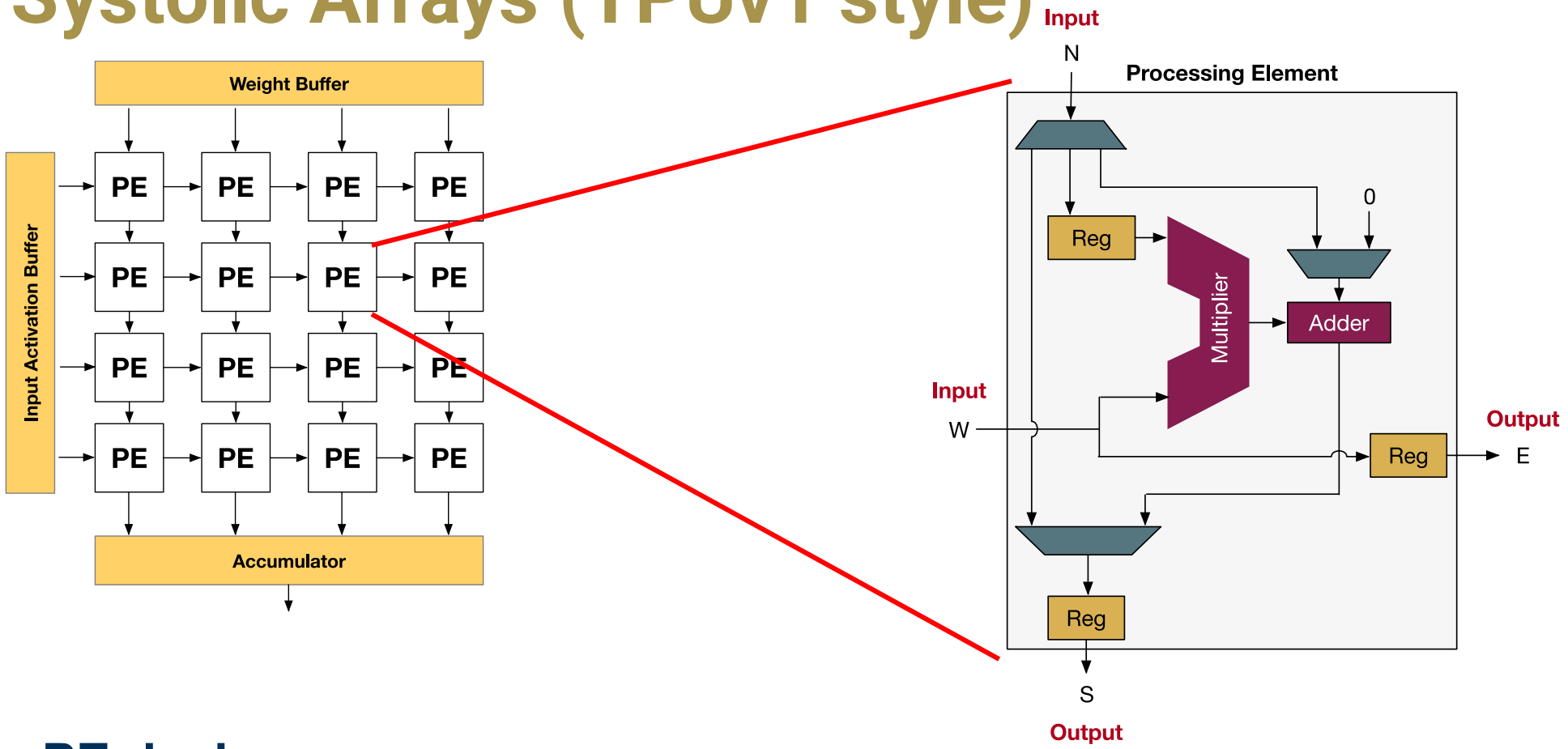
PEs in Systolic Arrays (TPUv1 style)



- **Simple PE design**
  - No local scratchpad; several one-slot registers
  - Enables to increase the number of PEs (small area for each PE -> more PEs within the same area)

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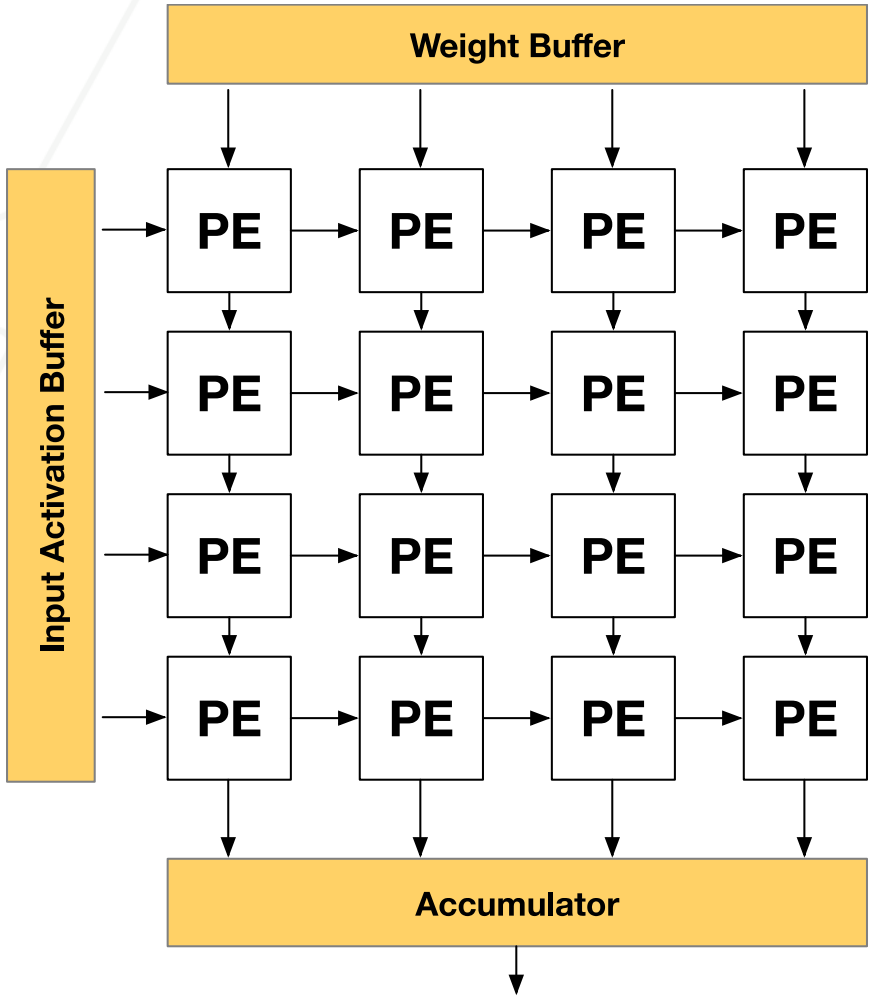
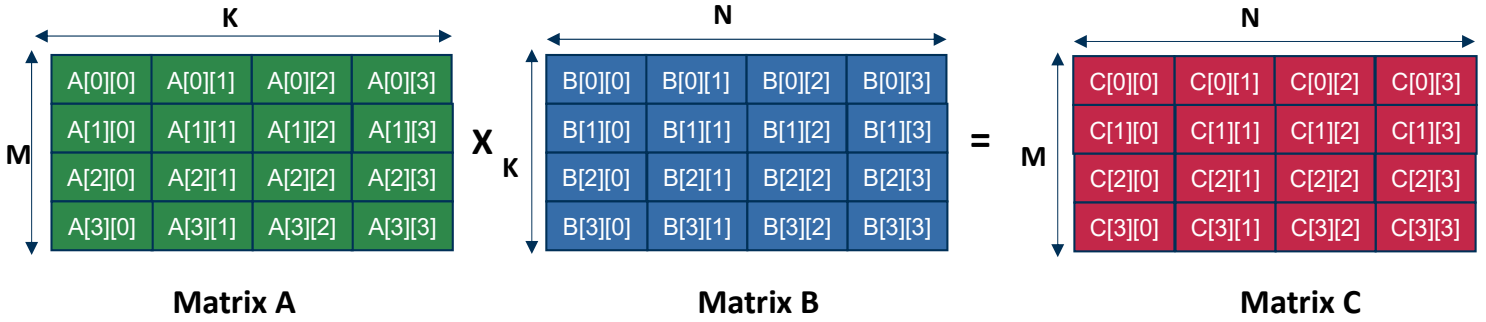
Matrix Multiplication on a Systolic Array






```
for m in range(4):
  for n in range(4):
    for k in range(4):
      C[m][n] += A[m][k] * B[k][n]
```

Assume following "mapping"

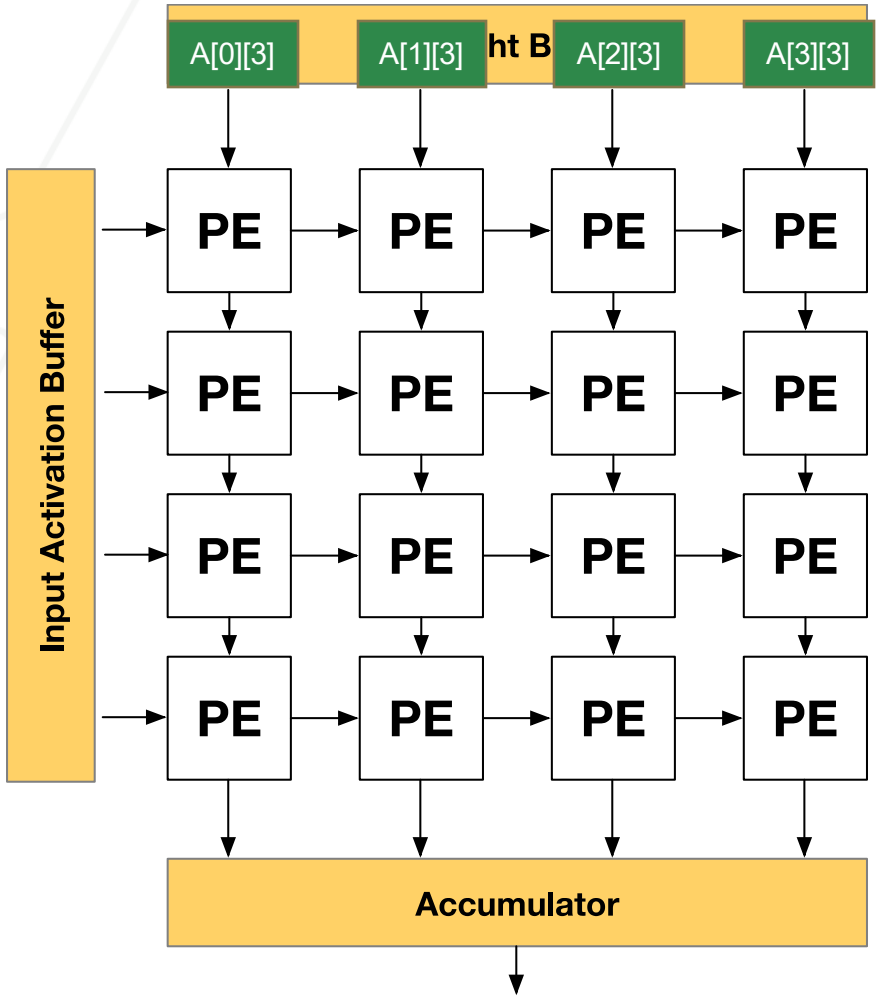
- Column: Reduced (*aka* contracted) Dimension (K)
- Row: Spatial Dimension (either M or N)

