Math 306 - HW6

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1)

Let $\theta = y_1$ so $y_1' = y_2$

$\theta_2 = y_2$ $\quad y_2' = \sin(t) - \sin(y_1) - y_2$

$y_1(0) = \pi/4, \quad y_2(0) = 0$

$\Delta t = \pi$

Euler method:

$0 \leq i \leq 4$

$$(y_1)_{i+1} = (y_1)_i + \pi (y_2)_i$$
$$(y_2)_{i+1} = (y_2)_i + \pi\left(\sin(t_i) - \sin((y_1)_i) - (y_2)_i\right)$$

$$(y_1)_0 = \frac{\pi}{4}, \quad (y_2)_0 = 0$$

$$(y_1)_1 = \frac{\pi}{4} + \pi(0) = \frac{\pi}{4}$$

$$(y_2)_1 = 0 + \pi\left(0 - \frac{1}{\sqrt{2}} - 0\right)$$

$$\boxed{(y_2)_1 = \frac{\pi}{\sqrt{2}}}, \quad \boxed{(y_2)_1 = \frac{\pi}{4}}$$

4)

$$h = (\Delta x) = \frac{10-5}{(n-1)} = \frac{5}{4} = 1.25$$

$$x = [5,\ 6.25,\ 7.5,\ 8.75,\ 10]$$

$$y' = \frac{dy}{dx} = \frac{y_{i+1} - y_{i-1}}{2h} = \frac{y_{i+1} - y_{i-1}}{2.5}$$

$$y'' = \frac{d^2}{dx^2} = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

equation becomes:

$$y'' + y' + y = x^2$$

$$\left(\frac{y_{i+1} - 2y_i + y_{i-1}}{(1.25)^2}\right) + \left(\frac{y_{i+1} - y_{i-1}}{2.5}\right) + y_i = x_i^2$$

$$y_i = \left(\frac{-(x_i)^2 + 6.0625\, y_{i+1} + 1.09375\, y_{i-1}}{(1.09375)}\right)$$

$i = 2$ to 4

$$y_1 = y(x_1) = y(5) = 0$$

$$y_5 = y(x_5) = y(10) = 1$$

We can get all the y values by solving the above equations for all i.