

# Laboratory 5: Rigid and Non-Rigid Structure from Motion

Jesus Miguel Adrian Matos

July 3, 2024

## Abstract

The aim of this laboratory concerns on rigid and non-rigid structure from motion, by completing a Matlab code provided previously.

In particular, the report is divided as follows:

- Section 1: we explain the inputs in order to define the motion structure.
- Section 2: we compute the structure if the results, that is the shape of the matrix
- Section 3: we explain the code to compute the rigid structure from motion by factorization.
- Section 4: we explain the code to compute the non-Rigid structure from motion by non-linear optimization and assuming a low- rank shape model.
- Section 5: we explain the code to compute the non-Rigid structure from motion by factorization and assuming a low-rank trajectory model.

## 1 Inputs:

Given an image in the frame  $k$ , calling  $p$  as the number of track point and  $f$  as the number of frames, then the image is described by a  $2 \times p$  matrix  $I_k$ , defined

$$I_k = \begin{pmatrix} x_1 & x_2 & \cdots & x_p \\ y_1 & y_2 & \cdots & y_p \end{pmatrix} \quad (1)$$

where  $(x_k, y_k)$  are the coordinated of the bidimensional space. As a consequence, the input matrix is expressed

$$Input = \begin{pmatrix} I_1 \\ I_2 \\ \vdots \\ I_f \end{pmatrix} \quad (2)$$

## 2 Outputs:

We want to obtain a representation in the 3D plane of the points tracked in the 2D images. The resulting matrix is

$$Output = \begin{pmatrix} x_1 & x_2 & \cdots & x_p \\ y_1 & y_2 & \cdots & y_p \\ z_1 & z_2 & \cdots & z_p \end{pmatrix} \quad (3)$$

Where  $(x, y, z)$  represents the 3D coordinates in the  $\mathbb{R}^3$  space.

### 3 First Part (Rigid structure from motion by factorization)

In this task, factorization for a rigid body must be implemented, as shown in the following slide figure 1. So, we should complete the orthogonal factorization function by means of the file **rigidfactorization\_ortho.m**. The completed code runs with a provided test data composed by a synthetic sequence with an Ogre face, and the 3D reconstruction error is reported.

## More on factorization Orthographic camera

Because  $\mathbf{R}$  is a  $2 \times 3$  matrix and  $\mathbf{X}$  is a  $3 \times P$  matrix, the rank of  $\mathbf{W}$  is 3. If we apply SVD to  $\mathbf{W}$ , we will have only **3 non-zero singular values**

However, measurements are normally noisy, and in practice the rank will not be 3. We have to impose it

Applying SVD factorization, we have:

$$\mathbf{W} = \mathbf{U}\mathbf{A}\mathbf{V}^T = [\mathbf{U}\sqrt{\mathbf{A}}][\sqrt{\mathbf{A}}\mathbf{V}^T] = [\mathbf{U}\sqrt{\mathbf{A}}\mathbf{Q}][\mathbf{Q}^{-1}\sqrt{\mathbf{A}}\mathbf{V}^T]$$

i.e.,  $\mathbf{R} = \mathbf{U}\sqrt{\mathbf{A}}\mathbf{Q}$  and  $\mathbf{X} = \mathbf{Q}^{-1}\sqrt{\mathbf{A}}\mathbf{V}^T$  (the two factors we look for)

Many solutions can be achieved by modifying  $\mathbf{Q}$ . Of course, for all invertible  $3 \times 3$   $\mathbf{Q}$  matrices

Figure 1: Factoring slide

#### 3.1 Code:

Now, we introduce the code 1. We will explain each line number:

For the line 1 and 2,  $Zc$  is defined as the difference between the input matrix and the average of all the points in each frame, and not as the input matrix. This way, we no longer have to consider translation in the computations.

Therefore, the average matrix would be the following, with dimensions  $2f \times p$

$$Mean = \begin{pmatrix} Mean_1 & Mean_1 & \cdots & Mean_1 \\ Mean_2 & Mean_2 & \cdots & Mean_2 \\ \vdots & \vdots & \ddots & \vdots \\ Mean_f & Mean_f & \cdots & Mean_f \end{pmatrix} \quad (4)$$

where

$$Media_k = \begin{pmatrix} \frac{x_1 + x_2 + \cdots + x_p}{p} \\ \frac{y_1 + y_2 + \cdots + y_p}{p} \end{pmatrix} \quad (5)$$

for any  $k$  between 1 and  $f$ . Therefore from 4 we have:

$$Zc = Input - Mean \quad (6)$$

In the line "3, we calculate the factorization called Singular Value Decomposition (SVD).

