QUESTION 2

Theorem the sum of any five consecutive integers is divisible by 5 (without remainder).

Proof: by consideration of algebraic cases.

Any sequence of five consecutive integers can be written as one of the following:

$$\begin{aligned} 5n-4, 5n-3, 5n-2, 5n-1, 5n \quad (1)\\ 5n-4+5n-3+5n-2+5n-1+5n &= 25n-10\\ &= 5(5n-2) \end{aligned}$$

$$\begin{aligned} 5n-3, 5n-2, 5n-1, 5n, 5n+1 \quad (2)\\ 5n-3+5n-2+5n-1+5n+5n+1 &= 25n-5\\ &= 5(5n-1) \end{aligned}$$

$$\begin{aligned} 5n-2, 5n-1, 5n, 5n+1, 5n+2 \quad (3)\\ 5n-2+5n-1+5n+5n+1+5n+2 &= 25n\\ &= 5(5n) \end{aligned}$$

$$\begin{aligned} 5n-1, 5n, 5n+1, 5n+2, 5n+3 \quad (4)\\ 5n-1+5n+5n+1+5n+2+5n+3 &= 25n+5\\ &= 5(5n+1) \end{aligned}$$

$$\begin{aligned} 5n, 5n+1, 5n+2, 5n+3, 5n+4 \quad (5)\\ 5n+5n+1+5n+2+5n+3+5n+4 &= 25n+10\\ &= 5(5n+2) \end{aligned}$$

The sum of each sequence is divisible by 5. Therefore the sum of any five consecutive integers is divisible by 5. □