UNIVERSITÀ DI TRENTO
Department of Information Engineering and Computer Science

Automated Reasoning and **Formal Verification**

Laboratory 7

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Outline

History of nuXmv

SMV

Symbolic Model Verifier developed by McMillan in 1993.

NuSMV

Open-source symbolic model checker for SMV models. It has been developed by FBK, Carnegie Mellon University, the University of Genoa, and the University of Trento.

nuXmv

Extends NuSMV for infinite state and timed (since v2) systems.
The binary is available for non-commercial or academic purposes only.[a]
Developed and maintained by the Formal Methods unit at FBK.

[a] https://nuxmv.fbk.eu/download.html

Application of nuXmv

- nuXmv allows for the verification of:
  - **finite-state systems** using SAT and BDD based algorithms
  - **infinite-state systems** (i.e., with *real* and *integer* variables) using SMT-based techniques running on top of MathSAT5
  - **timed systems** (i.e., with *clock* type) via reduction to infinite state model-checking.
- nuXmv supports synchronous composition of systems
- Asynchronous composition is no longer supported!

Outline

Interactive shell [1/3]

`nuXmv -int` (or `NuSMV -int`) activates an interactive shell

`read_model -i filename` reads the model from the input file.

`go`, `go_bmc`, `go_msat` initialize nuXmv for verification or simulation with a specific backend engine.

`help` shows the list of all commands

`help <command>` shows detailed information for that command

`<command> -h` shows the command line help

Interactive shell [2/3]

`pick_state [-v] [-r | -i [-a]]` picks a state from the set of initial states.

- `-v` prints the chosen state.
- `-r` randomly picks a state from the set of initial states.
- `-i` picks a state from the set of the initial states interactively.
- `-a` displays all state variables (requires `-i`).

`simulate [-p | -v] [-r | -i [-a]] -k N` generates a sequence of at most `N` transitions starting from the `current state`.

- `-p` prints the changing variables in the generated trace;
- `-v` prints changed and unchanged variables in the generated trace;
- `-r` at every step picks the next state randomly.
- `-i` at every step picks the next state interactively.
- `-a` prints all state variables (requires `-i`);

`print_current_state [-v]` prints out the current state.

- `-v` prints all the variables.

Interacting Shell [3/3]

`show_vars [-s] [-f] [-i] [-t] [-v]` prints the variables content and type

`-s`	print state variables;
`-f`	print frozen variables;
`-i`	print input variables;
`-t`	prints the number of variables;
`-v`	prints verbosely;

`reset` resets the whole system (so you can read another model to analyze).

`quit` stops the program.

Interacting Shell - Output Example

```
nuXmv > read_model -i example01.smv ; go
nuXmv > pick_state -v -r
Trace Description: Simulation Trace
Trace Type: Simulation
-> State: 1.1 <-
b0 = FALSE
b1 = FALSE
nuXmv > simulate -v -r -k 2
******  Simulation Starting From State 1.1
******
Trace Description: Simulation Trace
Trace Type: Simulation
-> State: 1.1 <-
b0 = FALSE
b1 = FALSE
-> State: 1.2 <-
b0 = TRUE
b1 = FALSE
-> State: 1.3 <-
b0 = FALSE
b1 = TRUE
```

Interacting Shell - Output Example

```
nuXmv > read_model -i example01.smv ; go
nuXmv > pick_state -v -r
Trace Description: Simulation Trace
Trace Type: Simulation
-> State: 1.1 <-
b0 = FALSE
b1 = FALSE
nuXmv > simulate -v -r -k 2
******  Simulation Starting From State 1.1
******
Trace Description: Simulation Trace
Trace Type: Simulation
-> State: 1.1 <-
b0 = FALSE
b1 = FALSE
-> State: 1.2 <-
b0 = TRUE
b1 = FALSE
-> State: 1.3 <-
b0 = FALSE
b1 = TRUE
```

Note

States are numbered as `trace_number.state_number`

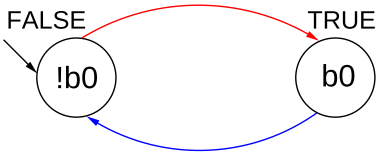
Outline

First SMV model

- An SMV model is composed of a number of **modules**;
- Each **module** can contain:
  - State variables declarations;
  - Formulas defining the valid initial states;
  - Formulas defining the transition relation;

