QUESTION 8

Theorem if the sequence $\{a_n\}_{n=1}^{\infty}$ tends to limit L as $n \to \infty$, then for any fixed number $M > 0$, the sequence $\{Ma_n\}_{n=1}^{\infty}$ tends to the limit LM.

Proof: from the definition of a limit.

Since $\lim_{n\to\infty} a_n = L$, for every real number $\epsilon > 0$, there exists a natural number N such that

$$(\forall n > N)(|a_n - L| < \epsilon)$$

Suppose $K = LM, M \in \mathbb{R}$ and $M > 0$. Then equivalently,

$$(\forall n > N)(|a_n - \frac{K}{M}| < \epsilon)$$

and therefore

$$(\forall n > N)(M|a_n - \frac{K}{M}| < M\epsilon)$$

Since the choice of ϵ and corresponding N are arbitrary, we can say for every real number $\epsilon_1 = M\epsilon$ there exists a natural number N_1 such that

$$(\forall n > N_1)(M|a_n - \frac{K}{M}| < \epsilon_1)$$

and hence,

$$(\forall n > N_1)(|Ma_n - K| < \epsilon_1)$$

This is the definition of a limit of the sequence $\{Ma_n\}_{n=1}^{\infty}$. Hence $\lim_{n\to\infty} Ma_n = K = LM$. $\square$