Robust Adaptive Beamforming

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***Abstract*—Adaptive beamforming algorithms can be extremely sensitive to slight errors in array characteristics. Errors which are uncorrelated from sensor to sensor pass through the beamformer like uncorrelated or spatially white noise. Hence, gain against white noise is a measure of robustness. A new algorithm is presented which includes a quadratic inequality constraint on the array gain against uncorrelated noise, while minimizing output power subject to multiple linear equality constraints. It is shown that a simple scaling of the projection of tentative weights, in the subspace orthogonal to the linear constraints, can be used to satisfy the quadratic inequality constraint. Moreover, this scaling is equivalent to a projection onto the quadratic constraint boundary so that the usual favorable properties of projection algorithms apply. This leads to a simple, effective, robust adaptive beamforming algorithm in which all constraints are satisfied exactly at each step and roundoff errors do not accumulate. The algorithm is then extended to the case of a more general quadratic constraint.**

I. Introduction

THE purpose of this paper is to present an improved recursive algorithm for adaptive beamforming which includes both multiple linear equality constraints and a quadratic inequality constraint. The quadratic inequality constraint may be used to ensure that the beamformer is robust, not highly sensitive to small amplitude, phase, or position errors. It limits signal suppression effects and limits the growth of the adaptive weights which is important in digital implementations. Thus, it controls sensitivity to tolerance errors. The significant new feature of the algorithm is the way in which this inequality constraint is implemented so that all constraints are satisfied exactly at each time step and roundoff errors do not accumulate. We call the new algorithm the "scaled projection algorithm" since it involves projection of the tentative weight updates onto the subspace which is orthogonal to the linear constraints followed by a scaling in that subspace, if necessary, to satisfy the quadratic inequality constraint.

The potential for using adaptive beamforming to improve the performance of sensor arrays was recognized in the early 1960's in the fields of sonar [1]–[6], radar [7]–[10], and seismic [11]–[16] signal processing. It soon became apparent that a variety of formulations of optimum detection and estimation problems gave rise to the same spatial processor [17]–[22]. The basic concept is to use measured background spatial correlation characteristics to reject noise and interference [23]–[27], thereby improving beam output signal-to-noise ratio. In order for the processor to reject noise without rejecting signal, it is necessary to make some assumptions about the signal characteristics. The problem of mismatch arises when the true signal characteristics differ from the assumed ones. A robust processor should be relatively insensitive to small errors in the assumed signal characteristics.

Many important sources of error which occur in physical systems are approximately uncorrelated from sensor to sensor and degrade system performance in a way which is similar to adding a corresponding amount of uncorrelated or spatially white noise to each sensor. Thus, the array gain against spatially white noise ("white noise gain") is a measure of robustness and its reciprocal is a measure of sensitivity to tolerance errors. A very similar problem had arisen much earlier in the context of superdirective transmitting arrays. Indeed, in the mid-1950's, Gilbert and Morgan [28] and Uzsoky and Solymár [29] specifically included a robustness constraint when maximizing the geometric gain of transmitting arrays. Geometric gain in a transmitting array is mathematically equivalent to array gain against spherically isotropic noise (directivity) in a receiving array.

The use of recursive algorithms for adaptive beamforming also dates back to the 1960's. Important early contributions were made by Shor [5], Widrow [30], Griffiths [32], and Lacoss [31]. Multiple linear constraints are used in adaptive beamforming to ensure unity response to a unit signal from the boresight signal direction, to control mainlobe shape, and to place beampattern nulls in selected directions. They were introduced by Booker and Ong [33] and were included by Frost [34] in his well-known adaptive beamforming algorithm. Kooij, in his doctoral thesis [35], used a penalty function approach in conjunction with Frost's algorithm in order to control the white noise gain in a recursive adaptive beamformer. The new scaled projection algorithm uses Frost's approach for handling the multiple linear equality constraints and includes a novel approach to handling the quadratic inequality constraint.

For a given problem, there is usually a close relationship between the optimum beamformer based on known covariances, and an adaptive beamformer which adjusts its parameters based on the input data without prior assumptions concerning the noise covariance structure. In a stationary situation, the steady-state performance of a good adaptive beamformer should be close to that of the

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optimum beamformer for the same problem. Also, the structure of an adaptive beamformer is frequently based on that of the corresponding optimum beamformer. Techniques for adaptive beamforming fall into two basic categories: "estimate and plug," and recursive algorithms. The estimate and plug procedure simply uses estimates in the place of known covariances in the optimum beamformer structure. Recursive algorithms for adaptive beamforming use a stochastic gradient or steepest descent approach to obtain weights in a manner similar to which a nonstochastic gradient algorithm can be used to obtain weights for the optimum beamformer with known covariance structure. Therefore, it is helpful to review results for optimum beamformers in order to establish the basis of the improved adaptive beamforming algorithm.

Stochastic algorithms [36], more sophisticated than steepest descent, may be used analogous to their use in deterministic optimization problems, but their consideration is beyond the scope of this paper. A number of authors [37]–[49] have discussed various aspects of the effects of errors on adaptive beamforming.

II. Notation

The general structure of the problem of interest is shown in the block diagram of Fig. 1. For simplicity, an element level frequency domain formulation of the problem will be used. The relationship to other formulations is discussed by Vural [50].

The array of interest consists of M sensors of known but arbitrary geometry. The complex narrow-band output of the mth sensor in a particular frequency bin is $x_m(t)$. The sensor outputs can be aggregated into a column vector $\boldsymbol{x}(t)$. The cross-spectral density matrix of the vector of sensor outputs multiplied by the bin width is

$$R = E(\boldsymbol{x}\boldsymbol{x}^*) \tag{1}$$

where E denotes expectation and the asterisk denotes complex conjugate transposition, so that $\boldsymbol{x}^*$ is the row vector $(x_1^*, x_2^*, \cdots, x_M^*)$. The complex scalar output of the beamformer at time t is $z(t)$. It is obtained from a weighted sum of the inputs. The complex weight applied to the mth sensor output is w_m^* and $\boldsymbol{w}^*$ is the row vector of weights $(w_1^*, w_2^*, \cdots, w_M^*)$. Thus,

$$z = \boldsymbol{w}^*\boldsymbol{x}. \tag{2}$$

The power output spectral density multiplied by the bin width is

$$E\left(|z|^2\right) = \boldsymbol{w}^*R\boldsymbol{w} = \sigma_z^2. \tag{3}$$

When a planewave signal of strength σ_s^2 from direction θ impinges on the array, R will include a term of the form $\sigma_s^2\boldsymbol{d}(\theta)\,\boldsymbol{d}^*(\theta)$, where $\boldsymbol{d}^*(\theta)$ is a row vector of phase delays to align the sensor outputs for a signal from direction θ for the specific array geometry under consideration. That is,

$$\boldsymbol{d}^*(\theta) = \left[\cdots, \exp\left(i\left(\frac{\omega}{c}\right)s(\theta)\cdot \boldsymbol{p}_m\right), \cdots\right] \tag{4}$$

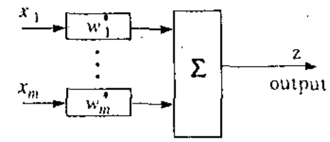


Fig. 1. General beamformer structure.

where $s(\theta)$ is a unit vector in the direction of propagation, and $\boldsymbol{p}_m$ is the vector of coordinates of the mth sensor. It is sometimes useful to decompose R into signal and noise components, as follows:

$$R = \sigma_s^2\boldsymbol{d}\boldsymbol{d}^* + \sigma_n^2 Q. \tag{5}$$

The noise cross-spectral matrix Q is normalized to have its trace equal to the number of sensors M so that σ_s^2/σ_n^2 is the input signal-to-noise spectral ratio averaged across the M sensors.

