QUESTION 7

Theorem $(\forall n \in \mathbb{N})(2 + 2^2 + 2^3 + ... + 2^n = 2^{n+1} - 2)$

Proof: by induction.

Base case: When $n = 1$, $2^n = 2^1 = 2 = 2^2 - 2$.

Inductive step: Assume the identity holds for n. Then:

$$\begin{aligned} 2 + 2^2 + 2^3 + ... + 2^{n+1} &= 2 + 2^2 + 2^3 + ... + 2^n + 2^{n+1} \\ &= 2^{n+1} - 2 + 2^{n+1} && \text{(by induction hypothesis)} \\ &= 2(2^{n+1}) - 2 \\ &= 2^{n+2} - 2 \end{aligned}$$

which establishes the identity for $n + 1$.

Therefore $(\forall n \in \mathbb{N})(2 + 2^2 + 2^3 + ... + 2^n = 2^{n+1} - 2)$. □