# 1-Modeling

## 1.5-Formal reasoning

UNIVERSITÀ
DI TRENTO

DataScientia
Unitas per Varietatem

# 1.5-Formal Reasoning

1. Logic(s)
2. Reasoning Problems
3. Choosing a logic

- Types of logics

# 1.5.1-Logic(s)

# Lecture index

# Logic, calculus and reasoning (notion)

**Notion 1 (Logic)** A **logic** L is a triple $\mathcal{L} = \langle L, I, \models \rangle$, where L is a formal language, $I : L \rightarrow D$ is an interpretation function, and $\models$ is an entailment relation $\models_L \subseteq M \times T$, where $T \subseteq L$ is a theory and $M \subseteq D^L$ is a model of $T$, with $D^L$ being generated starting from $I$)

**Notion 2 (Logical calculus)** A **logical calculus** $\mathcal{C}_\mathcal{L}$ is a pair $\mathcal{C}_\mathcal{L} = \langle \mathcal{L}, \mathcal{P} \rangle$, , where $\mathcal{L}$ is a logic and $\mathcal{P}$ is a set of **(reasoning) problems** to be solved in $\mathcal{L}$, defined as $\mathcal{P} = \{\langle \mathcal{Q}, \mathcal{A}(\mathcal{Q}) \rangle\}$, where $\mathcal{Q} = \{Q_i\}$ is a set of questions $Q_i$ and, for each $Q_i$, $\mathcal{A}(\mathcal{Q}) = \{A(Q_i)\}$ is a set of possible answers for $Q_i$.

**Notion 3 (Logical reasoning)** Given a logical calculus $\mathcal{C}_\mathcal{L}$, by **(logical) reasoning**, also called **(logical) inference** we mean the process by which a problem is solved via the application of a possibly not terminating **decision algorithm**.

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- A propositional language with only primitive propositions;

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- Examples of propositional connectives: $\neg$ (to be read "not", for negation), $\wedge$ (to be read "and", for conjunction), $\vee$ (to be read "or", for disjunction), $\Longrightarrow$ (to be read "implies", for implication), $\Longleftrightarrow$ (to be read "if and only if", for equivalence), $\uparrow$ (to be read "nand", for negative conjunction), $\downarrow$ (to be read "nor", for negative disjunction).

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- Examples of quantifiers are: $\forall$ (to be read "for all", for universal quantification over a set of terms, $\exists$ (to be read "there exists", for existential quantification over a set of terms).

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- The modal operator are: $\exists R$ (to be read "there exists an element of ... ", for existential quantification over the codomain of a role), $\forall R$ (to be read "for all elements of ", for universal quantification over the codomain of a role.

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- (Artificial Intelligence): **EML (Entity Modeling language and Logic): Graph Logic + agent interaction - Agent based information exchange in the Web**

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