5\Delta t^2 \cdot \sigma_{ax} \\ 0.5\Delta t^2 \cdot \sigma_{ay} \\ 0.5\Delta t^2 \cdot \sigma_{\alpha} \\ \Delta t \cdot \sigma_{ax} \\ \Delta t \cdot \sigma_{ay} \\ \Delta t \cdot \sigma_{\alpha} \end{bmatrix}^T \tag{10}$$

Q_t will be:

$$Q_t = \begin{bmatrix} \sigma_x^2 & 0 & 0 \\ 0 & \sigma_y^2 & 0 \\ 0 & 0 & \sigma_\theta^2 \end{bmatrix} \tag{11}$$

b.) The following figure will present the true values of each element we try to get.

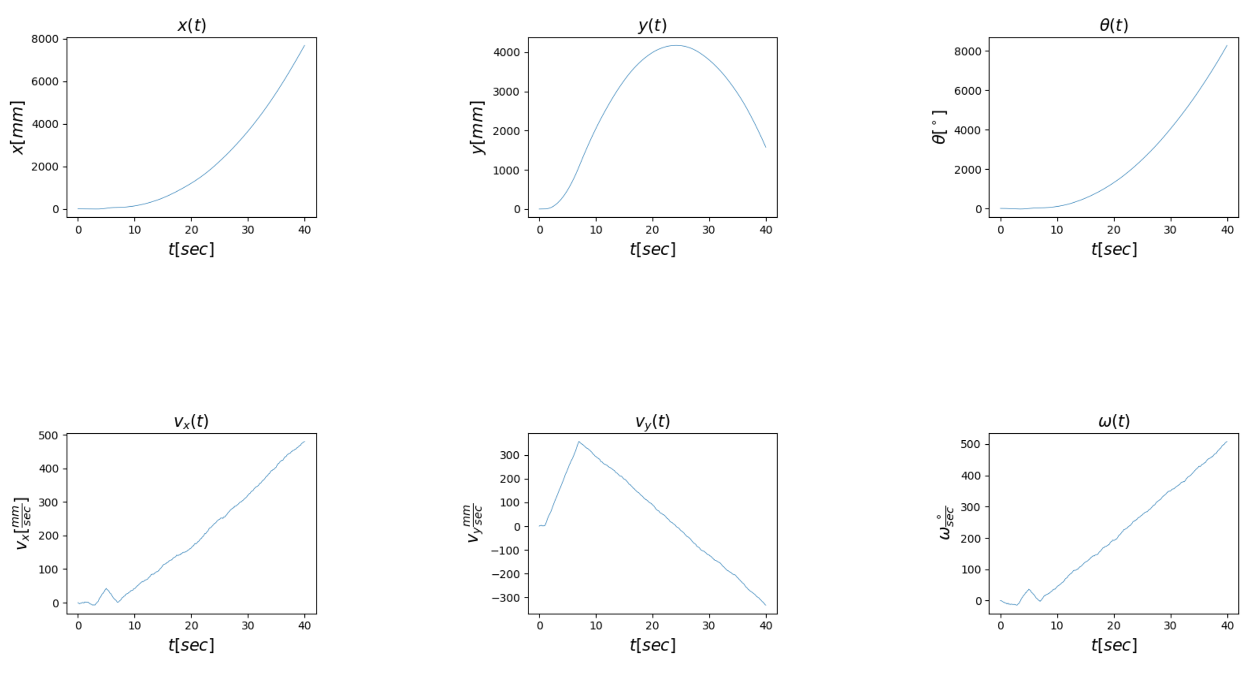

Figure 1: Figure 1 question b

The following figure will present the real xy-path and the measurements:

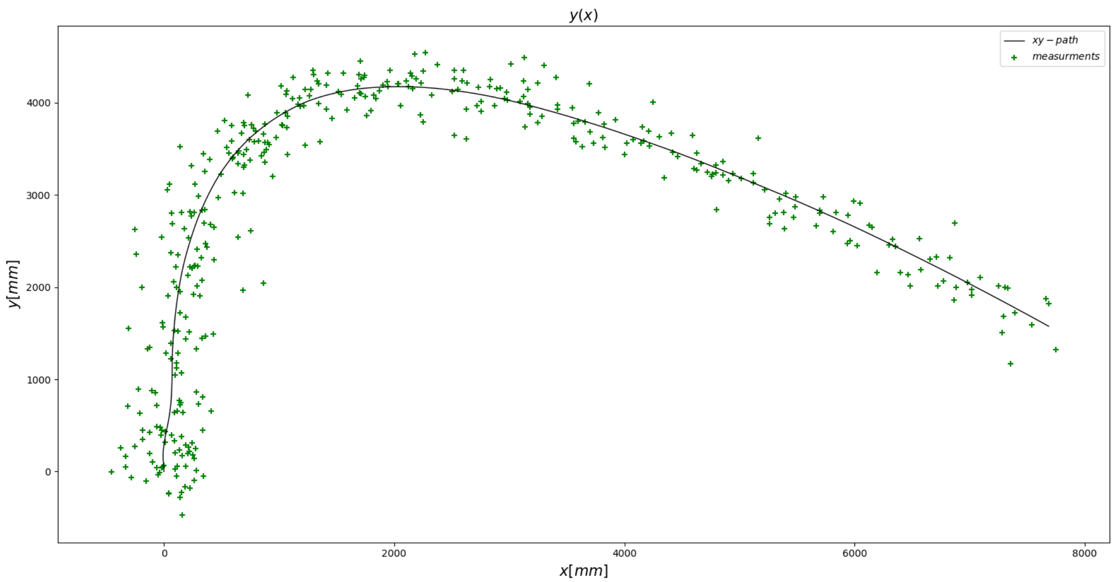

Figure 2: Figure 2 question b

c.) The following figure will present the estimated values of x_t:

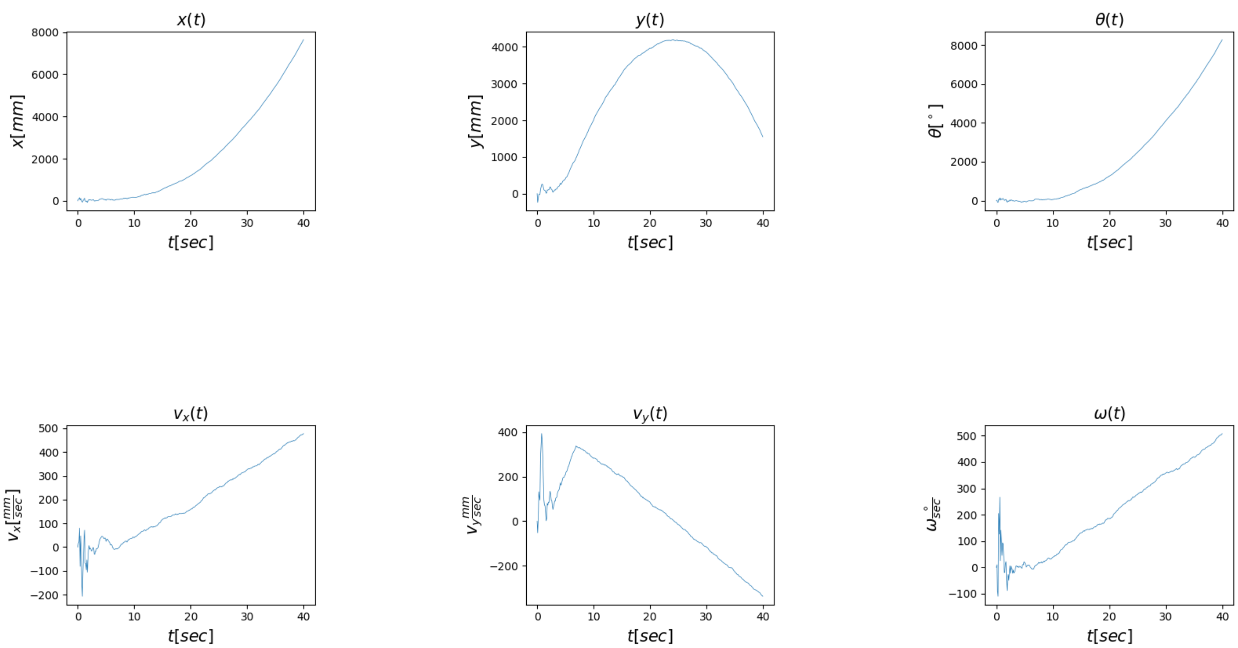

Figure 3: Figure 1 question c

The following figure will present the estimated xy-path and the measurements:

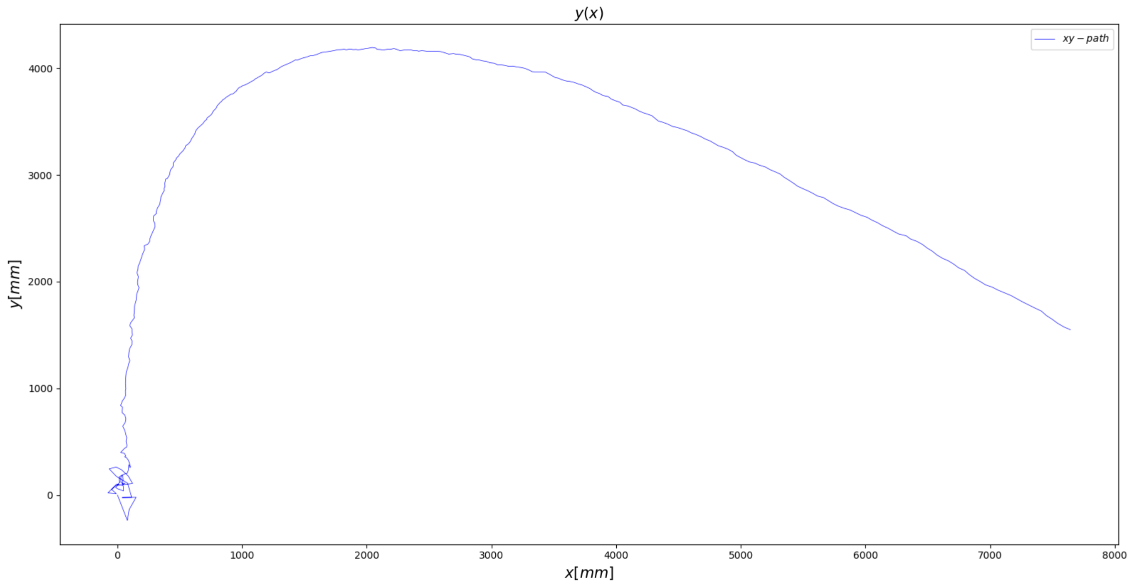

Figure 4: Figure 2 question c

d.) The following figure will present the estimated values of x_t:

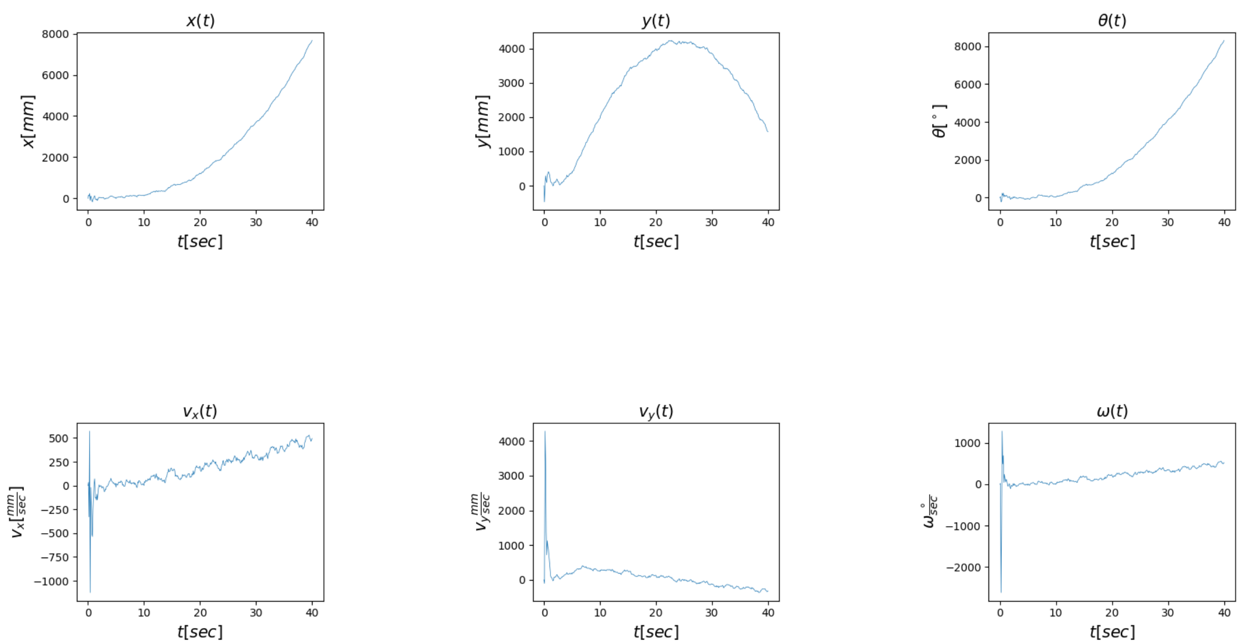

Figure 5: Figure 1 question d

The following figure will present the estimated xy-path and the measurements:

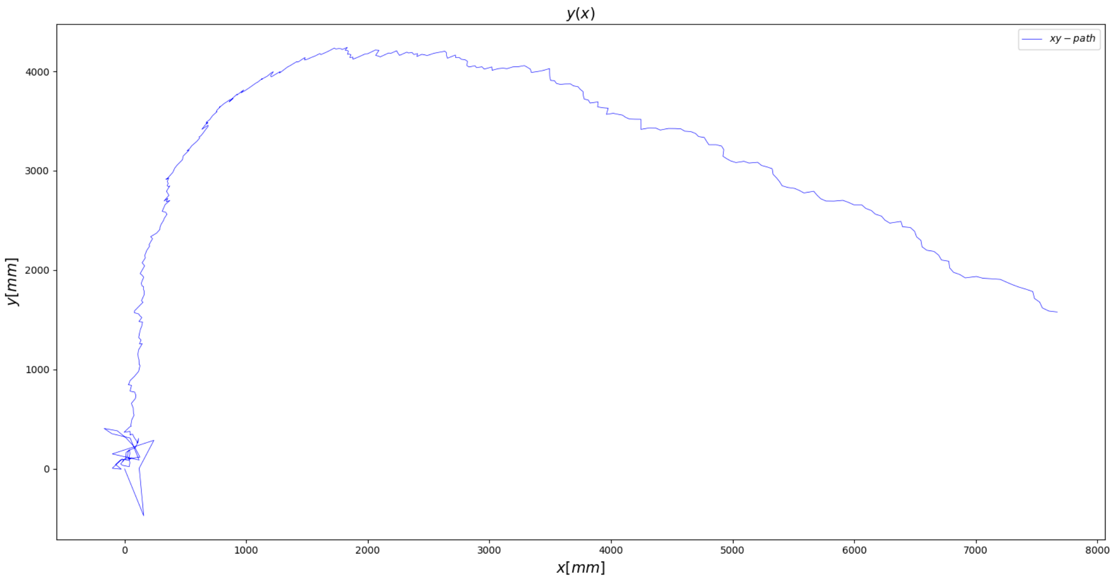

Figure 6: Figure 2 question d

We can see a difference from figure 6 to figure 4. We took in d.) smaller values for the covariance of the measurements than we did in c.). The meaning of this is that we give more value to the measurements than before. We could see from figure 2 that the measurements are very noisy and are not very reliable. This is why in Figures 3 and 4 we got a better estimation than we did in Figures 5 and 6.

e.) The following figure will present figure 4 with the variance for its location in the path:

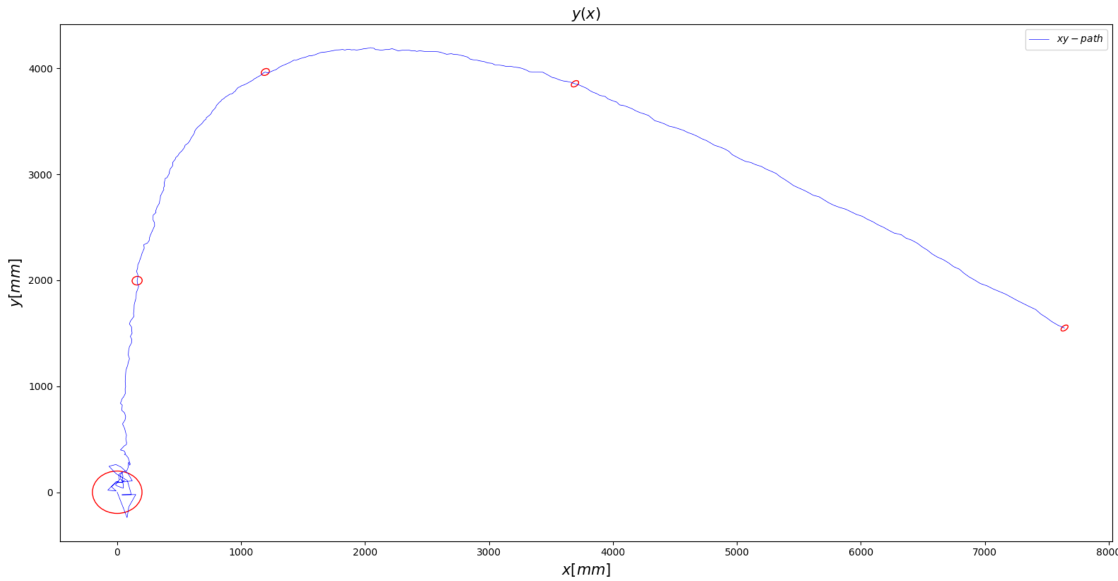

Figure 7: Figure 2 question c with the variance in red

The following figure presents figure 6 with the variance for its location in the path:

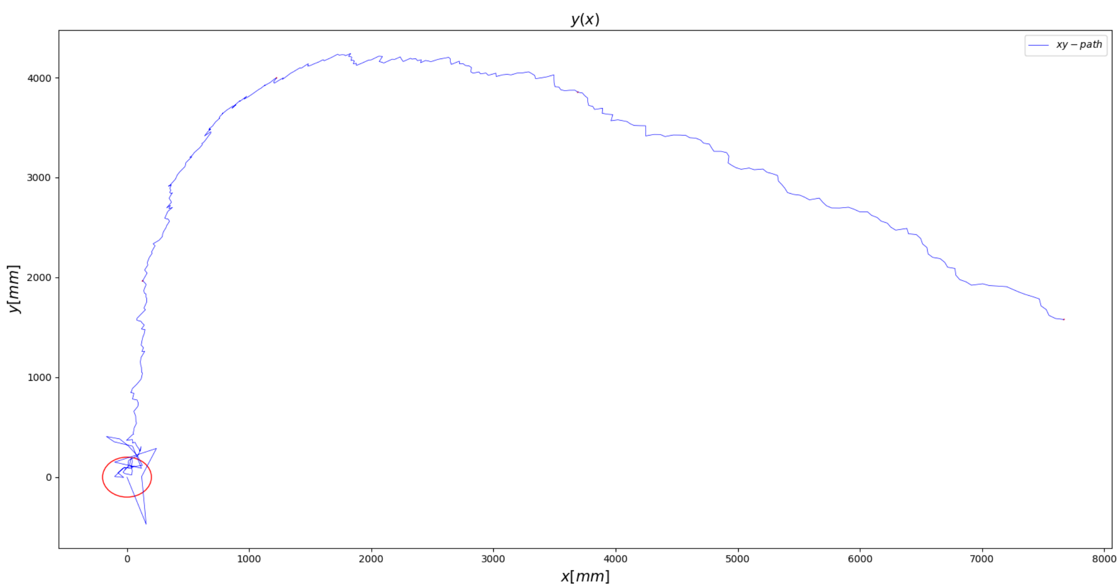

Figure 8: Figure 2 question d with the variance in red

We can see that we barley see the variance in figure 8 but in figure 7 it is more visible.

That happens because with what we did in c.), we took a bigger covariance for measurements so it gave us a bigger uncertainty of it location, but in d.) we took a smaller one so we are more certain about its location.