In the line 4 and 5, we approximate  $R = U(:, 1 : 3)D(1 : 3, 1 : 3)^{1/2}$  and  $S = D(1 : 3, 1 : 3)^{1/2}V(:, 1 : 3)'$  (where  $R$  represents rotation and  $S$  represents shape). Knowing that the rank of  $RS$  is 3, therefore the truth matrix  $D$  should have a dimension of  $3 \times 3$ , then we do the following:

- $D$ : We take the first 3 rows and the first 3 columns.
- $U$ : We take only the first 3 columns.
- $V$ : We take only the first 3 columns.

therefore in that way we can calculate  $S$  as  $U_{2I \times 3} \sqrt{D}_{3 \times 3}$  and  $R$  as  $\sqrt{D}_{3 \times 3} V'_{3 \times 3I}$

In the "while loop" of the line 10, we try to reduce  $\|Zc - RU\|$  to the threshold, which in the code is called epsilon. The idea to optimize the rotation matrix is use the rotation condition  $R_i R'_i = I$ , where  $I$  corresponds to the identity matrix. This rotation condition is used to calculate  $T$  in the code and thus be able to calculate the new matrices  $S$  and  $R$ .

```

1 T = mean(Zc,2);
2 Zc = Zc - repmat(T,1,size(Zc,2));
3 [U, D, V] = svd(Zc);
4 R = U(:, 1:3)*(D(1:3, 1:3) ^ 1/2);
5 S = (D(1:3, 1:3) ^ 1/2)*V(:, 1:3)';
6 % Metric Upgrade Step
7 F=size(Zc,1);
8 k=0;
9 thenorm=epsilon+1;
10 while thenorm>epsilon && k<n_iter
11     Rp=R;
12     M=[];
13     for f=1:2:F
14         Rf=R(f:f+1,:);
15         [U2,useless,V2]=svd(Rf,'econ');
16         T=U2*V2';
17         M=[M;T(1:2,:)];
18     end
19     S=pinv(M)*Zc;
20     R=Zc*pinv(S);
21     k=k+1;
22     thenorm=norm(R-Rp,'fro')/numel(M);
23 end

```

Listing 1: Factorization code for orthographic camera

Finally, ones that the optimization finish, we obtain a shape matrix  $S$  to compare as the truth matrix shape  $X$ .

### 3.2 Examples:

Here we will show how to reconstruct the face of an Ogre with the given frames and using the following input parameters:

- threholder:  $10^{-7}$
- maximum number of iterations: 200

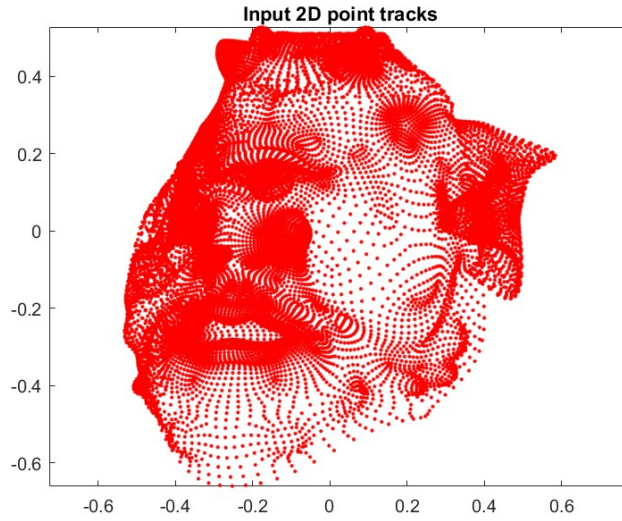


Figure 2: The original face of the Ogre.

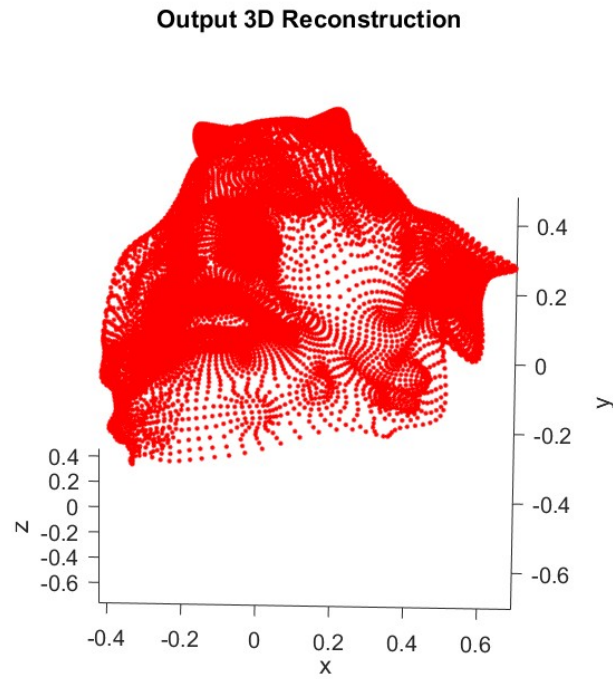


Figure 3: The Ogre's face reconstructed by the algorithm.

