6.1 Figure P6.1 shows a flywheel with moment of inertia $J = 0.5\,\text{kg-m}^2$ that is initially rotating at an angular velocity $\dot{\theta}_0 = 40\,\text{rad/s}$. The flywheel is subjected to friction, which is modeled by linear viscous friction torque $b\dot{\theta}$, with friction coefficient $b = 0.06\,\text{N-m-s/rad}$. Use Simulink to obtain the dynamic response and plot the angular position $\theta(t)$ (in rad) and angular velocity $\dot{\theta}(t)$ (in rad/s). In addition, use the simulation to integrate the rate of energy dissipation $\dot{\xi}$ and plot dissipated energy (in J) vs. time. Show that the total dissipated energy as computed by the simulation is equal to the system's initial energy.

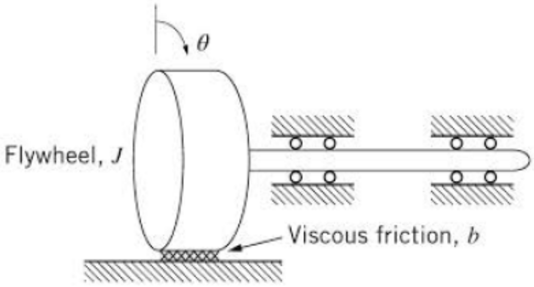

Figure P6.1

6.4 Figure P6.4 shows a mass–spring–damper mechanical system. The system is initially at rest. At time $t = 0$, a pulse force is applied to the mass. The pulse force is $F_a(t) = 8\,\text{N}$ for $0 \le t \le 0.05\,\text{s}$ and $F_a(t) = 0$ for $t > 0.05\,\text{s}$. The system parameters are $m = 2\,\text{kg}$, $b = 5\,\text{N-s/m}$, and $k = 200\,\text{N/m}$. Simulate the dynamic response using MATLAB commands and plot the response $z(t)$.

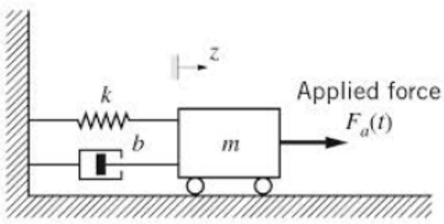

Figure P6.4

6.5 Given a system's SSR

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -34 & -2 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 2.5 \end{bmatrix}u \qquad y = \begin{bmatrix} 1 & 0 \end{bmatrix}\mathbf{x}$$

Use MATLAB commands to obtain the dynamic response for a step input $u(t) = 0.6U(t)$. The initial states are $x_1(0) = 2$ and $x_2 = -1.5$. Plot the output vs. time, $y(t)$.

6.10 A series RL circuit with a nonlinear inductor is shown in Fig. P6.10. Recall that the following nonlinear function for inductor current was used in Problem 3.11 in Chapter 3

$$I_L(\lambda) = 97.3\lambda^3 + 4.2\lambda \quad \text{(amps, A)}$$

where λ is the flux linkage. Use Simulink to obtain the dynamic response for current $I_L(t)$ if the source voltage is a 4-V step function, that is, $e_{\text{in}}(t) = 4U(t)$ V. The RL circuit has zero energy stored at time $t = 0$ and the resistance is $R = 1.2\ \Omega$. Plot current I_L vs. time.

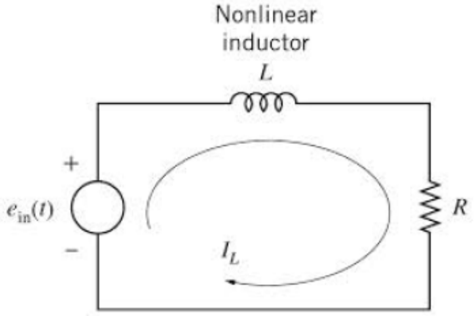

Figure P6.10

6.12 Consider again the nonlinear system from Problem 5.5 in Chapter 5:

$$\dot{x}_1 - x_2 = 0$$
$$\dot{x}_2 + 2x_1^{1/4} + 3x_2 = u$$

The initial conditions are $x_1(0) = 0.08$, $x_2(0) = 0.02$, and the input is $u = 1.01$ (constant).

a. Simulate the nonlinear system using Simulink to obtain the state responses $\mathbf{x}(t) = \begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}^T$. Plot $x_1(t)$ and $x_2(t)$ on the same figure.

b. Linearize the system about the static equilibrium state vector $\mathbf{x}^*$ that arises when the nominal input is $u^* = 1$ (see Problem 5.5). Use Simulink to simulate the linear model and obtain the approximate state response $\mathbf{x}(t) = \mathbf{x}^* + \delta\mathbf{x}(t)$. Plot the nonlinear state solutions [from part (a)] and linearized state solutions on the same figure. Comment on the accuracy of the linear solution.

