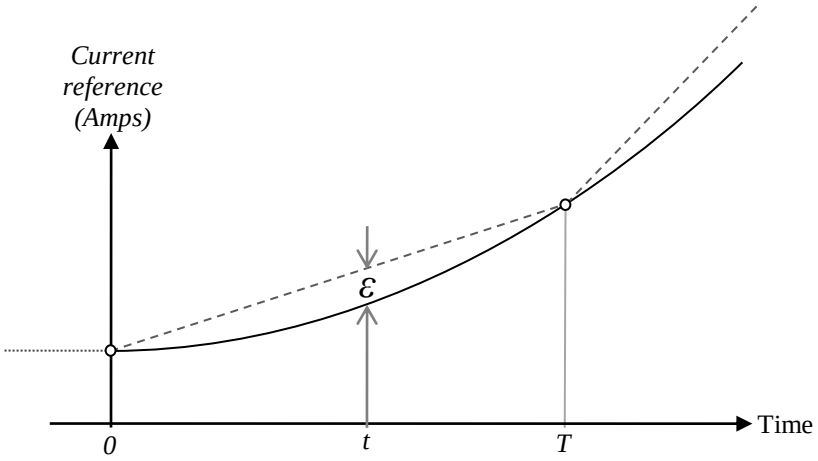
Interpolation Error

The current in the main magnet circuit in the Large Hadron Collider must follow the beam energy very precisely to ensure that the particles remain on the correct orbit. At the start of an acceleration ramp, the energy rises parabolically while the current reference is calculated using linear interpolation from a table of current versus time.

Using linear interpolation will result in an error between the actual linear reference and the ideal parabolic reference. This is illustrated for the first segment in this figure



Assuming that the second derivative of the ideal parabolic reference is *a*, obtain the formula for ε_{max} as a function of *a* and *T* where ε_{max} is the maximum interpolation error ε for the segment $0 \le t \le T$:

$$y_p'' = a$$

$$y_p' = at$$

$$y_p = \frac{at^2}{2}$$

$$y_l'' = 0$$

$$y_l' = b$$

$$y_l = bt$$

$$y_p(T) = y_l(T) \Rightarrow \frac{aT^2}{2} = bT \Rightarrow b = \frac{aT}{2}$$

$$\varepsilon = y_l - y_p = bt - \frac{at^2}{2} = \frac{aT}{2}t - \frac{at^2}{2} = \frac{at}{2}(T - t)$$

$$\varepsilon' = y_l' - y_p' = b - at$$

$$\varepsilon' = 0 \Rightarrow t_{\max} = \frac{b}{a} = \frac{T}{2}$$

$$\varepsilon_{\max} = \frac{a \cdot t_{\max}}{2}(T - t_{\max}) = \frac{aT^2}{8}$$