The array gain G is the improvement in signal-to-noise ratio due to beamforming, that is,

$$G = \frac{|\boldsymbol{w}^*\boldsymbol{d}|^2}{\boldsymbol{w}^*Q\boldsymbol{w}}. \tag{6}$$

The numerator of (6) is called the signal response, and the denominator is called the noise response. Several special cases of the noise cross-spectral matrix Q are of interest. When the noise is spatially white or uncorrelated from sensor to sensor, Q becomes the identity matrix I and the array gain becomes what is called the "white noise gain," that is,

$$G_w = \frac{|\boldsymbol{w}^*\boldsymbol{d}|^2}{\boldsymbol{w}^*\boldsymbol{w}} \le M. \tag{7}$$

The sensitivity of array gain to signal mismatch can be examined by considering the signal to be perturbed by small zero mean random errors with normalized covariance matrix A, so that the expected signal cross-spectral matrix becomes $\sigma_s^2[\boldsymbol{d}\boldsymbol{d}^* + \xi A]$ where ξ is a strength parameter [41]. The fractional sensitivity S of array gain to these random errors is

$$S = \left(\frac{dG/d\xi}{G}\right) = \frac{\boldsymbol{w}^*A\boldsymbol{w}}{|\boldsymbol{w}^*\boldsymbol{d}|^2} = \frac{1}{G_A}. \tag{8}$$

The fractional sensitivity is equal to the reciprocal of the array gain against noise with the covariance A of the random errors. When the errors are uncorrelated, the sensitivity is equal to the reciprocal of the white noise gain ($S_w = G_w^{-1}$). S_w is a classic measure of sensitivity to tolerance errors [28], [29], [40], [41]. The white noise gain is a useful and convenient measure of robustness.

When the noise is spherically isotropic, the noise matrix will be denoted by Q_g to emphasize its dependence on array geometry. The corresponding array gain obtained by using Q_g in (6) is called the geometric gain or directivity

$$G_g = \frac{|\boldsymbol{w}^*\boldsymbol{d}|^2}{\boldsymbol{w}^*Q_g\boldsymbol{w}}. \tag{9}$$

The Q-factor or supergain ratio [10], [29], [51] is an alternative measure of sensitivity to errors which arises naturally in the theory of superdirective arrays. It is defined as the ratio of geometric gain to white noise gain, that is,

$$\frac{G_g}{G_w} = \frac{w^*w}{w^*Q_g w}. \tag{10}$$

The quantity $\rho = G/G_w$ has been called the generalized supergain ratio [41] since it generalizes the supergain ratio to an arbitrary noise matrix. It arises in analyses of the effects of noise perturbations on beamformers with fixed weights.

III. Optimization

In this section, the relationships among a number of optimization problems are discussed. This provides the foundation for later discussions of recursive adaptive algorithms as well as providing historical perspective.

A. *Unconstrained Array Gain*

The beamformer, which maximizes the improvement in signal-to-noise ratio or array gain, is optimum for a variety of detection and estimation problems [17]–[21]. The problem is formulated as follows:

$$\operatorname*{Max}_{w} \frac{|w^*d|^2}{w^*Qw}.$$

The well-known solution is

$$w = \alpha Q^{-1} d \tag{11}$$

where "α" is an arbitrary complex constant. Choosing α to produce unity signal response with zero phase shift so that $w^*d = 1$ yields

$$w = \frac{Q^{-1}d}{d^*Q^{-1}d}. \tag{12}$$

This form of the weight vector was used in early seismic work [12], [15], where noise estimates were obtained before and after the signal arrival. The resulting array gain is

$$G = d^*Q^{-1}d. \tag{13}$$

The expression for G may be written in terms of the eigenvalues λ_j and eigenvectors e_j of Q as

$$G = \sum_{j=1}^{M} \frac{|d^*e_j|^2}{\lambda_j}. \tag{14}$$

The optimization is seen to emphasize projections of the direction vector d onto eigenvectors associated with small eigenvalues. When model imperfections exist, difficulties arise if too much reliance is placed on very small projections of the signal.

White noise gain is maximized by the conventional beamformer $w = d/M$, for which $G_w = M$, and the array gain is

$$G = \frac{M^2}{d^*Qd}. \tag{15}$$

B. *Single Boresight Look Direction Constraint*

A practical difficulty with unconstrained array gain optimization is that the signal and noise powers and the noise cross-spectral matrix Q are not known. In many applications, what is measurable is the signal-plus-noise matrix R. An important suggestion by Levin and others [13], [14], [21] was to minimize the total output power, which is directly measurable, subject to a constraint of unity undistorted signal response from the desired look direction. That is,

$$\operatorname*{Min}_{w} w^*Rw \quad \text{subject to } w^*d = 1.$$

This problem may be seen to be mathematically equivalent to the unconstrained array gain optimization problem since (w^*Rw) is a linear combination of the numerator and denominator of (6), and the numerator has been constrained. The solution is

$$w = \frac{R^{-1}d}{d^*R^{-1}d}. \tag{16}$$

When R is given by (5), the equivalence of (16) and (12) may be shown using matrix identities [43]. The inclusion of the signal in R leads to signal suppression in the face of tolerance errors, or mismatch between the true and assumed signal spatial characteristics. This effect was analyzed in detail in [43].

C. *White Noise Gain or Robustness Constraint*

This generalization to an arbitrary noise cross-spectral matrix Q of the problem addressed by Gilbert and Morgan [28] was applied to adaptive beamforming by Cox [41]. The problem is to maximize array gain subject to an equality constraint on the white noise gain, that is,

$$\operatorname*{Max}_{w} G \quad \text{subject to } G_w = \delta^2 \le M.$$

The constraining value δ^2 must be chosen less than or equal to the maximum possible white noise gain M for the problem to be self-consistent.

Equivalently, one can minimize $((1/G) + \epsilon(1/G_w))$ where ϵ is a Lagrange multiplier. This leads to

$$\operatorname*{Min}_{w} \frac{w^*Qw}{|w^*d|^2} + \frac{\epsilon w^*w}{|w^*d|^2} = \operatorname*{Min}_{w} \frac{w^*(Q+\epsilon I)w}{|w^*d|^2}. \tag{17}$$

The solution, which is normalized for unity response to a unit boresight signal, in analogy to (12), is

$$w = \frac{(Q+\epsilon I)^{-1}d}{d^*(Q+\epsilon I)^{-1}d} \tag{18}$$

where ϵ is adjusted to satisfy the white noise gain constraint. It is seen to involve adding ϵ to the diagonal elements of Q which adds ϵ to each eigenvalue without modifying the eigenvectors. The old ad hoc technique of adding a small amount to each diagonal element prior to matrix inversion is actually the optimum procedure for this problem. The Lagrange multiplier ϵ provides a continuous monotonic parameterization between the unconstrained optimum ($\epsilon = 0$) and the conventional ($\epsilon = \infty$) beamformers. Unfortunately, the relationship between ϵ and the constraint value δ^2 is not simple.

For example, consider an eight-element uniformly spaced line array in spherically isotropic noise. Fig. 2, due to Kooij [35], presents curves of array gain versus white noise gain for element spacings of 0.1, 0.2, 0.3, 0.4, and 0.5 wavelengths when the signal direction is endfire. The parameter ϵ varies from 0 to ∞ along the curves. For closely spaced elements, unconstrained beamforming is seen to result in extremely low values of white noise gain. This is a classical supergain situation. Curves such as those of Fig. 2 may be used to trade off array gain for robustness in setting the value of the white noise gain constraint. If we consider the spacing $s/\lambda = 0.2$, and constrain the white noise gain to be unity (0 dB), it is possible to achieve 13.4 dB of array gain with an 8-element array which is only 1.4 wavelengths long. The resulting beampattern is given in Fig. 3. The performance of the improved adaptive algorithm for this example will be presented later. Additional curves of the type given in Fig. 2, together with a discussion of the practicality of achieving high gain with short endfire arrays, are presented in [52].

The problem of maximizing array gain subject to an equality constraint on white noise gain involves three quadratic forms:

$$|w^*d|^2, \quad w^*Qw, \quad \text{and} \quad w^*w.$$

Since the quantities output power, array gain, white noise gain, and generalized supergain ratio each involve two of these quadratic forms, there exists a number of equivalent formulations of the optimization problem. Specifically, the following interesting problems are equivalent.