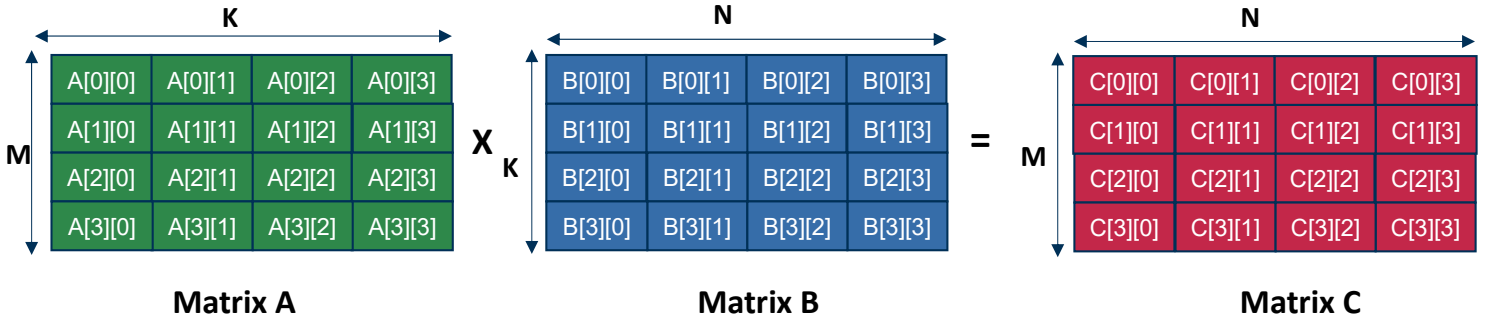
Note: alternate mapping styles possible

2

Matrix Multiplication on a Systolic Array

Phase 1:
Load Weight Tensor



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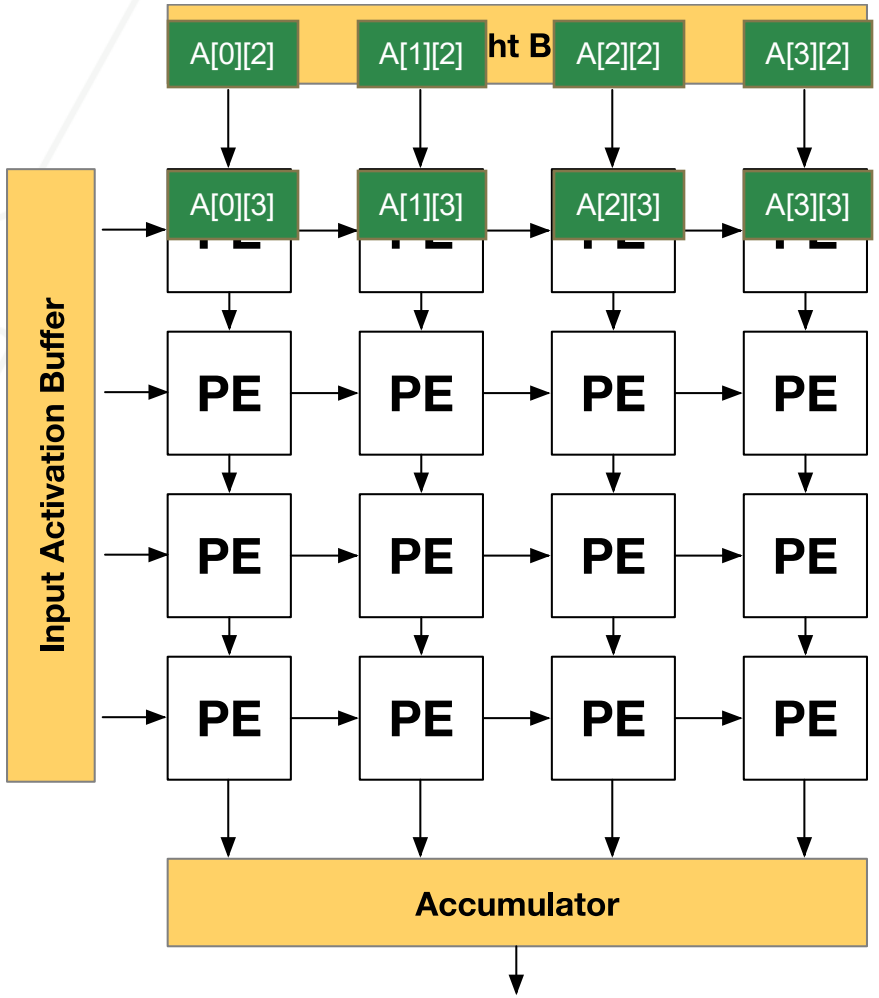
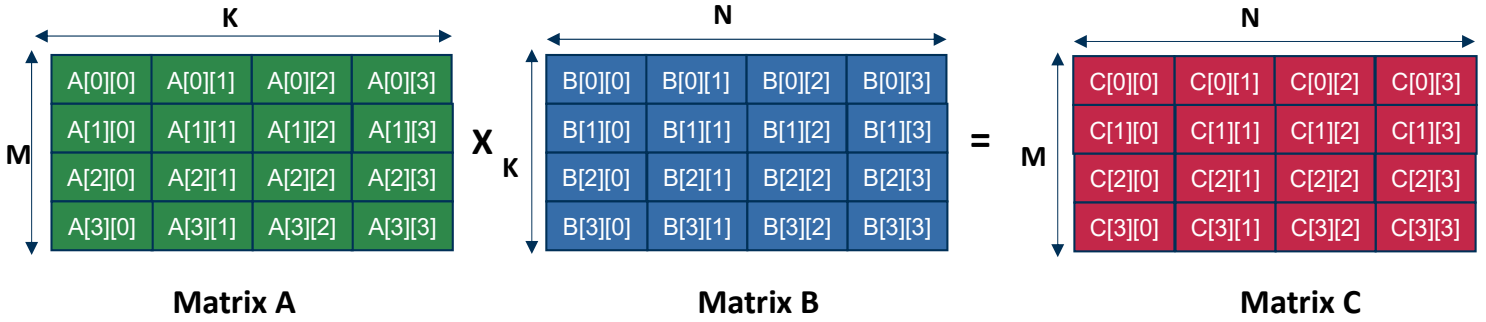
Matrix Multiplication on a Systolic Array

Phase 1:
Load Weight Tensor



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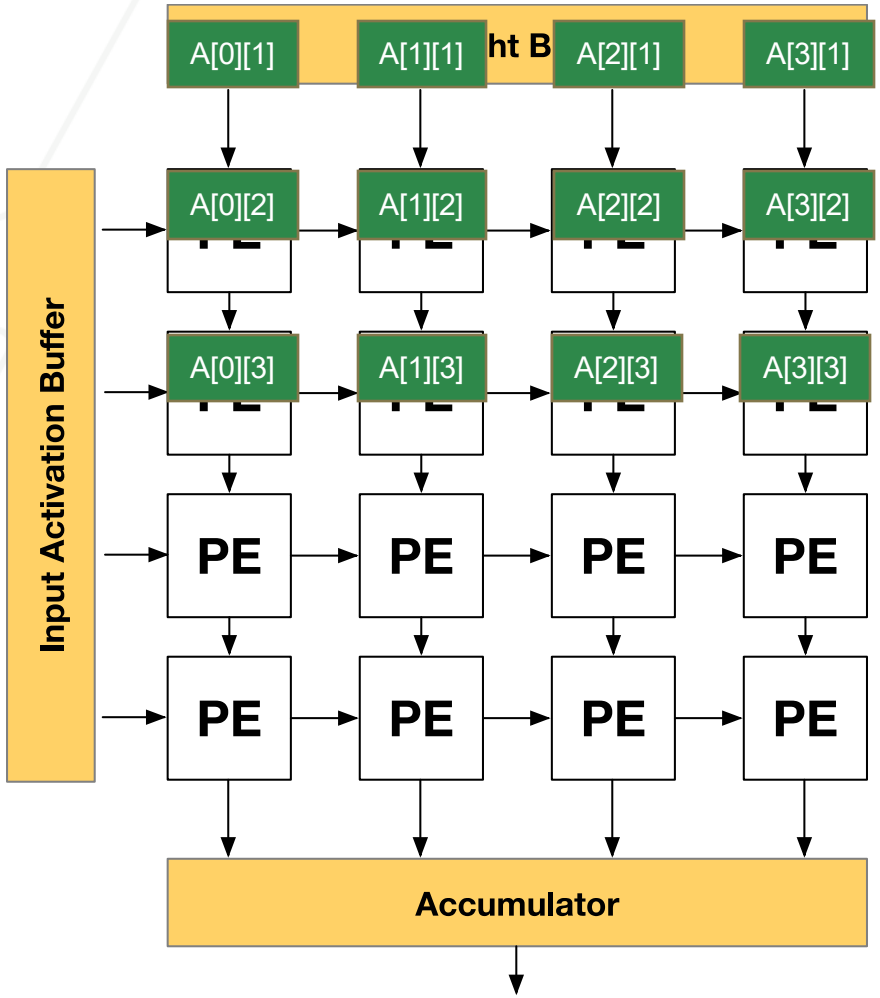
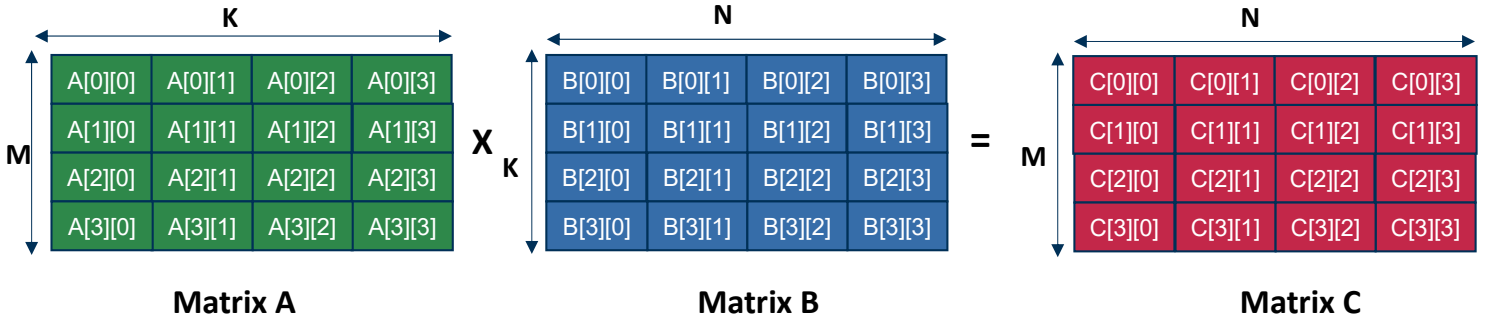
Matrix Multiplication on a Systolic Array