Example

```
MODULE main
VAR
  b0 : boolean;
ASSIGN
  init(b0) := FALSE;
  next(b0) := !b0;
```



Basic Types [1/3]

Boolean : `TRUE`, `FALSE`

```
x : boolean;
```

Enumerative :

```
s : {ready, busy, waiting, stopped};
```

Bounded integers : (within C/C++ `INT_MIN` and `INT_MAX`)

```
n : 1..8;
```

Basic Types [2/3]

integers : -1, 0, 1, ... (within C/C++ `INT_MIN` and `INT_MAX`)

```
n : integer;
```

rationals : `1.66`, `f'2/3`, `2e3`, `10e-1`, ...

```
r : real;
```

words : arrays of bits supporting bitwise logical and arithmetic operations.

```
unsigned word[3];
signed word[7];
```

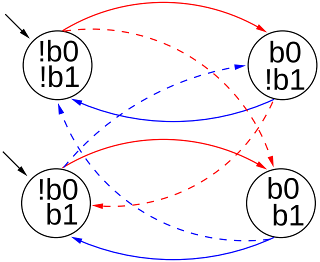
Basic Types [3/3]

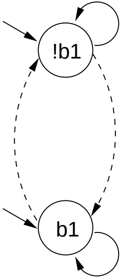
arrays : declared with a pair of lower/upper bounds for the index and a type.
Array indexes *must be constants*.

```
VAR
  x : array 0..10 of boolean; -- array of 11 items
  y : array -1..1 of {red, green, orange}; -- array of 3 items
  z : array 1..10 of array 1..5 of boolean; -- array of arrays
ASSIGN
  init(x[5]) := bool(1);
  init(y[0]) := {red, green}; -- any value in the set
  init(z[3][2]) := TRUE;
```

Adding a state variable

```
MODULE main
VAR
  b0 : boolean;
  b1 : boolean;
ASSIGN
  init(b0) := FALSE;
  next(b0) := !b0;
```





Remarks:

- the FSM is the result of the synchronous composition of the "subsystems" for `b0` and `b1`
- the new state space is the Cartesian product of variables' ranges.

Initial States [1/2]

Example

```
init(x) := FALSE;      -- x must be FALSE
init(y) := {1, 2, 3}; -- y can be either 1, 2 or 3
```

```
init(<variable>) := <simple_expression>;
```

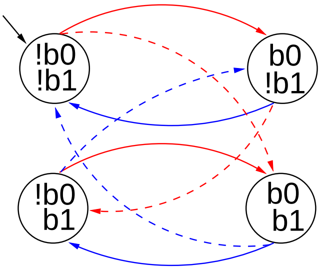
- constrains the **initial value** of <variable> to satisfy the <simple_expression>
- the initial value of an unconstrained variable can be any of those in its domain

The set of **initial states** is given by the set of states whose variables satisfy *all* the `init(...)` constraints in a module.

Initial States [2/2]

Example

```
MODULE main
VAR
  b0 : boolean;
  b1 : boolean;
ASSIGN
  init(b0) := FALSE;
  next(b0) := !b0;
  init(b1) := FALSE;
```



Expressions [1/3]

arithmetic operators :

`+` `-` `*` `/` `mod` `-` (unary)

comparison operators :

`=` `!=` `>` `<` `<=` `>=`

logic operators :

`&` `|` `xor` `!` (not) `->` `<->`

bitwise operators :

`<<` `>>`

set operators : `{v1,v2,...,vn}`

- `in` : tests a value for membership in a set *(set inclusion)*
- `union` : takes the union of 2 sets *(set union)*

count operator : counts number of true *Boolean* expressions

`count(b1, b2, ..., bn)`

Expressions [2/3]

case expression :

```
case
  c1   : e1;
  c2   : e2;
  ...
  TRUE : en;
esac
```

C/C++ equivalent:

```
if (c1) return e1;
else if (c2) return e2;
        ...
else return en;
```

if-then-else expression :

```
cond_expr ? basic_expr1 : basic_expr2
```

conversion operators : `toint`, `bool`, `floor`, and

`swconst`/`uwconst` : convert an integer to a signed/unsigned word.
`word1` : convert boolean to a single word bit.
`unsigned`/`signed` : convert signed to unsigned word and vice-versa.

