QUESTION 1

Theorem$(\exists m \in \mathbb{N})(\exists n \in \mathbb{N})(3m + 5n = 12)$

Disproof: By consideration of cases.

From the definition of the natural numbers $\mathbb{N}$, and since

$$m \geq 4 \Rightarrow 3m + 5n > 12$$
$$n \geq 3 \Rightarrow 3m + 5n > 12$$

it follows that in order for the theorem to be true, $0 < m \leq 3$ and $0 < n \leq 2$.

$m = 1 \wedge n = 1 \Rightarrow 3m + 5n = 8$
$m = 1 \wedge n = 2 \Rightarrow 3m + 5n = 13$
$m = 2 \wedge n = 1 \Rightarrow 3m + 5n = 11$
$m = 2 \wedge n = 2 \Rightarrow 3m + 5n = 16$
$m = 3 \wedge n = 1 \Rightarrow 3m + 5n = 14$
$m = 3 \wedge n = 2 \Rightarrow 3m + 5n = 19$

Therefore there are no m, n that satisfy the theorem. $\square$