Phase 1:
Load Weight Tensor



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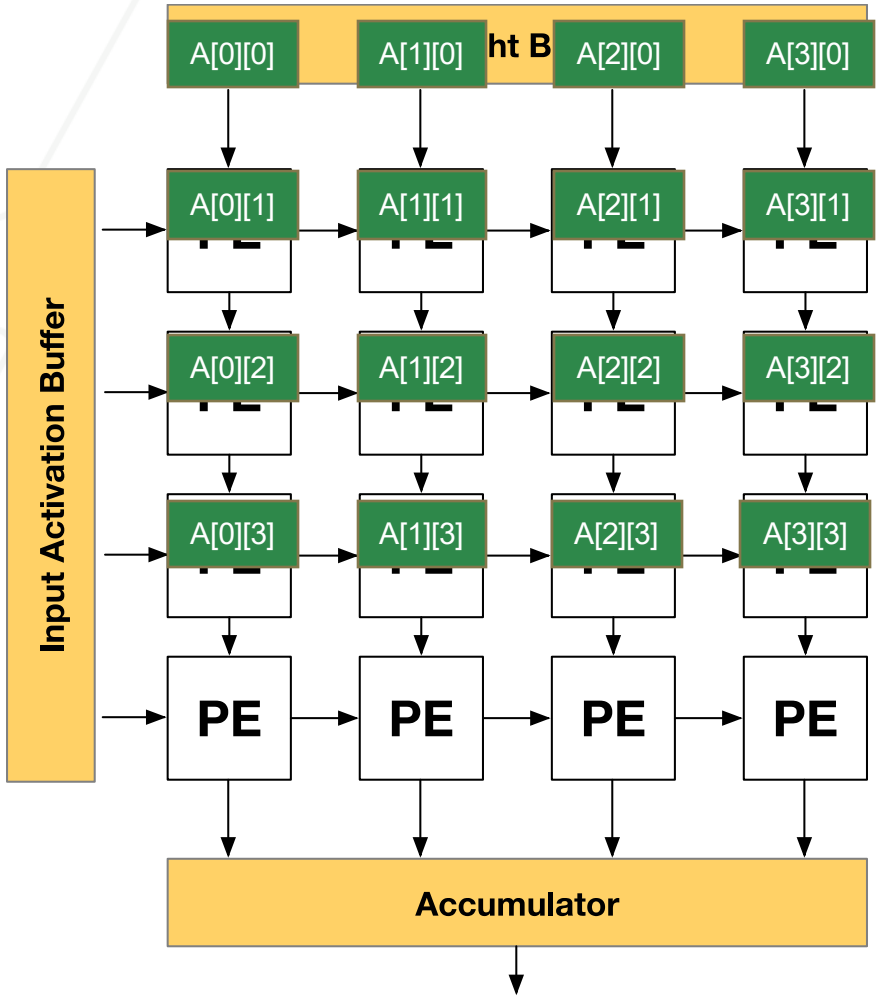
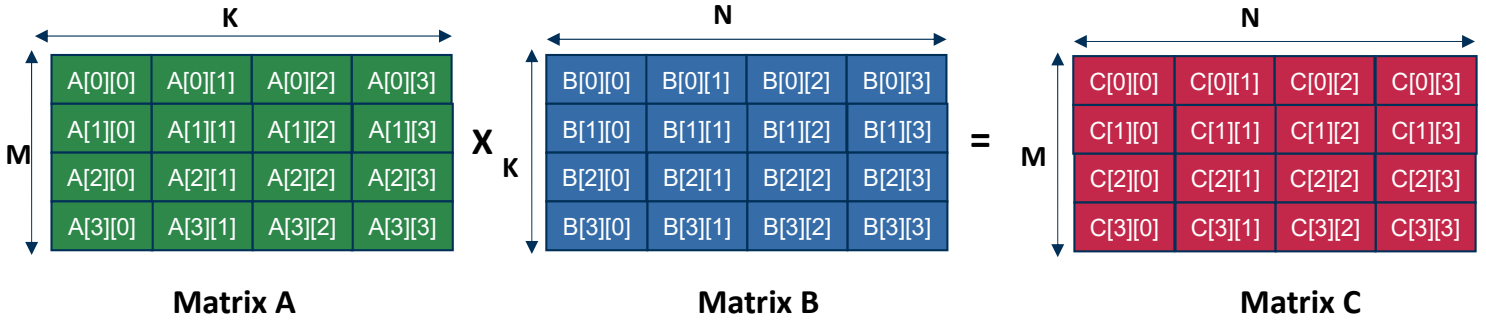
5

Matrix Multiplication on a Systolic Array

Phase 1:
Load Weight Tensor



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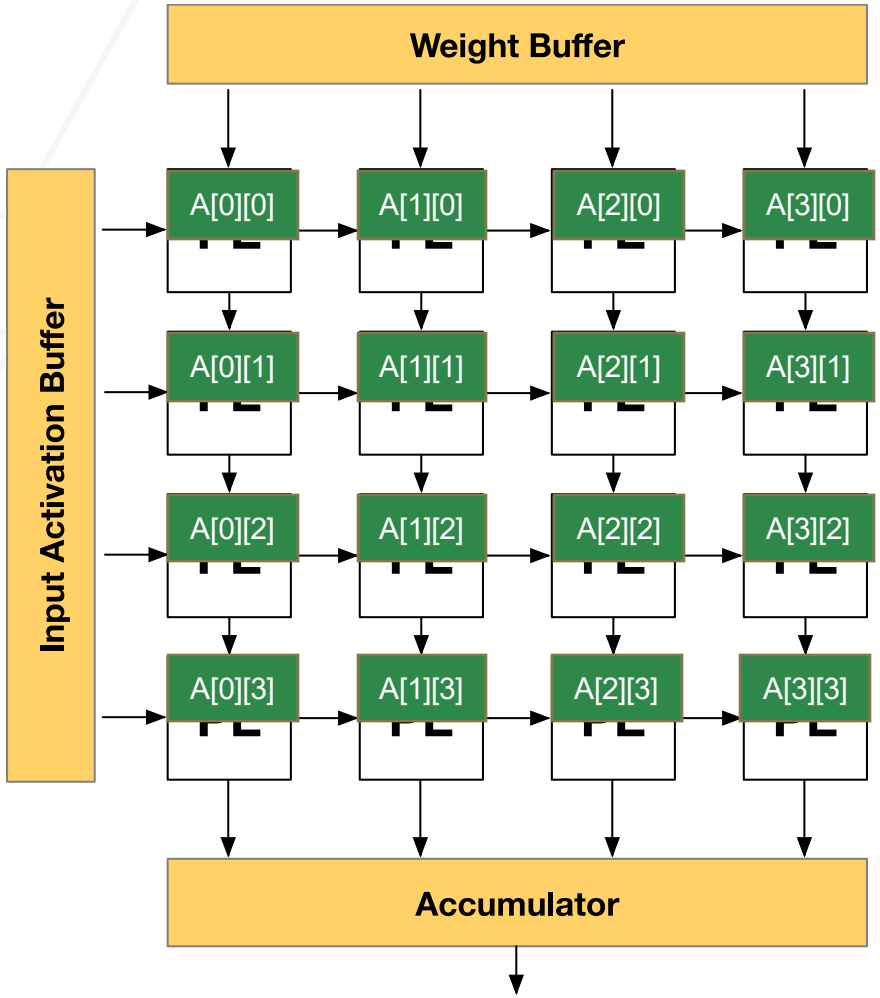
Note: alternate mapping styles possible

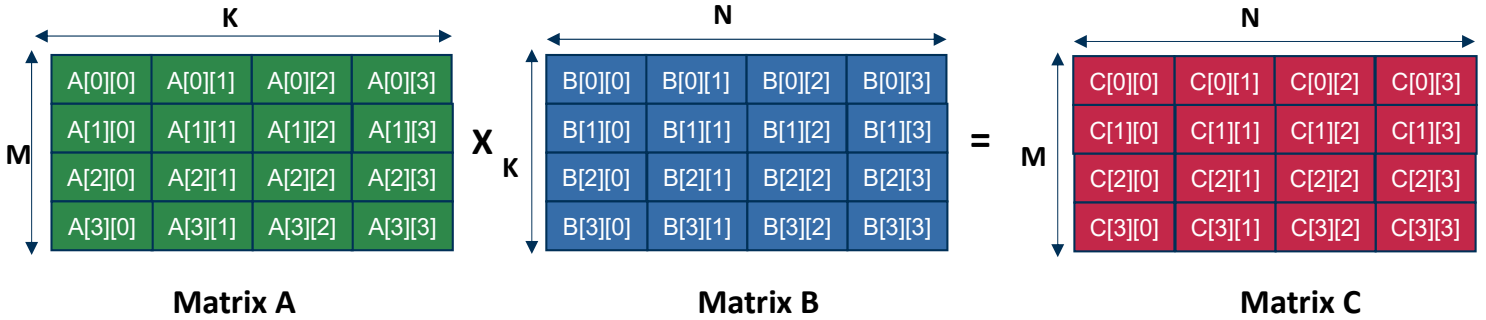
6

Matrix Multiplication on a Systolic Array

Phase 1:
Load Weight Tensor



Matrix A will now stay resident (i.e., "stationary") in the PEs

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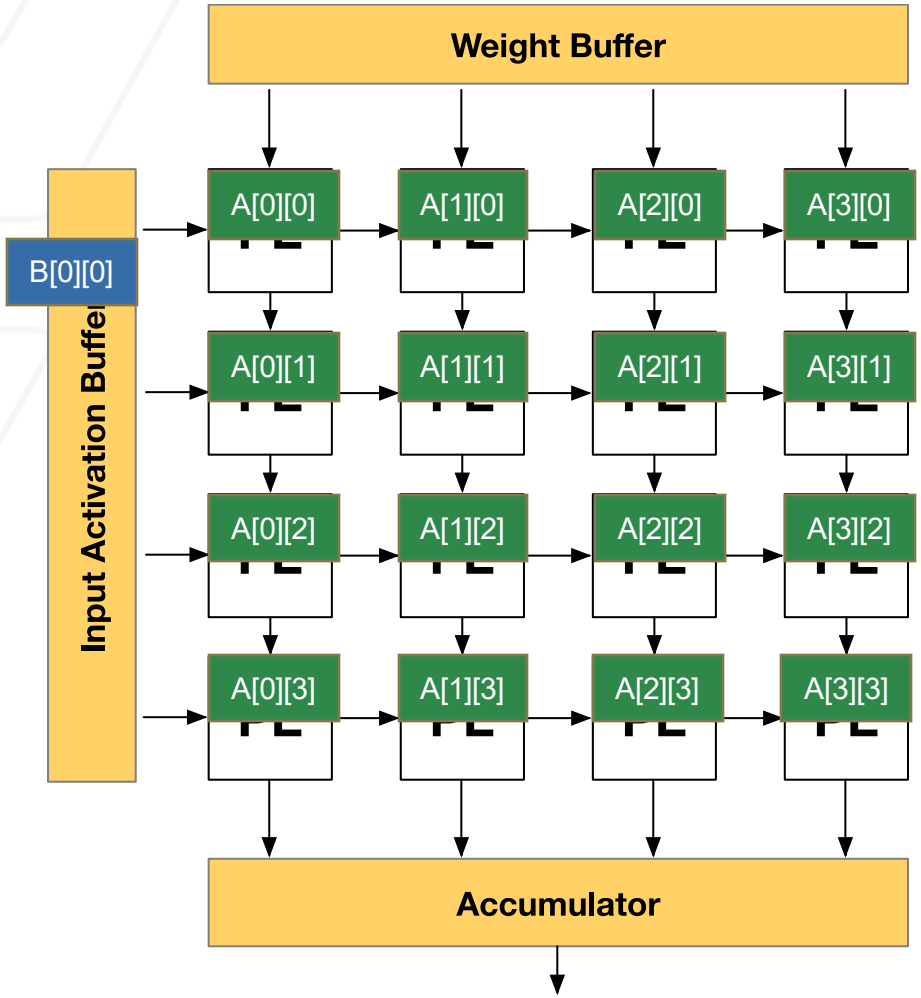
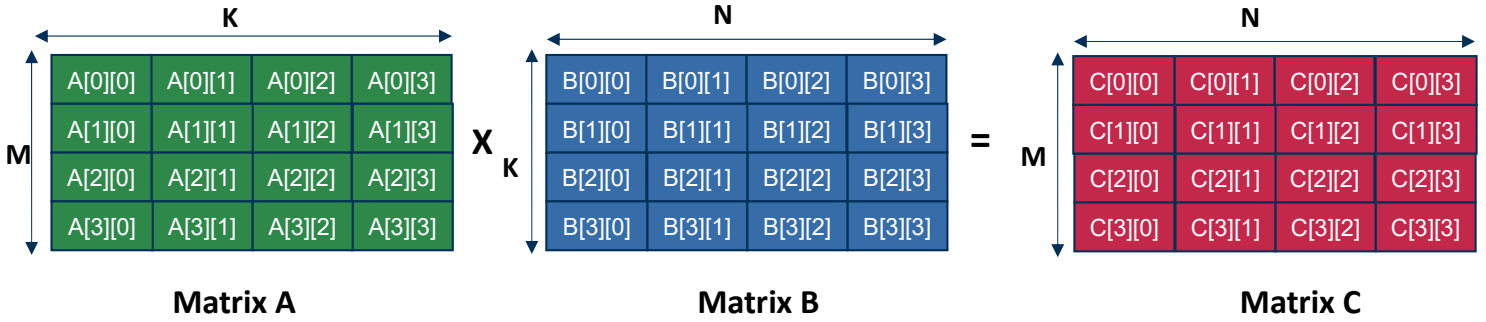
Note: alternate mapping styles possible