Problem A: Maximize array gain, constrain white noise gain, and signal response [41]

$$\operatorname*{Max}_{w} \frac{|w^*d|^2}{w^*Qw}, \quad \frac{|w^*d|^2}{w^*w} = \delta^2, \quad w^*d = 1.$$

Problem B: Maximize array gain, constrain norm of w, and signal response

$$\operatorname*{Max}_{w} \frac{|w^*d|^2}{w^*Qw}, \quad w^*w = \delta^{-2}, \quad w^*d = 1.$$

Problem C: Minimize output power, constrain white noise gain, and signal response [35]

$$\operatorname*{Min}_{w} w^*Rw, \quad \frac{|w^*d|^2}{w^*w} = \delta^2, \quad w^*d = 1.$$

Problem D: Minimize output power, constrain norm of w, and signal response [53]

$$\operatorname*{Min}_{w} w^*Rw, \quad w^*w = \delta^{-2}, \quad w^*d = 1.$$

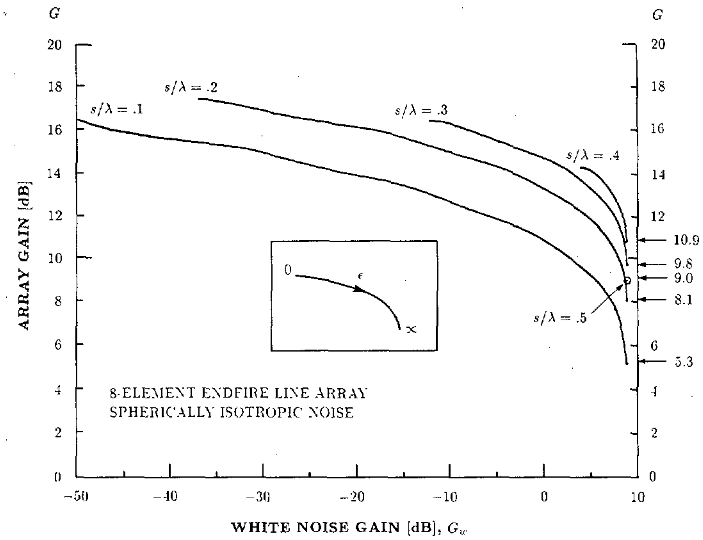


Fig. 2. Array gain versus white noise gain.

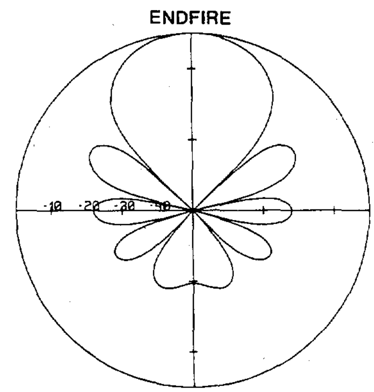


Fig. 3. Constrained optimum beampattern.

In analogy with (12) and (16), an equivalent to (18) which does not require knowledge of the noise matrix Q, and is a solution to Problems A–D, is

$$w = \frac{[R + \epsilon I]^{-1} d}{d^*[R + \epsilon I]^{-1} d}. \tag{19}$$

D. Constraint on Supergain Ratio

Uzsoky and Solymár [29] considered the problem of maximizing geometric gain subject to a constraint on the supergain ratio, that is,

$$\operatorname*{Max}_{w} \frac{|w^*d|^2}{w^*Q_g w}, \quad \frac{w^*w}{w^*Q_g w} = a.$$

They showed that this is equivalent to the problem of Gilbert and Morgan who constrained white noise gain. Again, only three quadratic forms are involved. By analogy, there are equivalents to Problems A–D which constrain the generalized supergain ratio.

Lo, Lee, and Lee [10], following the lead of Uzsoky and Solymár, considered the problem of maximizing array gain with a constraint on the supergain ratio. That is,

$$\operatorname*{Max}_{w} \frac{|w^*d|^2}{w^*Qw}, \quad \frac{w^*w}{w^*Q_g w} = a.$$

This problem involves four quadratic forms and is significantly more complicated than constraining white noise gain to ensure robustness. There is no apparent reward for the increased complication.

E. Multiple Linear Constraints

Multiple linear constraints are a generalization of the single boresight constraint [31]. They were introduced by Booker and Ong [33] and included by Frost [34] in his well-known recursive adaptive beamforming algorithm. This problem is formulated as one of minimizing the output power subject to multiple linear equality constraints, that is,

$$\underset{w}{\text{Min}}\ w^*Rw$$

subject to

$$C^*w = g. \tag{20}$$

The matrix C^* is assumed to have $K(K < M)$ linearly independent rows, one for each constraint, so that C^* is a $K \times M$ matrix. One row of C^* is usually d^*, and the corresponding element of g is 1, so that the unit boresight response constraint

$$d^*w = 1 \tag{21}$$

is usually included in (20). This important constraint permits the combination of various frequency bins at the beamformer output without amplitude and phase distortion. Other linear constraints may include multiple point constraints on the mainlobe [50], [54], or derivative constraints [50], [53], [55], [56] to control mainlobe shape, or null constraints [57] to reduce response in specific directions or to strongly control mainlobe width.

The problem may be solved by using a Lagrange multiplier to adjoin the constraints to the objective function. The solution is

$$w = R^{-1}C[C^*R^{-1}C]^{-1}g. \tag{22}$$

It is useful to consider this problem in terms of the complementary orthogonal linear subspaces associated with the constraint matrix C. The weight vector w may be decomposed into two orthogonal components:

$$w = w_c + v \tag{23}$$

where w_c is made up of a linear combination of the columns of C (w_c is the projection of w onto the range of C), and v is orthogonal to the rows of C^* (v is the projection of w onto the null space of C^*). The respective projection matrices are:

$$P_c = C[C^*C]^{-1}C^* \quad (\text{range of } C) \tag{24}$$

and

$$\tilde{P}_c = I - P_c \quad (\text{null space of } C^*) \tag{25}$$

so that

$$w_c = P_c w \tag{26}$$

and

$$v = \tilde{P}_c w. \tag{27}$$

Projecting the optimum solution for w of (22) onto the range of C yields

$$w_c = C[C^*C]^{-1}g \tag{28}$$

which does not depend on R. The matrix $C[C^*C]^{-1}$ may be recognized as the generalized inverse of C^*. Thus, (28) gives the minimum norm solution of the constraint equation (20). It is also useful to note from (28) that

$$w_c^* w_c = g^*[C^*C]^{-1}g. \tag{29}$$

Since w_c is independent of R, an equivalent optimization may be carried out in the subspace which is orthogonal to C^*.

F. Implementation Considerations

Since the K rows of C^* are assumed to be linearly independent, it is always possible to transform the constraint equation (20) such that the rows of C^* are orthogonal and $C^*C = I$. This simplifies (24), (28), and (29).

For line arrays of sensors, significant simplifications of mainlobe constraints can be accomplished if the outputs of the individual sensors are first delayed or phased to align signals from the boresight direction of each beam of interest. Then, multiple point constraints at corresponding positions (e.g., boresight, 3 dB down points) of different beams may be represented by a single constraint matrix C^*, which does not depend on frequency. Moreover, when the constraints are chosen symmetrically on the mainlobe, they occur in complex conjugate pairs and may be transformed into a pair of real constraints. Thus, the constraint matrix C^* can be reduced to a single real matrix which applies across beams and frequencies. Significant memory and computation savings result.

IV. Adaptation

Frost [34] presented an adaptive algorithm for minimizing output power subject to multiple linear equality constraints. In the notation of this paper, his algorithm is

$$w(t+1) = w_c + \tilde{P}_c[w(t) - \mu x(t)\, z^*(t)] \tag{30}$$

where μ is a small, positive, step-size parameter which controls the rate-of-change of w. The algorithm is closely related to deterministic gradient projection. Since $[x(t)\, x^*(t)]$ is an instantaneous estimate of R, the quantity

$$x(t)\, z^*(t) = [x(t)\, x^*(t)]\, w(t) \tag{31}$$

is an instantaneous estimate of the generalized gradient Rw of the output power σ_z^2, with respect to the weights. Frost's algorithm is an improvement over the gradient projection algorithms of Lacoss [31] (single boresight constraint) and Booker and Ong [33] (multiple linear constraints) which had the form

$$w(t+1) = w(t) - \mu z^*(t)\, \tilde{P}_c x(t). \tag{32}$$

Because gradient projection algorithms are known to wander off the constraints due to roundoff error, Frost projected the updated solution onto the null space of C^* at each step, and then added the component w_c, which does not depend on the input statistics.

