Eddy Currents Equivalent Model for Control

One Eddy Currents Circuit

The equations to consider are:

$$\begin{cases} R_e I_e + L_e \dfrac{dI_e}{dt} + L_{em} \dfrac{dI_m}{dt} = 0 \\ R_m I_m + L_m \dfrac{dI_m}{dt} + L_{me} \dfrac{dI_e}{dt} = V_m \end{cases}$$

where $L_{me} = L_{em}$ (mutual inductance) by reciprocity, V_m is the output voltage of the Power Converter and I_m is its current as usually measured by the DCCT.

The desired transfer function is the magnet admittance: $A_m(s) = \frac{I_m(s)}{V_m(s)}$

By means of a few manipulations and by introducing the following time constants:

$$\begin{cases} \tau_e = \dfrac{L_e}{R_e} \\ \tau_m = \dfrac{L_m}{R_m} \\ \tau_{em} = \dfrac{L_{em}^2}{R_e L_m} = k^2 \tau_e \end{cases}$$

the equivalent transfer function can be written as:

$$A_m(s) = \frac{1}{R_m} \frac{1 + s\tau_e}{1 + s(\tau_e + \tau_m) + s^2 \tau_m (\tau_e - \tau_{em})} = \frac{1}{R_m} \frac{1 + s\tau_e}{1 + s(\tau_e + \tau_m) + s^2 \tau_m \tau_e (1 - k^2)}$$

This transfer function is intrinsically stable. Indeed the term $(\tau_e - \tau_{em})$ is ≥ 0 by definition; $k^2 \leq 1$ where k is the coupling coefficient.

This can be shown by considering: $\tau_e = \frac{L_e}{R_e} \geq \frac{L_{em}^2}{R_e L_m} = \tau_{em}$ which implies $L_e L_m \geq L_{em}^2$ which fits the definition of mutual inductance.

In case of perfectly coupled circuits, $L_e L_m = L_{em}^2$, $k = 1$ the transfer function $A_m(s)$ is still of the first order. In that case, with respect to the usual RL model of the magnet, there is only an additional zero in the transfer function. This case can be modeled with a parallel resistor in the magnet model such as the one reported in Figure 1 (Load Model 2):

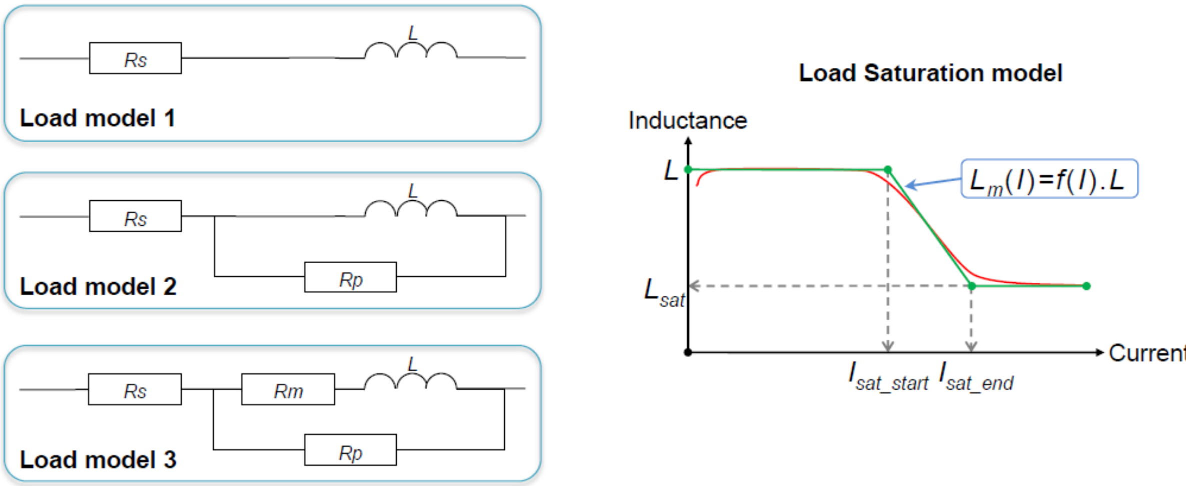

Figure 1 Load Models currently available for FGC3 RST controller

With reference to Figure 1 (Load Model 2) the following equations hold:

$$\begin{cases} R_s = R_m \ (plus\ cable\ resistance\ \dots) \\ R_p = \dfrac{L_m}{\tau_e} \\ k = 1 \end{cases}$$