7

Matrix Multiplication on a Systolic Array

Phase 2:
Stream Activation Tensor



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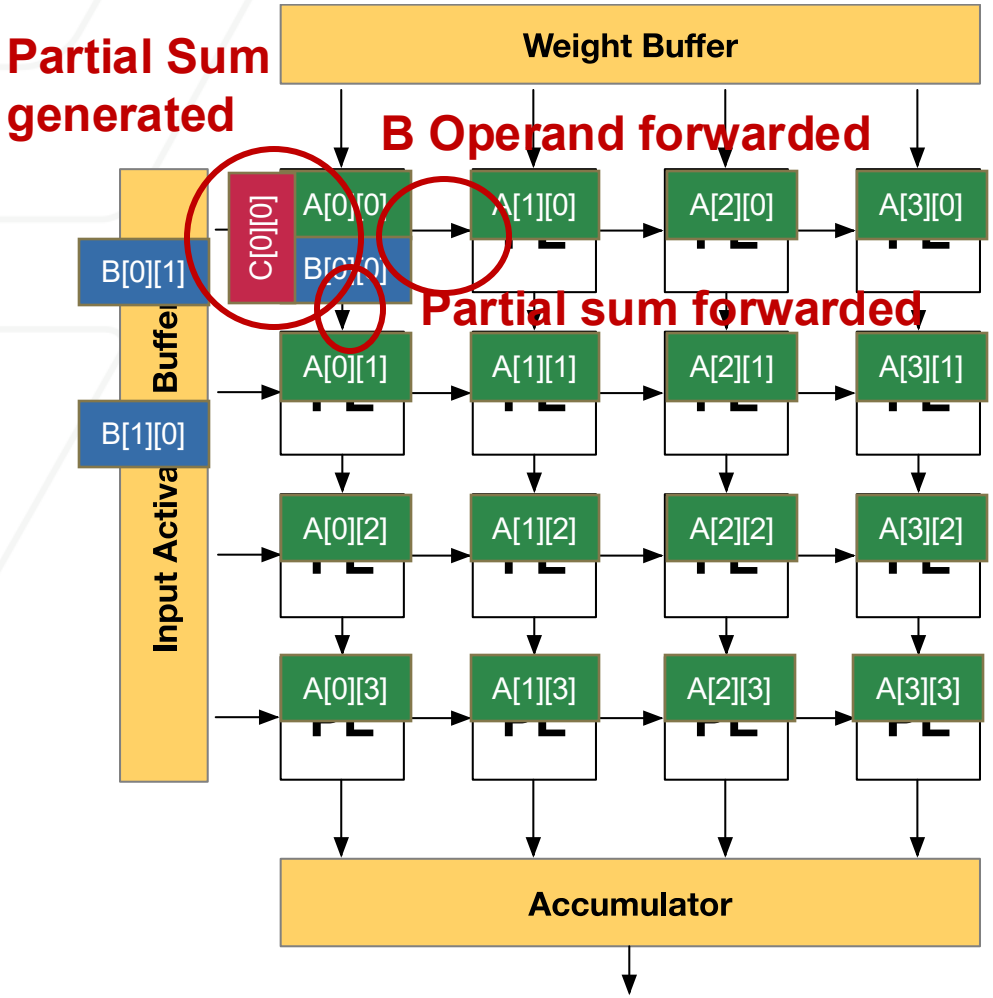
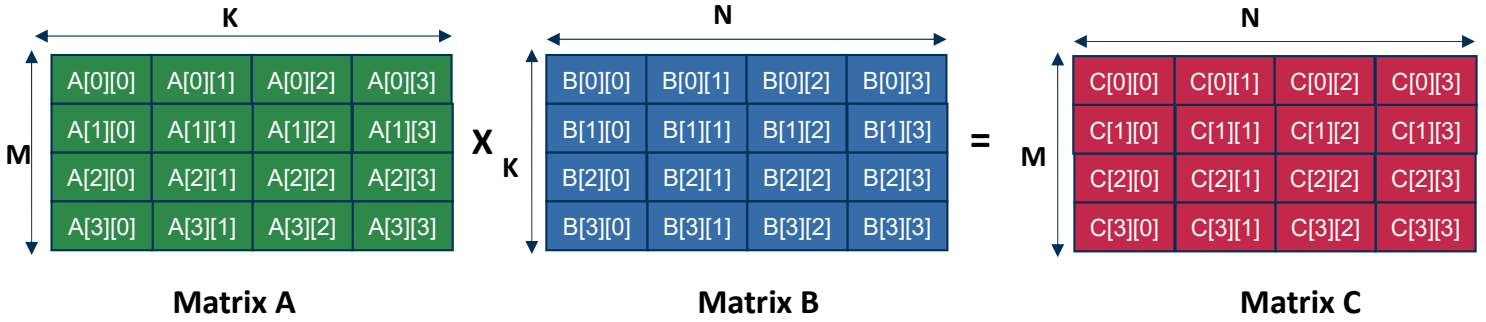
Matrix Multiplication on a Systolic Array

Phase 2:
Stream Activation Tensor



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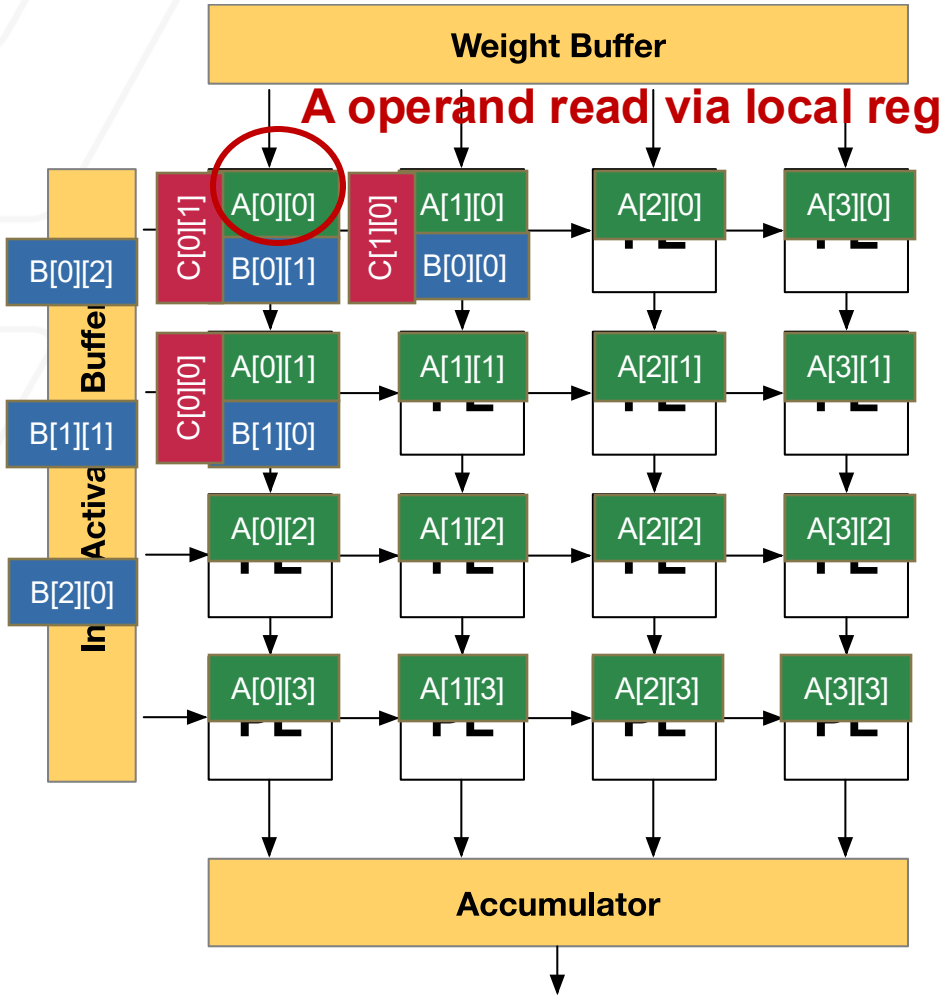
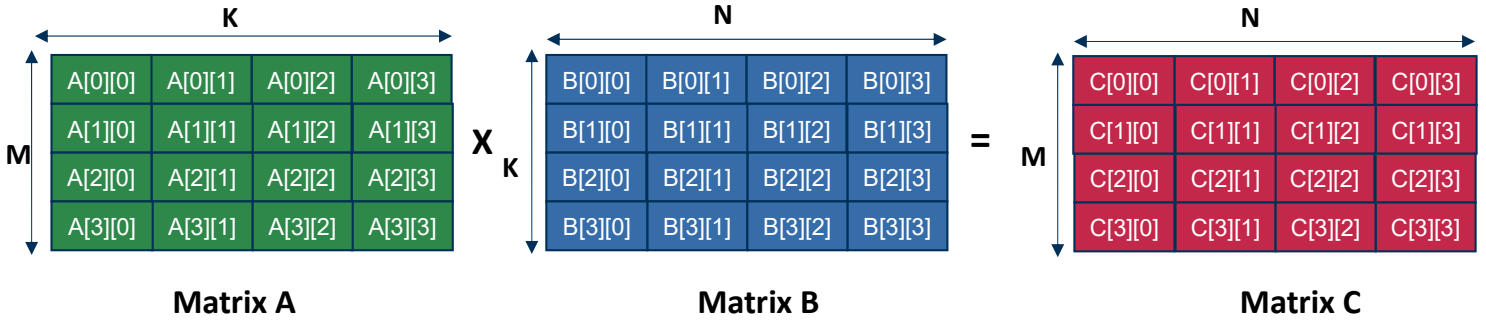
Matrix Multiplication on a Systolic Array