6.19 Figure P6.19 shows the wind turbine generator system from Example 2.8 in Chapter 2. The system inputs are aerodynamic torque T_{aero} (from the wind) and generator (or electrical) torque T_{gen}. The system parameters for a 3000 kW wind turbine generator are

Turbine moment of inertia $J_1 = 1.26(10^7)$ kg-m^2
Turbine radius (blade tip to hub) $R = 45.6$ m
Turbine friction coefficient $b_1 = 1100$ N-m-s/rad
Generator moment of inertia $J_2 = 240$ kg-m^2
Generator friction coefficient $b_2 = 0.1$ N-m-s/rad
Gear ratio $N = r_2/r_1 = 1/93$

The aerodynamic torque is a function of air density ρ (in kg/m^3), turbine radius R (in m), wind speed V_w (in m/s), and torque coefficient C_q

$$T_{aero} = \frac{1}{2}\rho\pi R^3 V_w^2 C_q$$

Assume that air density is $\rho = 1.225$ kg/m^3 and the torque coefficient is $C_q = 0.075$. At time $t = 0$, the wind speed is $V_w = 13$ m/s. The generator torque is a function of generator shaft speed: $T_{gen} = 158.7\dot{\theta}_2$ (N-m).

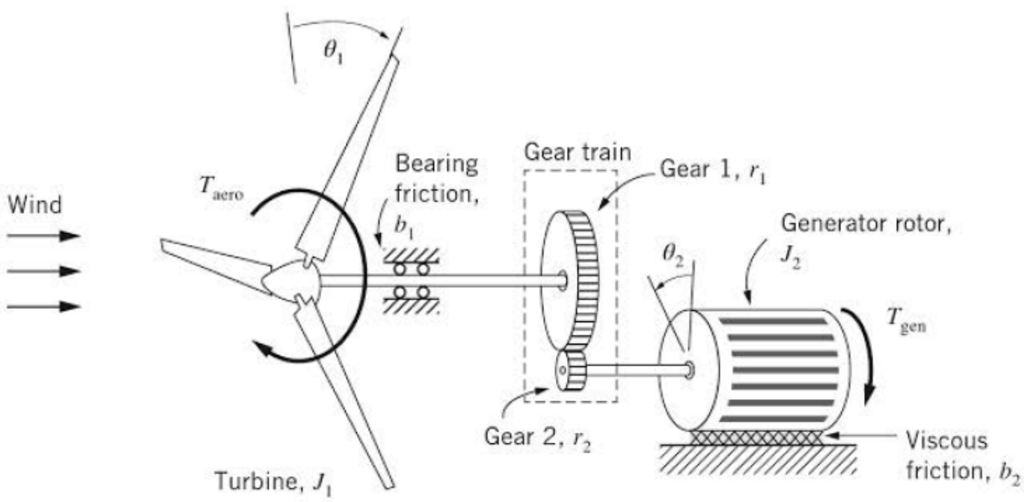

Figure P6.19

6.24 Figure P6.24 shows a pneumatic servomechanism [4]. The total length of the cylinder is 10 cm and the piston position x is measured relative to the mid-point of the cylinder (hence, when $x = 0$ the piston is at the middle of the cylinder). This highly nonlinear system has been linearized about a nominal pressure, volume, and piston position ($x = 0$) and the resulting system transfer function is

$$G(s) = \frac{7.003\,(10^6)}{s(s^2 + 28.3s + 15{,}890)} = \frac{X(s)}{U(s)}$$

where $x(t)$ is the piston/load mass position (in m) and $u(t)$ is the spool-valve position (in m). See Reference 4 for details in developing the linear model.

a. Use Simulink to obtain the dynamic response of the piston/load mass $x(t)$ for a pulse input for valve position: $u(t) = 0.001$ m (or 1 mm) for $0 < t \leq 0.05$ s, and $u(t) = 0$ for $0.05 < t \leq 0.5$ s. At time $t = 0$ the piston is initially at rest at the mid-point position. Plot the time response of the piston/load mass position (in cm) for the 1-mm pulse input.

b. Use your simulation to determine the maximum pulse duration of a 1-mm valve displacement (i.e., a feasible piston position response). Plot the piston response $x(t)$ for this maximum pulse input.