As can be seen in Figure 4, the reconstruction of the Ogre's face is quite good. This is demonstrated by the fact that the error between the original Ogre's face and its face reconstructed by the algorithm is 0,0026993% it means quite low, as depicted Figure 5.

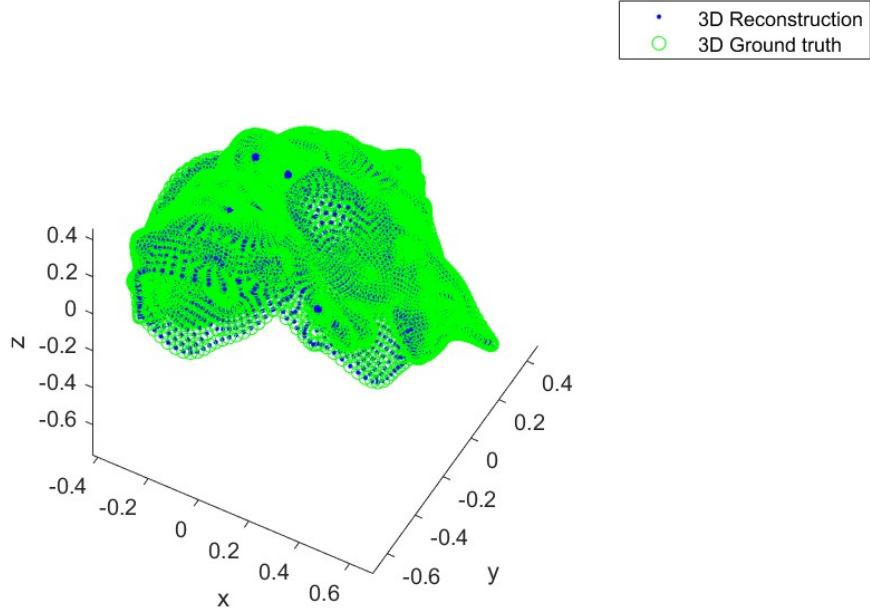


Figure 4: Comparison between the 3D representation of the Ogre's original face and its face reconstructed by the algorithm.

```
Running...
3D error: 0.0026993 %
;>>
```

Figure 5: Error between the ogre's face and the reconstructed face.

## 4 Second Part (Non-Rigid structure from motion by non-linear optimization and assuming a low-rank shape model)

In this task we should perform a nonlinear optimization for a non-rigid body, assuming a low rank. This code uses Levenberg-Marquardt algorithm to optimize the non-linear equation that it is shown to next:

$$\arg \min_{R^i, B_k, l_i^k, t^i} \sum_{i=1}^i \|\widehat{W}^i - R^i \sum_{k=1}^k l_i^k B_k - t^i\|^2 + \gamma \sum_{i=1}^{i-1} \|L^i - L^{i+1}\|^2 + \phi \sum_{i=1}^{i-1} \|R^i - R^{i+1}\|^2 \quad (7)$$

Where:  $B_k$  is a row of bases matrix.  $R^i$  is a rotation for a point in matrix column  $i$  of  $X$ , which is the 3D shape vector.  $l_i$  are the constants such that  $\sum_{k=1}^k l_i^k B_k = X$ .  $t^i$  is the translation.  $L^i$  is the vector of all constants in frame  $i$ . The norm used is the Frobenius norm.

In order to calculate the Levenberg-Marquardt optimization, we need to calculate the Jacobian form of the equation 7. We do this as shown in the following slide Figure 6

## Exercise

Let us assume a monocular video of 3 images, where 6 points are observed. Considering the map is non-rigid and the visibility is full, define the corresponding Jacobian matrix. A low-rank shape model of rank 2 can be considered

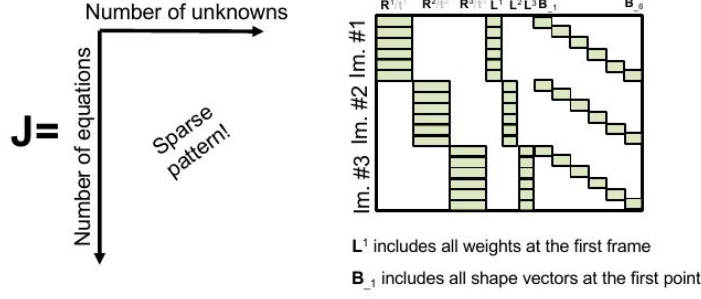


Figure 6: Jacobian form

### 4.1 Code:

Now we will show how this Jacobian form is calculated in the code.

