UNIVERSITÀ
DI TRENTO
Department of
Information Engineering and Computer Science

Automated Reasoning and **Formal Verification**

Laboratory 9

Gabriele Masina
gabriele.masina@unitn.it
https://github.com/masinag/arfv2025

Università di Trento

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Outline

Planning Problem

Planning Problem

Given $\langle I, G, A \rangle$, where

I : (representation of) initial states

G : (representation of) goal states

A : (representation of) actions leading from one state to another

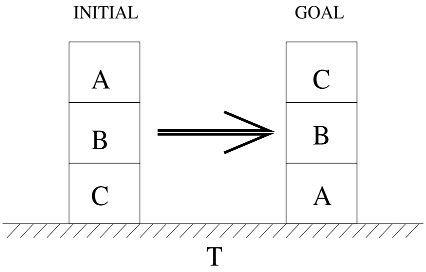
find a **plan** (sequence of actions) $a_1, \ldots, a_n$ leading from an initial state to a goal state.

Idea: from Planning to Model Checking

Encode a planning problem as a model checking problem:

1. Impose I as initial state(s)
2. Encode A as transition relation system
3. Verify the LTL property `(! F goal)`. The plan is a counter-example for it.

Example: Blocks [1/9]



$$
\begin{aligned}
&\mathit{Init}: && Above(A,B) \wedge Above(B,C) \wedge Above(C,T) \wedge Clear(A) \\
&\mathit{Goal}: && Above(C,B) \wedge Above(B,A) \wedge Above(A,T) \\
&Move(a,b,c) && \\
&\quad \mathit{Precond}: && Block(a) \wedge Clear(a) \wedge Above(a,b) \wedge (Clear(c) \vee Table(c)) \wedge \\
& && a \neq b \wedge a \neq c \wedge b \neq c \\
&\quad \mathit{Effect}: && Clear(b) \wedge \neg Above(a,b) \wedge Above(a,c) \wedge \neg Clear(c)
\end{aligned}
$$

Example: Blocks [2/9]

```
MODULE block(id, ab, bl)
VAR    above : {none, a, b, c}; -- the block above this one
       below : {none, a, b, c}; -- the block below this one
DEFINE clear := (above = none);
INIT   above = ab & below = bl
INVAR  below != id & above != id -- a block cannot be above/below itself

MODULE main
VAR    move : {move_a, move_b, move_c}; -- at each step one block moves
       block_a : block(a, none, b);
       block_b : block(b, a, c);
       block_c : block(c, b, none);
...
```

Example: Blocks [3/9]

- A block cannot move if it has some other block above itself

```
INVAR
  (!block_a.clear -> move != move_a) &
  (!block_b.clear -> move != move_b) &
  (!block_c.clear -> move != move_c);
```

Example: Blocks [3/9]

- A block cannot move if it has some other block above itself

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INVAR
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Q: Why `INVAR` and not `TRANS`?

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Q: Why `INVAR` and not `TRANS`?

A: `INVAR p` is equivalent to `INIT p` and `TRANS next(p)`.
With `TRANS` only, we could have in initial state with `move_b`.

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Q: What's wrong with following formulation?

```
INVAR
  (block_a.clear -> move = move_a) &
  (block_b.clear -> move = move_b) &
  (block_c.clear -> move = move_c);
```

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```
INVAR
  (!block_a.clear -> move != move_a) &
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With `TRANS` only, we could have in initial state with `move_b`.

Q: What's wrong with following formulation?

```
INVAR
  (block_a.clear -> move = move_a) &
  (block_b.clear -> move = move_b) &
  (block_c.clear -> move = move_c);
```

A: Two `clear` blocks would cause an inconsistency (`move` can have **one** valid value).
Moreover, any `non-clear block` would still be able to `move`.

Example: Blocks [4/9]

- A moving `block` changes location and remains clear

```
TRANS
  (move = move_a -> next(block_a.clear) &
                    next(block_a.below) != block_a.below) &
  (move = move_b -> next(block_b.clear) &
                    next(block_b.below) != block_b.below) &
  (move = move_c -> next(block_c.clear) &
                    next(block_c.below) != block_c.below);
```

- A non-moving `block` does not change its location

```
TRANS
  (move != move_a -> next(block_a.below) = block_a.below) &
  (move != move_b -> next(block_b.below) = block_b.below) &
  (move != move_c -> next(block_c.below) = block_c.below);
```

Example: Blocks [5/9]