The effect of the size of the parameter μ on the performance of recursive algorithms is discussed by Griffiths [32] and Frost [34]. A sufficient condition for bounded steady state "misadjustment" is

$$0 < \mu < \frac{2}{3E(x^*x)}. \tag{33}$$

To ensure that (33) is satisfied, it is common practice to adapt μ based on an estimate of input power.

The first explicit consideration of a robustness constraint in a recursive adaptive beamforming algorithm was apparently by Winkler and Schwartz [58]. They applied a gradient projection algorithm to the problem of Lo, Lee, and Lee of maximizing array gain subject to a constraint on the supergain ratio. That formulation did not lead to a simple algorithm. They later applied a penalty function with gradient projection to the same problem [59].

Kooij [35] investigated several algorithms which included a consideration of white noise gain directly in the algorithm. In particular, for a smoothed estimate of R of the form

$$\hat{R}(t) = (1 - \alpha)\,\hat{R}(t - 1) + \alpha x(t)\,x^*(t), \tag{34}$$

where α is a smoothing parameter $(0 < \alpha \leq 1)$, Kooij, motivated by (19), gave the following weight update equation:

$$w(t + 1) = w_c + \tilde{P}_c\big[w(t) - \mu\big[\hat{R}(t) + \epsilon(t)I\big]\,w(t)\big]. \tag{35}$$

In the special case of no smoothing $(\alpha = 1)$, (35) reduces to

$$w(t + 1) = w_c + \tilde{P}_c\big[w(t)(1 - \mu\epsilon(t)) - \mu x(t)\,z^*(t)\big] \tag{36}$$

which is very similar to (30), and involves a reduction of w prior to projection. This same algorithm with fixed ϵ was given recently by Takao and Kikuma [60], who noted that the step size parameter μ should satisfy the following modified condition for convergence:

$$0 < \mu < \frac{2/3}{E[x^*x] + M\epsilon}. \tag{37}$$

Because R is unknown, *a priori*, it is difficult to set ϵ to satisfy the constraint $G_w \geq \delta^2$. Kooij suggested adding a variable amount $\epsilon(t)$ to the diagonal of $\hat{R}$ given by the following equation:

$$\epsilon(t + 1) = (1 - \eta)\,\epsilon(t) + \Delta\epsilon\big[1 + \mathrm{sign}\big(\delta^2 - G_\omega(t)\big)\big] \tag{38}$$

where $\Delta\epsilon$ and η are real positive scalar constants with values less than or equal to unity. The variable amount $\epsilon(t)$ increases when G_w is too small, and decreases when G_w is sufficiently large. Here $\Delta\epsilon$ is the increment size, and $(1 - \eta)$ is the relaxation parameter. The maximum value of ϵ is limited to $\Delta\epsilon/\eta$. This maximum value of ϵ can be used in (37) in setting the size of μ.

This algorithm treats G_w using a penalty function approach, and involves side computations for $G_w(t)$ and $\epsilon(t)$. It has been found to work well in applications.

A. Scaled Projection Algorithm

We now describe the improved algorithm [61] which minimizes output power subject to both multiple linear equality constraints of the form given by (20), where the boresight constraint $d^*w = 1$ is included in (20), and the quadratic inequality constraint

$$G_w = \frac{|w^*d|^2}{w^*w} \geq \delta^2 \tag{39}$$

and where (20) and (39) are self-consistent. We wish to work with δ^2 directly and avoid using the intermediate parameter ϵ.

Writing w in terms of its orthogonal components and using the boresight unity response constraint of (21), (39) becomes

$$G_w = \frac{1}{w_c^* w_c + v^*v} \geq \delta^2 \tag{40}$$

where w_c, given by (28), has the smallest norm while satisfying the linear constraints. Using (29), the constraint (40) may be written as

$$v^*v \leq \delta^{-2} - g^*[C^*C]^{-1}\,g = b^2. \tag{41}$$

Thus, the white noise gain constraint can be replaced by a constraint on v, the projection of w onto the null space of C^*, where (41) defines b^2 in terms of the other constraint parameters. Since w_c is given by (28) independent of the data, the adaptation may be carried out in the orthogonal subspace to which v is restricted.

Consider the situation depicted in Fig. 4. Suppose that $v(t)$ of Frost's algorithm were on the boundary $v^*(t)\,v(t) = b^2$, and that the next iteration would place the tentative updated vector $\tilde{v}(t + 1)$ outside of the boundary so that (41) would not be satisfied. We would like to modify the algorithm to project the result of this iteration onto the constraint boundary, so that $v(t + 1)$ would step along the boundary in the correct direction. *Because the constraint region is a closed sphere centered at the origin, $\tilde{v}(t + 1)$ itself is normal to the constraint boundary surface, and the projection of $\tilde{v}(t + 1)$ onto the constraint boundary surface may be obtained by simply scaling $\tilde{v}(t + 1)$.* This important observation leads to the following simple algorithm.

Define a tentative update vector

$$\tilde{v}(t + 1) = \tilde{P}_c\big[v(t) - \mu\hat{R}(t)\,w(t)\big] \tag{42}$$

where $\hat{R}$ is given by (34). Then, update the weights by

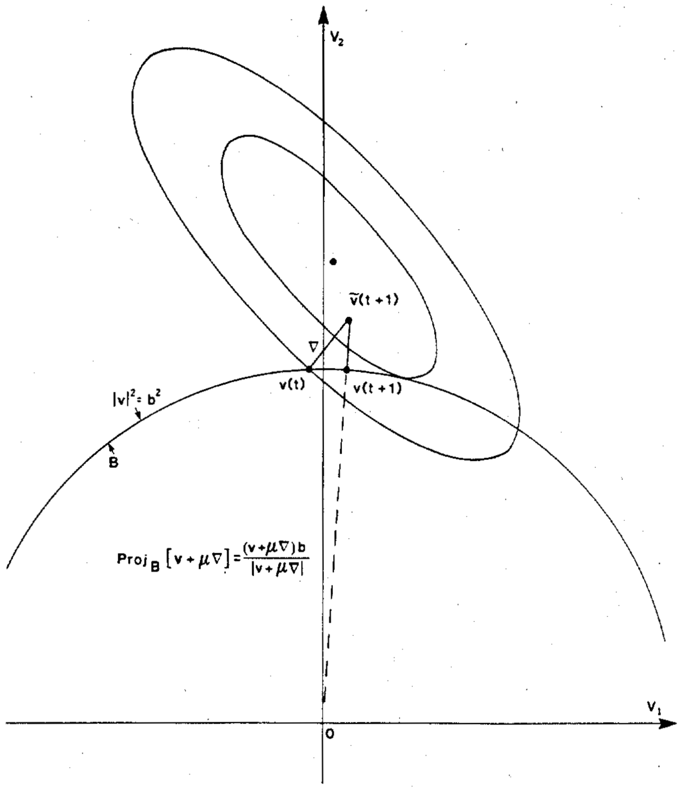


Fig. 4. Equivalence of gradient projection and scaling of the tentative update vector in the null space of C^*.

scaling $\tilde{v}$, as required to satisfy (41). That is,

$$w(t+1) = w_c + \begin{cases} \tilde{v}(t+1) & \text{for } |\tilde{v}|^2 \le b^2 \\ \dfrac{b\tilde{v}(t+1)}{|\tilde{v}(t+1)|} & \text{for } |\tilde{v}|^2 > b^2 \end{cases} \tag{43}$$

where w_c, b, and $\tilde{P}_c$ are given by (28), (41), and (25), respectively. For the case of $\alpha = 1$ in (34), (42) reduces to

$$\tilde{v}(t+1) = \tilde{P}_c[v(t) - \mu x(t) z^*(t)]. \tag{44}$$

The algorithm involves a projection of the tentative new weights onto the orthogonal subspace, followed by a scaling in that subspace, if necessary, to satisfy the inequality constraint, and finally, the addition of component w_c in the other subspace. All constraints are satisfied exactly at each step.

Not only is the new algorithm extremely simple, but it achieves the favorable properties [58] of gradient projection algorithms for handling nonlinear constraints without incurring their disadvantages. By effectively projecting the result of the iteration rather than the gradient, it achieves the same advantage that Frost's algorithm achieved over gradient projection algorithms for linear constraints. Namely, roundoff errors do not accumulate.

