

# Integrating Factors

## Goal: Solve First Order ODE's

Integrating factors is a way to solve ODE's of the form

$$y' + r(t)y = q(t)$$

Can do it only when we can compute some integrals (see next).

You can use your favorite integral calculator.

## Integrating factors - Theory

$$y' + r(t)y = q(t)$$

Let

$$\mu(t) = \exp \left( \int r(t) dt \right)$$

Then, note that

$$\frac{d}{dt} (\mu(t)y) = \mu(t)y' + y\mu'(t) = \mu(t)y' + r(t)\mu(t)y = \mu(t) (y' + r(t)y)$$

# Integrating factors - Theory

Use this observation to write

$$\mu(t)(y' + r(t)y) = \mu(t)q(t)$$

$$(\mu(t)y)' = \mu(t)q(t)$$

$$\mu(t)y = \int \mu(t)q(t)dt + C$$

$$y = \mu(t)^{-1} \int \mu(t)q(t)dt + C\mu(t)^{-1}$$

Get  $C$  from initial conditions.

- Good only for first order linear ODE's
- Must be able to evaluate integrals
- Very useful for simple problems

For more see

<https://tutorial.math.lamar.edu/classes/de/linear.aspx>

# Integrating factors - Examples

Solve the ODE

$$y' + \lambda y = k \quad y(0) = 1$$

How does the equation behaves for different  $\lambda$

# Homework

from

*<https://tutorial.math.lamar.edu/classes/de/linear.aspx>*

do examples 3-6