Phase 2:
Stream Activation Tensor



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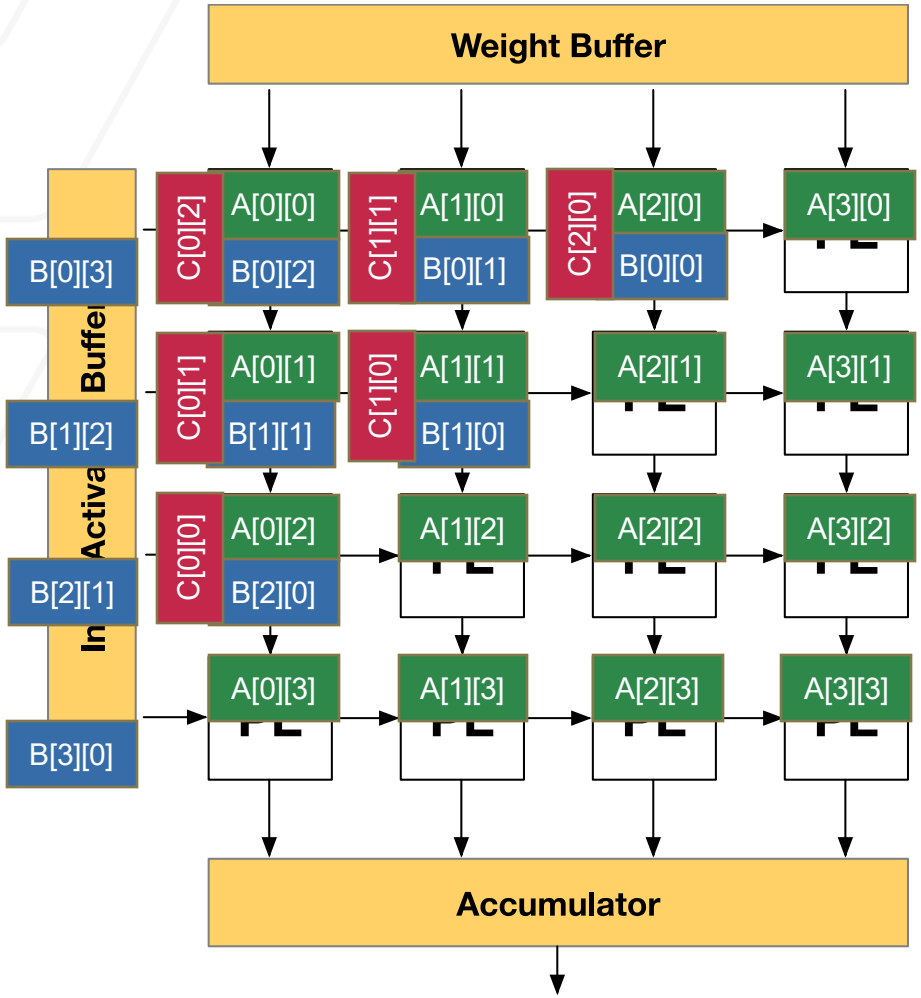
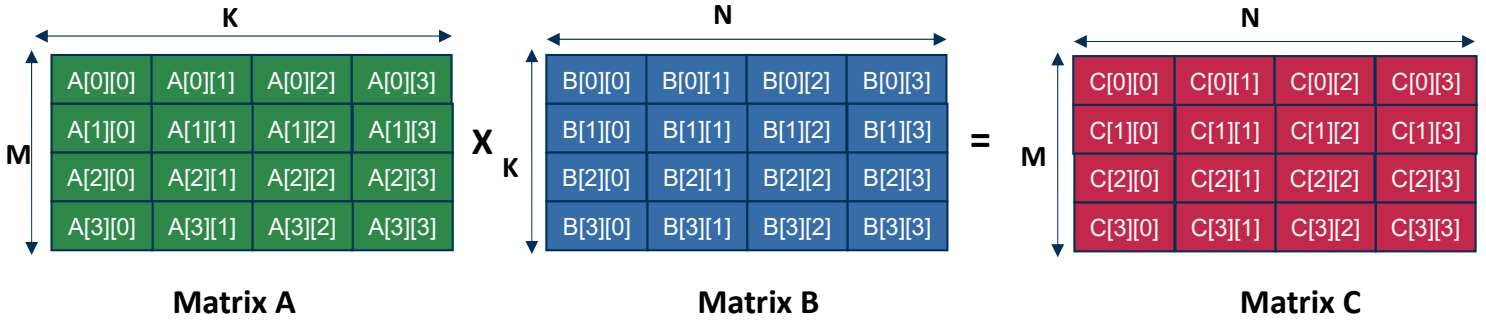
10

Matrix Multiplication on a Systolic Array

Phase 2:
Stream Activation Tensor



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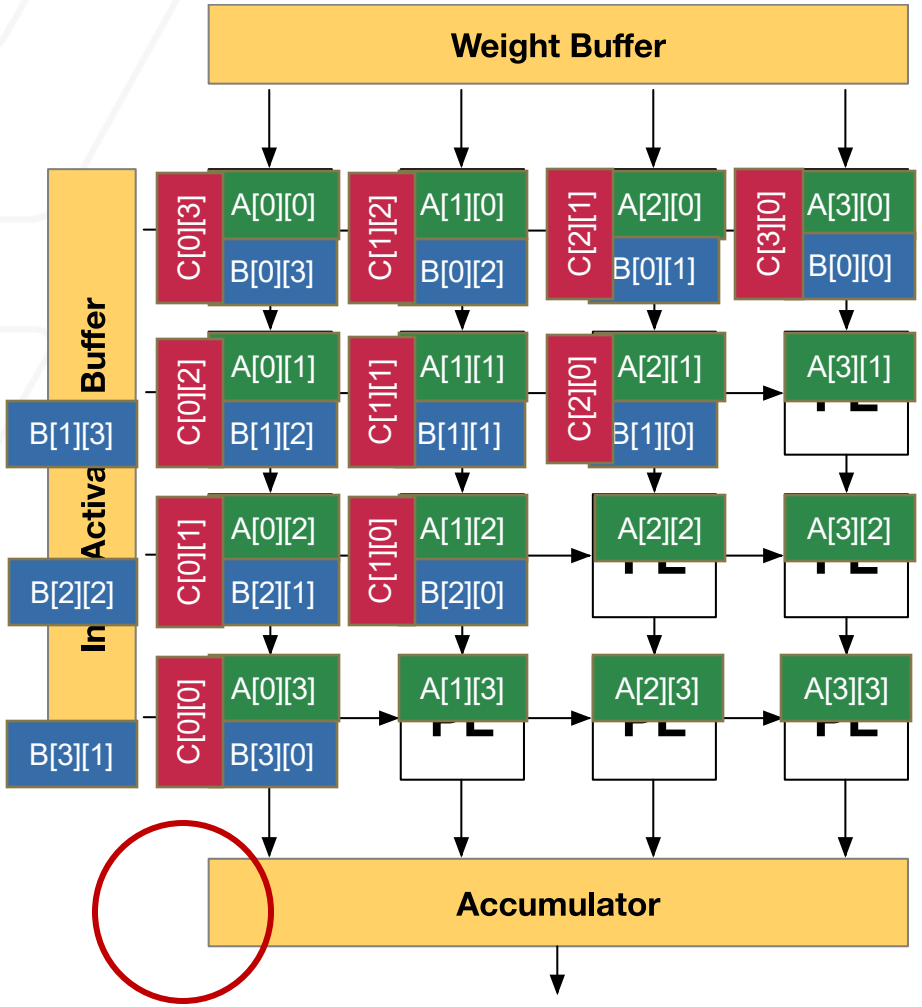
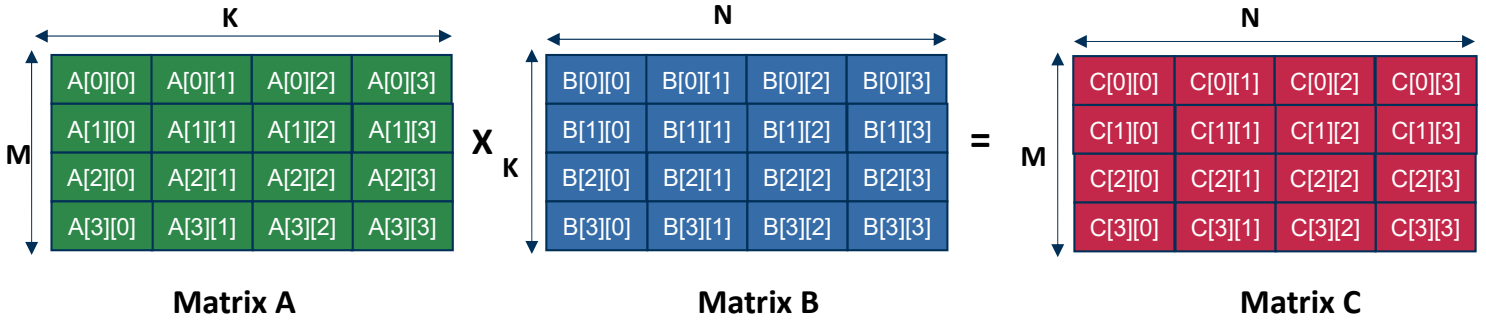
11