So first we will define the dimensions of Jacobian matrix. For this code we are using quaternions for the rotations, therefore  $R^i$  is  $2 \times 4$ .

Then the dimension of Jacobian matrix is defined in the following way

- number of rows of  $Dim(J^{rows}) = 2 \sum_{i=1}^f (number\ of\ points\ visible\ in\ frame\ i)$
- number of columns of  $Dim(J_{columns}) = 4f + 2f + kf + 3kp$

Where  $f$  is the number of frames,  $p$  is the total number of points tracked and  $k$  is the total number of bases.

#### Explaining row sizes:

**Green:** Represents the visible points per frame. When we multiply the sum of the visible points by two it is because the rotations are in quaternions. The quaternions are  $2 \times 4$  and then they are two vectors per each point and that is why it is multiplied by two.

#### Explaining column sizes:

**Red:** Represents the value of the columns of the rotation matrix, which is  $2 \times 4$  quaternions. We have a rotation matrix  $R^i$  for each frame  $i$ . Therefore, the dimension is  $4f$ , where  $f$  is the number of frames.

**Blue:** Represents the translation matrix  $t^i$ , whose dimension is  $1 \times 2$ , for each frame  $i$ . Therefore, the dimension is  $2f$ , where  $f$  is the number of frames.

**Orange:** Represents the matrix  $L_i$ , whose dimension is  $1 \times k$  for each frame  $i$ . Therefore, the dimension is  $kf$ , where  $f$  is the number of frames.

**Yellow:** Represents the matrix  $B_b$ , whose dimension is  $3 \times p$  for each base  $b$ . Therefore, the dimension is  $3pk$ , where  $k$  is the number of bases.

Well, then we need to define the shape of our Jacobian matrix by putting 1 where there are values and leaving 0 in the spaces that do not exist, as shown in the following code.

```

1 for i=1:n_frames
2     for j=1:nnz(vij(i,:))
3         J(2*j-1+computed_points:2*j+computed_points,6*j-5:6*j)=ones
4             (2,6);
5         J(2*j-1+computed_points:2*j+computed_points,K*j-K+1+6*n_frames
6             :K*j+6*n_frames)=ones(2,K);
7         J(2*j-1+computed_points:2*j+computed_points,3*K*j-3*K+1+(6+K)*
8             n_frames:3*K*j+(6+K)*n_frames)=ones(2,3*K);
9     end
10    computed_points=computed_points+2*nnz(vij(i,:));
11 end

```

In the first for loop each frame is iterated, in the second for loop only the points visible in that frame are iterated.

$J(2*j-1+computed\_points:2*j+computed\_points,6*j-5:6*j)=ones(2,6);$

Here, the Jacobian is calculated for the rotation part  $R$ , in quaternions, and the translation part  $t$ , having this

$$\begin{pmatrix} R_{2 \times 4} & | & t_{2 \times 2} \end{pmatrix}_{2 \times 6} \quad (8)$$

we can add a matrix of ones of  $2 \times 6$

Now, the Jacobian is calculated for the constants part  $L^i$ , where  $L^i = [l_1^i, l_2^i, \dots, l_k^i]$  such that  $i$  is a frame.

$J(2*j-1+computed\_points:2*j+computed\_points,K*j-K+1+6*n\_frames:K*j+6*n\_frames)=ones(2,K);$

and for that, in the code is added a matrix of ones from  $2 \times k$ .

Then, the Jacobian is calculated for the bases part  $B_i$ , where

$$B_i = \begin{pmatrix} [b_1^i]_{3 \times 1} & [b_2^i]_{3 \times 1} & \cdots & [b_k^i]_{3 \times 1} \end{pmatrix}_{3 \times k} \quad (9)$$

$J(2*j-1+computed\_points:2*j+computed\_points,3*K*j-3*K+1+(6+K)*n\_frames:3*K*j+(6+K)*n\_frames)=ones(2,3*K);$

and for that, in the code is added a matrix of ones from  $2 \times 3k$ .

So, we calculate the Jacobian part of the priors.

$$\arg \min_{R^i, B_k, l_k^i, t^i} \gamma \sum_{i=1}^{i-1} \|L^i - L^{i+1}\|^2 + \phi \sum_{i=1}^{i-1} \|R^i - R^{i+1}\|^2 \quad (10)$$

from the equation we have

- **The prior with respect to the rotation  $R^i$ :** This means that the rotation between two consecutive frames  $\sum_{i=1}^{i-1} \|L^i - L^{i+1}\|^2$  do not have abrupt changes, it is a smoothing of the rotations.

- **The prior with respect to the constants  $L^i$ :** This means that the constants, which multiply the bases, such that  $X^i = \sum_{j=1}^k l_j^i B_j$  form the shape matrix. Therefore, if the constants for two consecutive frames  $\sum_{i=1}^{i-1} \|L^i - L^{i+1}\|^2$  do not have abrupt changes, then the shapes do not have abrupt changes, and for this reason, it is a smoothing of the shapes.