Expressions [3/3]

- Expressions in SMV do not necessarily evaluate to one value. In general, they can represent a set of possible values.
  `init(var) := {a,b,c} union {x,y,z};`
- The meaning of `:=` in assignments is that the `lhs` can non-deterministically be assigned to any value in the set of values represented by the `rhs`.
- A constant `c` is a syntactic abbreviation for `{c}` (the singleton containing `c`).

Transition Relation [1/2]

Transition Relation

It specifies a constraint on the values that a variable can assume in the **next state**, given the value of variables in the **current state**.

```
next(<variable>) := <next_expression>;
```

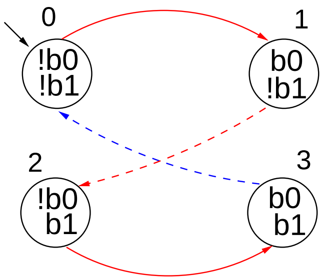
- <next_expression> can depend both on "current" and "next" variables:

```
next(a) := { a, a+1 };
next(b) := b + (next(a) - a);
```

- <next_expression> must evaluate to values in the domain of <variable>;
- the **next** value of an unconstrained variable evolves non-deterministically;

Example: Modulo-4 Counter

```
MODULE main
VAR
  b0 : boolean;
  b1 : boolean;
ASSIGN
  init(b0) := FALSE;
  init(b1) := FALSE;
  next(b0) := !b0;
  next(b1) := case
                b0   : !b1;
                TRUE : b1;
              esac;
```



Output Variable [1/2]

Output Variable

A variable whose value deterministically depends on the value of other "current" state variables and for which no `init()` or `next()` are defined.

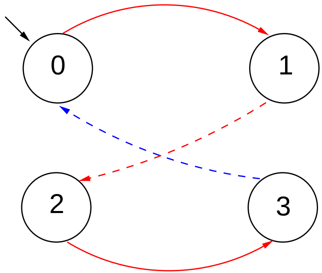
```
<variable> := <simple_expression>;
```

- `<simple_expression>` must evaluate to values in the domain of the `<variable>`.
- used to model *outputs* of a system;

Output Variable [2/2]

Example: Modulo-4 Counter + Output

```
MODULE main
VAR
  b0 : boolean;
  b1 : boolean;
  out : 0..3;
ASSIGN
  init(b0) := FALSE;
  next(b0) := !b0;
  init(b1) := FALSE;
  next(b1) := ((!b0 & b1) | (b0 & !b1));
  out      := toint(b0) + 2 * toint(b1);
```



Assignment Rules (:=)

Single assignment rule :Each variable may be **assigned only once**. Illegal examples.

```
init(var) := ready;
init(var) := busy;
```

```
var := ready;
var := busy;
```

```
next(var) := ready;
var := busy;
```

```
next(var) := ready;
next(var) := busy;
```

```
init(var) := ready;
var := busy;
```

Assignment Rules (:=)

Single assignment rule :Each variable may be **assigned only once**. Illegal examples.

```
init(var) := ready;
init(var) := busy;
```

```
var := ready;
var := busy;
```

```
next(var) := ready;
var := busy;
```

```
next(var) := ready;
next(var) := busy;
```

```
init(var) := ready;
var := busy;
```

Circular dependency rule :A set of equations must not form *cycles* in their dependency graph, unless broken by delays. Illegal examples:

```
next(x) := next(y);
next(y) := next(x);
```

```
x := (x + 1) mod 2;
```

```
next(x) := x & next(x);
```

Instead, the following is legal:

```
next(x) := next(y);
next(y) := y & x;
```

DEFINE declarations

```
DEFINE <id> := <simple_expression>;
```

- Each occurrence of the defined symbol is replaced with the body of the definition
- Alternative way to define *output variables*;

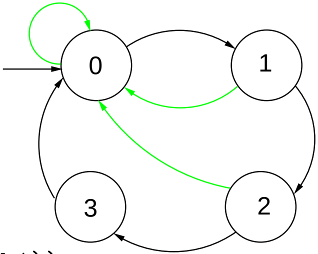
Example

```
MODULE main
VAR
  b0 : boolean;
  b1 : boolean;
ASSIGN
  init(b0) := FALSE; next(b0) := !b0;
  init(b1) := FALSE; next(b1) := ((!b0 & b1) | (b0 & !b1));
DEFINE
  out := toint(b0) + 2 * toint(b1);
```