The algorithm has been implemented by combining the outputs of two beamformers, as shown in Fig. 5. One beamformer is fixed and uses w_c^* as fixed weights. When the constraints are all chosen on the beampattern of the conventional beamformer, w_c^* reduces to d^*/M, the unshaded conventional beamformer weights. The other

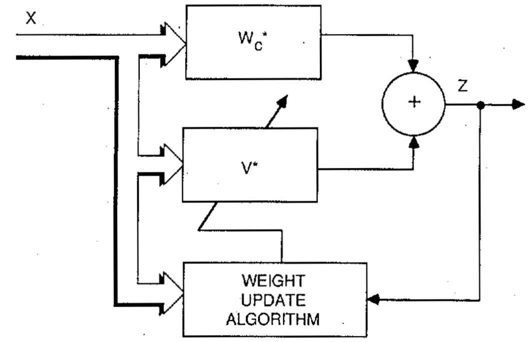


Fig. 5. Adaptive beamformer structure.

beamformer is adaptive and uses $v(t)$ as its weights. The weights $v(t)$ are sometimes called perturbation weights. This configuration has the advantage of providing a conventional beamformer output as a byproduct. This beamforming structure was called a generalized sidelobe canceller by Griffiths and Jim [62] because of its similarity to radar sidelobe cancellers [7], [8].

Recently, Jablon [63] discussed adding artificial noise in a generalized sidelobe canceller configuration to obtain an algorithm similar to (36) with fixed ϵ.

Ahmed and Evans [64] have considered a different formulation of the robustness problem in which the constraint matrix C^* was perturbed by errors which were bounded in absolute value. An absolute value constraint was applied to the amount which each linear constraint could be violated. They presented an algorithm based on techniques of mathematical programming.

V. Examples

The performance of the algorithm in a weak signal situation is illustrated in the simulations shown in Figs. 6–8. The line array consists of eight elements spaced at $s/\lambda = 0.2$ in a spherically isotropic noise field. It is steered to endfire with a single boresight constraint. The noise power σ_n^2 is 0 dB and the signal power σ_s^2 is -10 dB. The white noise gain constraint is set at unity (0 dB) based on the performance curves of Fig. 2. Fig. 6 presents the array gain as a function of time. The step size constant μ for this simulation is 0.02, and there is no extra smoothing ($\alpha = 1$). The array gain increases to a steady level of about 13 dB, less than 1 dB below the constrained optimum for known covariance R.

Fig. 7 presents the white noise gain as a function of time. It shows a steady decrease in white noise gain until the constraint of 0 dB comes into play. The fluctuations in both G and G_w, due to the finite step size, are evident. The constraints are fully satisfied at each time step.

Fig. 8 shows the beampattern of the weights at a snapshot in time at the 4000th time step. It may be compared to Fig. 3. The mainlobe is faithfully reproduced. In the absence of strong interferers, sidelobes are not critical to the optimization and are not faithfully reproduced in the adaptation.

To illustrate signal suppression effects in a situation of a strong signal and array imperfections, consider an eight-element array whose elements are nominally spaced uni-

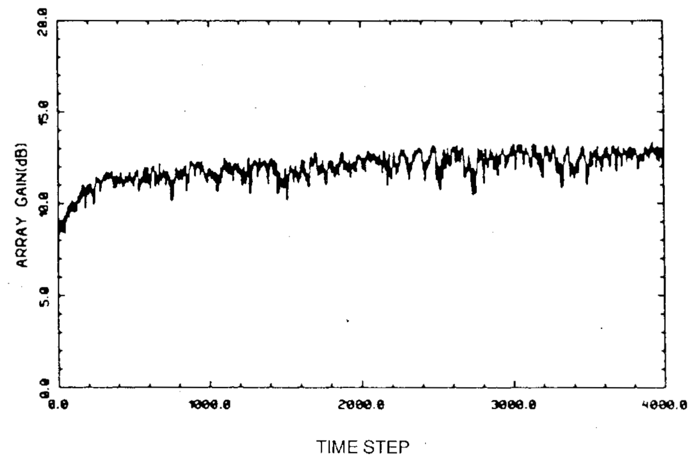


Fig. 6. Array gain as a function of time for endfire example.

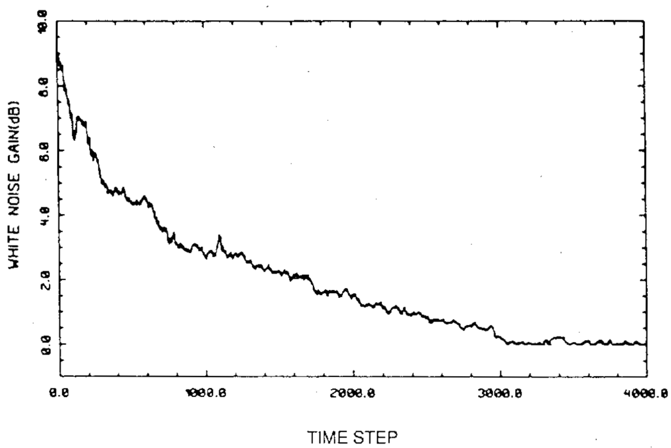


Fig. 7. White noise gain versus time for endfire example.

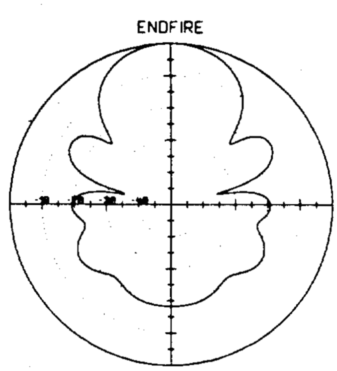


Fig. 8. Adaptive beampattern (t = 4000).

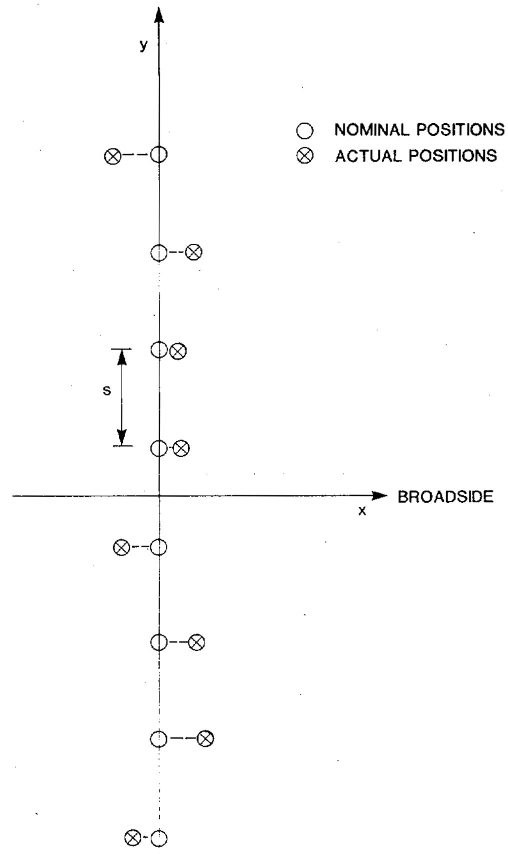


Fig. 9. Imperfect array geometry.

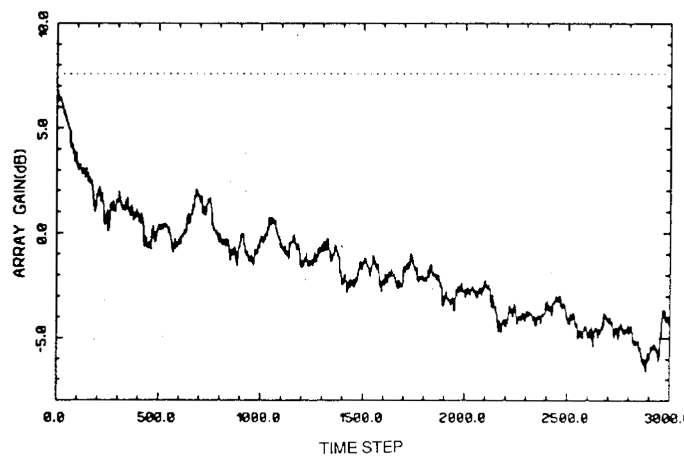


Fig. 10. Broadside beam array gain as a function of time. The Frost algorithm was used for adaptation and does not include a white noise gain constraint.

formly at $s/\lambda = 0.3$ in a spherically isotropic noise field. The actual element positions differ from the nominal element positions in the broadside (x-axis) direction, as indicated in Fig. 9. The actual element positions in the x-axis direction were chosen from a zero-mean Gaussian distribution with a standard deviation $\sigma_x = 0.03\lambda$. The noise power σ_n^2 is 0 dB, and the planewave arriving from broadside signal power σ_s^2 is 0 dB. The array is steered to broadside based on the nominal element positions.

