QUESTION 5

Theorem for any integer n, at least one of the integers n, $n+2$, $n+4$ is divisible by 3.

Proof: by consideration of cases.

Any $n \in \mathbb{Z}$ can be written as $3k$, $3k+1$ or $3k+2$, $k \in \mathbb{Z}$.

(i) Assume $n = 3k$. Then (trivially) n is divisible by 3.

(ii) Assume $n = 3k+1$. Then:

$$\begin{aligned} n+2 &= 3k+1+2 \\ &= 3k+3 \\ &= 3(k+1) \end{aligned}$$

Therefore, $n+2$ is divisible by 3.

(iii) Assume $n = 3k+2$. Then:

$$\begin{aligned} n+4 &= 3k+2+4 \\ &= 3k+6 \\ &= 3(k+2) \end{aligned}$$

Therefore, $n+4$ is divisible by 3.

Therefore for any integer n, at least one of the integers n, $n+2$, $n+4$ is divisible by 3. $\square$