Example: Modulo-4 Counter with Reset

The counter can be reset by an external "uncontrollable" signal.

```
MODULE main
VAR
  b0 : boolean; b1 : boolean; reset : boolean;
ASSIGN
  init(b0) := FALSE; init(b1) := FALSE;
  next(b0) := case reset : FALSE;
                   TRUE  : !b0;
              esac;
  next(b1) := case reset : FALSE;
                   TRUE  : ((!b0 & b1) | (b0 & !b1));
              esac;
DEFINE out := toint(b0) + 2 * toint(b1);
```



Requests Simulation

Excercise 7.1

Simulate the system with nuXmv and draw the FSM.

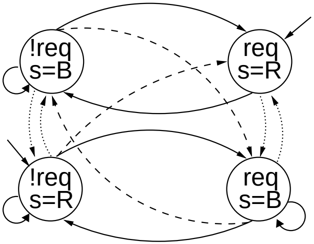
```
MODULE main
VAR
  request : boolean;
  state   : { ready, busy };
ASSIGN
  init(state) := ready;
  next(state) :=
    case
      state = ready & request : busy;
      TRUE                    : { ready, busy };
    esac;
```

Requests Simulation

Excercise 7.1

Simulate the system with nuXmv and draw the FSM.

```
MODULE main
VAR
  request : boolean;
  state   : { ready, busy };
ASSIGN
  init(state) := ready;
  next(state) :=
    case
      state = ready & request : busy;
      TRUE                    : { ready, busy };
    esac;
```



Constraint Style Modeling [1/4]

- Up to now, we have seen how to define a model in **assignment style**:

```
MODULE main
VAR request : boolean;  state : {ready,busy};
ASSIGN
  init(state) := ready;
  next(state) := case state = ready & request : busy;
                      TRUE                     : {ready,busy};
                 esac;
```

- Every program can be alternatively defined in a **constraint style**:

```
INIT
  state = ready
TRANS
  (state = ready & request) -> next(state) = busy
```

Constraint Style Modeling [2/4]

- A model can be specified by zero or more **constraints** on:
  - **initial states**:
    `INIT <simple_expression>`
  - **transitions**:
    `TRANS <next_expression>`
  - **invariants**:
    `INVAR <simple_expression>`
- Any propositional or SMT formula can be used as constraint;
- Constraints can be **mixed** with assignments;
- Not all constraints can be easily rewritten in terms of assignments!

```
TRANS
  next(b0) + 2 * next(b1) + 4 * next(b2) =
    (b0 + 2 * b1 + 4 * b2 + tick) mod 8
```

Constraint Style Modeling [3/4]

Assignment Style

- By construction, there is always **at least one initial state**
- By construction, all states have **at least one next state**
- **Non-determinism is apparent** (unassigned variables, set assignments. . .)

Constraint Style Modeling [4/4]

Constraint Style

- INIT constraints **can be inconsistent** $\Longrightarrow$ no initial state!
  - Any specification (also SPEC 0) is vacuously true.
- TRANS constraints **can be inconsistent** $\Longrightarrow$ deadlock state!

```
MODULE main
VAR b : boolean;
TRANS b -> FALSE;
```

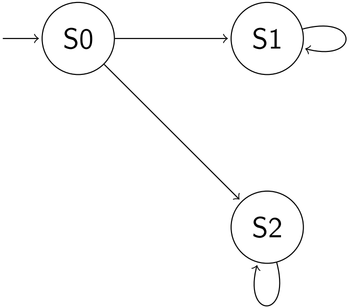
tip: use check_fsm to detect deadlock states

- **non-determinism is hidden**

```
TRANS (state = ready & request) -> next(state) = busy
```

Example: Constraint Style & Case

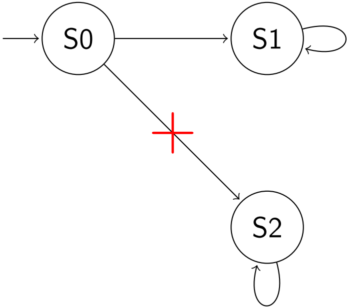
```
MODULE main
VAR
  state : {S0, S1, S2};
DEFINE
  go_s1 := state != S2;
  go_s2 := state != S1;
INIT
  state = S0;
TRANS
case
  go_s1 : next(state) = S1;
  go_s2 : next(state) = S2;
esac;
```



Q: does it correspond to the FSM?

