



UNIVERSITÀ
DI TRENTO

Department of
Information Engineering and Computer Science

Automated Reasoning and **Formal Verification**

Laboratory 9

Gabriele Masina

gabriele.masina@unitn.it

<https://github.com/masinag/arfv2025>

Università di Trento

May 07, 2025

These slides are derived from those by Stefano Tonetta, Alberto Griggio, Silvia Tomasi, Thi Thieu Hoa Le, Alessandra Giordani, Patrick Trentin, Giuseppe Spallitta for FM lab 2005-2024.



Outline

1. Planning problem

Blocks Example

2. Exercises

3. Homeworks

Planning Problem

Given $\langle I, G, A \rangle$, where

I : (representation of) initial states

G : (representation of) goal states

A : (representation of) actions leading from one state to another

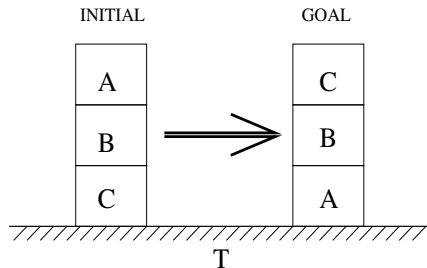
find a **plan** (sequence of actions) a_1, \dots, a_n leading from an initial state to a goal state.

Idea: from Planning to Model Checking

Encode a planning problem as a model checking problem:

1. Impose I as initial state(s)
2. Encode A as transition relation system
3. Verify the LTL property $(! F \text{ goal})$. The plan is a **counter-example for it**.

Example: Blocks [1/9]



Init : $Above(A, B) \wedge Above(B, C) \wedge Above(C, T) \wedge Clear(A)$

Goal : $Above(C, B) \wedge Above(B, A) \wedge Above(A, T)$

Move(a, b, c)

Precond : $Block(a) \wedge Clear(a) \wedge Above(a, b) \wedge (Clear(c) \vee Table(c)) \wedge a \neq b \wedge a \neq c \wedge b \neq c$

Effect : $Clear(b) \wedge \neg Above(a, b) \wedge Above(a, c) \wedge \neg Clear(c)$

Example: Blocks [2/9]

```
MODULE block(id, ab, bl)
VAR   above : {none, a, b, c}; -- the block above this one
      below : {none, a, b, c}; -- the block below this one
DEFINE clear := (above = none);
INIT   above = ab & below = bl
INVAR  below != id & above != id -- a block cannot be above/below itself

MODULE main
VAR   move : {move_a, move_b, move_c}; -- at each step one block moves
      block_a : block(a, none, b);
      block_b : block(b, a, c);
      block_c : block(c, b, none);

...

```

Example: Blocks [3/9]

- ▶ A block cannot move if it has some other block above itself

INVAR

```
(!block_a.clear -> move != move_a) &  
(!block_b.clear -> move != move_b) &  
(!block_c.clear -> move != move_c);
```

Example: Blocks [3/9]

- ▶ A block cannot move if it has some other block above itself

INVAR

```
(!block_a.clear -> move != move_a) &  
(!block_b.clear -> move != move_b) &  
(!block_c.clear -> move != move_c);
```

Q: Why INVAR and not TRANS?

Example: Blocks [3/9]

- ▶ A block cannot move if it has some other block above itself

INVAR

```
(!block_a.clear -> move != move_a) &  
(!block_b.clear -> move != move_b) &  
(!block_c.clear -> move != move_c);
```

Q: Why **INVAR** and not **TRANS**?

A: **INVAR** p is equivalent to **INIT** p and **TRANS** next(p).

With **TRANS** only, we could have in initial state with move_b.

Example: Blocks [3/9]

- ▶ A block cannot move if it has some other block above itself

INVAR

```
(!block_a.clear -> move != move_a) &  
(!block_b.clear -> move != move_b) &  
(!block_c.clear -> move != move_c);
```

Q: Why **INVAR** and not **TRANS**?

