

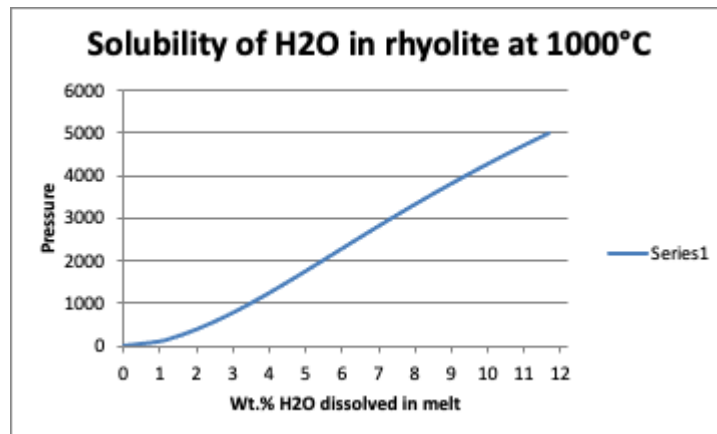
Problem 3: Will it ooze or will it solidify.

Why is the volatile phase taken to be the driver of eruptions?

The volatile phase is taken to be a driver of eruptions to the solubility of the volatiles in the magma. For rhyolitic magma we know that it erupts at relatively lower temperature (compared to basaltic magmas) ranges of 800-1000 celsius and due its high silicate composition it also has high viscosity. Volatiles have high vapor pressures so they will form a gaseous phase as pressure decreases. This of course will depend on the solubility of the water present. If it were to dissolve into the melt then gas will not form. For water to dissolve the following must be true: $\text{H}_2\text{O}(\text{melt}) + \text{O}^{2-}(\text{melt}) \rightleftharpoons 2\text{OH}^-(\text{melt})$, which will also reduce viscosity. This indicates that the proportion of water in silicate melts is $K = (X_{\text{OH}})^2 / (X_{\text{H}_2\text{O}})(X_{\text{O}})$. This can hold for ideal mixtures and small wt%. If we consider solubility as a function of pressure, temperature and composition than an empirical expression exist to obtain measurements of water solubility. Using the software program VolatileCalc (from USGS) we can determine this measurement. The equation used to model is

$$2 \ln X_{\text{H}_2\text{O}}[\text{melt}] = \frac{a}{T} + \sum b_i X_i \left(\frac{P}{T} \right) + c \ln f_{\text{H}_2\text{O}} + d$$

In this equation $a = 2565$, b_i = molar volume of H_2O in the melt (dependent on composition), $c = 1.171$ (modifies linear dependence of X on effective partial pressure), $d = -14.21$ (entropy change). Using VolatileCalc I get graph below of wt% H_2O vs pressure at 1000°C with 100 bar (10MPa) increments.



Fig(1)

We can see that for smaller pressures solubility will be low and 5.45%Wt H_2O is at 200 MPa.

Since at lower pressure water is not dissolving as much as it could be at higher pressures it is therefore exsolving. Gas bubbles form and rise. Volume will increase as a result of the ideal gas law equation.

$$V = \frac{nRT}{P}$$

We can assume a water amount of 5.55 moles, $T = 1000^{\circ}\text{C}$, and R is the ideal gas constant in SI units ($\text{m}^3\text{PaK}^{-1}\text{mol}^{-1}$). If we vary the pressure we can determine the pressure gradient related to the change in volume. Doing this we see for $\Delta P = 100\text{MPa}$ the volume increases from $2.30\text{e-}4$ to $46.20\text{e-}4$. Below are various plots of pressure vs volume.

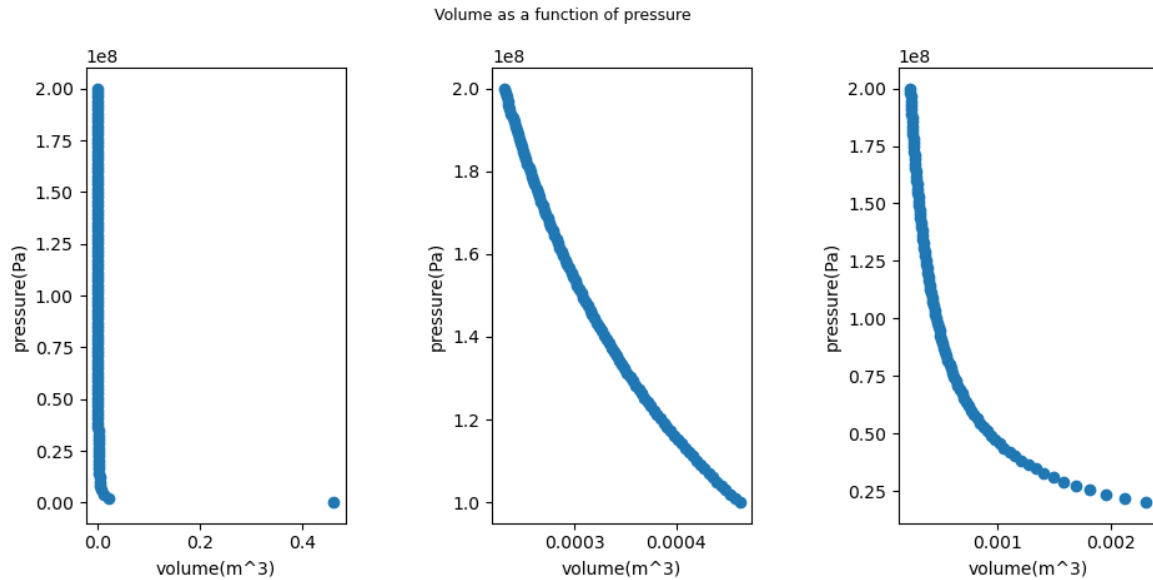


Fig2

The first plot shows the change in volume from 200MPa to atmospheric pressure. The plot in the middle shows the change in volume from 200MPa to 100MPa . The leftmost plot tracks the volume as the pressure is reduced 10 fold. All plots show increments of 100 evenly spaced values from starting pressure to minimum pressure.

