Università di Genova

Corso di Laurea Magistrale in Ingegneria Meccanica Progettazione e Produzione
Robotics engineering

Smart coupled systems for sensing and actuation

Exercise 01 – Piezoelectric shunt

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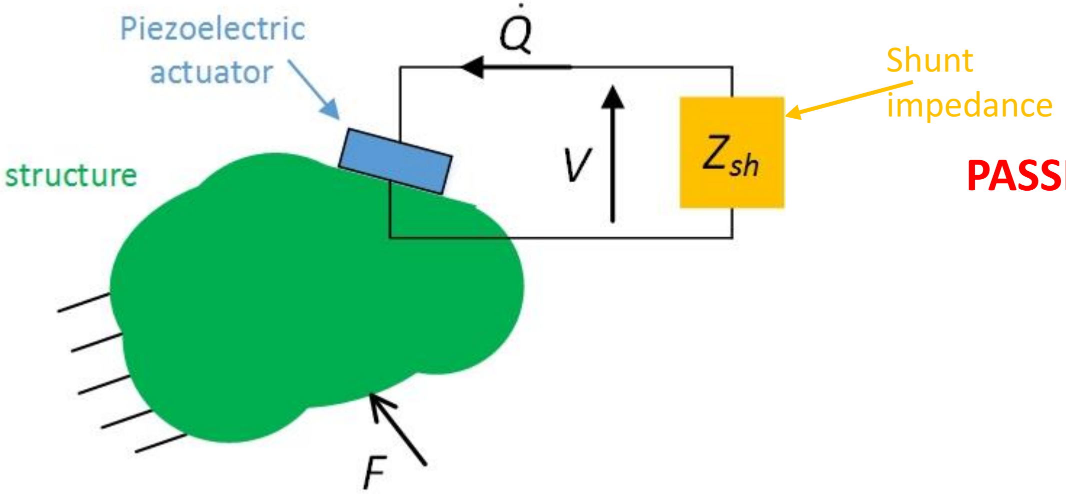
Introduction

Vibration control by means of piezoelectric shunt

Vibration control of structures by means of piezoelectric actuators connected to properly designed electrical networks

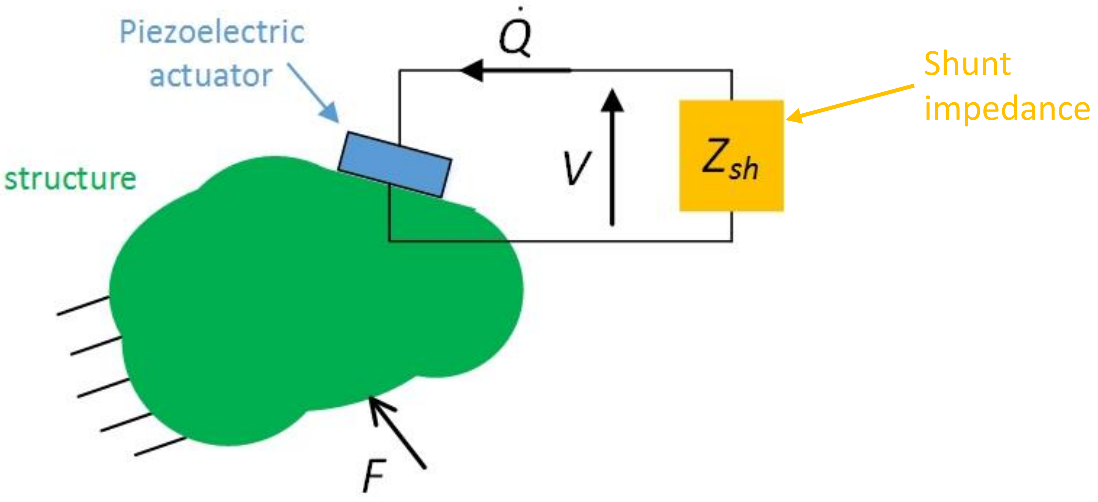


PASSIVE

Depending on the impedance layout, it is possible to focus the control action on a specific target

