1-Modeling

1.6-Exercises

October 6, 2022

UNIVERSITÀ DI TRENTO

DataScientia
Unitas per Varietatem

1.6-Exercises

Mental representation reprise

Notion 1 (Mental representation - 1) A **mental representation** is a part of the world which describes it (the world itself), meaning that there is correspondence between what is the case in the world and the contents of a mental representation.

Notion 2 (Mental representation - 2) A mental representation is the key human artifact which allows humans to **act** in the world, to **reason** about it and to **communicate** about it to other humans.

Notion 3 (Mental representation - 3) A mental representation is constructed by the mind, inside the mind of each and any human, and it is not accessible to anybody else.

Analogical and linguistic mental representations

Notion 4 (Analogical mental representation) **An analogical mental representation** is a mental representation depicting (in Italian "*raffigurante*") the world as we perceive it (see, hear, touch, taste, smell).

Notion 5 (Linguistic mental representation) A **Linguistic mental representation** is a mental representations describing the contents of an analogical mental representation.

Non-mental representation reprise

Notion 6 (Representation) A representation has two main properties:

- multiple humans perceive it (thus generating corresponding mental representations);
- it is a part of the world which describes it (the world itself), meaning that there is a correspondence between what is the case in the world and the contents of the mental representations generated by perceiving it.

Notion 7 (Analogical representation) Analogical representations **depict** the world. By depicting we mean that there is a **one-to-one mapping** between their contents and what is the case in the world.

Notion 8 (Linguistic representation) Linguistic representations **describe** the world. By describing we mean that there is a **one-to-one mapping** between their contents and what is the case in analogical representations. They are said to **denote** the analogical representation they represent.

Theory reprise

Notion 9 (Sentence) A theory $T = \{s\}$ is a set of **sentences** s, where a sentence is a linguistic representation of a set of facts f.

Notion 10 (Theory) A linguistic representation produced by a modeling activity is called a **Theory**.

Model reprise

Notion 11 (Fact) A model $M = \{f\}$ is a set of **facts** f, where a fact is an analogical representation of a part of the part of the world described by M.

Notion 12 (Model) An analogical representation represented by a theory is called a **semantic model** or, simply, a **model**. We also say that this is the theory's **intended model**.

Denotation relation reprise

Notion 13 (Denotation, Semantics) We say that a theory T **denotes** its intended model M, and write $T = Den(M)$. Alternatively, we say that a model M is the intended **semantics** of T, and write $M = Sem(T)$.

Notion 14 (Theory and model) Let $M = \{f\}$ be a set of facts and $T = \{s\}$ be a set of sentences. Let M_T be smaller than M. Then T **is a theory of model** M_T if and only if, for all $s \in T$ we have $s = Den(f)$ for some $f \in M_T$. We also say that M_T is a **model of** T.

Language and domain reprise

Notion 15 (Language) A **language** $L = \{T\}$ is a set of theories T.

Notion 16 (Domain) A **domain** $D = \{M\}$ is a set of models M.

Notion 17 (Denotation, Semantics extended) We say that a language $L = \{T\}$ **denotes** a domain $D = \{M\}$ if it describes all its models, and write $L = Den(D)$. Alternatively, we say that a domain D is the intended **semantics** of L, and write $D = Sem(L)$.

Correctness and completeness reprise

Notion 18 (Correctness) Let $M \in D$, $T \in L$ with $L = Den(T)$. Then a theory T of a model M is **correct** with respect to M if and only if for every $s \in T$ there is a fact $f \in M$ such that $f = Sem(s)$, **incorrect** otherwise.

Notion 19 (Completeness) Let $M \in D$, $T \in L$ with $L = Den(T)$. Then a **theory** T of a model M is **complete** with respect to M if and only if for every $f \in M$ there is a sentence $s \in T$ such that $s = Den(f)$, **incompleteness** otherwise.

Notion 20 (Correctness and completeness) Let $M \in D$, $T \in L$ with $L = Den(T)$. Then a **theory** T of a model M is **correct and complete** with respect to M if it is both correct and complete.

Informal language reprise

Notion 21 (Formal syntax) The syntax of a language is **formal** if

- The alphabet is recognizable
- The set of formation rules is finite
- There exists correctness checking algorithm which takes in input a sentence and checks whether it is well formed.

