Crystals and Bubbles in a Melt

Edgar Carrillo

January 25, 2022

1 Introduction

The question to consider is: Are crystal sinking and bubble rising important processes during the cooling and crystallization of lavas, considering the physical properties of magmas and their components?

For this problem, we consider a basaltic lava flow that is 40 ft (12.192 m) thick. We first assume the lava temperature is approximately 1200°C. Then we consider the properties of lava that can affect its cooling: viscosity, density, and thermal conductivity. Basaltic lava has low viscosity and therefore flows faster. The viscosity can range from 10 to 100 Pa·s and density can range from 2650 to 2800 kg/m^3. We also consider Fourier's law for heat conduction.

2 Fourier's Law for Heat Conduction

$$q = -k\frac{dT}{dx} \tag{1}$$

where k is thermal conductivity, q is heat flux, and $\frac{dT}{dx}$ is the thermal gradient.

Assuming the heat flux is constant and using a temperature gradient (i.e. $q = -k\Delta T$), we assume a thermal conductivity of approximately 0.6 W/(m·K). We also know that the lava will fully solidify at 800°C. We also consider Newton's law of cooling.

3 Newton's Law of Cooling

Newton's law of cooling can be expressed as:

$$\frac{dT}{dt} = -k(T - T_{\text{env}}) \tag{2}$$

where T is the temperature of the object, T_{env} is the ambient temperature, and k is a positive constant, and $h = \frac{q}{\Delta T}$.

So then,

$$\int dT = \int -k(T - T_{\text{env}})dt \tag{3}$$

Finally,

$$T(t) = T_{\text{env}} + (T_0 - T_{\text{env}})e^{-kt} \tag{4}$$

where:

- $T(t)$ is the temperature of the object at time t.
- T_{env} is the ambient (or surrounding) temperature.
- T_0 is the initial temperature of the object at time $t = 0$.
- k is a positive constant related to the cooling rate.

4 Cooling of Magma

Considering the equations and the values above, we see that the cooling of magma is related to the thermal conductivity and is on the order of minutes to days for basaltic lava flows. This indicates fast cooling.

5 Stokes' Law and Particle Settling

We now consider Stokes' law and the excess gravitational force due to weight and buoyancy of a particle in the melt. The drag force F_d and the gravitational force F_g are given by:

$$F_d = 6\pi\eta r v \tag{5}$$

$$F_g = \frac{4}{3}\pi r^3(\rho_s - \rho_l)g \tag{6}$$

where η is the dynamic viscosity, r is the radius of the particle, v is the velocity of the particle, ρ_s is the density of the solid, ρ_l is the density of the liquid, and g is the acceleration due to gravity.

For lava flow not on a steep slope, we can assume a speed of 0.27 m/s. We can, therefore, assume $F_d = F_g$ and

$$v = \frac{2}{9}\frac{\rho_p - \rho_l}{\mu}gR^2 \tag{7}$$

6 Settling Velocity of Crystals

From the settling equation, we can infer that the crystals (assuming $\rho_p = 3500$ kg m^3 and $\rho_l = 2600$ kg m^3) will take about a month to settle in a 10 m thick melt. The sinking velocity is proportional to the density contrast between the solid and the liquid (denser minerals sink faster than less dense ones), inversely proportional to the viscosity (a crystal would sink faster in basaltic magma than in more viscous granitic magma), and proportional to the square of the diameter of the grain. Thus, if the crystals have time to sink, the cooling was slow and the crystals will be larger, resulting in coarser-grained rock.

7 Viscosity and Crystal Growth

If we see more crystals (and bubbles) in the melt, it is because the melt had high viscosity. For flows with lower viscosity, we will see less crystal growth (and we know basaltic lava flow has low viscosity). Bubbles in magma will increase viscosity and therefore facilitate crystallization.