Piezo-shunt: optimisation

- Work on the controller: shunt impedance

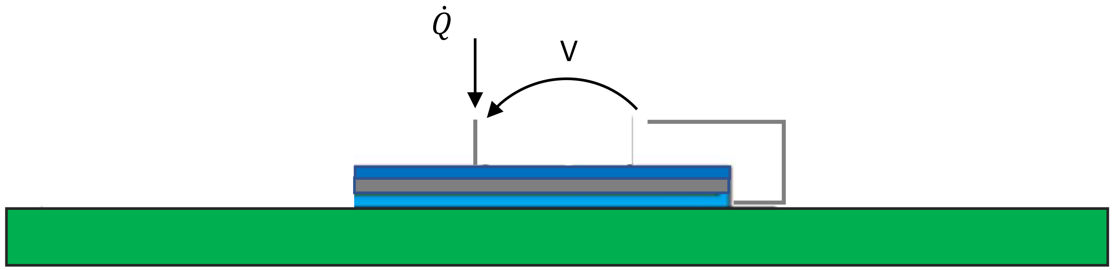


Single mode control

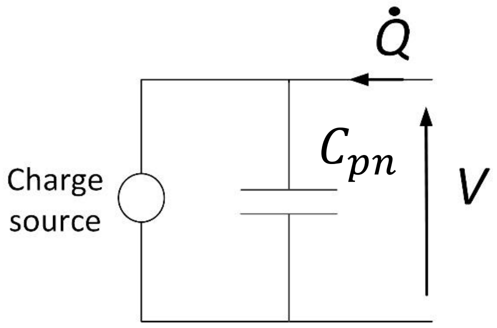
- resistive — R
- resonant — R+L

Introduction

Modal model



Piezoelectric actuator



C_{pn} Modal capacitance:
blocked capacitance C_∞ + a static correction

$$
\begin{cases}
\ddot{q}_i + 2\xi_i\omega_i\dot{q}_i + \omega_i^2 q_i - \chi_i V = F_i \qquad \forall i \in \{1, \dots, n\} \\
C_{pn}V - Q + \sum_{i=u}^{n} \chi_i q_i = 0
\end{cases}
$$

Displacement U of any point of the structure

$$U(x,t) = \sum_{i=1}^{\infty} \phi_i(x) q_i(t)$$

Introduction

Modal model

$$\begin{cases} \ddot{q}_i + 2\xi_i\omega_i\dot{q}_i + \omega_i^2 q_i - \chi_i V = F_i \qquad \forall i \in \{1, \dots, n\} \\ C_{pn}V - Q + \sum_{i=u}^{n} \chi_i q_i = 0 \end{cases}$$

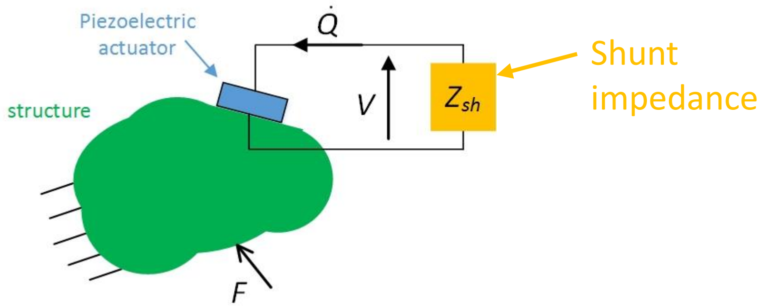
Displacement U of any point of the structure