- A `block` remains connected to any *non-moving* `block`

```
TRANS
    (move != move_a & block_b.above = a -> next(block_b.above) = a) &
    (move != move_a & block_c.above = a -> next(block_c.above) = a) &
    (move != move_b & block_a.above = b -> next(block_a.above) = b) &
    (move != move_b & block_c.above = b -> next(block_c.above) = b) &
    (move != move_c & block_a.above = c -> next(block_a.above) = c) &
    (move != move_c & block_b.above = c -> next(block_b.above) = c);
```

Example: Blocks [5/9]

- A `block` remains connected to any *non-moving* `block`

```
TRANS
   (move != move_a & block_b.above = a -> next(block_b.above) = a) &
   (move != move_a & block_c.above = a -> next(block_c.above) = a) &
   (move != move_b & block_a.above = b -> next(block_a.above) = b) &
   (move != move_b & block_c.above = b -> next(block_c.above) = b) &
   (move != move_c & block_a.above = c -> next(block_a.above) = c) &
   (move != move_c & block_b.above = c -> next(block_b.above) = c);
```

Q: What about "`below`" block?

Example: Blocks [5/9]

- A `block` remains connected to any *non-moving* `block`

```
TRANS
   (move != move_a & block_b.above = a -> next(block_b.above) = a) &
   (move != move_a & block_c.above = a -> next(block_c.above) = a) &
   (move != move_b & block_a.above = b -> next(block_a.above) = b) &
   (move != move_b & block_c.above = b -> next(block_c.above) = b) &
   (move != move_c & block_a.above = c -> next(block_a.above) = c) &
   (move != move_c & block_b.above = c -> next(block_b.above) = c);
```

Q: What about "`below`" block?

A: Covered in previous slide!

Example: Blocks [6/9]

- Positioning of blocks: above and below relations must be symmetric.

```
INVAR
  (block_a.above = b <-> block_b.below = a) &
  (block_a.above = c <-> block_c.below = a) &
  (block_b.above = a <-> block_a.below = b) &
  (block_b.above = c <-> block_c.below = b) &
  (block_c.above = a <-> block_a.below = c) &
  (block_c.above = b <-> block_b.below = c);
```

Notice that the above handles also the case of `none` blocks!

Example: Blocks [7/9]

Remark

A **plan** is a sequence of transitions/actions leading from the initial state to an accepting/goal state.

Idea

- Assert property p: "goal state is not reachable"
- If a plan **exists**, nuXmv produces a counter-example for p
- The counterexample for p is a plan to reach the goal

Example: Blocks [8/9]

Examples

- Get a plan for reaching "goal state"

```
LTLSPEC
  ! F(block_a.below = none & block_a.above = b    &
      block_b.below = a    & block_b.above = c    &
      block_c.below = b    & block_c.above = none);
```

Example: Blocks [8/9]

Examples

- Get a plan for reaching "goal state"

```
LTLSPEC
  ! F(block_a.below = none & block_a.above = b    &
      block_b.below = a    & block_b.above = c    &
      block_c.below = b    & block_c.above = none);
```

- Get a plan for reaching a configuration in which all blocks are placed on the table

```
LTLSPEC
  ! F(block_a.below = none & block_b.below = none &
      block_c.below = none);
```

Example: Blocks [9/9]

Examples

- At any given time, at least one block is placed on the table

```
INVARSPEC
  block_a.below=none | block_b.below=none | block_c.below=none;
```

Examples

- At any given time, at least one block is placed on the table

```
INVARSPEC
  block_a.below=none | block_b.below=none | block_c.below=none;
```

- At any given time, at least one block has nothing above

```
INVARSPEC
  block_a.above=none | block_b.above=none | block_c.above=none;
```

Outline

1. Planning problem

2. Exercises
   - Tower of Hanoi
   - Ferryman
   - Tic-Tac-Toe

3. Homeworks

Tower of Hanoi [1/5]

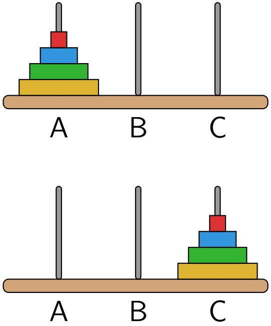
Exercise 9.1: Tower of Hanoi

Given 3 `poles` and N `disks` of different sizes:

Init A stack of disks with decreasing sizes is placed on pole A

Goal Move the stack to pole C

Rules
- Only one disk can be moved per step
- Only the upper disk can be moved
- A disk cannot be put above a smaller disk



Tower of Hanoi [2/5]

- Base system model

```
MODULE main
VAR d1   : {left,middle,right}; -- smallest
    d2   : {left,middle,right};
    d3   : {left,middle,right};
    d4   : {left,middle,right}; -- largest
    move : 1..4;                -- possible moves
```

Tower of Hanoi [2/5]