Matrix Multiplication on a Systolic Array

Phase 2:
Stream Activation Tensor



First output generated

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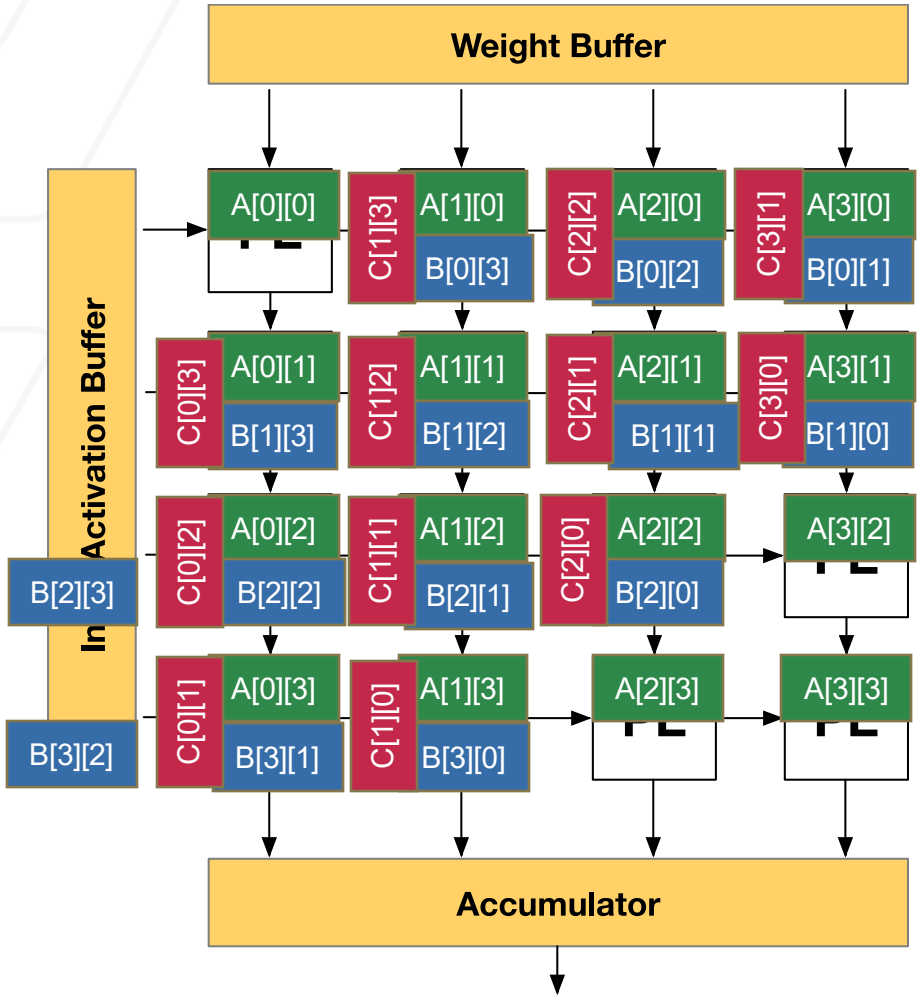
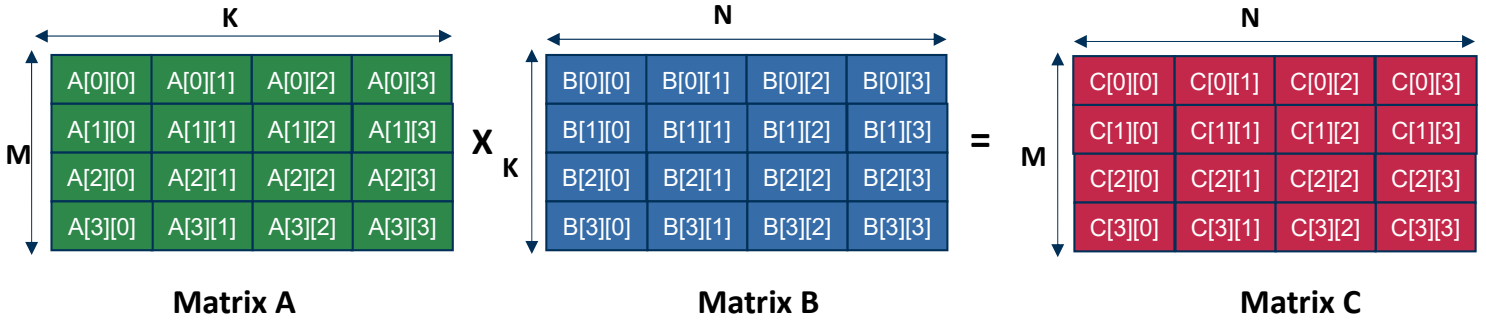
Matrix Multiplication on a Systolic Array

Phase 2:
Stream Activation Tensor



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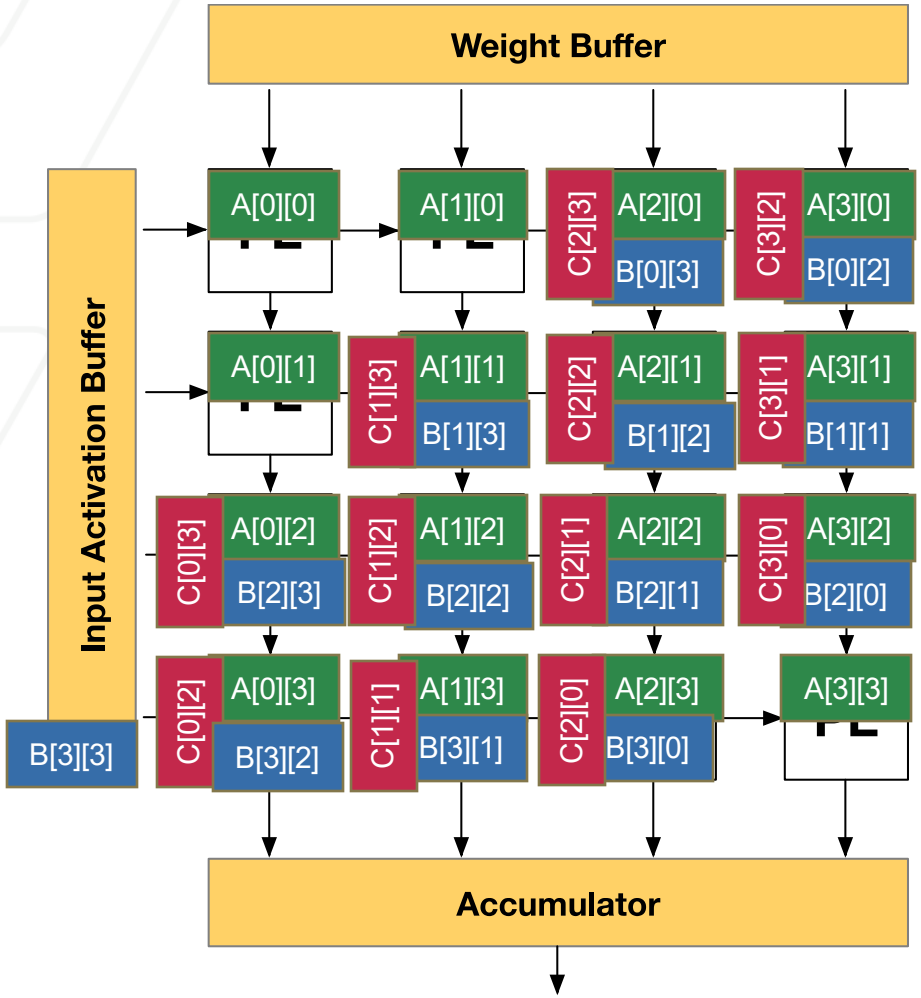
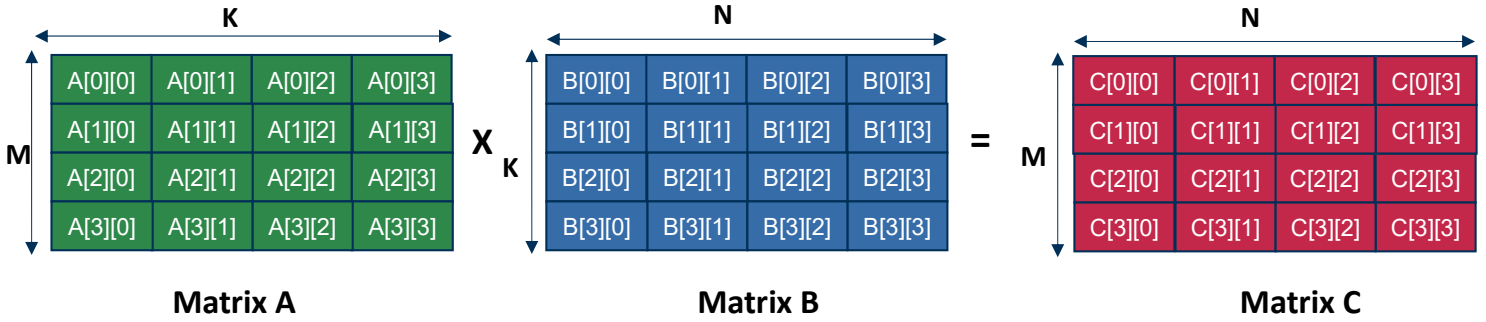
Matrix Multiplication on a Systolic Array

Phase 2:
Stream Activation Tensor



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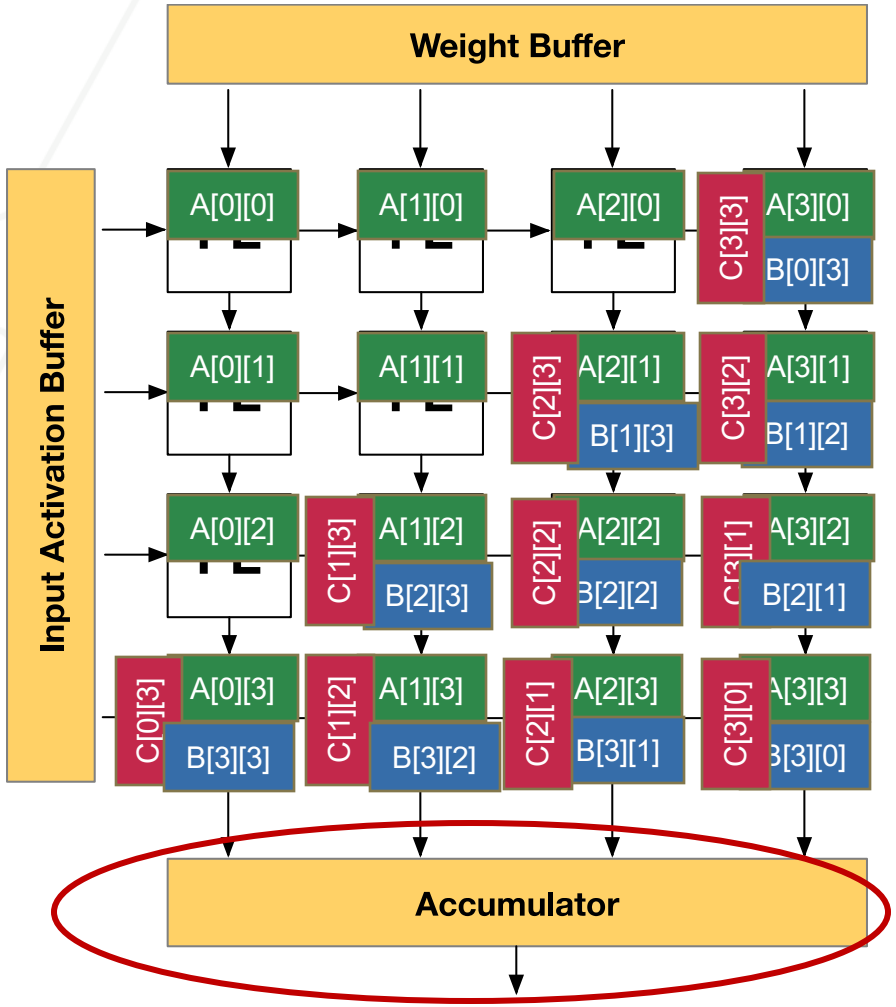
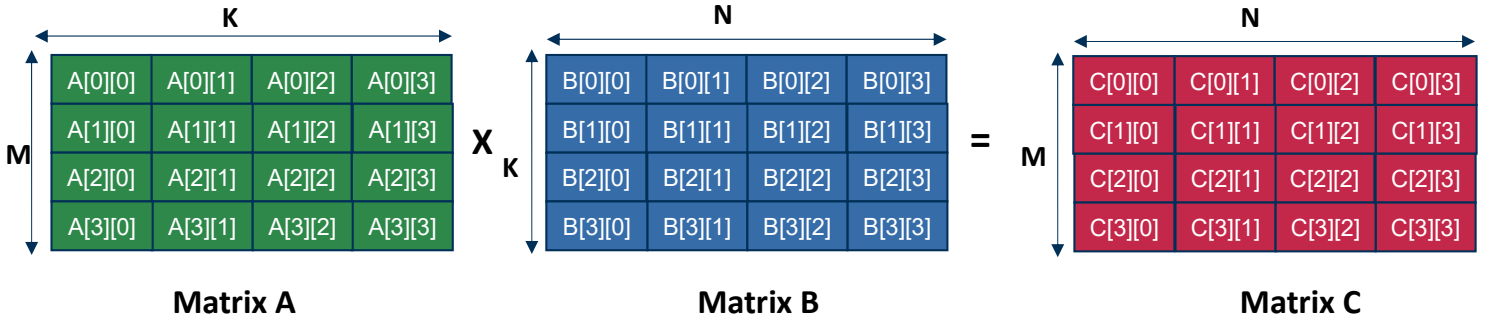
Matrix Multiplication on a Systolic Array