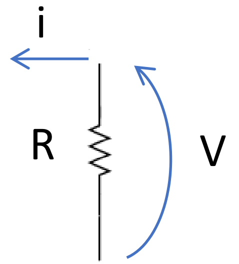
$$U(x,t) = \sum_{i=1}^{\infty} \phi_i(x) q_i(t)$$

$$\alpha_{jk}(\omega) = \frac{x_j}{F_k(\omega)} = \sum_{r=1}^{n} \frac{\varphi_{j,r}\varphi_{k,r}}{({\omega_{0,r}}^2 - \omega^2) + i(2\zeta_r\omega_{0,r}\omega)}$$

j= measurement point
k= forcing point
r= mode

Piezo-shunt: resistive





Made from a single resistance *R*

$$Z_{sh} = \frac{V}{I} = -R \qquad \bar{V} = \dot{\bar{Q}} R C_{pi} = -\tau \dot{\bar{Q}}$$

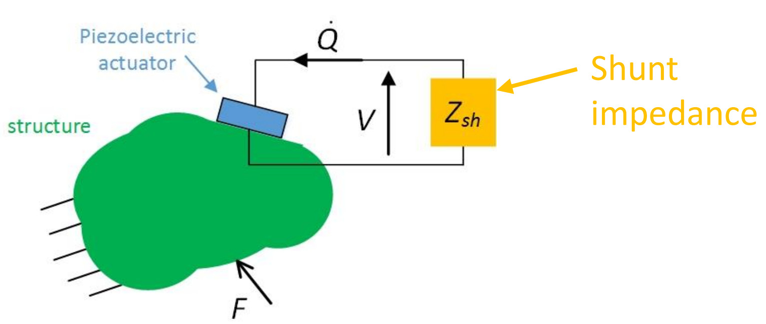
$$\begin{cases} \ddot{q}_i + 2\xi_i\omega_i\dot{q}_i + \omega_i^2 q_i - \omega_i k_i \bar{V} = F_i \\ \ddot{q}_i + 2\xi_i\omega_i\dot{q}_i + \omega_{oc}^2 - \omega_i k_i \bar{Q} = F_i \\ \bar{V} - \bar{Q} + \omega_i k_i q_i = 0 \end{cases}$$

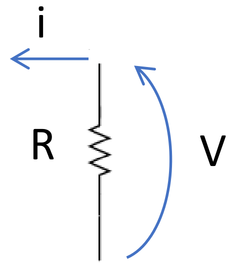
$$\omega_{oc}^2 = \widehat{\omega}_i^2 = \omega_i^2\left(1 + k_i^2\right)$$

