DL exercises

December 11, 2022

Exercise 0.1. *Formulate ALC concepts: for each of the following concepts, build a suitable ALC concept description, using only the concept names*

Person, Happy, Animal, Cat,Old, Fish

and the role name

owns

1. *Happy person*
2. *Happy pet owner*
3. *Person who owns only cats*
4. *Unhappy pet owners who own an old cat*
5. *Pet owners who own only cat and fish*

Then, draw a set representation that depicts the described situation

Exercise 0.2. *Given the knowledge base*

- $Car \equiv Vehicle \sqcap \exists hasPart.Wheel \sqcap \exists poweredBy.Engine$
- $Bicycle \equiv Vehicle \sqcap \exists hasPart.Wheel \sqcap \exists poweredBy.Human$
- $Boat \equiv Vehicle \sqcap \exists travelsOn.Water$
- $Boat \sqsubseteq \forall hasPart.\neg Wheel$
- $Car \sqcup Bicycle \sqsubseteq \forall travelsOn.\neg Water$
- $Wheel \equiv Device \sqcap \exists hasPart.Axle \sqcap \exists capableOf.Rotation$
- $Driver \equiv Human \sqcap \exists controls.Vehicle$
- $Driver \sqcap \exists controls.Car \sqsubseteq Adult$
- $Human \sqsubseteq \neg Vehicle$
- $Wheel \sqcup Engine \sqsubseteq \neg Human$

- $Human \sqsubseteq Adult \sqcup Child$
- $Adult \sqsubseteq \neg Child$
- Bob :($\exists controls.Car$)
- Bob : $Human$
- (Bob, $QE2$) : $controls$
- $QE2$:($Vehicle \sqcap \exists travelsOn.Water$)

draw a possible interpretation of the given knowledge base as a Schema knowledge graph

Exercise 0.3. *Translate the sentences in knowledge base $\mathcal{K}$ (the one in exercise 0.2) to First Order Logic.*

Exercise 0.4. *Extend the Knowledge Base $\mathcal{K}$ in exercise 0.2 to a Knowledge Base $\mathcal{K}'$ with a translation of the following sentences:*

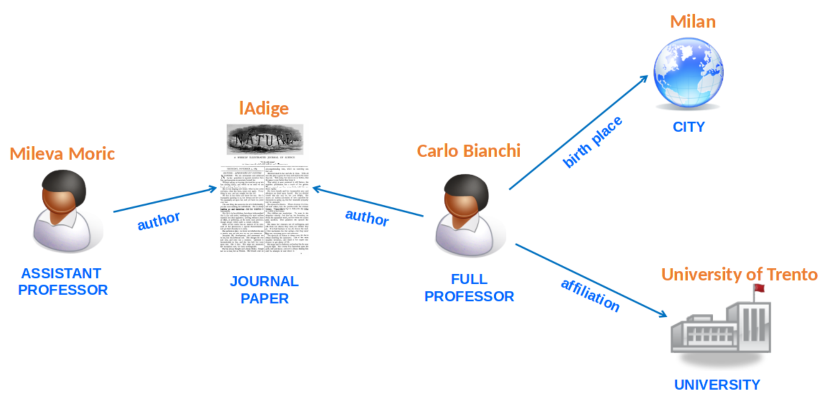
- *A human who legally controls a car holds a driving license and is an adult*
- *A car with a broken part is broken*
- *Bob controls a car with a wheel that has a broken axle*

Then, say whether the following statements are true or false:

- $\mathcal{K}'$ *is consistent*
- $\exists legallyControls.\top$ *is subsumed by* $\exists controls.\top$ *w.r.t.* $\mathcal{K}'$
- *Bob is an instance of* $controls.(Car \sqcap Broken)$ *w.r.t.* $\mathcal{K}'$

Exercise 0.5. *Definire formalmente i 4 "reasoning services" di una ABOX in una Logical Descrittiva (max 20 righe)*

Exercise 0.6. *Sia dato il Data Knowledge Graph in figura*



dove:

- *i nomi sopra le figure sono i nomi delle istanze considerate*
- *i nomi sotto le figure rappresentano "role assertions" cioè il fatto che quelle istanze soddisfano quel concetto*
- *i link taggati rappresentano "role assertions" ossia il fatto che quella coppia di istanze soddisfano quel ruolo*

Scrivere la teoria, ossia l'insieme di assiomi che formalizza la conoscenza rappresentata nel knowledge graph. Disegnare prima la TBox e dopo la ABox. Nella scrittura della TBox dire se e quali quantificazioni esistenziali e Universali esistono nei ruoli utilizzati.