The performance of the basic Frost algorithm, given by (30), with a single (boresight) constraint, is illustrated in the simulations shown in Figs. 10 and 11. This algorithm does not include the constraint on the white noise gain. Fig. 10 presents the array gain as a function of time. The step size constant μ for this simulation is 0.01, and there is no extra smoothing ($\alpha = 1$). The array gain rapidly decreases becoming less than unity. Performance is totally unsatisfactory. The mismatch between the nominal and actual element positions, coupled with a strong signal, has resulted in substantial signal suppression and poor performance. Fig. 11 presents the corresponding white noise gain as a function of time. The nominal element

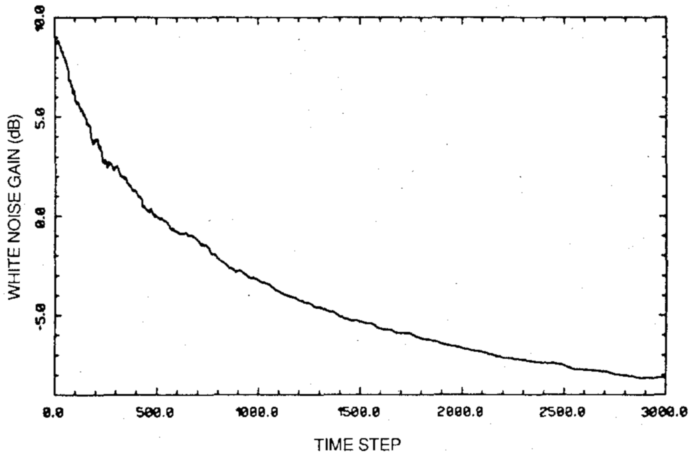


Fig. 11. White noise gain as a function of time resulting from use of the Frost algorithm.

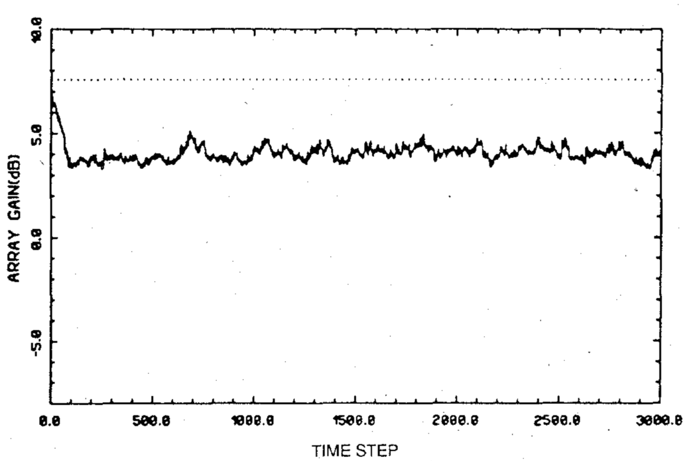


Fig. 12. Broadside beam array gain as a function of time using the new algorithm with white noise gain constraint.

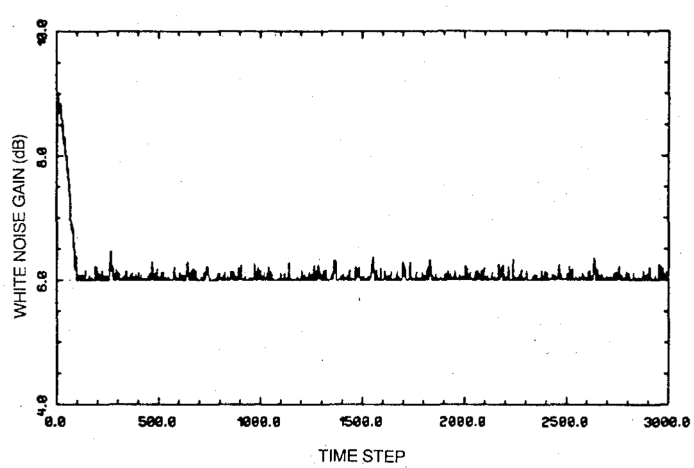


Fig. 13. White noise gain as a function of time using the new algorithm.

positions were used to compute $\boldsymbol{d}$ in the numerator of (7). It shows a steady decrease in white noise gain, indicating the loss of robustness.

The performance of the new robust algorithm given by (43) and (44) is illustrated in the simulations shown in Figs. 12 and 13. The parameters for this simulation are exactly the same as for the Frost algorithm discussed above. The white noise gain constraint is 4.0 (6 dB). The white noise gain is set higher in this example since there is no significant supergain potential at broadside. The setting is 3 dB below the maximum possible value of 9 dB. Fig. 12 presents the array gain as a function of time. The array gain initially decreases as before, due to the signal suppression caused by position mismatch, but soon the decrease is halted when the white noise gain constraint comes into play at about the 100th time step. This can be seen in Fig. 13, which presents the corresponding white noise gain. It shows an initial rapid decrease in white noise gain until the constraint of 6 dB is reached. The white noise gain is maintained at or above the constraint value at each time step of the simulation. The effectiveness of the white noise gain constraint and the behavior of the constrained projection algorithm are clearly illustrated.

VI. Generalization

A more general problem arises when the constraint $\boldsymbol{w}^*\boldsymbol{w} \le \delta^{-2}$ is replaced by a quadratic inequality constraint of the following form:

$$\boldsymbol{w}^*BB^*\boldsymbol{w} \le \gamma^2 \tag{45}$$

where B is M by M and nonsingular so that BB^* is positive definite. Consider the problem

$$\min_{\boldsymbol{w}} \boldsymbol{w}^*R\boldsymbol{w}$$

subject to the quadratic constraint (45) and the linear constraints (20). Simple scaling as in (43) no longer has the projection property and would, in general, converge to a nonoptimum solution. This problem is similar to one discussed by Owsley [65]. The matrix BB^* may be used, for example, to represent correlated errors as in (8), isotropic noise for directivity control, or regions of controlled sidelobe response. Recently, Er and Cantoni [66] discussed the use of a quadratic constraint to control mainlobe response.

The scaled projection algorithm can be applied to the above problem by introducing a transformation such that the quadratic constraint boundary is spherical in the new coordinate space.

Let

$$\boldsymbol{y} = B^{-1}\boldsymbol{x} \tag{46}$$

and

$$\boldsymbol{u} = B^*\boldsymbol{w} \tag{47}$$

so that

$$E[\boldsymbol{y}\boldsymbol{y}^*] = B^{-1}RB^{*-1} = R_y \tag{48}$$

$$\boldsymbol{w}^*\boldsymbol{x} = \boldsymbol{u}^*\boldsymbol{y} = z \tag{49}$$

and

$$\boldsymbol{w}^*R\boldsymbol{w} = \boldsymbol{u}^*R_y\boldsymbol{u}. \tag{50}$$

Then the problem is transformed into the proper form for the application of the scaled projection algorithm. That is,

$$\min_{\boldsymbol{u}} \boldsymbol{u}^*R_y\boldsymbol{u}$$

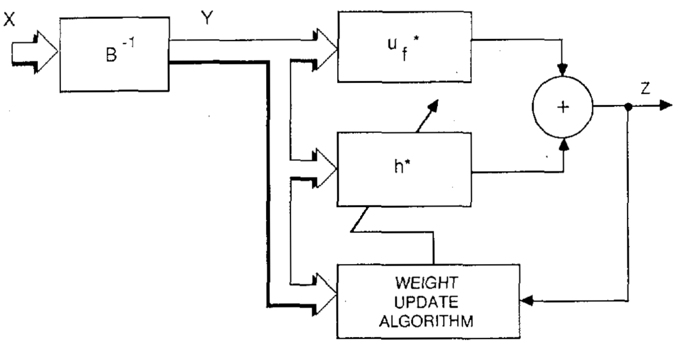


Fig. 14. Adaptive beamformer structure for a general quadratic constraint.

