Computer Engineering 4DK4
Lab 3
Call Blocking in Circuit Switched Networks

This lab first investigates call blocking in a circuit switched network. Call arrivals occur to a cellular basestation containing N communication channels. If a channel is available when a call arrives, it is assigned to the call for its duration, otherwise the call is blocked. A computer simulation is used to assess the blocking behaviour of the system with Poisson process call arrivals and exponentially distributed call holding times. The simulation results are compared to theory using the Erlang B formula. The case is then considered where incoming calls are not blocked but permitted to queue up until service is available. This system is simulated and compared to theory using the Erlang C formula. Finally, code is written that simulates a company that operates a fleet of N taxis that service incoming call requests. Results are obtained that assess the behaviour of the system.

1 Preparation

Download an electronic copy of the simulation code.

2 Experiments

1. As in the other labs, first familiarize yourself with the code and with running the simulation. Make sure that you understand how this simulation works. As before, the parameters for the simulation runs are in `simparameters.h`. *In all the experiments make sure you include runs using your McMaster student ID number as the random number generator seed.*

2. Using the provided simulation code, generate a set of curves that show the tradeoffs between blocking probability, offered load (in Erlangs) and the number of channels. Curves of this kind were presented earlier in class. (See the #6 pdf lecture notes, "Circuit Switching Performance", 2nd last page.)

   Compare your results to that obtained using the Erlang B formula. According to Erlang B, the probability that a call is blocked is given by

   $$P_B = \frac{A^N/N!}{\sum_{i=0}^{N} A^i/i!}$$

   where A is the offered load (in Erlangs) and is given by

   $$A = \lambda h$$

where λ is the call arrival rate (in calls per unit time, e.g., calls/minute), h is the average call holding time (in the same time units, e.g., minutes), and N is the total number of channels. Write a program to perform this computation (Feel free to use any programming language you want, e.g., Matlab, C, etc.) Include a listing of your program in your writeup. Check the results of your program using one of many online Erlang B calculators. Just google search for "Erlang B calculator". Include the URL of the calculator that you used.

3. Cellular networks are often designed so that under busy traffic conditions, the highest acceptable probability of blocking is about 1% (i.e., the probability of rejecting a cell-phone call is 1% because the cellular network has no available channels.) Using either the provided simulation code or the Erlang B formula, generate a graph of maximum offered loading (in Erlangs) versus the number of cellular channels needed to achieve a blocking probability of 1.5%.

4. In a cellular system such as that considered in Part 2, assume that callers never hang up when all channels are found busy, but instead, they wait until their call can be served. Make some changes to the supplied code so that you can simulate this case.

   In this case, the probability of an arriving call having to wait for service can be derived as

$$P_w = \frac{A^N/N!}{A^N/N! + (1-\rho)\sum_{i=0}^{N-1} A^i/i!}. \tag{1}$$

   where $\rho = A/N$. Note that since there is no blocking in this system, we must have $\rho < 1$ for stability. Equation 1 is referred to as the Erlang C Formula and is commonly used to design telephone call centres. It can be shown that the average caller waiting time is given by

$$T_w = \frac{P_w h}{N(1 - A/N)} \tag{2}$$

   and the probability that a call has to wait less than t seconds is given by

$$W(t) = 1 - P_w e^{-(N-A)t/h}. \tag{3}$$

   Select a set of system parameters and compare output from your simulation to that obtained from $W(t)$.

5. Consider a company that operates a fleet of N taxis. We can assume that customer calls arrive according to a Poisson process. When a taxi is assigned to a new call, assume that the time that it takes to arrive at the customer is random and exponentially distributed with a mean of W minutes (i.e., waiting time).

   Once the taxi arrives at a waiting customer, assume that the time to transport the customer to his/her destination is random and exponentially distributed with a mean of D minutes (i.e., delivery time). Note that a taxi is only assigned to a customer if the taxi is free, i.e., it has not been assigned to a customer. Otherwise, the call is rejected.

   If a called taxi takes too long to arrive, the customer may have given up and is no longer there when the taxi arrives. Assume that the time from when the call is made until a customer gives up and leaves without service is random and exponentially distributed with a mean of G minutes (i.e., give-up time).

Simulate the above system and show results for the probability of blocking and the probability of a taxi arriving to find that a customer has left. Use some reasonable parameter values to illustrate the behaviour of the system.

3 Submission

1. Submission format: Each group (maximum of 2 students) must submit a single report.
2. Methodology and data: The writeup must clearly describe all steps performed, include all collected data, and list the random number generator seeds used to produce the graphs.
3. Plots and code: The writeup must include all required plots and a listing of any code modifications.
4. Coverage of results: Unless otherwise specified, performance data and plots must cover the full range of system operation (from low to high traffic load) and show all asymptotes.
5. Analysis of results: For each part, the writeup must include (i) observations from the generated curves and (ii) clear explanations of the trends shown.
6. Plot quality: Plots must be smooth and representative; piecewise linear plots and overfitted curves are not acceptable.