General Magnet Model

Parameter	
sampling period	T
magnet inductance	L_m
magnet resistance	R_m
series resistance	R_s
parallel resistance	R_p

$$\tau = \frac{L_m}{R_m + \frac{R_p R_s}{R_p + R_s}} = \frac{L_m}{R_m + \frac{R_s}{1 + R_s R_p^{-1}}} \tag{1}$$

$$g_0 = \frac{1}{R_s + R_p} = \frac{R_p^{-1}}{1 + R_s R_p^{-1}} \tag{2}$$

$$g_1 = \frac{1}{R_s + \frac{R_p R_m}{R_p + R_m}} - \frac{1}{R_s + R_p} = \frac{1}{R_s + \frac{R_m}{1 + R_m R_p^{-1}}} - \frac{R_p^{-1}}{1 + R_s R_p^{-1}} \tag{3}$$

$$R_m = 0 \Rightarrow \qquad \tau = \frac{L_m}{R_p} + \frac{L_m}{R_s}, \qquad g_0 = \frac{1}{R_s + R_p}, \qquad g_1 = \frac{1}{R_s} - \frac{1}{R_s + R_p}, \tag{4}$$

$$R_s = 0 \Rightarrow \qquad \tau = \frac{L_m}{R_m}, \qquad g_0 = \frac{1}{R_p}, \qquad g_1 = \frac{1}{R_m}, \tag{5}$$

$$R_p = \infty \Rightarrow \qquad \tau = \frac{L_m}{R_m + R_s}, \qquad g_0 = 0, \qquad g_1 = \frac{1}{R_m + R_s}. \tag{6}$$

Zero-order hold:

$$H(z) = \frac{b_0 z + b_1}{z(z + a_1)}, \quad \begin{cases} a_1 = -e^{-T/\tau} & \approx -(1 - \frac{T}{\tau}) \\ b_0 = g_0 + g_1(1 - e^{-T/\tau}) & \approx g_0 + g_1 \frac{T}{\tau} \\ b_1 = -g_0 e^{-T/\tau} & \approx -g_0(1 - \frac{T}{\tau}) \end{cases} \tag{7}$$

First-order hold:

$$H(z) = \frac{b_0 z + b_1}{z + a_1}, \quad \begin{cases} a_1 = -e^{-T/\tau} & \approx -(1 - \frac{T}{\tau}) \\ b_0 = g_0 + g_1(1 - \frac{1 - e^{-T/\tau}}{T/\tau}) & \approx g_0 + g_1 \frac{T}{2\tau} \\ b_1 = -g_0 e^{-T/\tau} - g_1(e^{-T/\tau} - \frac{1 - e^{-T/\tau}}{T/\tau}) & \approx -g_0(1 - \frac{T}{\tau}) + g_1 \frac{T}{2\tau} \end{cases} \tag{8}$$

General Magnet Simulation

Magnet Saturation:

$$L_m(i_m) = l_m(i_m) L_m(0) \quad \Rightarrow \quad \tau(i_m) = l_m(i_m)\tau(0) \tag{9}$$

Fast voltage source ($T \gg 1/\omega_b$, zero-order hold of load) and slow load ($T \ll \tau$):

$$i'(k) \approx i'(k-1) + l_m^{-1}(i_m(k-1)) \tfrac{T}{\tau(0)} [g_1 u(k-1) - i'(k-1)] \tag{10}$$

$$i(k) = i'(k) + g_0 u(k-1) \tag{11}$$

$$i_m(k) = (1 + R_s R_p^{-1}) i'(k) \tag{12}$$

Slow voltage source ($T \ll 1/\omega_b$, first-order hold of load) and slow load ($T \ll \tau$):

$$i'(k) \approx i'(k-1) + l_m^{-1}(i_m(k-1)) \tfrac{T}{\tau(0)} [g_1 \tfrac{u(k)+u(k-1)}{2} - i'(k-1)] \tag{13}$$

$$i(k) = i'(k) + g_0 u(k) \tag{14}$$

$$i_m(k) = (1 + R_s R_p^{-1}) i'(k) \tag{15}$$