subject to

$$u^*u \le \gamma^2 \tag{51}$$

$$F^*u = g \tag{52}$$

where

$$F^* = C^*B^{*-1}. \tag{53}$$

The solution can be expressed in the form

$$u(t) = u_f + h(t) \tag{54}$$

where in analogy with (28)

$$u_f = F[F^*F]^{-1}g. \tag{55}$$

Let

$$\beta^2 = \gamma^2 - g^*[F^*F]^{-1}g. \tag{56}$$

Defining the projection matrix $\tilde{P}_f$ in analogy with (25) as follows:

$$\tilde{P}_f = I - F[F^*F]^{-1}F^*, \tag{57}$$

the scaled projection algorithm becomes

$$\tilde{h}(t+1) = \tilde{P}_f[h(t) - \mu y(t) z^*(t)] \tag{58}$$

$$h(t+1) = \begin{cases} \tilde{h}(t+1) & \text{for } |\tilde{h}|^2 \le \beta^2 \\ \dfrac{\beta \tilde{h}(t+1)}{|\tilde{h}(t+1)|} & \text{for } |\tilde{h}|^2 > \beta^2. \end{cases} \tag{59}$$

This algorithm may be implemented by a beamformer with the structure shown in Fig. 14. This structure is similar to the one of Fig. 5, but involves an initial transformation B^{-1} to obtain y from the input x.

VII. Conclusions

An improved adaptive beamforming algorithm has been presented which permits simultaneous linear equality constraints and a quadratic inequality constraint on the gain against spatially white noise. The algorithm involves a simple scaling of the weights in a subspace, if necessary, to satisfy the inequality constraint. Hence, we call it the scaled projection algorithm. The scaling is equivalent to projecting the tentative updated weights onto the boundary of the quadratic constraint surface. This projection property stems from the fact that the white noise gain constraint can be expressed as a sphere centered at the origin in the subspace which is orthogonal to the linear constraints.

Its performance has been illustrated in two examples. This performance is typical of what has been observed in extensive simulations. The algorithm is simple, reliable, and leads to systems which are robust in the face of the inevitable finite tolerances of physical systems.

The algorithm has been generalized to handle a more general quadratic constraint by introducing a linear transformation to convert the general quadratic constraint to a spherical constraint.

References

[1] F. Bryn, "Optimum signal processing of three-dimensional arrays operating on Gaussian signals and noise," *J. Acoust. Soc. Amer.*, vol. 34, no. 3, pp. 289–297, Mar. 1962.

[2] V. Vanderkulk, "Optimum processing for acoustic arrays," *J. Brit. IRE*, vol. 26, no. 4, pp. 286–292, Oct. 1963.

[3] H. Mermoz, "Filtrage adapté et utilisation optimale d'une antenne," in *Proc. NATO Advanced Study Inst. Signal Processing Emphasis Underwater Acoust.*, Grenoble, France, 1964.

[4] D. Middleton and H. I. Groginski, "Detection of random acoustic signals by receivers with distributed elements. Optimum receiver structures for normal signal and noise fields," *J. Acoust. Soc. Amer.*, vol. 38, pp. 727–737, Nov. 1965.

[5] S. Shor, "Adaptive technique to discriminate against coherent noise in a narrow-band system," *J. Acoust. Soc. Amer.*, vol. 39, no. 1, pp. 74–78, Jan. 1966.

[6] D. J. Edelblute, J. M. Fisk, and G. L. Kinnison, "Criteria for optimum-signal-detection theory for arrays," *J. Acoust. Soc. Amer.*, vol. 41, no. 1, pp. 199–205, Jan. 1967.

[7] P. W. Howells, "Intermediate frequency side-lobe canceller," General Electric Co., Patent 3, 202, 990, May 1959.

[8] S. P. Applebaum, "Adaptive arrays," Syracuse Univ. Res. Corp., SPL TR 66-1, Aug. 1966. Also, *IEEE Trans. Antennas Propagat.*, vol. AP-24, pp. 585–598, Sept. 1976.

[9] H. N. Kritikos, "Optimal signal-to-noise ratio for linear arrays by the Schwartz inequality," *J. Franklin Inst.*, vol. 276, no. 4, pp. 295–304, Oct. 1963.

[10] Y. T. Lo, S. W. Lee, and Q. H. Lee, "Optimization of directivity and signal-to-noise ratio of an arbitrary antenna array," *Proc. IEEE*, vol. 54, pp. 1033–1045, Aug. 1966.

[11] J. P. Burg, "Three-dimensional filtering with an array of seismometers," *Geophysics*, vol. 29, no. 5, pp. 693–713, Oct. 1964.

[12] E. J. Kelly, Jr., and M. J. Levin, "Signal parameter estimation for seismometer arrays," M.I.T. Lincoln Lab., Lexington, MA, Tech. Rep. 339, MIT DDC 435-489, Jan. 1964.

[13] M. J. Levin, "Maximum-likelihood array processing," Seismic Discrimination Semi-Annu. Tech. Summary Rep., M.I.T. Lincoln Lab., Lexington, MA, Tech. Rep. DDC 455743, Dec. 1964.

[14] P. E. Green, E. J. Kelly, Jr., and M. J. Levin, "A comparison of seismic array processing methods," *Geophys. J. Roy. Astron. Soc.*, vol. 11, pp. 67–84, 1966.

[15] J. Capon, R. J. Greenfield, and R. J. Kolker, "Multidimensional maximum-likelihood processing of a large aperture seismic array," *Proc. IEEE*, vol. 55, pp. 192–211, Feb. 1967.

[16] J. Capon, "High-resolution frequency-wavenumber spectrum analysis," *Proc. IEEE*, vol. 57, pp. 1408–1418, Aug. 1969.

[17] H. L. Van Trees, "A unified theory for optimum array processing," A. D. Little, Inc., Rep. 4160866, Aug. 1966.

[18] L. J. Griffiths, "A comparison of multidimensional Wiener and maximum-likelihood filters for antenna array," *Proc. IEEE*, vol. 55, pp. 2045–2047, Nov. 1967.

[19] H. Cox, "Interrelated problems in estimation and detection I and II," in *Proc. NATO Advanced Study Inst. Signal Processing with Emphasis on Underwater Acoust.*, Enschede, The Netherlands, Aug. 1968.

[20] ——, "Optimum arrays and the Schwartz inequality," *J. Acoust. Soc. Amer.*, vol. 45, no. 1, pp. 228–232, Jan. 1969.

[21] A. H. Nuttal and D. W. Hyde, "A unified approach to optimum and suboptimum processing for arrays," U.S. Navy Underwater Sound Lab., New London, CT, USL Rep. 992, Apr. 1969.

[22] W. J. Bangs and P. M. Schultheiss, "Space–time processing for optimal parameter estimation," in *Signal Processing (Proc. NATO Ad-*

vanced Study Inst. Signal Processing with Particular Reference to Underwater Acoust., Loughborough, U.K., Aug. 1972), J. W. L. Griffiths, P. L. Stocklin, and C. Van Schooneveld, Eds. London and New York: Academic, 1973.

[23] P. M. Schultheiss, "Passive sonar detection in the presence of interference," *J. Acoust. Soc. Amer.*, vol. 43, pp. 418–425, Mar. 1968.

[24] V. C. Anderson, "DICANNE, a realizable adaptive process," *J. Acoust. Soc. Amer.*, vol. 45, no. 2, pp. 398–405, Feb. 1969.

[25] V. C. Anderson and P. Rudnick, "Rejection of a coherent arrival at an array," *J. Acoust. Soc. Amer.*, vol. 45, no. 2, pp. 406–410, Feb. 1969.

[26] H. Cox, "Array processing against interference," in *Proc. Purdue Centennial Year Symp. on Inform. Processing*, vol. 2, Apr. 1969, pp. 453–463.

[27] J. B. Lewis and P. M. Schultheiss, "Optimum and conventional detection using a linear array," *J. Acoust. Soc. Amer.*, vol. 49, no. 4, pp. 1083–1091, Apr. 1971.