A: **INVAR** p is equivalent to **INIT** p and **TRANS** next(p).

With **TRANS** only, we could have in initial state with move_b.

Q: What's wrong with following formulation?

INVAR

```
(block_a.clear -> move = move_a) &  
(block_b.clear -> move = move_b) &  
(block_c.clear -> move = move_c);
```

Example: Blocks [3/9]

- ▶ A block cannot move if it has some other block above itself

INVAR

```
(!block_a.clear -> move != move_a) &  
(!block_b.clear -> move != move_b) &  
(!block_c.clear -> move != move_c);
```

Q: Why INVAR and not TRANS?

A: INVAR p is equivalent to INIT p and TRANS next(p).

With TRANS only, we could have in initial state with move_b.

Q: What's wrong with following formulation?

INVAR

```
(block_a.clear -> move = move_a) &  
(block_b.clear -> move = move_b) &  
(block_c.clear -> move = move_c);
```

A: Two clear blocks would cause an **inconsistency** (move can have **one** valid value).
Moreover, any non-clear block would still be able to move.

Example: Blocks [4/9]

- ▶ A **moving** block changes location and remains clear

TRANS

```
(move = move_a -> next(block_a.clear) &  
                        next(block_a.below) != block_a.below) &  
(move = move_b -> next(block_b.clear) &  
                        next(block_b.below) != block_b.below) &  
(move = move_c -> next(block_c.clear) &  
                        next(block_c.below) != block_c.below);
```

- ▶ A **non-moving** block does not change its location

TRANS

```
(move != move_a -> next(block_a.below) = block_a.below) &  
(move != move_b -> next(block_b.below) = block_b.below) &  
(move != move_c -> next(block_c.below) = block_c.below);
```

- ▶ A block remains connected to any *non-moving* block

TRANS

```
(move != move_a & block_b.above = a -> next(block_b.above) = a) &  
(move != move_a & block_c.above = a -> next(block_c.above) = a) &  
(move != move_b & block_a.above = b -> next(block_a.above) = b) &  
(move != move_b & block_c.above = b -> next(block_c.above) = b) &  
(move != move_c & block_a.above = c -> next(block_a.above) = c) &  
(move != move_c & block_b.above = c -> next(block_b.above) = c);
```

- ▶ A block remains connected to any *non-moving* block

TRANS

```
(move != move_a & block_b.above = a -> next(block_b.above) = a) &  
(move != move_a & block_c.above = a -> next(block_c.above) = a) &  
(move != move_b & block_a.above = b -> next(block_a.above) = b) &  
(move != move_b & block_c.above = b -> next(block_c.above) = b) &  
(move != move_c & block_a.above = c -> next(block_a.above) = c) &  
(move != move_c & block_b.above = c -> next(block_b.above) = c);
```

Q: What about “below” block?

- ▶ A block remains connected to any *non-moving* block

TRANS

```
(move != move_a & block_b.above = a -> next(block_b.above) = a) &  
(move != move_a & block_c.above = a -> next(block_c.above) = a) &  
(move != move_b & block_a.above = b -> next(block_a.above) = b) &  
(move != move_b & block_c.above = b -> next(block_c.above) = b) &  
(move != move_c & block_a.above = c -> next(block_a.above) = c) &  
(move != move_c & block_b.above = c -> next(block_b.above) = c);
```

Q: What about “below” block?

A: Covered in previous slide!

- Positioning of blocks: above and below relations must be symmetric.

INVAR

```
(block_a.above = b <-> block_b.below = a) &  
(block_a.above = c <-> block_c.below = a) &  
(block_b.above = a <-> block_a.below = b) &  
(block_b.above = c <-> block_c.below = b) &  
(block_c.above = a <-> block_a.below = c) &  
(block_c.above = b <-> block_b.below = c);
```

Notice that the above handles also the case of none blocks!

Remark

A **plan** is a sequence of transitions/actions leading from the initial state to an accepting/goal state.