Notion 22 (Informal language) A syntax which is not formal is called **informal**. A language which does not have a formal syntax is said to be an **informal language**.

Notion 23 (Well formed formula (wff)) A **sentence** generated from a formal syntax is called a **formula** or also a **well formed formula** (**wff**). A wff w_1 is a **subformula** of a wff w_2, if w_1 has been used in the construction of w_2 (via the formula formation rules).

Semi-formal language reprise

Notion 24 (Semi-formal, Formal language) A language defined by a formal syntax which is not a formal language is a **semi-formal language**.

Formal language reprise

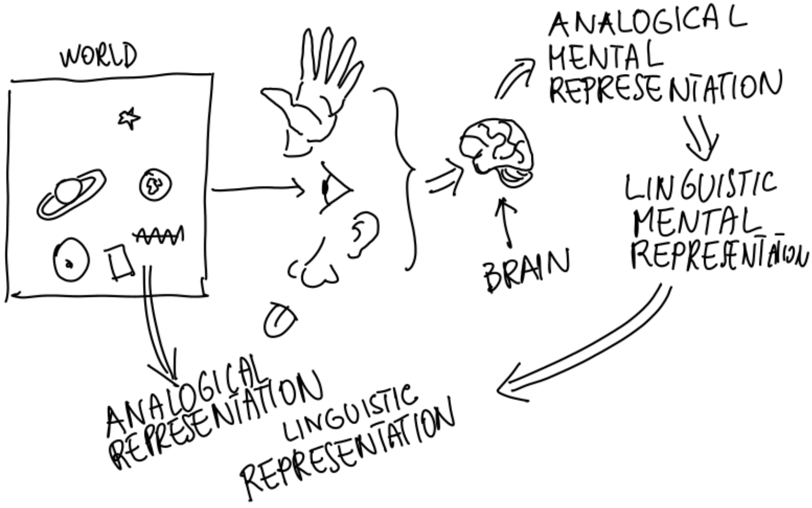
Notion 25 (Formal language) A language L is said to be **formal** if the following conditions apply:

- The formulas and terms of L are defined via a **formal syntax**;
- The domain D denoted by L is formally defined. We call D the **Domain of Interpretation** of L;
- The Denotation Den of L, with $L = Den(D)$ is a function I assigning to each and every **atomic formula** of L one and only one element of the domain, in formulas

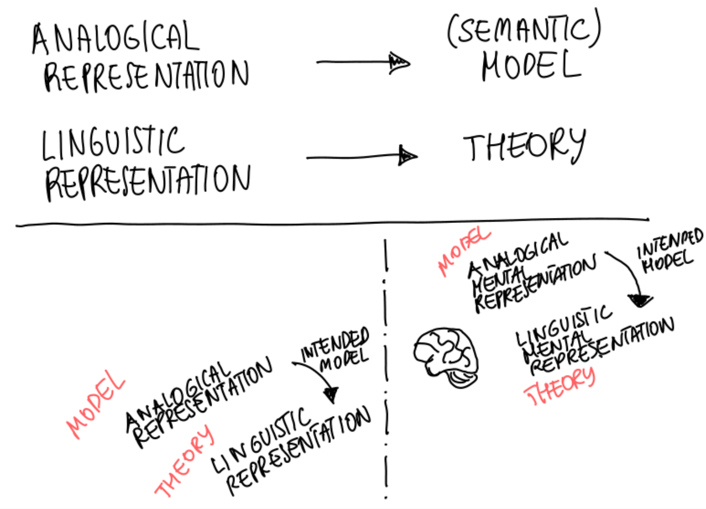
$$I : L \to D$$

  I is called the **Interpretation function** of L.

Representations

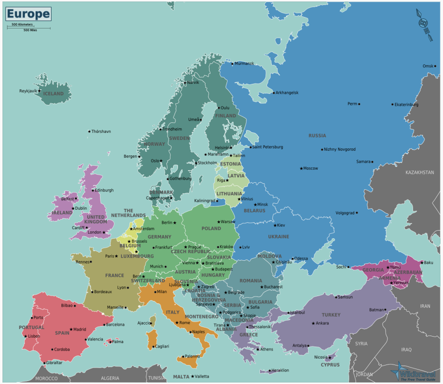


Model and Theory



Exercise 1. Statement

What is a theory of the following model?