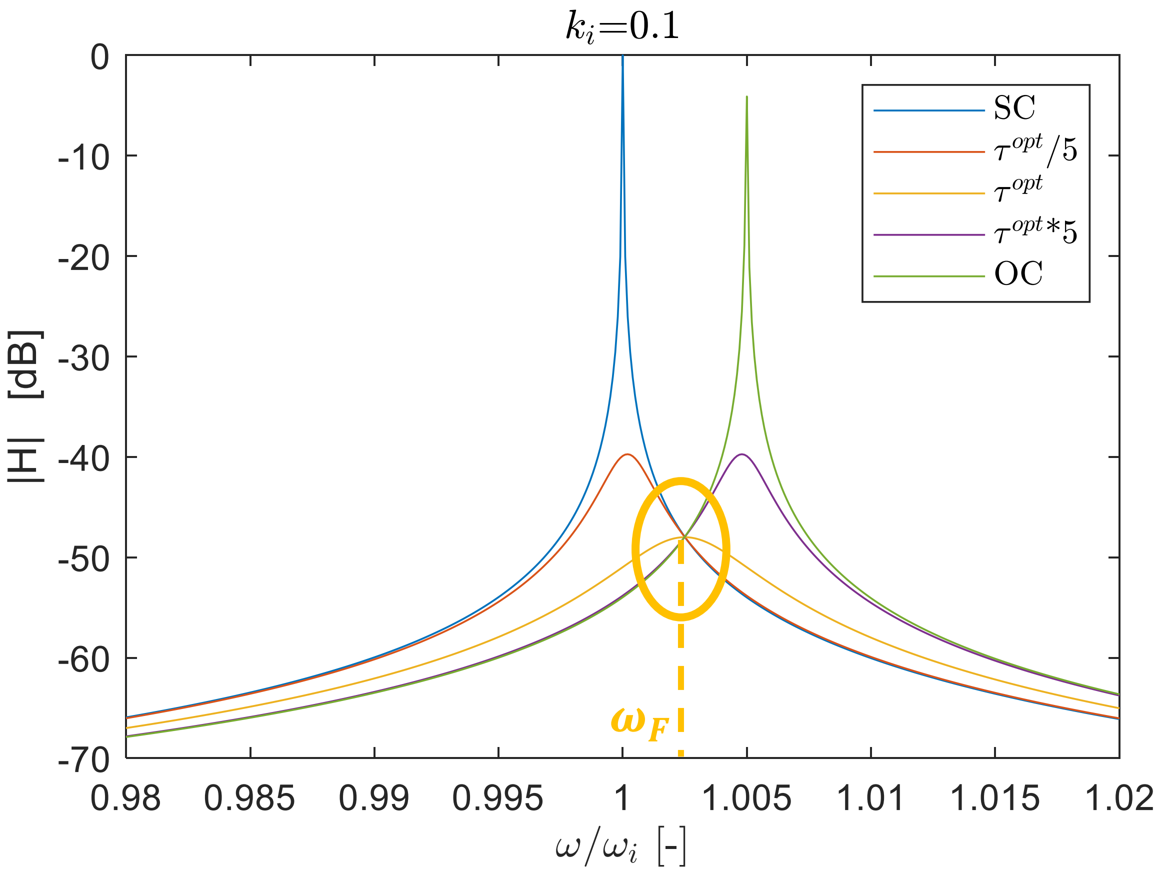
$$H(\mathrm{j}\omega) = \frac{1 + \mathrm{j}\omega\tau}{\omega_i^2 - \omega^2(1 + 2\tau\xi_i\omega_i) + \mathrm{j}\omega(\tau\widehat{\omega}_i^2 + 2\xi_i\omega_i - \tau\omega^2)}$$

Piezo-shunt: resistive

$$H(\mathrm{j}\omega) = \frac{1 + \mathrm{j}\omega\tau}{\omega_i^2 - \omega^2(1 + 2\tau\xi_i\omega_i) + \mathrm{j}\omega(\tau\widehat{\omega}_i^2 + 2\xi_i\omega_i - \tau\omega^2)}$$





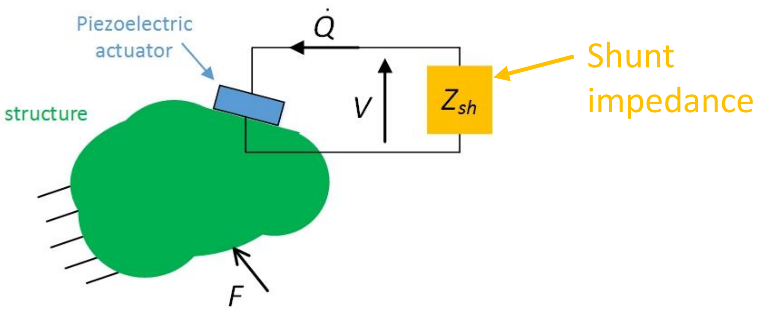


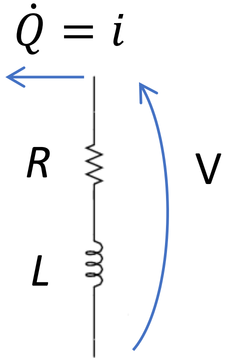
$$\boxed{\tau^{opt} = \frac{1}{\omega_F} = \frac{1}{\omega_i\sqrt{1 + k_i^2/2}}}$$