Idea

- ▶ Assert property p : “goal state is not reachable”
- ▶ If a plan **exists**, nuXmv produces a counter-example for p
- ▶ The counterexample for p is a plan to reach the goal



Example: Blocks [8/9]

Examples

- ▶ Get a plan for reaching “goal state”

Examples

- Get a plan for reaching “goal state”

LTLSPEC

```
! F(block_a.below = none & block_a.above = b    &  
    block_b.below = a    & block_b.above = c    &  
    block_c.below = b    & block_c.above = none);
```

Examples

- Get a plan for reaching “goal state”

LTLSPEC

```
! F(block_a.below = none & block_a.above = b    &  
    block_b.below = a    & block_b.above = c    &  
    block_c.below = b    & block_c.above = none);
```

- Get a plan for reaching a configuration in which all blocks are placed on the table

Examples

- Get a plan for reaching “goal state”

LTLSPEC

```
! F(block_a.below = none & block_a.above = b    &  
    block_b.below = a    & block_b.above = c    &  
    block_c.below = b    & block_c.above = none);
```

- Get a plan for reaching a configuration in which all blocks are placed on the table

LTLSPEC

```
! F(block_a.below = none & block_b.below = none &  
    block_c.below = none);
```



Example: Blocks [9/9]

Examples

- ▶ At any given time, at least one block is placed on the table



Example: Blocks [9/9]

Examples

- ▶ At any given time, at least one block is placed on the table

INVARSPEC

```
block_a.below=none | block_b.below=none | block_c.below=none;
```

Examples

- ▶ At any given time, at least one block is placed on the table

INVARSPEC

`block_a.below=none | block_b.below=none | block_c.below=none;`

- ▶ At any given time, at least one block has nothing above

Examples

- ▶ At any given time, at least one block is placed on the table

INVARSPEC

`block_a.below=none | block_b.below=none | block_c.below=none;`

- ▶ At any given time, at least one block has nothing above

INVARSPEC

`block_a.above=none | block_b.above=none | block_c.above=none;`



Outline

1. Planning problem

2. Exercises

Tower of Hanoi

Ferryman

Tic-Tac-Toe

3. Homeworks

Exercise 9.1: Tower of Hanoi

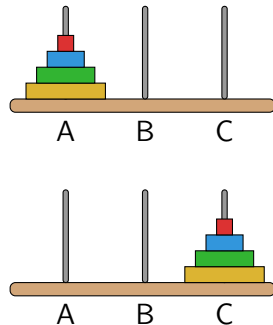
Given 3 poles and N disks of different sizes:

Init A stack of disks with decreasing sizes is placed on pole A

Goal Move the stack to pole C

Rules

- ▶ Only one disk can be moved per step
- ▶ Only the upper disk can be moved
- ▶ A disk cannot be put above a smaller disk



► Base system model

```
MODULE main
```

```
VAR d1    : {left,middle,right}; -- smallest  
    d2    : {left,middle,right};  
    d3    : {left,middle,right};  
    d4    : {left,middle,right}; -- largest  
    move  : 1..4;                -- possible moves
```

► Base system model

```
MODULE main
VAR d1    : {left,middle,right}; -- smallest
    d2    : {left,middle,right};
    d3    : {left,middle,right};
    d4    : {left,middle,right}; -- largest
    move  : 1..4;                -- possible moves
```

► A disk cannot move if a smaller disk is above it (i.e. they share the same column)

```
DEFINE clear1 := TRUE;
    clear2 := d2 != d1;
    clear3 := d3 != d1 & d3 != d2;
    clear4 := d4 != d1 & d4 != d2 & d4 != d3;
```



Tower of Hanoi [3/5]

► Initial state

`INIT d1 = left & d2 = left & d3 = left & d4 = left;`

Tower of Hanoi [3/5]

► Initial state

```
INIT d1 = left & d2 = left & d3 = left & d4 = left;
```

► Moves descriptions: one disk is moved at a time

TRANS

```
(move = 1 <-> next(d1) != d1) &  
(move = 2 <-> next(d2) != d2) &  
(move = 3 <-> next(d3) != d3) &  
(move = 4 <-> next(d4) != d4);
```

