Parameter Estimation

Given a system of ODE's that depend on parameters, find the parameters given the date

Example

predator pray model

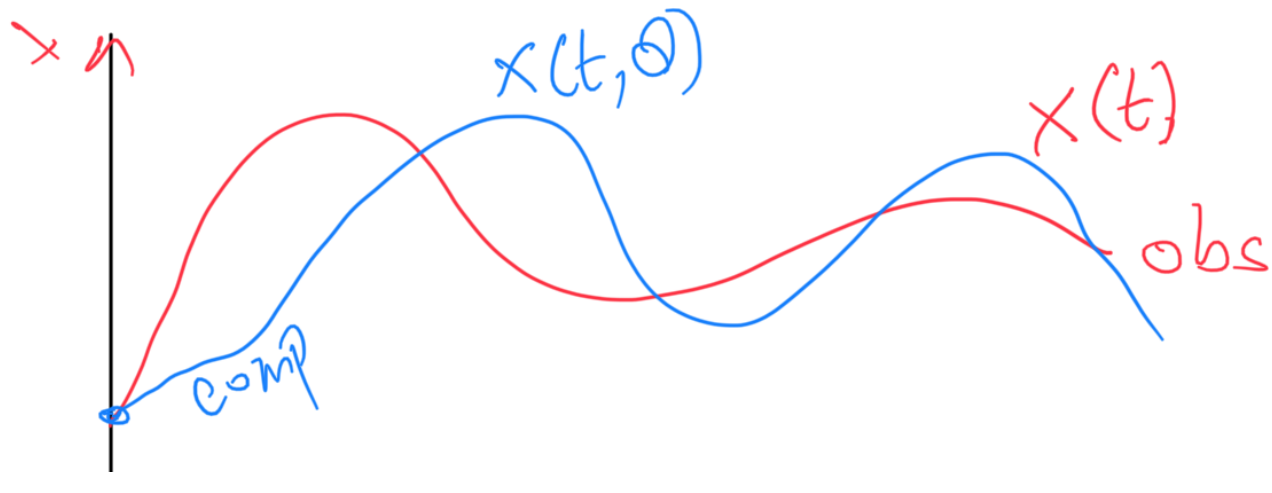
$$\begin{cases} \frac{dx}{dt} = \alpha x - \beta xy \\ \frac{dy}{dt} = \delta xy - \gamma y \end{cases}$$

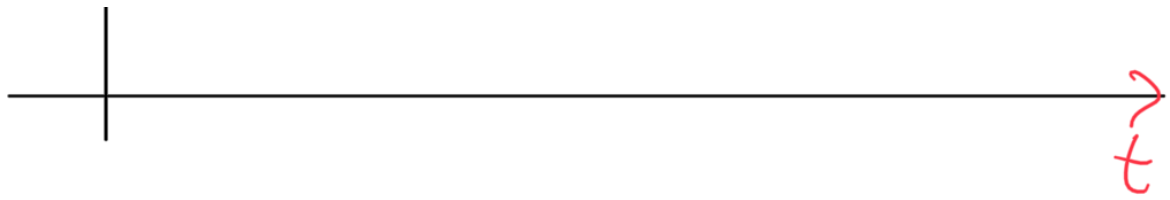
x - rabbits

y - foxes

Given a curve $x(t)$ $y(t)$

estimate $\alpha, \beta, \gamma, \delta$





$$\text{loss} = \frac{1}{2}\sum_{j=1}^{n} \left| x(t_j, \theta) - x^{obs}(t_j) \right|^2$$