Example: Constraint Style & Case

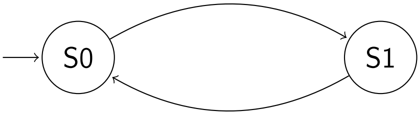
```
MODULE main
VAR
  state : {S0, S1, S2};
DEFINE
  go_s1 := state != S2;
  go_s2 := state != S1;
INIT
  state = S0;
TRANS
case
  go_s1 : next(state) = S1;
  go_s2 : next(state) = S2;
esac;
```



Q: does it correspond to the FSM? No: cases are evaluated in order!

Example: Constraint Style & Swap

```
MODULE main
VAR
   arr: array 0..1 of {1,2};
   i  : 0..1;
ASSIGN
  init(arr[0]) := 1;
  init(arr[1]) := 2;
  init(i) := 0;
  next(i) := 1-i;
TRANS
  next(arr[i]) = arr[1-i] &
  next(arr[1-i]) = arr[i];
```



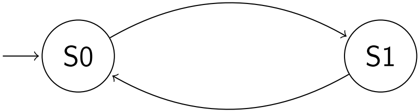
$arr[0] = 1$
$arr[1] = 2$
$i = 0$

$arr[0] = 2$
$arr[1] = 1$
$i = 1$

Q: does it correspond to the FSM?

Example: Constraint Style & Swap

```
MODULE main
VAR
   arr: array 0..1 of {1,2};
   i  : 0..1;
ASSIGN
  init(arr[0]) := 1;
  init(arr[1]) := 2;
  init(i) := 0;
  next(i) := 1-i;
TRANS
  next(arr[i]) = arr[1-i] &
  next(arr[1-i]) = arr[i];
```



$arr[0] = 1$
$arr[1] = 2$
$i = 0$

$arr[0] = 1$
$arr[1] = 2$
$i = 1$

Q: does it correspond to the FSM? No: everything inside the `next()` operator is evaluated within the next state, indexes included!

Outline

1. Introduction
2. nuXmv interactive shell
3. nuXmv Modeling
4. Modules
    - Modules Definition
    - Modules Composition
5. Homework

Modules [1/3]

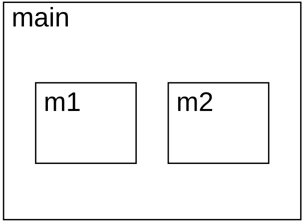
A nuXmv program is composed of a `main` module plus 0 or *more* `other` modules:

- a module can be **instantiated** as a VAR in `other` modules
- variables **local** to a module instance are accessed via **dot notation** (e.g., `m1.out`).

Example

```
MODULE counter
VAR out: 0..9;
ASSIGN next(out) := (out + 1) mod 10;

MODULE main
VAR m1  : counter; m2 : counter;
    sum : 0..18;
ASSIGN sum := m1.out + m2.out;
```



Modules [2/3]

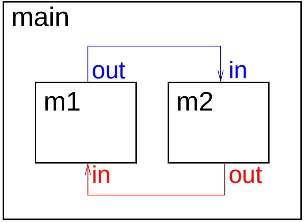
A module declaration can be *parametric*:

- a parameter is passed *by reference*;
- any expression can be used as parameter;

Example

```
MODULE counter(in)
VAR out: 0..9;
...

MODULE main
VAR m1 : counter(m2.out);
    m2 : counter(m1.out);
...
```



Modules [3/3]

- Modules can be **composed**
- Modules *without parameters and assignments* can be seen as simple **records**

Example

```
MODULE point
VAR
x: -10..10;
y: -10..10;

MODULE circle
VAR
center: point;
radius: 0..10;
```

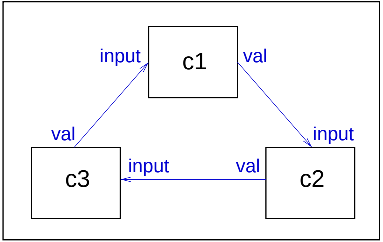
```
MODULE main
VAR c: circle;
ASSIGN
init(c.center.x) := 0;
init(c.center.y) := 0;
init(c.radius)   := 5;
```