► Moves descriptions: a disk cannot be moved on top of a smaller disk

TRANS

```
(move = 2 -> next(d2) != d1) &  
(move = 3 -> next(d3) != d1 & next(d3) != d2) &  
(move = 4 -> next(d4) != d1 & next(d4) != d2 & next(d4) != d3);
```

- ▶ A non-clear disk cannot move

INVAR

```
(!clear1 -> move != 1) & (!clear2 -> move != 2) &  
(!clear3 -> move != 3) & (!clear4 -> move != 4);
```

- ▶ A non-clear disk cannot move

INVAR

```
(!clear1 -> move != 1) & (!clear2 -> move != 2) &  
(!clear3 -> move != 3) & (!clear4 -> move != 4);
```

- ▶ If all columns are being used, don't move the largest disk (or we would reach a deadlock).

INVAR

```
((clear1 & clear2 & clear3) -> move != 3) &  
((clear1 & clear2 & clear4) -> move != 4) &  
((clear1 & clear2 & clear3) -> move != 4) &  
((clear2 & clear3 & clear4) -> move != 4);
```


- Get a plan for a reaching “goal state”:

LTLSPEC

`! F (d1 = right & d2 = right & d3 = right & d4 = right);`

INVARSPEC

`! (d1 = right & d2 = right & d3 = right & d4 = right);`

Exercise 9.2: Ferryman

A ferryman has to bring a sheep, a cabbage, and a wolf safely across a river:

Init all the items are on the right side

Goal all the items are on the left side

Actions

- ▶ The ferryman can have at most one passenger on his boat
- ▶ The cabbage and the sheep cannot be left unattended on the same side of the river
- ▶ The sheep and the wolf cannot be left unattended on the same side of the river

Q: Can the ferryman transport all the items to the other side safely?

► Base system model

```
MODULE main
```

```
VAR
```

```
  cabbage : {right,left};  sheep : {right,left};
```

```
  wolf    : {right,left};  man    : {right,left};
```

```
  move    : {c, s, w, e};  -- possible moves
```

```
DEFINE
```

```
  carry_cabbage := (move = c); carry_sheep := (move = s);
```

```
  carry_wolf    := (move = w); no_carry    := (move = e);
```

► Initial state

```
INIT cabbage = right & sheep = right & wolf = right & man = right;
```

- The ferryman can move to the other side of the river with one item at a time

TRANS

```
next(man) != man &  
(carry_cabbage <-> next(cabbage) != cabbage) &  
(carry_sheep <-> next(sheep) != sheep) &  
(carry_wolf <-> next(wolf) != wolf);
```

- If the man is not in the same side of an item, we cannot choose it for the next movement (otherwise deadlock).

INVAR

```
(man != cabbage -> !carry_cabbage) &  
(man != sheep   -> !carry_sheep) &  
(man != wolf     -> !carry_wolf);
```

- Get a plan for reaching a “goal state”

DEFINE

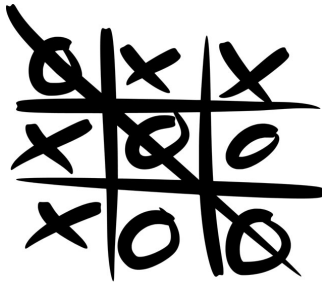
```
safe_state := (sheep = wolf | sheep = cabbage) -> sheep = man;  
goal := cabbage = left & sheep = left & wolf = left;
```

LTLSPEC

```
! (safe_state U goal);
```

Exercise 9.3: Tic-Tac-Toe

Tic-tac-toe is a turn-based game for two adversarial players (\times and \circ) marking the squares of a 3×3 grid. The player who succeeds in placing three respective marks in a horizontal, vertical or diagonal row wins the game.