Two Eddy Currents Circuits

The equations to consider are:

$$\begin{cases} R_{e1}I_{e1} + L_{e1}\dfrac{dI_{e1}}{dt} + L_{em1}\dfrac{dI_m}{dt} = 0 \\ R_{e2}I_{e2} + L_{e2}\dfrac{dI_{e2}}{dt} + L_{em2}\dfrac{dI_m}{dt} = 0 \\ R_m I_m + L_m\dfrac{dI_m}{dt} + L_{me1}\dfrac{dI_{e1}}{dt} + L_{me2}\dfrac{dI_{e2}}{dt} = V_m \end{cases}$$

Where:

$$\begin{cases} L_{emX} = L_{meX} \\ \tau_{e1} = \dfrac{L_{e1}}{R_{e1}} \\ \tau_{e2} = \dfrac{L_{e2}}{R_{e1}} \end{cases}$$

The equations can be suitably manipulated in Matlab by using a matrix notation:

$$L\dot{x} + \acute{A}x + \acute{b}u = 0$$

$$L = \begin{bmatrix} L_{e1} & 0 & L_{em1} \\ 0 & L_{e2} & L_{em2} \\ L_{em1} & L_{em2} & L_m \end{bmatrix}$$

$$\acute{A} = \begin{bmatrix} R_{e1} & 0 & 0 \\ 0 & R_{e2} & 0 \\ 0 & 0 & R_m \end{bmatrix}$$

$$\acute{b} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

$$x = \begin{bmatrix} I_{e1} \\ I_{e2} \\ I_m \end{bmatrix}$$

$$u = V_m$$

Doing all the calculations it can be shown that the transfer function has the following expression:

$$A_m(s) = \frac{1}{R_m} \frac{1 + s(\tau_{e1} + \tau_{e2}) + s^2\tau_{e1}\tau_{e2}}{1 + s(\tau_m + \tau_{e1} + \tau_{e2}) + s^2\left[\tau_{e1}\tau_{e2} + \tau_m[(1 - k_1^2)\tau_{e1} + (1 - k_2^2)\tau_{e2}]\right] + s^3\tau_m\tau_{e1}\tau_{e2}(1 - k_1^2 - k_2^2)}$$

Where:

$$\begin{cases} k_1^2 = \dfrac{L_{em1}^2}{L_{e1}L_m} \\ k_2^2 = \dfrac{L_{em2}^2}{L_{e2}L_m} \end{cases}$$

In this case the transfer function has two zeros and three poles unless $k_1^2 + k_2^2 = 1$

General consideration 1

With the proposed formulation it is not necessary to estimate the "eddy current resistance" R_{eX}.
If the coupling coefficient(s) and the "eddy current time constant(s)" can be efficiently estimated or measured, the transfer function of the magnet used by the current regulator can be written directly with these values.

General consideration 2

For the case in which only one eddy current circuit is considered a simple equivalent circuit is available as shown in Figure 2:

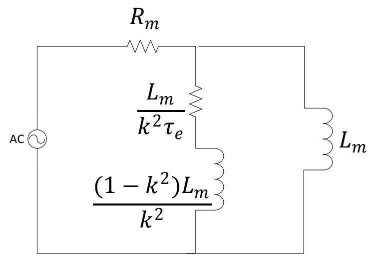

Figure 2 Equivalent circuit for one eddy current circuit

It might be possible to find an equivalent circuit for two eddy currents but it probably going to include additional circuital elements in addition to resistors and inductors in order to properly take into account the interaction between the “two eddy current circuits”. Its usefulness is not very high though, a complete transfer function has been already obtained indeed.