UNIVERSITÀ
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Automated Reasoning and Formal Verification

Laboratory 6

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These slides are derived from those by Giuseppe Spallitta for FM lab 2021-2024.

Outline

1. Advanced OMT
   - Multi-Objective Optimization
   - OMT properties: MinMax/MaxMin

Pareto vs Lexicographic Multi-Optimization

Exercise 6.1: Multi-objective optimization

A small business promotes itself using two methods: traditional media ads and personal appearances.

A **traditional media ad campaign** costs \$2000, generating 2 new customers and 1 positive rating per month. Each ad campaign takes 1 hour.

A **personal appearance** costs \$500, generating 2 new customers and 5 positive ratings. Each personal appearance takes 2 hours.

The company wants at least 16 new customers and 28 positive ratings per month.

Try to **minimize both costs and time**.

Multi-objective optimization: problem

Let's identify variables, constraints, and cost functions (our goals to minimize):

- **Variables**: number of ads and number of personal appearance.
- **Constraints**: number of minimum customers and positive ratings (these are mandatory to be satisfied).
- **Goals**: minimize time and money.

We must encode time and money as functions depending on the variables!

Multi-objective optimization

OptiMathSAT supports multi-objective optimization with different *priorities*:

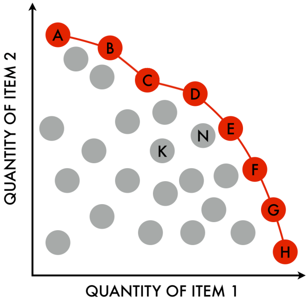
- Boxed : optimizes **all the objective independently**. A model for each objective is returned (default behavior).
- Lexicographic : optimizes **the first objective, then the second, and so on**. A single model is returned.
- Pareto : Computes the **Pareto front**: a set of non-dominated models. Each `check-sat` finds one, until all are found (possibly infinite).

You can change the type of optimization from the option

```
(set-option :opt.priority=box|lex|par)
```

Pareto front

- The Pareto front is a set of solutions that are not dominated by any other solution.
- A solution A is dominated by another solution B if B is better in at least one objective and not worse in all others.
- The Pareto front represents the trade-offs between different objectives.



Minimize Worst-Case Delivery Cost

Exercise 6.2: Delivery

A company must deliver goods from a central depot to three warehouses: A, B, and C. There are two transport scenarios: **Normal Traffic** and **Heavy Traffic**.

Delivery Costs (per unit):

Warehouse	Normal Traffic	Heavy Traffic
A	\$10	\$21
B	\$15	\$18
C	\$20	\$15

Let x_A, x_B, x_C be the number of units sent to warehouses A, B, and C.
The company requires that: $100 \leq x_A + x_B + x_C \leq 150$

Use OptiMathSAT to find the number of units to minimize the worst-case delivery cost.

Delivery: Hints

- The problem is to minimize the cost of the worst-case scenario.
- We should *minimize* the *maximum* cost of delivery.
- OptiMathSAT has a built-in function for this: `(minmax obj1 obj2 ...)`.

Delivery: Encoding

Variables:
- Define the variables x_A, x_B, x_C as `Int` constants.
- Define two derived `Int` constants `cost-normal` and `cost-heavy`

Constraints:
- Total number of units must be between 100 and 150
- Non-negativity (hidden): $x_A, x_B, x_C \geq 0$

Objectives:
- Minimize the maximum of the two scenario costs:

```
(minmax cost-normal cost-heavy)
```