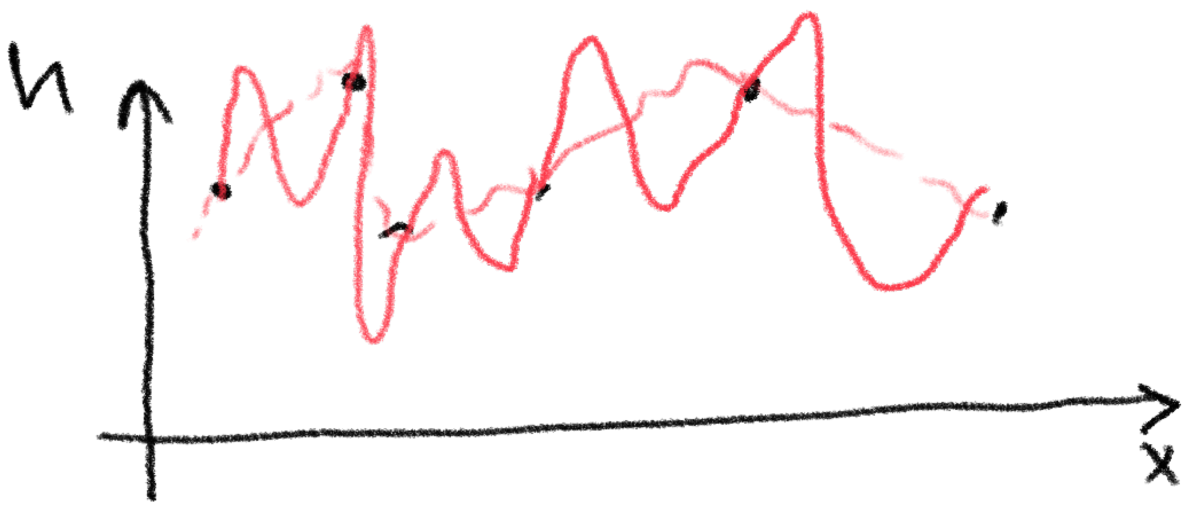
Radial Function Interpolation

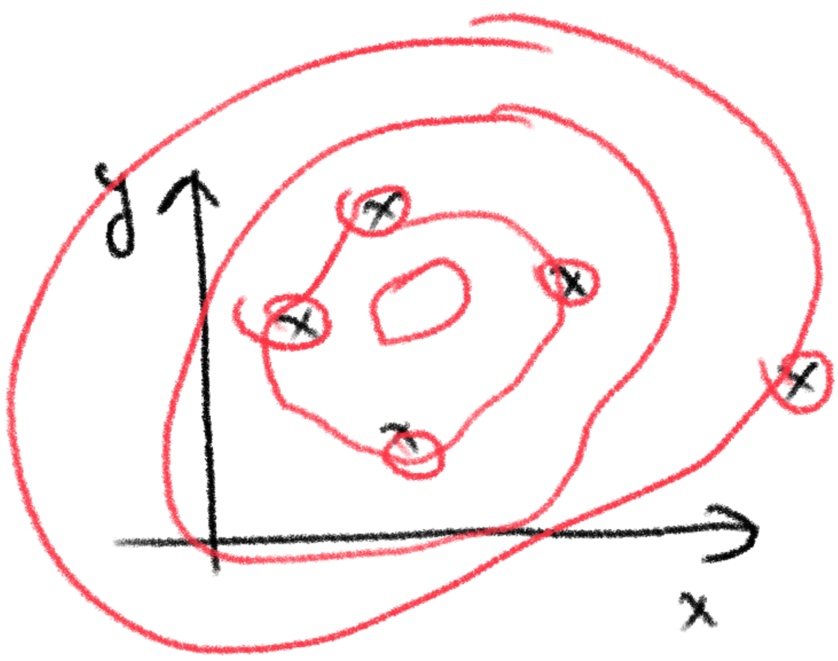
in 1D

$x_1 \quad x_2 \quad , \ldots \quad x_n$

$u_1 \quad u_2 \quad \ldots \quad u_n$



In 2D



Basis function (we choose)

$\phi_1(x,y) \quad \phi_2(x,y) \quad \ldots \quad \phi_r(x,y)$

Assume $u(x,y)$ is a linear

combination

$$u(x,y) = c_1 \phi_1(x,y) + c_2 \phi_2(x,y) + \dots + c_n \phi_n(x,y)$$

Interpolation conditions

$$u(x_1, y_1) = \sum c_j \phi_j(x_1, y_1)$$

$$u(x_2, y_2) = \sum_j c_j \phi_j(x_2, y_2)$$

$$\vdots$$

$$u(x_n, y_n) = \sum_j c_j \phi_j(x_n, y_n)$$

$$\begin{bmatrix} u(x_1, y_1) \\ u(x_2, y_2) \\ \vdots \\ u(x_n, y_n) \end{bmatrix} = \begin{bmatrix} \phi_1(x_1, y_1) & \phi_2(x_1, y_1) & \dots & \phi_n(x_1, y_1) \\ \phi_1(x_2, y_2) & \phi_2(x_2, y_2) & \dots & \phi_n(x_2, y_2) \\ \vdots & & & \\ \phi_1(x_n, y_n) & & & \phi_n(x_n, y_n) \end{bmatrix} \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix}$$

$$u = \Phi(x,y)\, c$$

$$c = \Phi^{-1} u$$

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1) Basis func

2) Build the Vandermonde Matrix

$$\Phi = \begin{pmatrix} \phi_1(x_1, y_1) & \cdots & \phi_n(x_1, y_1) \\ \vdots & & \\ \phi_1(x_n, y_n) & & \phi_n(x_n, y_n) \end{pmatrix}$$

3) Solve linear system

$$\Phi c = u$$

4) Given new x, y

$$u(x,y) = \sum c_j \phi_j(x, y)$$

Radial Basis Function

$$\phi_j(x, y, x_j, y_j) =$$

$$\exp\left(-\frac{1}{\sigma^2}\left((x - x_j)^2 + (y - y_j)^2\right)\right)$$