We use the expressions in equation 10 to calculate the dimension of the Jacobian for the priors.

So from the equation 10 and the orange color of the Explaining column sizes for the Jacobian matrix, we deduce that the Jacobian dimension for the prior  $L$  would be

$$Dim(J_{rows}) = f - 1Dim(J_{columns}) = kf \quad (11)$$

and the blue and red color of the Explaining column sizes for the Jacobian matrix, we can deduce the Jacobian dimension for the prior  $R$  as

$$Dim(J_{rows}) = f - 1Dim(J_{columns}) = 6f \quad (12)$$

Now we will show how we fill the indices that have a value in the Jacobian Matrix of the  $L$  prior with 1 and otherwise hold with zero.

```

if (priors.coeff_prior == 1)
    % prior terms on L
    L=spalloc(n_frames-1,(K+6)*n_frames + K*3*n_points,2*K*(n_frames-1));
    for i=1:n_frames-1
        L(i,K*i-K+1+6*n_frames:K*i+K+6*n_frames)=ones(1,2*K);
    end
    if K==1
        J(2*nnz(vij)+1:size(J,1), :) = L;
    else
        J(2*nnz(vij)+1:2*nnz(vij)+n_frames-1, :) = L;
    end
end
end

```

We do the following, we take the  $L_i$  and  $L_{i+1}$  and join them as shown in the equation 13

$$([L_i]_{1 \times k} \quad | \quad [L_{i+1}]_{1 \times k})_{1 \times 2k} \quad (13)$$

obtaining a merged matrix with dimensions  $1 \times 2K$ , whose elements are all ones.

For the Jacobian Matrix of the  $R$  prior is the similar way, in this case we joint  $R^i$  and  $R^{i+1}$  like show in the equation 14

$$([R^i]_{1 \times 6} \quad | \quad [R_{i+1}]_{1 \times 6})_{1 \times 12} \quad (14)$$

obtaining a merged matrix with dimensions  $1 \times 12$ , whose elements are all ones.

Joining the Jacobian and the two Jacobians of the priors we have the following graph figure 7.



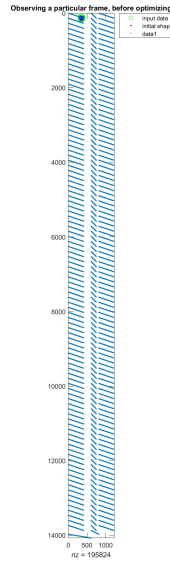


Figure 7: Jacobian result

## 4.2 Examples:

Here we will show how to reconstruct the human face with the given frames and using the following input parameters:

- low-rank  $K$  (it is the number of bases): 2
- camera\_prior (it means that the  $R$  prior is active): 1
- coeff\_prior (it means that the  $L$  prior is active): 1
- number of optimizer iterations: 50

In Figure 8 we can see the comparison of the input points of a frame in green against the estimated points of that frame in blue color.

In Figure 9 we can see the estimated 3D points for a frame.

In Figure 10 we see the calculation of each of the 50 optimizations, this method has a high cost in performance, but a good result is achieved as shown by the low error at the end.

For this model, if we increase the rank of the shape basis  $k$ , the approximation can improve, but also the performance cost increases. On the other hand, the factorization method, assuming a low-rank trajectory model, has a lower performance cost than the non-linear optimization method, assuming a low-rank shape model.