Phase 2:
Stream Activation Tensor



Four outputs every cycle in steady state

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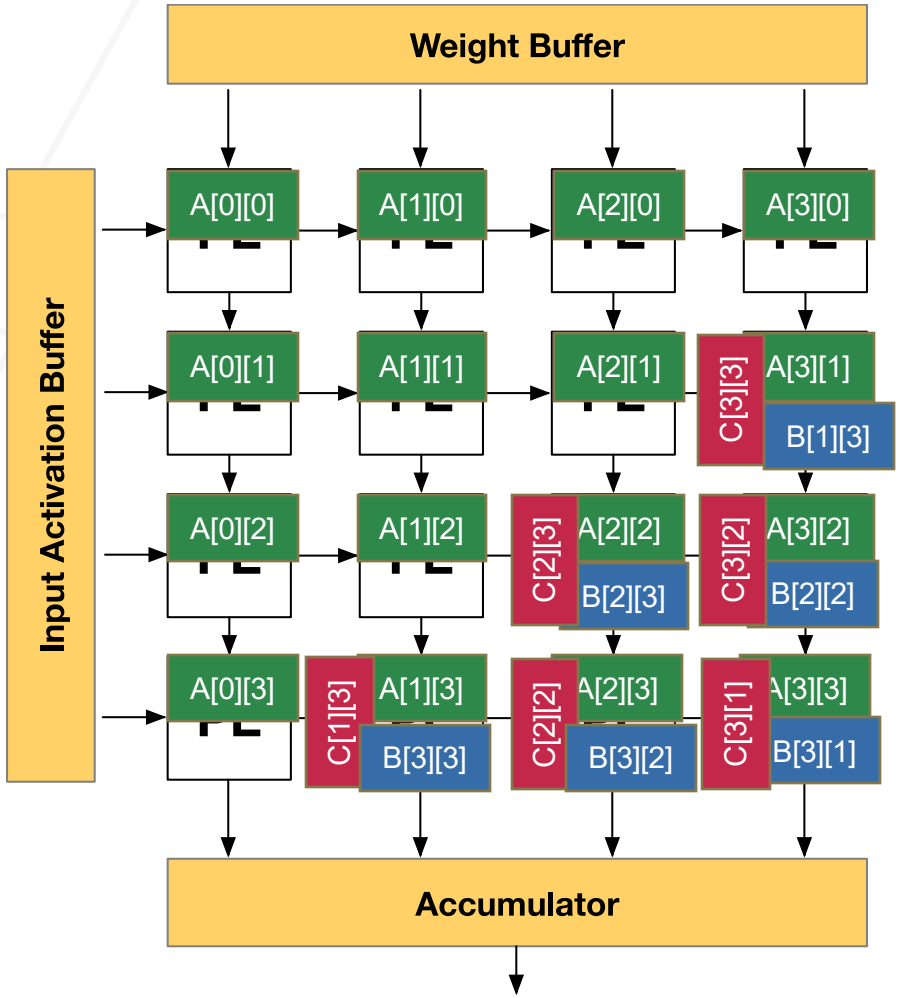
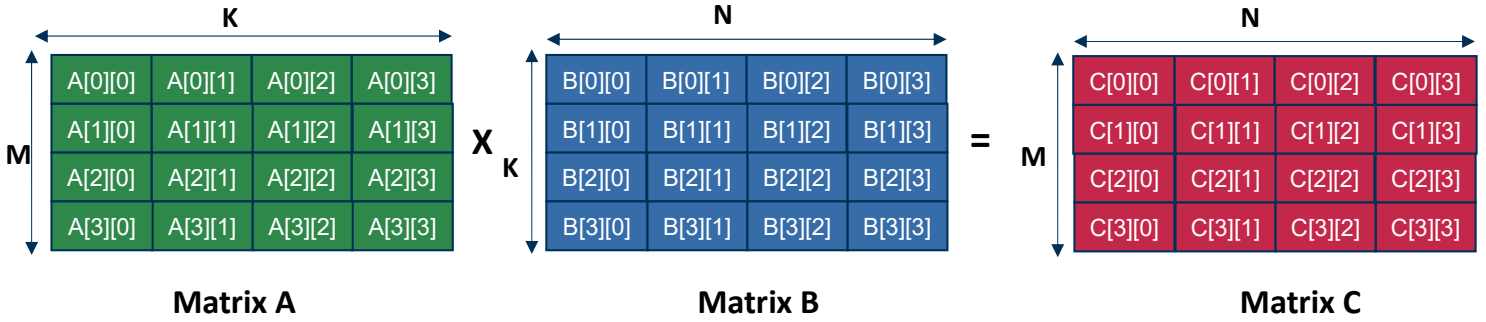
Matrix Multiplication on a Systolic Array

Phase 2:
Stream Activation Tensor



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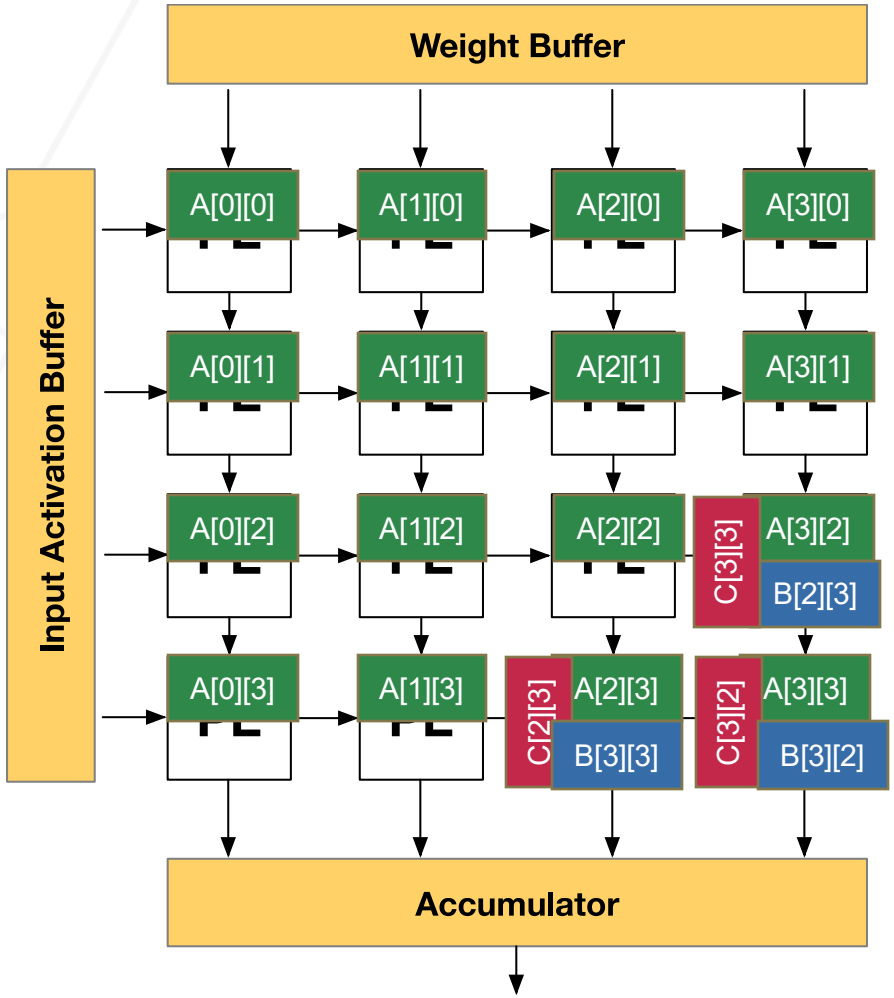
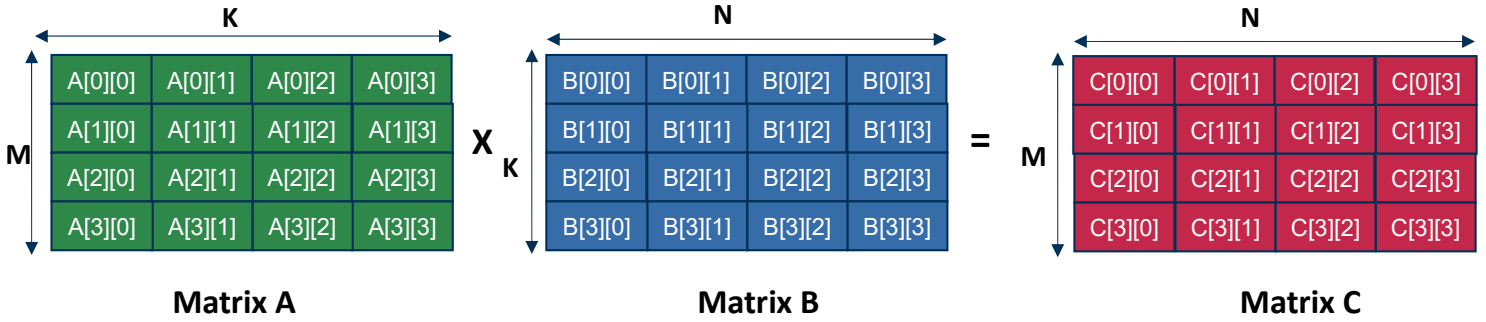
Matrix Multiplication on a Systolic Array