Synchronous composition [1/2]

The composition of modules is **synchronous**: *all modules move at each step*.

```
MODULE cell(input)
VAR
  val : {red, green, blue};
ASSIGN
  next(val) := input;

MODULE main
VAR
  c1 : cell(c3.val);
  c2 : cell(c1.val);
  c3 : cell(c2.val);
```



Synchronous composition [2/2]

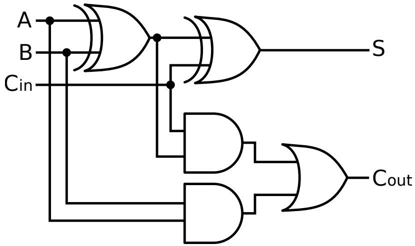
A possible execution:

step	`c1.val`	`c2.val`	`c3.val`
0	red	green	blue
1	blue	red	green
2	green	blue	red
3	red	green	blue
4	...	...	...
5	red	green	blue

Exercise: Adder [1/3]

Exercise 7.2: Binary Adder

Implement a binary adder that takes into account two 4-bits numbers and returns their sum using an output variable. Implement both a bit-adder and the general adder as two separate modules.



Exercise: Adder [2/3]

```
MODULE bit-adder(in1, in2, cin)
VAR sum : boolean; cout : boolean;
ASSIGN next(sum) := (in1 xor in2) xor cin;
       next(cout) := (in1 & in2) | ((in1 xor in2) & cin);

MODULE adder(in1, in2)
VAR bit[0] : bit-adder(in1[0], in2[0], bool(0));
    bit[1] : bit-adder(in1[1], in2[1], bit[0].cout);
    bit[2] : bit-adder(in1[2], in2[2], bit[1].cout);
    bit[3] : bit-adder(in1[3], in2[3], bit[2].cout);
DEFINE sum[0] := bit[0].sum;
       sum[1] := bit[1].sum;
       sum[2] := bit[2].sum;
       sum[3] := bit[3].sum;
       overflow := bit[3].cout;
```

Exercise: Adder [3/3]

```
MODULE main
VAR in1 : array 0..3 of boolean;
    in2 : array 0..3 of boolean;
    a   : adder(in1, in2);
ASSIGN next(in1[0]) := in1[0]; next(in1[1]) := in1[1];
       next(in1[2]) := in1[2]; next(in1[3]) := in1[3];
       next(in2[0]) := in2[0]; next(in2[1]) := in2[1];
       next(in2[2]) := in2[2]; next(in2[3]) := in2[3];
DEFINE op1 := toint(in1[0]) + 2*toint(in1[1]) + 4*toint(in1[2]) +
              8*toint(in1[3]);
       op2 := toint(in2[0]) + 2*toint(in2[1]) + 4*toint(in2[2]) +
              8*toint(in2[3]);
       sum := toint(a.sum[0]) + 2*toint(a.sum[1]) + 4*toint(a.sum[2]) +
              8*toint(a.sum[3]) + 16*toint(a.overflow);
```

Outline

Homework

Homework 7.1: playing with Adder

- Simulate a random execution of the "Adder" system;
- After how many steps the adder stores the computed final `sum` value? Is this number constant? Can you explain its behaviour?
- What happens if we initialize both `sum` and `cout` inside the bit-adder to `FALSE`? Can you tell the main difference with respect to the original algorithm?
- Can you modify the model in a simple way so that the sum is obtained after a single iteration? (PS: simple means you must modify/add less than 5 lines of code)
- Add a `reset` control which changes the values of the operands and restarts the computation of the sum

Homework

Homework 7.2: Random Calculator

Use nuXmv to create a "random" calculator:

- it creates two random arrays of 3 integers numbers in the range [1,10]
- it randomly chooses an operator to apply to each pair of items in the arrays (sum, subtraction and multiplication), storing the result in an output array of 3 elements called `res`
- the results must be defined in 3 steps:
  - in the first iteration you'll store the random operation between elements with index 0
  - in the second iteration the random operation between elements with index 1
  - and the same for the last index

Use an additional variable, `index`, to take into account this evolution.