Intelligent Robotic Systems

Exercise 1: Kalman Filter

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1. Calculation of Kalman filter steps (25 pts)

The motion and measurement model of a one-dimensional system are given by the following set of equations:

$$
\begin{aligned}
x_k &= x_{k-1} + \epsilon_t \quad \text{with} \quad \epsilon_t = \mathcal{N}(0,2) \\
z_k &= x_k + \delta_t \quad \text{with} \quad \delta_t = \mathcal{N}(0,1)
\end{aligned}
$$

where $\mathcal{N}(a,b)$ denotes a Gaussian random variable with mean a and variance b.
The following measurements are taken:

$$z_1 = 4, z_2 = 2$$

The initial state is given by

$$
\begin{aligned}
\mu_0 &= 2 \\
\Sigma_0 &= 8
\end{aligned}
$$

a.) Calculate two Kalman filter steps ($k = 1, 2$) by completing the following table (15 pts):

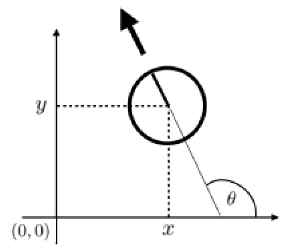
k	$\bar{\mu}_k$	$\bar{\Sigma}_k$	K_k	μ_k	Σ_k
1					
2					

b.) Explain what these parameters describe (5 pts).

c.) What is the steady-state covariance matrix $\Sigma_\infty := \lim_{k \to \infty} \Sigma_k$ (5 pts)?

2. ***2D-tracking of a mobile robot*** (75 pts)

A mobile robot is moving in the plane. The pose of the robot is described by (x, y, θ), where x and y define the center of the robot and θ is the direction of moving with respect to the x-axis (see below). Assume that the pose is measured every $\Delta t = 0.1$s with a sensor that is noisy with variances $\sigma_x^2, \sigma_y^2, \sigma_\theta^2$. Track the pose (x, y, θ) and velocities (v_x, v_y, ω) of the robot by implementing a Kalman filter ($\omega = \dot{\theta}$ is the angular velocity). Assume that the dynamics can be described by a constant acceleration model. The controls are noisy and are described by the variances σ_{ax}^2, σ_{ay}^2 and σ_α^2 ($\alpha = \dot{\omega}$ is the angular acceleration).



a.) Define first the state vector of the problem and derive the motion and measurement models. Provide all matrices needed for the implementation of the Kalman filter. (20 pts)

b.) The pose measurements (`pose.txt`), the controls (`controls.txt`) and the ground truth (`ground_truth.txt`) are provided in an attachment for $T = 40$s (all length units are in mm). Note that the ground-truth is often not available. Generate two figures: Figure 1 contains six subplots showing the ground truth of the state vector as a function of time. Figure 2 shows the robot xy-path (in *black*) and the xy measurements (use '+' marker in *green*). (10 pts)

c.) Run the Kalman filter with $\sigma_{ax} = 10$, $\sigma_{ay} = 10$, $\sigma_\alpha = 10$ and $\sigma_x = 200$, $\sigma_y = 200, \sigma_\theta = 200$ and plot the results of the state estimate in Figure 1 (in *blue*) and the robot path in Figure 2 (in *blue*). Assume that the initial state vector is $\boldsymbol{x}_0 = \mathbf{0}$ and there is some initial uncertainty described by the covariance matrix $\boldsymbol{\Sigma}_0 = diag(200^2, 200^2, 200^2, 300^2, 300^2, 400^2)$, where *diag* is a diagonal matrix. (25 pts)

d.) Run the Kalman filter with $\sigma_{ax} = 10$, $\sigma_{ay} = 10$, $\sigma_\alpha = 10$ and $\sigma_x = 10$, $\sigma_y = 10, \sigma_\theta = 10$ using the same initial conditions as in c.). Generate another Figure 1 and 2 with the new Kalman estimate. Which difference to c.) do you observe? Explain! (10 pts)

e.) Add to Figure 2c.) and 2d.) the variance of your state estimate at times $t = 0, 10, 20, 30, 40$s. *Hint*: The variance can be represented as an ellipse using the covariance matrix. What do you observe? (10 pts)

Remark: You can use `Matlab` or `Python` for the implementation of the Kalman Filter.