[28] E. N. Gilbert and S. P. Morgan, "Optimum design of directive antenna arrays subject to random variables," *Bell Syst. Tech. J.*, vol. 34, pp. 637–663, May 1955.

[29] M. Uzsoky and L. Solymár, "Theory of superdirective linear arrays," *Acta Phys. Acad. Sci. Hungary*, vol. 6, no. 2, pp. 185–205, 1956.

[30] B. Widrow, P. E. Mantey, L. J. Griffiths, and B. B. Goode, "Adaptive antenna systems," *Proc. IEEE*, vol. 55, pp. 2143–2159, Dec. 1967.

[31] R. T. Lacoss, "Adaptive combining of wideband array data for optimal reception," *IEEE Trans. Geosci. Electron.*, vol. GE-6, pp. 78–86, May 1968.

[32] L. J. Griffiths, "A simple adaptive algorithm for real-time processing in antenna arrays," *Proc. IEEE*, vol. 57, pp. 1696–1704, Oct. 1969.

[33] A. Booker and C. Y. Ong, "Multiple constraint adaptive filtering," *Geophysics*, vol. 36, pp. 498–509, June 1971.

[34] O. L. Frost, III, "An algorithm for linearly constrained adaptive array processing," *Proc. IEEE*, vol. 60, pp. 926–935, Aug. 1972.

[35] T. Kooij, "Adaptive array processors for sensitivity constrained optimization," Ph.D. dissertation, Catholic Univ. Amer., Washington, DC, June 1977.

[36] H. Kobayashi, "Iterative synthesis methods for a seismic array processor," *IEEE Trans. Geosci. Electron.*, vol. GE-8, pp. 169–178, July 1970.

[37] D. K. Cheng and F. I. Tseng, "Optimum spatial processing in a noisy environment for arbitrary antenna arrays subject to random errors," *IEEE Trans. Antennas Propagat.*, vol. AP-16, pp. 164–171, Mar. 1968.

[38] C. D. Seligson, "Comments on high resolution frequency wavenumber spectrum analysis," *Proc. IEEE*, vol. 58, pp. 947–949, June 1970.

[39] L. E. Brennan, E. L. Pugh, and I. S. Reed, "Control loop noise in adaptive array antennas," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-7, pp. 254–262, Mar. 1971.

[40] R. N. McDonough, "Degraded performance of nonlinear array processors in the presence of data modeling errors," *J. Acoust. Soc. Amer.*, vol. 51, no. 4, pp. 1186–1193, 1972.

[41] H. Cox, "Sensitivity considerations in adaptive beamforming," in *Signal Processing* (*Proc. NATO Advanced Study Inst. Signal Processing with Particular Reference to Underwater Acoust.*, Loughborough, U.K., Aug. 1972.) J. W. R. Griffiths, P. L. Stocklin, and C. Van Schooneveld, Eds. New York and London: Academic, 1973.

[42] C. L. Zahm, "Effects of errors in the direction of incidence on the performance of an adaptive array," *Proc. IEEE*, vol. 60, pp. 1008–1009, Aug. 1972.

[43] H. Cox, "Resolving power and sensitivity to mismatch of optimum array processors," *J. Acoust. Soc. Amer.*, vol. 54, no. 3, pp. 771–785, 1973. Also, in *Selected Papers in Multidimensional Signal Processing*. New York: IEEE Press, 1986.

[44] ——, "Line array performance when the signal coherence is spatially dependent," *J. Acoust. Soc. Amer.*, vol. 54, no. 6, pp. 1743–1796, 1973.

[45] A. M. Vural, "Effects of perturbations on the performance of optimum/adaptive arrays," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-15, pp. 76–87, Jan. 1979.

[46] R. T. Compton, Jr., "Multiplier offset voltages in adaptive arrays," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-12, pp. 616–627, Sept. 1976.

[47] ——, "Pointing accuracy and dynamic range in a steered beam adaptive array," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-16, pp. 280–287, May 1980.

[48] ——, "The effect of random steering vector errors in the Applebaum adaptive array," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-18, pp. 392–400, Sept. 1982.

[49] L. C. Godara, "Error analysis of the optimal antenna array processors," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-22, July 1986.

[50] A. M. Vural, "A comparative performance study of adaptive array processors," in *IEEE ICASSP'77 Rec.*, May 1977, pp. 695–700.

[51] T. T. Taylor, "Design of line-source antennas for narrow beam width and low sidelobes," *IRE Trans. Antennas Propagat.*, vol. AP-3, pp. 16–28, 1955.

[52] H. Cox, R. M. Zeskind, and T. Kooij, "Practical supergain," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-34, pp. 393–398, July 1986.

[53] J. E. Hudson, *Adaptive Array Principles*. New York: Peter Peregrinus for IEE, 1981.

[54] K. Takao, H. Fujita, and T. Nishi, "An adaptive array under directional constraint," *IEEE Trans. Antennas Propagat.*, vol. AP-24, pp. 662–669, Sept. 1976.

[55] S. P. Applebaum and D. J. Chapman, "Adaptive arrays with main beam constraints," *IEEE Trans. Antennas Propagat.*, vol. AP-24, pp. 650–662, Sept. 1976.

[56] M. H. Er and A. Cantoni, "Derivative constraints for broad-band element space antenna array processors," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-31, pp. 1378–1393, Dec. 1983.

[57] A. Fenwick, "An array processor optimum subject to a constraint," *J. Acoust. Soc. Amer.*, vol. 49, no. 1, pp. 1108–1110, 1971.

[58] L. P. Winkler and M. Schwartz, "Adaptive nonlinear optimization of the signal-to-noise ratio of an array subject to a constraint," *J. Acoust. Soc. Amer.*, vol. 52, no. 1, pp. 39–51, July 1972.

[59] ——, "Constrained array optimization by penalty function techniques," *J. Acoust. Soc. Amer.*, vol. 55, no. 5, pp. 1042–1048, May 1974.

[60] K. Takao and N. Kikuma, "Tamed adaptive antenna array," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-34, pp. 388–394, Mar. 1986.

[61] H. Cox, "An adaptive beamforming algorithm with simultaneous mainlobe response and tolerance sensitivity constraints," in *Proc. 19th IEEE Asilomar Conf. Circuits, Syst., Comput.*, Nov. 6–8, 1985.

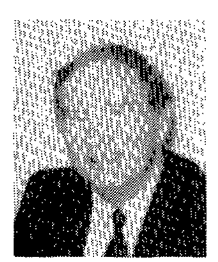
[62] L. J. Griffiths and C. W. Jim, "An alternative approach to linearly constrained adaptive beamforming," *IEEE Trans. Antennas Propagat.*, vol. AP-30, pp. 27–34, Jan. 1982.

[63] N. K. Jablon, "Adaptive beamforming with the generalized sidelobe canceller in the presence of array imperfections," *IEEE Trans. Antennas Propagat.*, vol. AP-34, pp. 996–1012, Aug. 1986.

[64] K. M. Ahmed and R. J. Evans, "An adaptive array processor with robustness and broad-band capabilities," *IEEE Trans. Antennas Propagat.*, vol. AP-32, pp. 944–950, Sept. 1984.

[65] N. L. Owsley, "A recent trend in adaptive spatial processing for sensor arrays: Constrained adaptation," in *Signal Processing* (*Proc. NATO Advanced Study Inst. Signal Processing with Particular Reference to Underwater Acoust.*, Loughborough, U.K., Aug. 1972), J. W. L. Griffiths, P. L. Stocklin, and C. Van Schooneveld, Eds. London and New York: Academic, 1973.

[66] M. H. Er and A. Cantoni, "An alternative formulation for an optimum beamformer with robustness capability," *IEE Proc.*, vol. 132 F, pp. 447–460, Oct. 1985.



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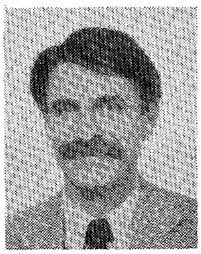

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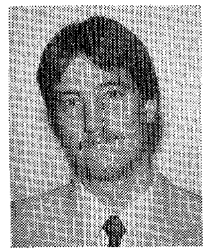
Dr. Zeskind is a member of Sigma Xi.



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