Scenario where \circ wins

- Base system model: we model the grid as an array of size 9:

1		2		3
----- ----- -----				
4		5		6
----- ----- -----				
7		8		9

```
MODULE main
```

```
VAR
```

```
  B : array 1..9 of {0,1,2};
```

```
  player : 1..2;
```

```
  move : 0..9;
```

- Base system model: we model the grid as an array of size 9:

1		2		3
---		---		---
4		5		6
---		---		---
7		8		9

```
MODULE main
```

```
VAR
```

```
  B : array 1..9 of {0,1,2};
```

```
  player : 1..2;
```

```
  move : 0..9;
```

- Initial state

```
INIT
```

```
  B[1] = 0 & B[2] = 0 & B[3] = 0 & B[4] = 0 & B[5] = 0 &
```

```
  B[6] = 0 & B[7] = 0 & B[8] = 0 & B[9] = 0 & move = 0;
```




Tic-Tac-Toe [3/5]

► Turns Modeling

ASSIGN

```
init(player) := 1;
```

```
next(player) := player = 1 ? 2 : 1;
```



► Turns Modeling

ASSIGN

```
init(player) := 1;  
next(player) := player = 1 ? 2 : 1;
```

► Moves Modeling

TRANS

```
B[1] != 0 -> next(move) != 1;
```

TRANS

```
next(move) = 1 -> next(B[1]) = player &  
  next(B[2])=B[2] & next(B[3])=B[3] & next(B[4])=B[4] &  
  next(B[5])=B[5] & next(B[6])=B[6] & next(B[7])=B[7] &  
  next(B[8])=B[8] & next(B[9])=B[9];
```

...

► “End” state

DEFINE

```
win1 := (B[1]=1 & B[2]=1 & B[3]=1) | (B[4]=1 & B[5]=1 & B[6]=1) |
        (B[7]=1 & B[8]=1 & B[9]=1) | (B[1]=1 & B[4]=1 & B[7]=1) |
        (B[2]=1 & B[5]=1 & B[8]=1) | (B[3]=1 & B[6]=1 & B[9]=1) |
        (B[1]=1 & B[5]=1 & B[9]=1) | (B[3]=1 & B[5]=1 & B[7]=1);
win2 := (B[1]=2 & B[2]=2 & B[3]=2) | (B[4]=2 & B[5]=2 & B[6]=2) |
        (B[7]=2 & B[8]=2 & B[9]=2) | (B[1]=2 & B[4]=2 & B[7]=2) |
        (B[2]=2 & B[5]=2 & B[8]=2) | (B[3]=2 & B[6]=2 & B[9]=2) |
        (B[1]=2 & B[5]=2 & B[9]=2) | (B[3]=2 & B[5]=2 & B[7]=2);
draw := !win1 & !win2 &
        !(B[1]=0 | B[2]=0 | B[3]=0 | B[4]=0 | B[5]=0 |
          B[6]=0 | B[7]=0 | B[8]=0 | B[9]=0);
```

INVAR

```
(win1 | win2 | draw) <-> next(move)=0;
```

- We can easily check if there is a way to reach every end state using the typical formulation:

LTLSPEC

! (F draw);

LTLSPEC

! (F win1);

LTLSPEC

! (F win2);

For each property, a trace satisfying the property is returned as a counterexample.



Outline

1. Planning problem
2. Exercises
3. Homeworks



Homework 9.1: Tower of Hanoi

Extend the Tower of Hanoi to handle five disks, and check that the goal state is reachable.

Homework 9.2: Ferryman

Another ferryman has to bring a fox, a chicken, a caterpillar and a crop of lettuce safely across a river.

Init All the items are on the right side

Goal All the items are on the left side

Rules

- ▶ The ferryman can cross the river with at most **two** passengers on his boat
- ▶ The fox eats the chicken if left unattended on the same side of the river
- ▶ The chicken eats the caterpillar if left unattended on the same side
- ▶ The caterpillar eats the lettuce if left unattended on the same side

Can the ferryman bring every item safely on the other side?