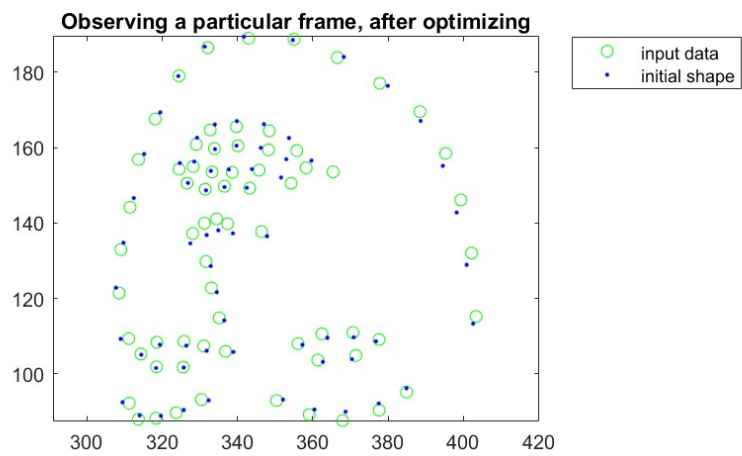


Figure 8: Input points compared to optimized points

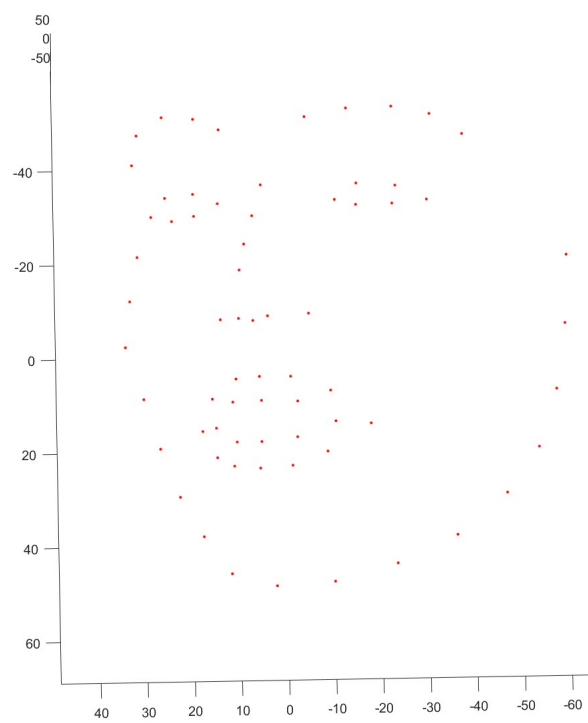


Figure 9: 3D reconstruction

Iteration	Func-count	Resnorm	First-order optimality	Lambda	Norm of step
0	1225	1.84649e+06	9.51e+04	0.01	
1	2450	26864.9	2.09e+03	0.001	25.0136
2	3678	16728	1.07e+03	1	15.9044
3	4903	12361	487	0.1	24.5625
4	6129	11207.5	837	1	15.9301
5	7354	10545.2	661	0.1	12.7065
6	8580	10245.9	717	1	7.64863
7	9805	10085.3	465	0.1	5.64436
8	11031	9981.24	272	1	4.18336
9	12256	9910.45	164	0.1	3.76857
10	13482	9851.74	138	1	3.28008
11	14707	9806.24	108	0.1	2.81746
12	15932	9769.25	165	0.01	5.31267
13	17158	9728.06	114	0.1	3.42729
14	18383	9704.76	96.1	0.01	3.27792
15	19609	9687.54	81.7	0.1	2.7978
16	20834	9675.29	48.5	0.01	2.4206
17	22060	9666.78	42.2	0.1	2.06087
18	23285	9660.79	36.6	0.01	1.83145
19	24511	9656.37	30.5	0.1	1.78
20	25736	9652.86	25.5	0.01	1.81386
21	26962	9649.82	22.3	0.1	1.9704
22	28187	9646.98	20.2	0.01	2.11821
23	29413	9644.15	19.7	0.1	2.32606
24	30638	9641.23	19.1	0.01	2.48918
25	31864	9638.15	19.8	0.1	2.69185
26	33089	9634.88	19.7	0.01	2.83915
27	34315	9631.4	21	0.1	3.02158
28	35540	9627.71	20.9	0.01	3.14215
29	36766	9623.8	22.1	0.1	3.29737
30	37991	9619.73	22.9	0.01	3.38541
31	39217	9615.48	22.4	0.1	3.50923
32	40442	9611.11	24.7	0.01	3.56288
33	41668	9606.61	22.1	0.1	3.65309
34	42893	9602.06	26.4	0.01	3.67613
35	44119	9597.42	21.1	0.1	3.73425
36	45344	9592.76	28	0.01	3.73824
37	46570	9588.01	20.6	0.1	3.76665
38	47795	9583.3	29.4	0.01	3.76896
39	49021	9578.49	23.2	0.1	3.76972
40	50246	9573.86	40.4	0.01	3.79026
41	51472	9569.16	30.5	0.1	3.76321
42	52697	9565.07	73.9	0.01	3.81738
43	53923	9560.89	73.9	0.1	3.76668
44	55148	9558.3	153	0.01	3.86225
45	56374	9555.13	168	0.1	3.80734
46	57601	9550.17	104	10	0.546409
47	58826	9547.64	80.2	1	0.302035
48	60051	9546.47	34.9	0.1	0.763707
49	61276	9542.47	44.1	0.01	3.79643
50	62502	9539.54	86.7	0.1	3.62819

[Solver stopped prematurely.](#)

lsqnonlin stopped because it exceeded the iteration limit,  
[options.MaxIterations](#) = 5.000000e+01.