- Base system model

```
MODULE main
VAR d1   : {left,middle,right}; -- smallest
    d2   : {left,middle,right};
    d3   : {left,middle,right};
    d4   : {left,middle,right}; -- largest
    move : 1..4;                -- possible moves
```

- A disk cannot move if a smaller disk is above it (i.e. they share the same column)

```
DEFINE clear1 := TRUE;
       clear2 := d2 != d1;
       clear3 := d3 != d1 & d3 != d2;
       clear4 := d4 != d1 & d4 != d2 & d4 != d3;
```

Tower of Hanoi [3/5]

- Initial state

```
INIT d1 = left & d2 = left & d3 = left & d4 = left;
```

Tower of Hanoi [3/5]

- Initial state

```
INIT d1 = left & d2 = left & d3 = left & d4 = left;
```

- Moves descriptions

```
TRANS
  (move = 4 ->
    -- disks location changes
    next(d1) = d1 & next(d2) = d2 & next(d3) = d3 & next(d4) != d4 &
    -- d4 can not move on top of smaller disks
    next(d4) != d1 & next(d4) != d2 & next(d4) != d3) &
  (move = 3 -> ...)
  (move = 2 -> ...)
  (move = 1 -> ...);
```

Tower of Hanoi [4/5]

- A non-clear disk cannot move

```
INVAR
  (!clear1 -> move != 1) & (!clear2 -> move != 2) &
  (!clear3 -> move != 3) & (!clear4 -> move != 4);
```

Tower of Hanoi [4/5]

- A non-clear disk cannot move

```
INVAR
  (!clear1 -> move != 1) & (!clear2 -> move != 2) &
  (!clear3 -> move != 3) & (!clear4 -> move != 4);
```

- If all columns are being used, don't move the largest disk (or we would reach a deadlock).

```
INVAR
  ((clear1 & clear2 & clear3) -> move != 3) &
  ((clear1 & clear2 & clear4) -> move != 4) &
  ((clear1 & clear2 & clear3) -> move != 4) &
  ((clear2 & clear3 & clear4) -> move != 4);
```

Tower of Hanoi [5/5]

- Get a plan for a reaching "goal state":

```
LTLSPEC
  ! F (d1 = right & d2 = right & d3 = right & d4 = right);
INVARSPEC
  !   (d1 = right & d2 = right & d3 = right & d4 = right);
```

Ferryman [1/4]

Exercise 9.2: Ferryman

A ferryman has to bring a `sheep`, a `cabbage`, and a `wolf` safely across a river:

Init all the items are on the right side

Goal all the items are on the left side

Actions
- The ferryman can have at most one passenger on his boat
- The `cabbage` and the `sheep` cannot be left unattended on the same side of the river
- The `sheep` and the `wolf` cannot be left unattended on the same side of the river

Q: Can the ferryman transport all the items to the other side safely?

Ferryman [2/4]

- Base system model

```
MODULE main
VAR
  cabbage : {right,left};  sheep : {right,left};
  wolf    : {right,left};  man   : {right,left};
  move    : {c, s, w, e}; -- possible moves
DEFINE
  carry_cabbage := (move = c); carry_sheep := (move = s);
  carry_wolf    := (move = w); no_carry    := (move = e);
```

- Initial state

```
INIT cabbage = right & sheep = right & wolf = right & man = right;
```

Ferryman [3/4]

- The ferryman can move to the other side of the river with one item at a time

```
TRANS
  next(man) != man &
  (carry_cabbage ->
   (next(cabbage)!=cabbage & next(sheep) =sheep & next(wolf) =wolf)) &
  (carry_sheep ->
   (next(cabbage) =cabbage & next(sheep)!=sheep & next(wolf) =wolf)) &
  (carry_wolf ->
   (next(cabbage) =cabbage & next(sheep) =sheep & next(wolf)!=wolf)) &
  (no_carry ->
   (next(cabbage) =cabbage & next(sheep) =sheep & next(wolf) =wolf));
```

Ferryman [4/4]

- If the man is not in the same side of an item, we cannot choose it for the next movement (otherwise deadlock).

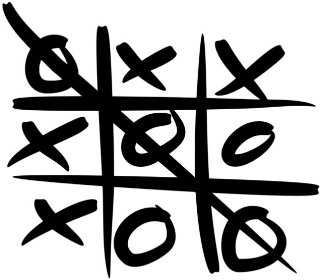
```
TRANS
  (next(man) != next(cabbage) -> !next(carry_cabbage)) &
  (next(man) != next(sheep)   -> !next(carry_sheep)) &
  (next(man) != next(wolf)    -> !next(carry_wolf));
```

- Get a plan for reaching a "goal state"

```
DEFINE
  safe_state := (sheep = wolf | sheep = cabbage) -> sheep = man;
  goal := cabbage = left & sheep = left & wolf = left;
LTLSPEC
  ! (safe_state U goal);
```

Exercise 9.3: Tic-Tac-Toe

Tic-tac-toe is a turn-based game for two adversarial players ($\times$ and $\circ$) marking the squares of a 3×3 grid. The player who succeeds in placing three respective marks in a horizontal, vertical or diagonal row wins the game.