to find θ

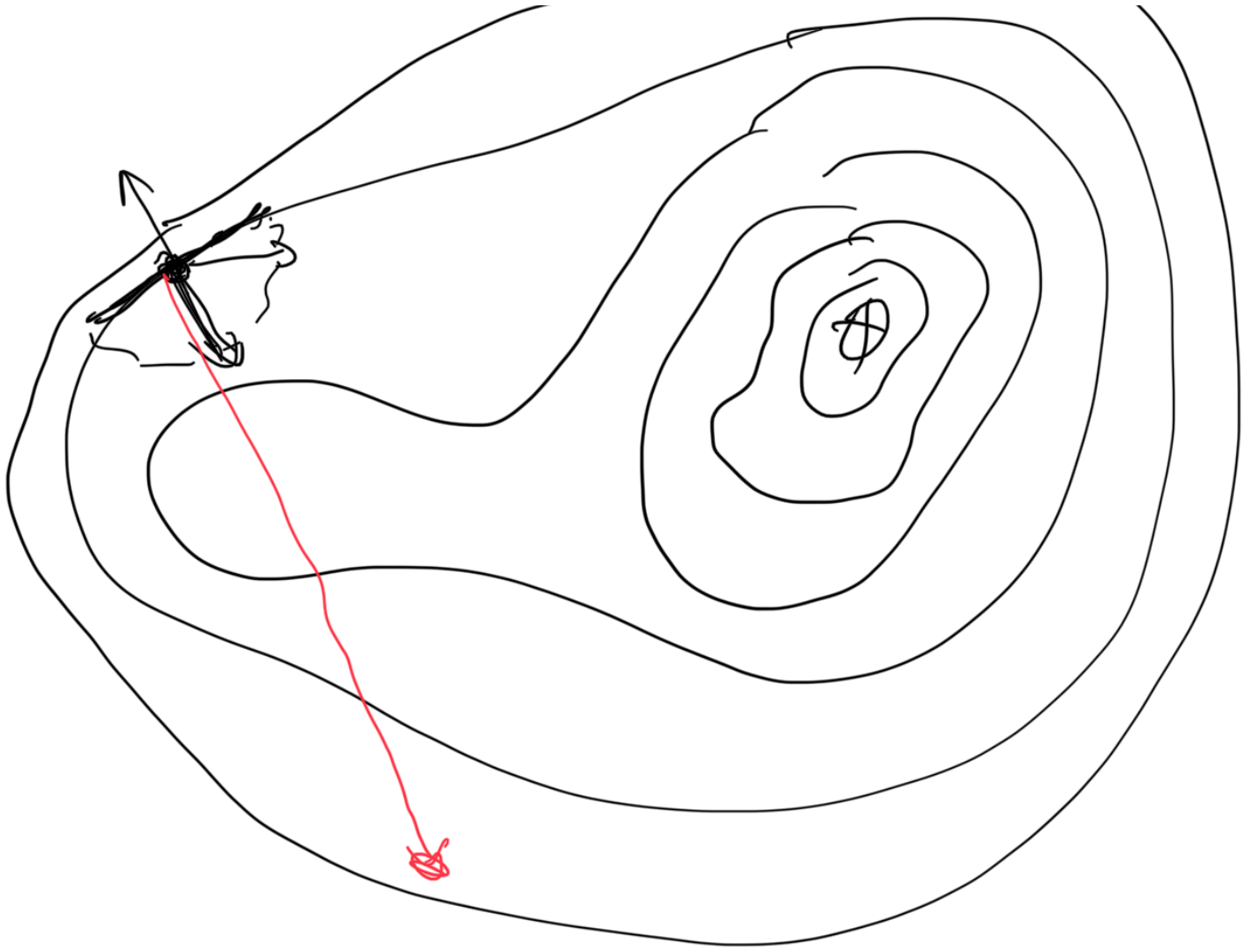
$$\min_{\theta} \frac{1}{2}\sum_{j=1}^{n} \left| x(t_j, \theta) - x^{obs}(t_j) \right|^2$$

$$\frac{\partial}{\partial \theta} \text{loss}(\theta) = 0 \quad \text{(approx)}$$

Solving an optimization problem

$$\min_{\theta} f(\theta)$$

$$\nabla_\theta f(\theta) = \begin{pmatrix} \frac{\partial f}{\partial \theta_1} \\ \frac{\partial f}{\partial \theta_2} \\ \vdots \\ \frac{\partial f}{\partial \theta_n} \end{pmatrix} = 0$$



$$f(\theta + \delta\theta) \approx f(\theta) + \nabla f(\theta)^T \delta\theta + \text{HOT}$$

Choose $\delta\theta$ that makes $f(\theta + \delta\theta)$ as small as possible

Choose the direction (because the approx is linear)

direct determined by

$$\nabla_\theta f^T \delta\theta =$$

$$\frac{\partial E}{\partial \theta_1}\delta\theta_1 + \frac{\partial E}{\partial \theta_2}\delta\theta_2 + \cdots + \frac{\partial E}{\partial \theta_n}\delta\theta_n$$

if $\min \nabla f^T \delta\theta \Rightarrow \delta\theta = -\frac{\nabla f}{\|\nabla f\|}$

$\|\delta\theta\| = 1$

Algorithm for solving an optimization problem

$\min f(\theta)$

1) Choose θ_0

2) For $i = 1 \ldots$

compute $\nabla f(\theta_i)$

set $\theta_{i+1} = \theta_i - \mu_i \cdot \nabla f(\theta_i)$

until ???