Automated Reasoning and Formal Verification
Laboratory 6

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Pareto vs Lexicographic Multi-Optimization

Exercise 6.1: Multi-objective optimization

A small business promotes itself using two methods: traditional media ads and personal appearances.

A **traditional media ad campaign** costs $2000, generating 2 new customers and 1 positive rating per month. Each ad campaign takes 1 hour.

A **personal appearance** costs $500, generating 2 new customers and 5 positive ratings. Each personal appearance takes 2 hours.

The company wants at least 16 new customers and 28 positive ratings per month.

Try to **minimize both costs and time**.

Let's identify variables, constraints, and cost functions (our goals to minimize):

- ▶ **Variables**: number of ads and number of personal appearance.
- ▶ **Constraints**: number of minimum customers and positive ratings (these are mandatory to be satisfied).
- ▶ **Goals**: minimize time and money.

We must encode time and money as functions depending on the variables!

1. Advanced OMT

Multi-objective optimization

OptiMathSAT supports multi-objective optimization with different *priorities*:

Boxed : optimizes **all the objective independently**. A model for each objective is returned (default behavior).

Lexicographic : optimizes **the first objective, then the second, and so on**. A single model is returned.

Pareto : Computes the **Pareto front**: a set of non-dominated models. Each check-sat finds one, until all are found (possibly infinite).

You can change the type of optimization from the option

```
(set-option :opt.priority=box|lex|par)
```

Pareto front

- The Pareto front is a set of solutions that are not dominated by any other solution.
- A solution A is dominated by another solution B if B is better in at least one objective and not worse in all others.
- The Pareto front represents the trade-offs between different objectives.
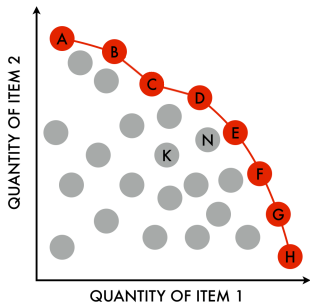
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Exercise 6.2: Delivery

A company must deliver goods from a central depot to three warehouses: A, B, and C.
There are two transport scenarios: **Normal Traffic** and **Heavy Traffic**.

Delivery Costs (per unit):

Warehouse	Normal Traffic	Heavy Traffic
A	$10	$21
B	$15	$18
C	$20	$15

Let x_A, x_B, x_C be the number of units sent to warehouses A, B, and C.
The company requires that: $100 \leq x_A + x_B + x_C \leq 150$

Use OptiMathSAT to find the number of units to minimize the worst-case delivery cost.

- The problem is to minimize the cost of the worst-case scenario.
- We should *minimize* the *maximum* cost of delivery.
- OptiMathSAT has a built-in function for this: (minmax obj1 obj2 ...).

1. Advanced OMT

Variables:
- Define the variables x_A, x_B, x_C as Int constants.
- Define two derived Int constants cost-normal and cost-heavy

Constraints:
- Total number of units must be between 100 and 150
- Non-negativity (hidden): $x_A, x_B, x_C \geq 0$

Objectives:
- Minimize the maximum of the two scenario costs:

  (minmax cost-normal cost-heavy)