Piezo-shunt: resonant

Made from the series of a resistance *R* and an inductance *L*

$$Z_{sh} = \frac{V}{i} = -(R + j\omega L)$$





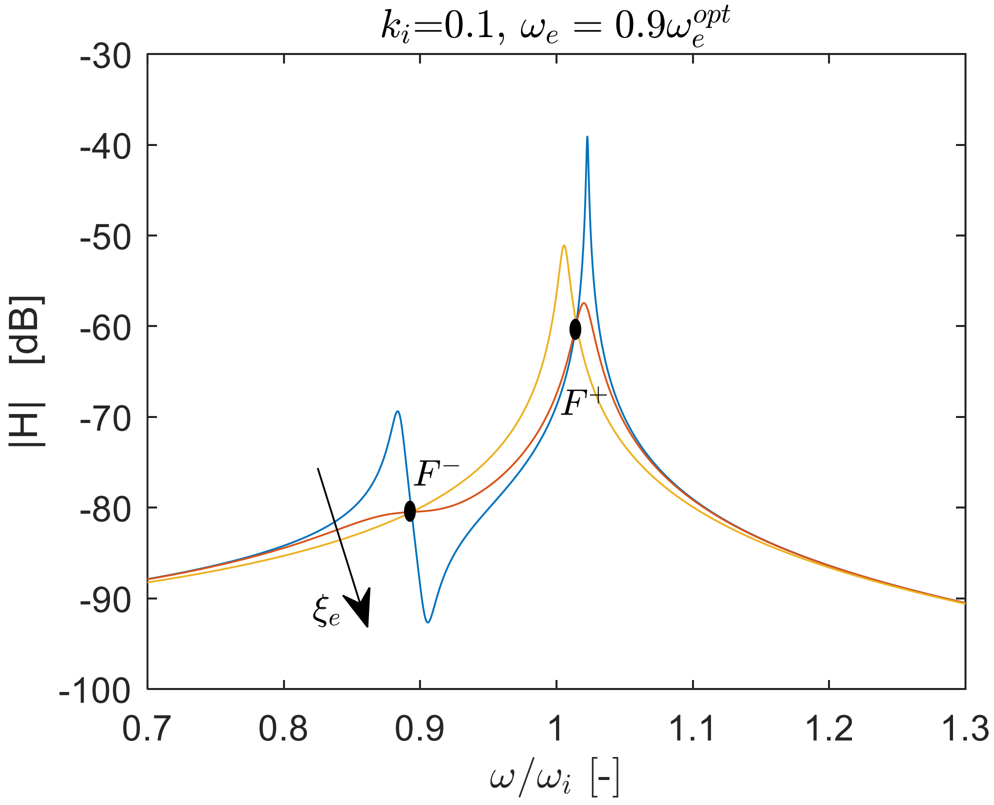
$$\begin{cases} \ddot{q}_i + 2\xi_i\omega_i\dot{q}_i + \omega_i^2 q_i - \omega_i k_i \bar{V} = F_i \\ \ddot{q}_i + 2\xi_i\omega_i\dot{q}_i + \widehat{\omega}_i q_i - \omega_i k_i \bar{Q} = F_i \\ \bar{V} - \bar{Q} + \omega_i k_i q_i = 0 \end{cases}$$

$$\begin{cases} \omega_e = \dfrac{1}{\sqrt{LC_{pi}}} \\ \xi_e = \dfrac{RC_{pi}\omega_e}{2} \end{cases}$$

$$H(\omega) = \frac{q_i}{F_i} = \frac{-\omega^2 + \omega_e^2 + 2j\xi_e\omega_e\omega}{\omega^4 - \omega^2\left(\omega_e^2 + 4\xi_i\xi_e\omega_i\omega_e + \widehat{\omega}_i^2\right) + j\omega\left(2\xi_e\omega_e\left(\widehat{\omega}_i^2 - \omega^2\right) + 2\xi_i\omega_i\left(\omega_e^2 - \omega^2\right)\right) + \omega_i^2\omega_e^2}$$