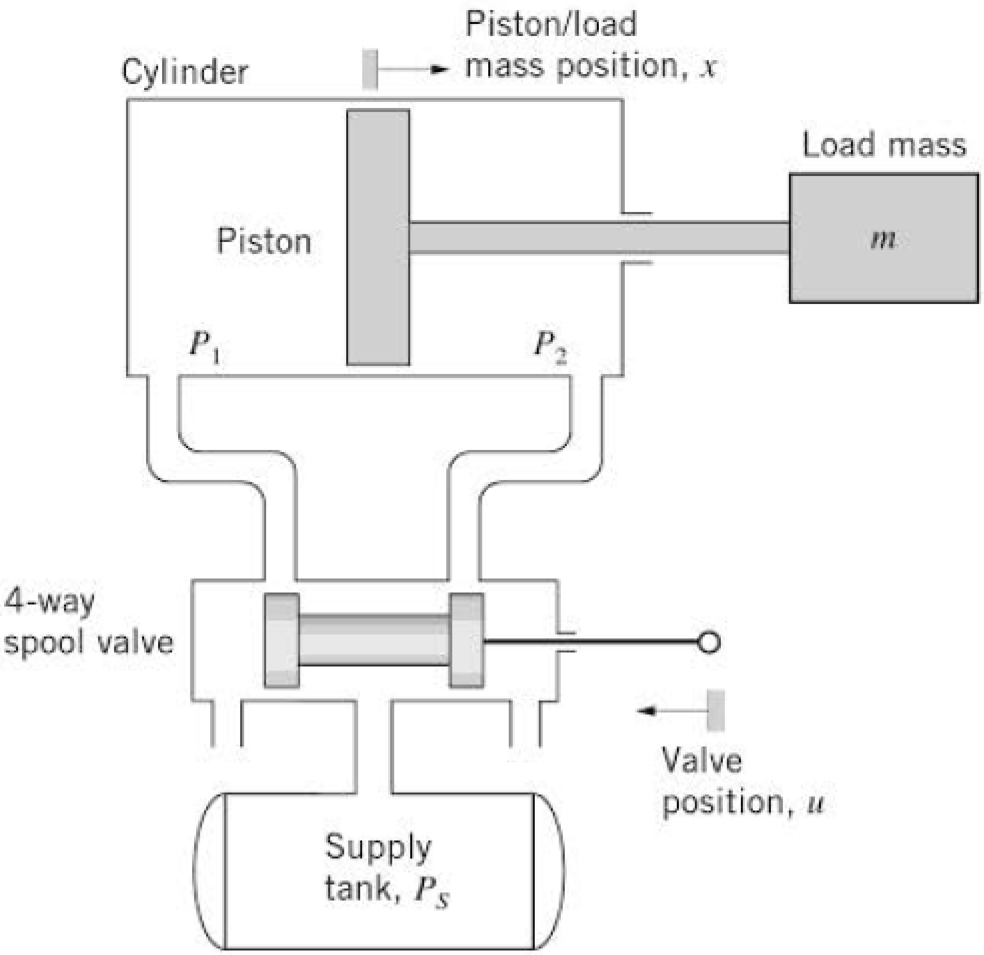

Figure P6.24

6.25 Figure P6.25 shows the thermal system (interior office room with baseboard heater) from Example 4.7 in Chapter 4. The system parameters are [5]

Total thermal capacitance $C = 4.83(10^5)$ J/°C
North wall thermal resistance $R_1 = 0.041$ °C-s/J
South wall thermal resistance $R_2 = 0.151$ °C-s/J
East wall thermal resistance $R_3 = 0.108$ °C-s/J
West wall thermal resistance $R_4 = 0.209$ °C-s/J
Ceiling thermal resistance $R_5 = 0.240$ °C-s/J
Floor thermal resistance $R_6 = 0.159$ °C-s/J

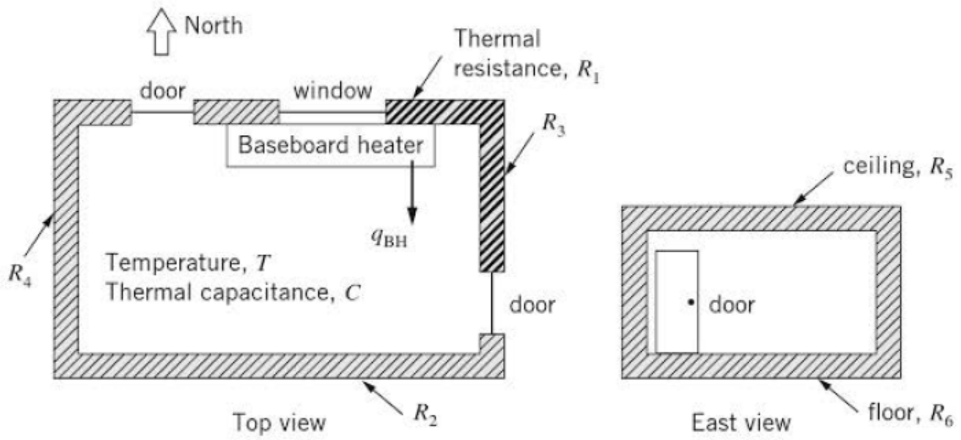

Figure P6.25

The interior room temperature is initially equal to the ambient temperature, which is $T_a = 10$ °C (or, 50 °F). At time $t = 0$, the baseboard heater supplies a constant heat input $q_{BH} = 1000$ W(1 kW). Obtain the temperature response using Simulink and plot room temperature T vs. time (in hrs) for a simulation time of 18 hrs. What is the room temperature (in °C and °F) at 18 hrs?