Exercise 0.7. *Given the following Knowledge Base:*

$$\mathcal{T} = \begin{cases} A \equiv B \sqcap C \\ C \equiv D \sqcap E \\ E \sqsubseteq F \sqcap G \end{cases}$$

$$\mathcal{A} = \Big\{ A(1)$$

Provide $\mathcal{A}'$ obtained by extending $\mathcal{A}$ with respect to $\mathcal{T}$.

Exercise 0.8. *Considera la seguente tabella di un database relazionale che descrive le istanze, e relativi attributi del concetto "Persona".*

Nome	$Vive_{in}$	$Amico_{di}$	*Nazionalità*	*Uomo*	*Età*
Fausto	*Trento*	*Adolfo*	*Italiano*	*Vero*	*50*
MaryAnne	*Milano*	*Fausto*	*Americano*	*Falso*	*35*
Adolfo	*Italia*	*MaryAnne*	*Italiano*	*Vero*	*40*

Quali delle seguenti affermazioni sono vere?

- *Un Data Scientist che dovesse formalizzare la conoscenza implicita codificata nella base di dati sopra rappresentata utilizzando le parole nel loro significato inteso, potrebbe codificare la seguente TBOX ?*

  $TBOX = \{$

  $Uomo \sqsubseteq Persona,$

  $\neg\ Uomo \sqsubseteq Persona$

  $\}$

- *Il data scientist di cui al punto precedente potrebbe arricchire la TBOX di al punto precedente con il seguente fatto?*

{ $\exists$ *Nazionalita.Italiano* $\sqsubseteq$ *Persona*}

- *Il Data scientist di cui al punto precedente potrebbe arricchire la TBOX di cui al punto precedente con il seguente fatto?*

  { *Età* $\sqsubseteq$ *Persona* }

- *Assumendo Open World assumption, la seguente ABOX rappresenta correttamente ma non completamente i contenuti del data base?*

  ABOX = {
  $Amico_{di}$*(Fausto, Adolfo),*
  $Amico_{di}$*(Adolfo, MaryAnne),*
  Uomo(Fausto),
  Uomo (MaryAnne)
  }

- *Nel processo di formalizzazione della ABOX, un data scientist potrebbe aggiungere i seguenti due fatti alla ABOX, intendendo per* $part_{of}$ *il fatto che il primo elemento è geograficamente incluso nel secondo?*

  { $Part_{of}$*(Trento, Italia),* $part_{of}$*(Milano, Italia)* }

Exercise 0.9. *Dati lo Schema Knowledge Graph (SKG) e il Data Knowledge Graph (DKG) in figura 1 e 2, quali delle seguenti affermazioni sono vere?*

- *Il Data Knowledge Graph è consistente con lo Schema Knowledge Graph*
- *Se uno aggiunge nella TBOX l'assioma Disjoint(Man, Woman), il DKG è consistente con lo SKG*
- *Se uno aggiunge nella ABOX l'asserzione ¬MotherOf(Mary,Fred) il DKG è consistente con lo SKG*
- *Se uno aggiunge nella ABOX l'asserzione ¬MotherOf(Mary,Fred) il DKG è consistente*
- *Il Data Knowledge Graph non è consistente*

Exercise 0.10. *Given the Tbox*

$$\mathcal{T} = \begin{cases} Anything \equiv abstractObjects \sqcup generalizedEvents \\ Numbers \sqcup representationalObjects \equiv abstractObjects \\ representationalObjects \equiv sentences \sqcup measurements \\ measurements \equiv time \sqcup weight \end{cases}$$

and the Abox $\mathcal{A}$

$$\mathcal{A} = \Big\{ Anything(Thurs\ 01-01-2009\ 6:00)$$

expand $\mathcal{A}$ *with respect to* $\mathcal{T}$

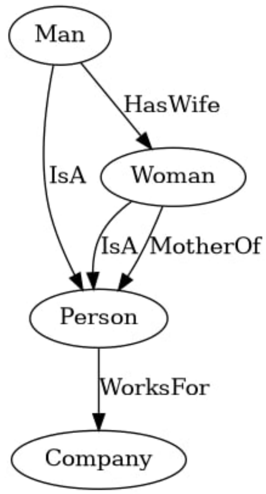


Figure 1: Schema knowledge Graph

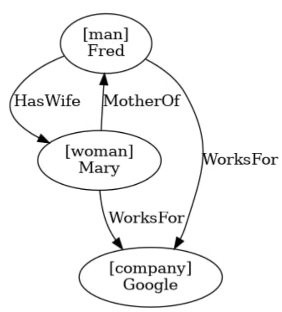


Figure 2: Data knowledge graph

Exercise 0.11. *Given the Tbox*

$$\mathcal{T} = \begin{cases} Animal \sqsubseteq livingThing \\ Donkey \equiv Animal \sqcap \forall hasParent.Donkey \\ Horse \equiv Animal \sqcap \forall hasParent.Horse \\ Mule \equiv Animal \sqcap \exists hasParent.Horse \sqcap \exists hasParent.Donkey \\ \exists hasParent.Mule \sqsubseteq \bot \end{cases}$$

and the Abox $\mathcal{A}$

$$\mathcal{A} = \begin{cases} Horse(Mary) \\ Mule(Peter) \\ Donkey(Sven) \\ hasParent(Peter, Mary) \\ hasParent(Peter, Carl) \end{cases}$$

expand $\mathcal{A}$ *with respect to* $\mathcal{T}$

Exercise 0.12.