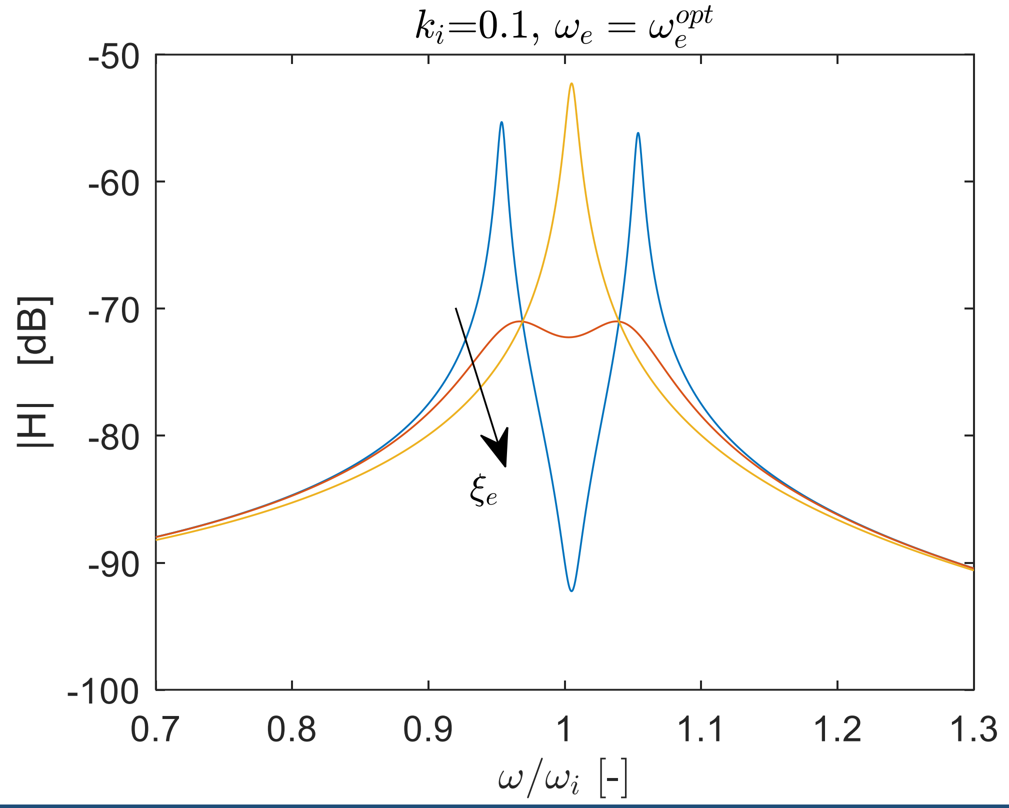
Piezo-shunt: resonant

$$H(\omega) = \frac{-\omega^2 + \omega_e^2 + 2j\xi_e\omega_e\omega}{\omega^4 - \omega^2\left(\omega_e^2 + 4\xi_i\xi_e\omega_i\omega_e + \widehat{\omega}_i^2\right) + j\omega\left(2\xi_e\omega_e\left(\widehat{\omega}_i^2 - \omega^2\right) + 2\xi_i\omega_i\left(\omega_e^2 - \omega^2\right)\right) + \omega_i^2\omega_e^2}$$