Final reprojection (error: 0.0030556%)

Figure 10: Optimizer iterations

## 5 Third Part (Non-Rigid structure from motion by factorization and assuming a low-rank trajectory model)

In this section, we calculate the reconstruction with the low-rank trajectory method. This method is a little simpler because we already know in advance the matrix of base trajectories, since we defined them with the value of  $k$ . We will show the structure in Equation 15.

$$\theta = \begin{pmatrix} \theta_1^1 & \cdots & \theta_k^1 \\ \vdots & \ddots & \vdots \\ \theta_1^f & \cdots & \theta_k^f \end{pmatrix}_{f \times k}$$

$$[a^x] = \begin{pmatrix} a_{11}^x & \cdots & a_{1p}^x \\ \vdots & \ddots & \vdots \\ a_{k1}^x & \cdots & a_{kp}^x \end{pmatrix}_{k \times p}, [a^y] = \begin{pmatrix} a_{11}^y & \cdots & a_{1p}^y \\ \vdots & \ddots & \vdots \\ a_{k1}^y & \cdots & a_{kp}^y \end{pmatrix}_{k \times p} \text{ and } [a^z] = \begin{pmatrix} a_{11}^z & \cdots & a_{1p}^z \\ \vdots & \ddots & \vdots \\ a_{k1}^z & \cdots & a_{kp}^z \end{pmatrix}_{k \times p}$$

$$\underbrace{\begin{pmatrix} x_1^1 & x_2^1 & \cdots & x_p^1 \\ y_1^1 & y_2^1 & \cdots & y_p^1 \\ z_1^1 & z_2^1 & \cdots & z_p^1 \\ \vdots & \vdots & \ddots & \vdots \\ x_1^f & x_2^f & \cdots & x_p^f \\ y_1^f & y_2^f & \cdots & y_p^f \\ z_1^f & z_2^f & \cdots & z_p^f \end{pmatrix}}_X = \underbrace{\begin{pmatrix} \theta_{f \times k} & 0_{f \times k} & \cdots & 0_{f \times k} \\ 0_{f \times k} & \theta_{f \times k} & \vdots & 0_{f \times k} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{f \times k} & 0_{f \times k} & \cdots & \theta_{f \times k} \end{pmatrix}}_C \underbrace{\begin{pmatrix} [a^x]_{k \times p} \\ [a^y]_{k \times p} \\ [a^z]_{k \times p} \end{pmatrix}}_D \quad (15)$$

Where  $C$  is called trajectory bases and  $D$  is called coefficients.

The low-rank trajectory method is optimized using the factorization method with SVD as shown in the lecture slide in Figure 11.

### Algorithm

Orthographic camera

Because  $\mathbf{RC}$  is a  $2l \times 3K$  matrix and  $\mathbf{D}$  is a  $3K \times p$  matrix, the rank of  $\mathbf{W}$  is  $3K$ . If we apply SVD to  $\mathbf{W}$ , we will have only  **$3K$  non-zero singular values**

However, measurements are normally noisy, and in practice the rank will not be  $3K$ . We have to impose it

Applying SVD factorization, we have:

$$\mathbf{W} = \mathbf{U}\mathbf{A}\mathbf{V}^T = [\mathbf{U}\sqrt{\mathbf{A}}][\sqrt{\mathbf{A}}\mathbf{V}^T] = [\mathbf{U}\sqrt{\mathbf{A}}\mathbf{Q}][\mathbf{Q}^{-1}\sqrt{\mathbf{A}}\mathbf{V}^T]$$

i.e.,  $\mathbf{RC} = \mathbf{U}\sqrt{\mathbf{A}}\mathbf{Q}$  and  $\mathbf{D} = \mathbf{Q}^{-1}\sqrt{\mathbf{A}}\mathbf{V}^T$  (the two factors we look for). As  $\mathbf{C}$  is known, we can directly obtain  $\mathbf{R}$ , and finally,  $\mathbf{X} = \mathbf{CD}$

Metric upgrade to compute  $\mathbf{Q}$  by enforcing the orthogonality constraints

We need to tune the rank  $K$  a priori

Figure 11: Factoring slide

As shown in Figure 11, in this case, the rank of the matrix  $W$  (measurement matrix) is  $3k$ , since  $RC$  has dimensions  $2I \times 3k$  and  $D$  has dimensions  $3k \times p$ . Therefore, we should calculate  $Q$  by enforcing the orthogonality constraints for  $RC$ , defined  $(RC)(RC)' = I_{2I \times 2I}$ , and since  $C$  is already known, we can calculate  $X$  in the form  $X = CD$ .

## 5.1 Code:

Now we will show how to update  $Q$  to find the shape matrix  $X$ .

```

1      [U,D,V]=svd(W,0);
2      RHat=U(:,1:3*K)*sqrt(D(1:3*K,1:3*K));
3      Xhat=sqrt(D(1:3*K,1:3*K))*V(:,1:3*K)';
4
5      % Metric Upgrade step
6      [Q] = metricUpgrade(RHat);
7      Rsh = RHat*Q;
8      % Computing final motion and shape
9      R = recoverR(Rsh);
10     C = DCT_basis(n_frames,K);
11     G = R*C;
12     D = inv(G'*G)*G'*W;
13     X=C*D;
```

Listing 2: Factorization method

In line 1 in Listing 2, we compute the SVD for  $W$  (measurement matrix).