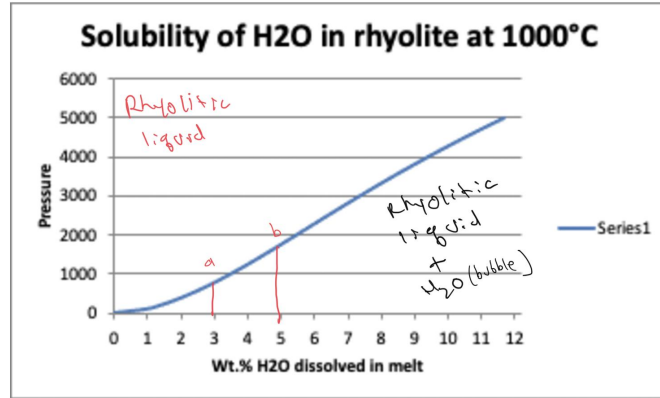
We can see that once the bubble reaches atmospheric pressure it “erupts”.

The presence of water favors the formation of granites rather than the eruption to the surface due to water decreasing the melting temperature of the magma. Therefore there will be partial melting in the crust and eventually fractional crystallization.

Obsidian bypasses explosive eruption because of rhyolitic melt with low wt% H_2O . If this were the case then most of the volatiles would have had to leave the melt. Since we know obsidian is glassy, this indicates a lack of crystals that formed.

Post Discussion Report

Looking back at the pressure vs wt% graph we can see the line that separates the melt mixture with vapor, where H₂O is exsolved.



We also see how much of the H₂O is exsolved between two points, where in this case it's 2wt% H₂O now in bubble form. If we integrate between the two points we can also see how much work energy is required to exsolve the gas.

$$W = Fd = \int_a^b P dX_{H_2O} = nRT \int_a^b \frac{dX_{H_2O}}{V}$$

We can see that work increases greatly as pressure continues to decrease and this excess energy will eventually create a more explosive eruption (assuming conservation of mass in the magma body for a closed system).

From the definition of density $\rho_i = \frac{m_i}{V_i}$ (where ρ_i is the density of the compound/mixture) we can figure out volume%. Given wt% we can say that

$$\sum_i^n \rho_i = \frac{\sum_i^n X_i}{V_{total}}$$

We hold V_{total} constant in order to determine its value and let $X_i = wt\%_i$, and $v_i = \frac{X_i}{\rho_i}$.

The volume fraction will be

$$\frac{1}{V_{total}} \sum_i^n \frac{X_i}{\rho_i} = 1$$

So then we can determine vol% using

$$vol\% = \frac{v_i}{V_{total}} \times 100$$

Where $V_{total} = \sum_i^n v_i$. Given that $X_{1,2} = 5.5\% H_2O, 94.5\% RAH$ (where RAH is the mixture of rhyolite without H_2O) and $\rho_{1,2} = 1100 \frac{kg}{m^3} H_2O, 2500 \frac{kg}{m^3} Rhyolite$ we can determine that the vol% is approximately 14.20% H_2O and 89% RAH.

A few things to consider though, in this calculation I'm assuming the density of water to be $1100 \frac{kg}{m^3}$ at 200MPa in the shallow crust. Considering ρ_{H_2O} is approximately $1000 \frac{kg}{m^3}$, at atmospheric pressure, I believe this assumption is reasonable. With this information however, we can see that the vol% of the exsolved gas would be anywhere between 5-10% for a decrease in pressure of 100 MPa!

We also know that molar fraction by definition is

$$m_i = \frac{X_i/M_i}{\sum_j^n X_j/M_j},$$

Where $\sum_i^n m_i = 1$, and M is molecular mass. Then $mol\%_i = m_i \times 100$. From this we can determine that the mol% of H_2O in the Rhyolite mixture is about 13.5%. The composition values used are below.

compound	H2Or	sio2	tio2	al2o3	fe2o3	feo	mgo	cao	na2o
wt%(mass)	5.5	73.49	.18	13.55	0.36	0.976	0.5	1.43	3.36
Molar mass	18	46	113	74	100	42	28	36	38
density(g/cm^3)	1.1	2.65	4.23	3.95	5.24	5.74	3.58	3.34	2.27

From these equations we can see that if we had 1wt% H_2O then with a 100 MPa decrease in pressure there will be volume flux of much less than 1%. If we assume a much higher density for water at 200MPa or an even smaller initial wt% H_2O the volume flux becomes even smaller. Then under these conditions bubbles formed will be minimal therefore buoyancy will decrease overall and the magma will take its time to ascend and ultimately not erupt violently. These conditions favor the formation of obsidian. Another scenario to consider would be the obsidian forming post-eruption after most of the H_2O in the melt has exsolved.

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