Phase 2:
Stream Activation Tensor



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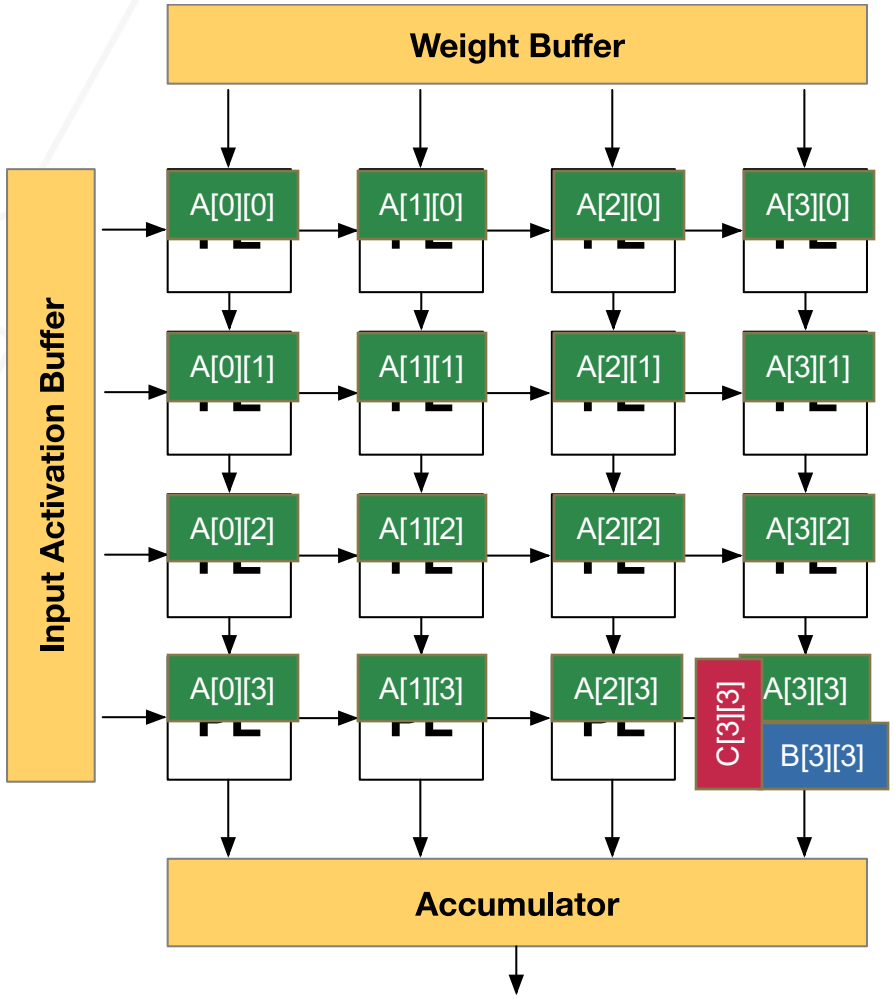
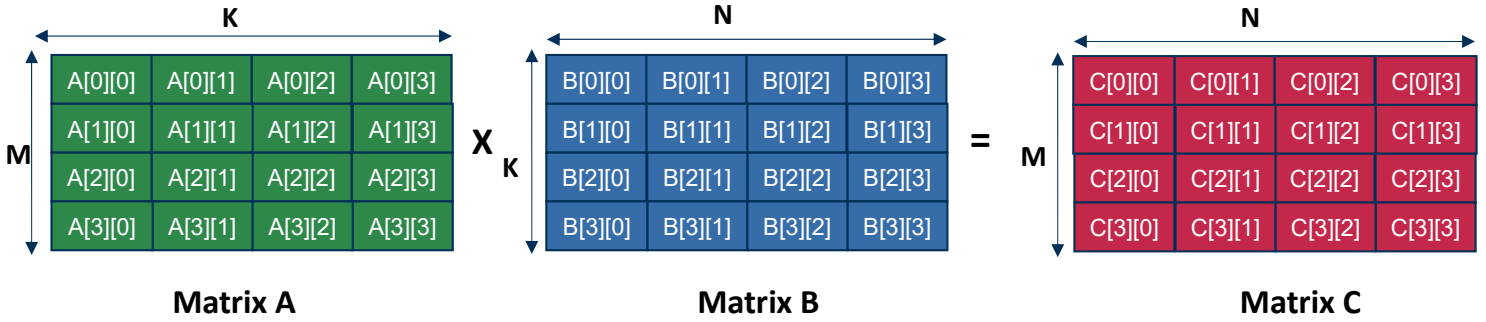
Matrix Multiplication on a Systolic Array

Phase 2:
Stream Activation Tensor



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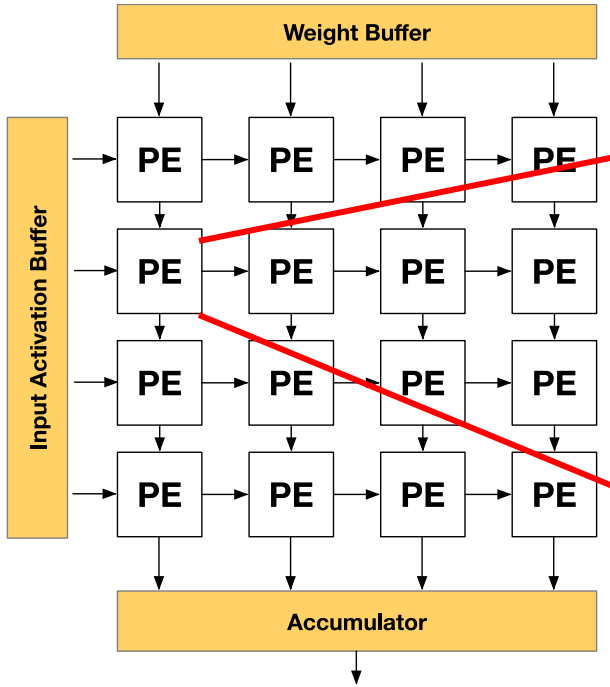
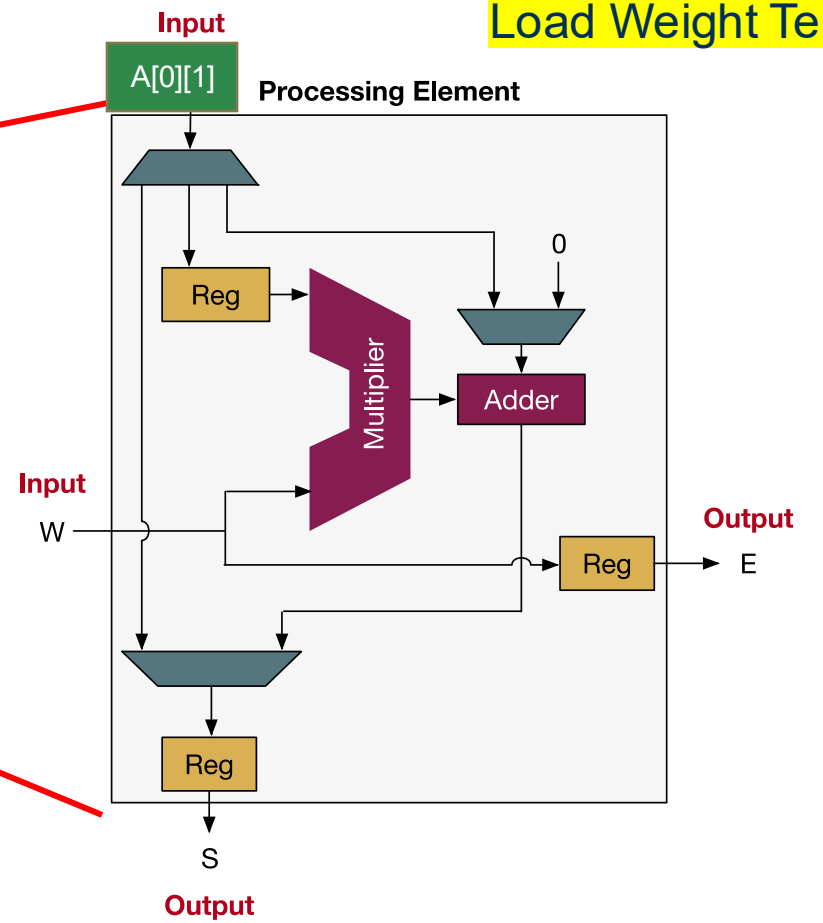
Recap: Zooming into PE

Phase 1:
Load Weight Tensor





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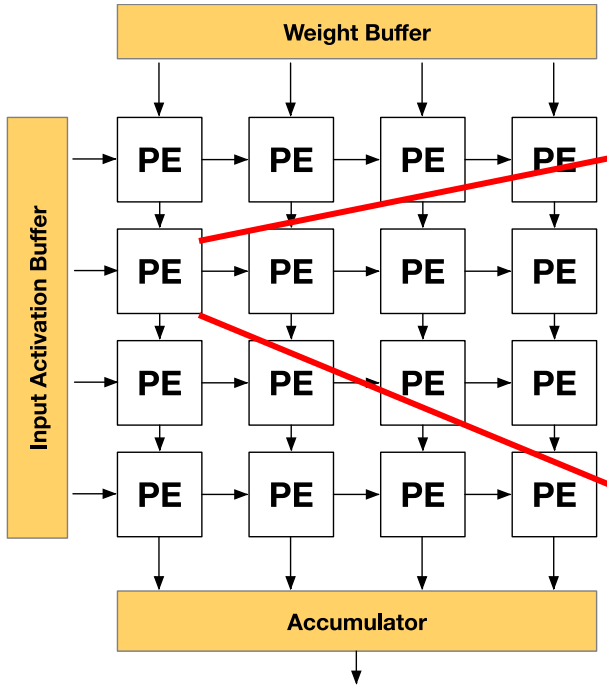
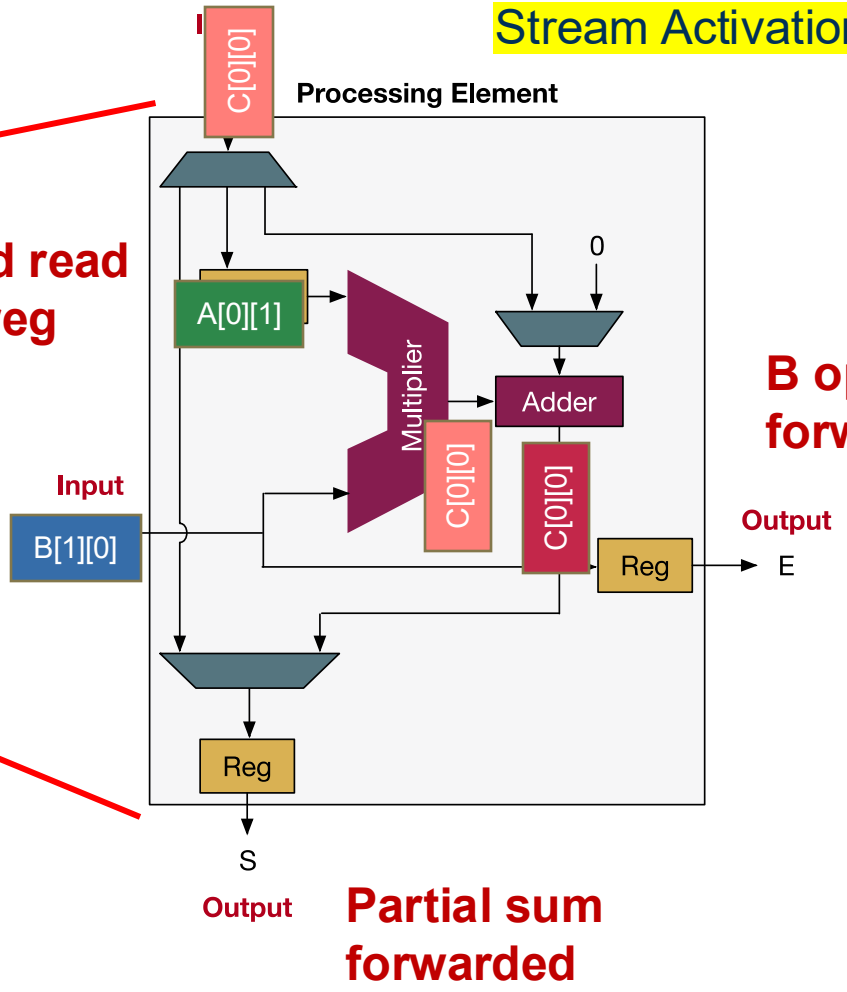
Recap: Zooming into PE



Phase 2:
Stream Activation Tensor

A operand read via local reg



B operand forwarded

Partial sum forwarded

Georgia Tech

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