In lines 2 and 3 in Listing 2, since we know the rank of the matrix is  $3k$ , then we restrict the dimensions of the matrices  $U$ ,  $D$ , and  $V$  as follows:

- For  $U$ , we take only its first  $3k$  columns.
- For  $D$ , we take only its first  $3k$  rows and first 3 columns.
- For  $V$  we take only the last  $3k$  columns

And with these new restrictions for  $U$ ,  $D$  and  $V$ , we calculate the initials matrix  $RC = U\sqrt{D}$  and  $D = \sqrt{D}V'$ . In line 6 in Listing 2, we update  $Q$  by using the orthogonality conditions for the matrix  $RHat$ , initialized as  $RHat = \text{initial } RC$ .  $Q$  is defined as follows.

```

1  function [Q] = metricUpgrade(LambdaHat,q0)
2
3  k = size(LambdaHat,2)/3;
4  F = size(LambdaHat,1)/2;
5
6  if(~exist('q0'))
7  q0 = zeros(3*k,3);
8  q0(0*k+1,1) = 1;
9  q0(1*k+1,2) = 1;
10 q0(2*k+1,3) = 1;
11 end
12 options = optimset('Diagnostics','off','MaxFunEval',100000,'MaxIter',2000,
13 'TolFun',1e-10,'TolX',1e-10);
14 [q, fval] = fminunc(@evalQ,q0,options,LambdaHat);
```

```
15 Q = reshape(q,3*k,3);
```

Listing 3: Update function

We can see inside the function *metricUpgrade(LambdaHat,q0)* on line 13 of Listing 3, where we compute the  $Q$  such that  $Q$  minimizes the function *evalQ* that computes the orthogonality condition.

In line 10 in Listing 2, we compute the trajectory bases.

In line 12 in Listing 2, we calculate  $D$  from  $RC$ , solving the equation  $\|W - RCD\| = 0$ .

In line 13 in Listing 2, finally we calculate the shape matrix  $X = CD$ .

## 5.2 Examples:

In this part, we show some results of the code for the low-rank trajectory model.

We start with the file **drink\_mocap.mat** that provides the key points whose represent a person is holding a bottle. The inputs for the code are

- Loaded data: **drink\_mocap.mat**
- Rank of the trajectory basis  $K$ : 11

In the Figure 12, we can notice the accordance between the truth 3D points and the reconstructed 3D points in a fixed frame  $i$  ( $\|X_{\text{truth shape}}^i - X_{\text{reconstructed shape}}^i\|$ ).

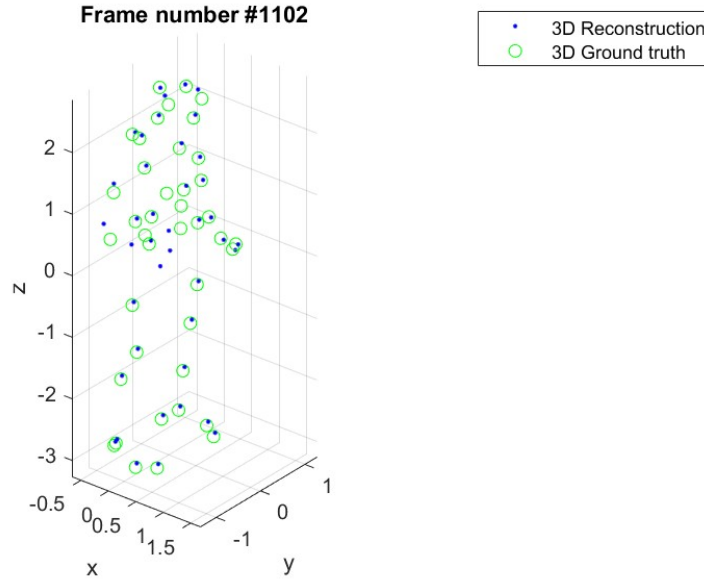


Figure 12: Comparison between real 3D points (green) and 3D points reconstructed (blue) for *drink\_mocap.mat*.

In the Figure 13, the first error of 0.0034452 represents the difference between the truth rotation matrix and the reconstructed rotation matrix. The second error of 0.0052503 represents the difference between the truth shape matrix and the reconstructed shape matrix.

```
<stopping criteria details>  
3D error: 0.034452  
3D error: 0.0052503
```

Figure 13: Error between real points and reconstruction points for **drink\_mocap.mat**. The first error is for the shape matrix, and the second is for the rotation matrix.