- *Write down all the reasoning problems in a TBox*
- *Show how the reasoning problems n a TBox can be reduced to a single reasoning problem, which one?*
- *Write down all the reasoning problem of an ABox with respect to a TBox*
- *Write down all the reasoning problems of an ABox*
- *Can the reasoning problems of an ABox be reduced to one or more reasoning problems of an ABox? Which one(s) if any?*

Exercise 0.13. *Given a Tbox* $\mathcal{T}$ *and two w.f.f.* P,Q, *which of the following statements are true?*

1. *if* $\mathcal{T} \models P$ *and* $\mathcal{T} \models Q$ *then* $\mathcal{T} \models P \sqcup Q$
2. *if* $\mathcal{T} \models P$ *and* $\mathcal{T} \models Q$ *then* $\mathcal{T} \models P \sqcap Q$
3. $\mathcal{T} \models \neg\, (P \sqsubseteq Q)$ *follows from* $\mathcal{T} \models P \sqcap Q \sqsubseteq \bot$
4. $\mathcal{T} \models P \sqcap Q \sqsubseteq \bot$ *follows from* $\mathcal{T} \models P \sqsubseteq Q$
5. $\mathcal{T} \models P \sqsubseteq Q$ *follows from* $\mathcal{T} \models P \sqcap Q \sqsubseteq \bot$
6. $\mathcal{T} \models \neg((Q \sqsubseteq P) \sqcup (P \sqsubseteq Q))$ *follows from* $\mathcal{T} \models P \sqcap Q \sqsubseteq \bot$

Exercise 0.14. *Consider the following Knowledge base*

$$\mathcal{T} = \{\ A \equiv B\}$$
$$\mathcal{A} = \{B(u_1),\ C(u_2)\}$$

State which of the following sentences are true:

1. *$A(u_1)$ can be deduced from $\mathcal{A}$ assuming the OWA*
2. *Every formula that can be derived under a CWA can also be derived under an OWA*
3. *Every formula that can be derived under a OWA can also be derived under an CWA*
4. *$\neg A(u_2)$ can be deduced from $\mathcal{A}$ under CWA*
5. *$\neg A(u_2)$ can be deduced from $\mathcal{A}$ under OWA*

Exercise 0.15. *Given $\mathcal{T}$ a terminology in Description Logics written in a language L, and I the interpretation function that maps $\mathcal{T}$ to the domain Δ. Having C, C_1, C_2 in $\mathcal{T}$, say which of the following statements are true:*

1. *if $\mathcal{T} \not\models C_1$ then $\mathcal{T} \models C_1 \sqsubseteq C_2$ for every concept description C_2*
2. *$I(\exists R.\top)=\{a \in \Delta$ — there exists b so that $(a,b) \in I(R)\}$*

Exercise 0.16. *Given the knowledge base $\mathcal{K}$*

$$\mathcal{A} = \begin{cases} Willy : Whale \\ PacificOcean : Habitat \\ Krill : Food \\ (Willy, Krill) : feedsOf \\ (Willy, PacificOcean) : livesIn \\ Simba : Lion \\ Africa : Habitat \\ Springbok : Food \\ (Simba, Springbok) : feedsOf \\ (Simba, Africa) : livesIn \end{cases}$$

Draw a Knowledge Graph representation of $\mathcal{K}$.

Exercise 0.17. *Given the knowledge base $\mathcal{K}$*

$$\mathcal{T} = \begin{cases} Lion \sqsubseteq Mammal \\ Whale \sqsubseteq Mammal \\ Mammal \sqsubseteq Animal \\ Mammal \equiv livesIn.Habitat \\ Animal \equiv feedsOf.Food \end{cases}$$

Draw a Knowledge Graph representation of $\mathcal{K}$.

Exercise 0.18. *Given*

$$\mathcal{T} = \begin{cases} Male \sqcap Female \sqsubseteq \bot \\ DaughterParent \equiv \forall hasChild.Female \end{cases}$$

$$\mathcal{A} = \begin{cases} hasChild(Anna, Bob) \\ \neg Female(Bob) \end{cases}$$

Is DaughterParent satisfiable?