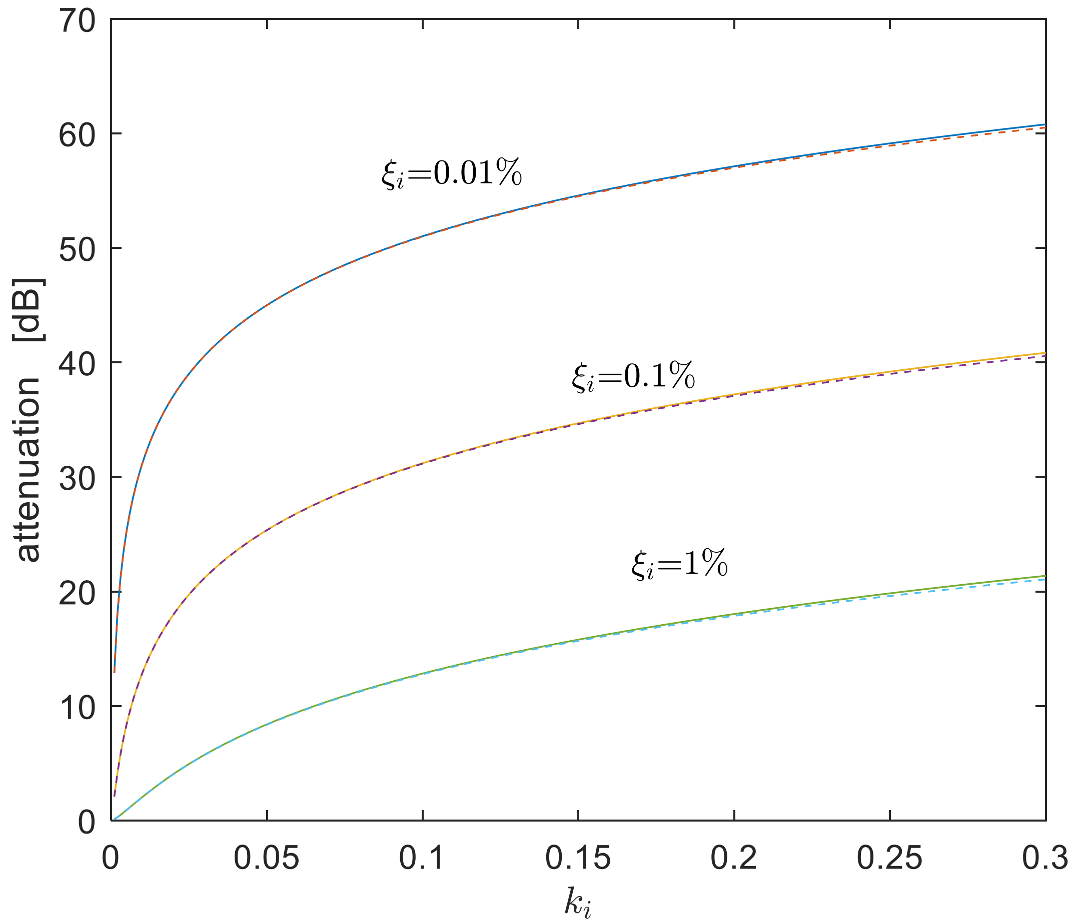
$\boldsymbol{\omega_e}$ / *L*

$\boldsymbol{\xi_e}$ / *R*



Piezo-shunt: resonant

Resonant impedances



	Series connection
$\omega_e =$	$\sqrt{\dfrac{1}{LC_{\text{pi}}}}$
$\xi_e =$	$\dfrac{R}{2}\sqrt{\dfrac{C_{\text{pi}}}{L}}$

	Series connection
$\omega_e^{opt} =$	$\widehat{\omega}_i$
$\xi_e^{opt} =$	$\dfrac{\sqrt{3}}{2}\sqrt{\dfrac{\widehat{\omega}_i^2 - \omega_i^2}{\widehat{\omega}_i^2 + \omega_i^2}}$

Piezo-shunt: data

File containing the data of the considered system:

- Data.mat

The numbers 1, 2 and 3 associated to the vaiables refer to three different systems. For each system the following variables are provided.

As an example, for system 1:

- PHI: matrix of the mode shapes (equal for the 3 systems)
- Cpi_tot: modal capacitance associated to each mode (equal for the 3 systems)
- nf1_sc: natural frequencies in short circuit expressed in Hz
- nf1_oc: natural frequencies in open circuit expressed in Hz
- csi1: adimensional damping ratios

Piezo-shunt: exercise

1. Plot the FRF of the system. On the same plot represent:
   - the FRF of each single mode
   - the FRF obtained as the sum of the modes

2. Plot the FRF of the system in short and open circuit on the same figure

3. Calculate the electro-mechanical coupling coefficient for each mode

4. For each mode of the system (one at time):
   - Tune a resistive shunt
   - Tune a resonant shunt
   - Plot the controlled FRF, the SC FRF and the OC FRF on the same graph

5. Repeat point from 2 to 4 for the second and third set of data