Scenario where $\circ$ wins

Tic-Tac-Toe [2/5]

- Base system model: we model the grid as an array of size 9:

```
  1 | 2 | 3
----|---|----
  4 | 5 | 6
----|---|----
  7 | 8 | 9
    |   |
```

```
MODULE main
VAR
  B : array 1..9 of {0,1,2};
  player : 1..2;
  move : 0..9;
```

Tic-Tac-Toe [2/5]

- Base system model: we model the grid as an array of size 9:

```
 1 | 2 | 3
___|___|___
 4 | 5 | 6
___|___|___
 7 | 8 | 9
   |   |
```

```
MODULE main
VAR
  B : array 1..9 of {0,1,2};
  player : 1..2;
  move : 0..9;
```

- Initial state

```
INIT
  B[1] = 0 & B[2] = 0 & B[3] = 0 & B[4] = 0 & B[5] = 0 &
  B[6] = 0 & B[7] = 0 & B[8] = 0 & B[9] = 0 & move = 0;
```

Tic-Tac-Toe [3/5]

- Turns Modeling

```
ASSIGN
  init(player) := 1;
  next(player) := player = 1 ? 2 : 1;
```

Tic-Tac-Toe [3/5]

- Turns Modeling

```
ASSIGN
  init(player) := 1;
  next(player) := player = 1 ? 2 : 1;
```

- Moves Modeling

```
TRANS
  B[1] != 0 -> next(move) != 1;
TRANS
  next(move) = 1 -> next(B[1]) = player &
    next(B[2])=B[2] & next(B[3])=B[3] & next(B[4])=B[4] &
    next(B[5])=B[5] & next(B[6])=B[6] & next(B[7])=B[7] &
    next(B[8])=B[8] & next(B[9])=B[9];
  ...
```

Tic-Tac-Toe [4/5]

- "End" state

```
DEFINE
  win1 := (B[1]=1 & B[2]=1 & B[3]=1) | (B[4]=1 & B[5]=1 & B[6]=1) |
          (B[7]=1 & B[8]=1 & B[9]=1) | (B[1]=1 & B[4]=1 & B[7]=1) |
          (B[2]=1 & B[5]=1 & B[8]=1) | (B[3]=1 & B[6]=1 & B[9]=1) |
          (B[1]=1 & B[5]=1 & B[9]=1) | (B[3]=1 & B[5]=1 & B[7]=1);
  win2 := (B[1]=2 & B[2]=2 & B[3]=2) | (B[4]=2 & B[5]=2 & B[6]=2) |
          (B[7]=2 & B[8]=2 & B[9]=2) | (B[1]=2 & B[4]=2 & B[7]=2) |
          (B[2]=2 & B[5]=2 & B[8]=2) | (B[3]=2 & B[6]=2 & B[9]=2) |
          (B[1]=2 & B[5]=2 & B[9]=2) | (B[3]=2 & B[5]=2 & B[7]=2);
  draw := !win1 & !win2 &
    !(B[1]=0 | B[2]=0 | B[3]=0 | B[4]=0 | B[5]=0 |
      B[6]=0 | B[7]=0 | B[8]=0 | B[9]=0);
INVAR
  (win1 | win2 | draw) <-> next(move)=0;
```

Tic-Tac-Toe [5/5]

- We can easily check if there is a way to reach every end state using the typical formulation:

```
LTLSPEC
    ! (F draw);
LTLSPEC
    ! (F win1);
LTLSPEC
    ! (F win2);
```

For each property, a trace satisfying the property is returned as a counterexample.

Outline

Homeoworks

Homework 9.1: Tower of Hanoi

Extend the Tower of Hanoi to handle five disks, and check that the goal state is reachable.

Homework 9.2: Ferryman

Another ferryman has to bring a fox, a chicken, a caterpillar and a crop of lettuce safely across a river.

Init All the items are on the right side

Goal All the items are on the left side

Rules

- The ferryman can cross the river with at most **two** passengers on his boat
- The fox eats the chicken if left unattended on the same side of the river
- The chicken eats the caterpillar if left unattended on the same side
- The caterpillar eats the lettuce if left unattended on the same side

Can the ferryman bring every item safely on the other side?