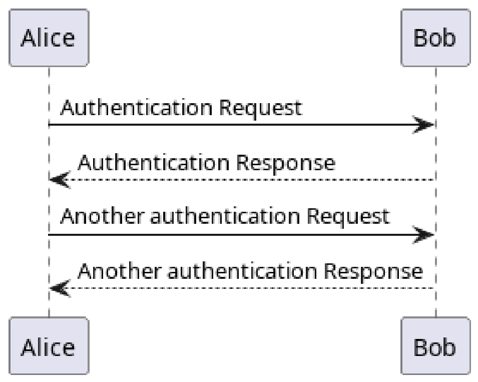


Exercise 1. Approach

1. Decide what you want your theory to represent
2. Define language and formation rules:
   - atomic terms
   - terms formation rules
   - atomic sentences
   - sentences formation rules
3. Provide an interpretation function

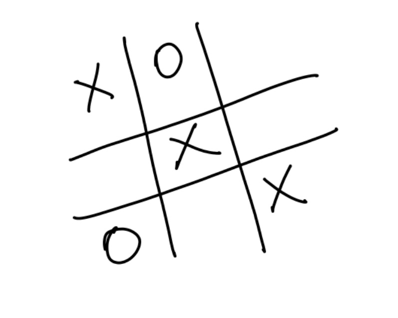
Exercise 2. Statement

What is a theory of the following model?



Exercise 3. Statement

What is a theory of the following model? (Consider the status of the board only; no rules, no dynamics.)



Exercise 1. Statement

Build a model of the following theory:

```
pet_of(bob, molly)
pef_of(alice, murphy)
```

Exercise 1. Approach

1. Decide how you want to represent your model (set, diagram, graph)
2. Identify the syntactic elements of the formal language in which the theory is expressed:
   - which are the atomic terms?
   - are there terms formation rules?
   - are there atomic sentences?
   - are there sentences formation rules?
3. Map the elements of the of the theory to elements of the model
4. Write the interpretation function

Exercise 2. Statement

```
pet_of(bob, molly)
pef_of(alice, murphy)

person(bob) and male(bob)
person(alice) and female(alice)

animal(molly) and female(molly)
animal(murphy) and male(murphy)

sister(alice, bob)
brother(bob, alice)
```

Exercise 3. Statement

```
age(carl, 25)
age(bob, 33)
age(alice, 28)
age(denise, 32)

older(bob, carl)
older(denise, alice)
```

Exercise 3. Statement

```
age(carl, 25)
age(bob, 33)
age(alice, 28)
age(denise, 32)

older(carl, bob)
older(denise, marta)
```

Exercise 4. Statement

What is a model of the following theory?

```
father(adam, bob)
father(bob, adam)
```

Exercise 1. Statement

Given the following set of rules:

- `a`, `b`, `c` are primitive sentences
- if `t` is a sentence, so is `( t )`
- if `t` and `q` are sentences, so is `( t q )`

Determine whether the following sentences are well formed or not.

- `d`
- `a`
- `( a )`
- `( a ( b ) )`
- `( a b a )`
- `( a b ( c ) )`
- `( ( a b ) ( c a ) a )`
- `( ( ( b b ) ( c b ) ) a )`

Exercise 1. Approach

- Check whether there is a sequence of applications of the formation rules which yields the sentence you are checking.
- This is simplified by tokenization and parsing:
  - identify the primitive symbols and make the string into a sequence of tokens
  - given the sequence of tokens identify the way in which a specific subset of the tokens could have been built by a rule
  - compose the tokens into a tree

Exercise 2. Statement

Given the following set of rules:

- a natural number is a primitive term
- if `t`, `q`, `r` are terms, so is `t + q + r`
- if `t` is a term, so is `( t )`

Determine whether the following sentences are well formed or not.

- `1 + 2`
- `1 + 4 + 9`
- `1 + + 1`
- `1 + 5 + 12 + 12`
- `1 + 2 + 3 + 4 + 5 + 6`
- `1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9`
- `(1 + 2 + 3) + (4 + 5) + (6 + 7 + 8) + 9`
- `(1 + 2 + 3) + (4 + 5 + 6)`
- `(1 + 2 + 3) + (4 + 5 + 6) + (1 + 2 + 3)`
- `(1 + 2) + (4 + 5 + 6) + (1 + 2 + 4 + 3)`

Exercise 3. Statement

Defines a language to represent genealogical trees.

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