Emergent Stock Market Behaviour via Agent-Based Simulation

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October 21, 2025

1 Introduction

This project implements a simplified agent-based financial market, a numerical experiment in which the market price of a single asset evolves as the result of interactions between many different types of traders, called agents. The purpose is to model a market driven by endogenous behavioural rules rather than by an externally imposed random process. That is, the price emerges from simulated supply and demand rather than being forced to follow a pre-specified formula.

This approach belongs to the field of agent-based computational economics (ACE) and more specifically to heterogeneous agent models. Such models have been widely studied since the 1990s in the works of Lux and Marchesi (1999), LeBaron (2000), and others who investigated market fluctuations as emergent phenomena. It differs from the classical finance view in which prices follow an exogenous stochastic process such as geometric Brownian motion (GBM). The hypothesis is that, given sufficiently realistic proportions and parameter weights for the agents, the resulting stock price dynamics would reproduce the statistical characteristics of real markets.

Exogenous vs. Endogenous Models

A **geometric Brownian motion** defines the price P_t as a continuous-time random process satisfying

$$dP_t = \mu P_t \, dt + \sigma P_t \, dW_t,$$

where μ is the drift, σ the volatility, and W_t a standard Wiener process (a random walk). That is, the randomness is injected externally by the modeller, and the market's collective behaviour is not explained by underlying agents.

In contrast, an **agent-based model** like ours treats the market as a system of interacting participants, each following a behavioural rule. The stochasticity arises endogenously from their decisions, feedback, and composition. Hence, the price dynamics are emergent, not imposed.

2 Theoretical Framework

We simulate one asset traded among several classes of agents. At each discrete tick $t \in \{1, 2, \ldots, T\}$, every agent i submits a signed order $q_{i,t} \in \mathbb{R}$, representing its desired net position change (positive = buy, negative = sell).

2.1 Market Clearing Mechanism

The total net order flow is

$$Q_t = \sum_i w_i \, q_{i,t},$$

where $w_i \in [0, 1]$ represents the relative population weight or market share of agent type i.

The price is then updated multiplicatively according to a simple **market impact function**:

$$P_t = P_{t-1} \left(1 + \kappa \, \frac{Q_t}{L_{t-1}} + \epsilon_t \right),$$

where:

- κ (impact coefficient) controls how sensitively price reacts to order flow.
- L_t (liquidity) measures market depth or capacity to absorb trades.
- $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$ adds small microstructure noise.

That is, when the total buy pressure Q_t is positive, price increases, and when it is negative, price decreases.

2.2 Liquidity and Volatility Feedback

Liquidity decreases when volatility increases, creating the well-known reflexive feedback of real markets:

$$L_t = \frac{L_0}{1 + 10\sigma_t},$$

where L_0 is the base liquidity.

Volatility is measured as an exponentially weighted moving average (EWMA) of absolute log returns:

$$\sigma_t = \alpha\sigma_{t-1} + (1 - \alpha) \, | \ln(P_t/P_{t-1})| \sqrt{252},$$

where $\alpha \in (0, 1)$ controls persistence. That is, large price jumps temporarily raise volatility and reduce liquidity, magnifying future shocks.

2.3 Fundamental Value Process

A latent "fundamental" value V_t evolves slowly:

$$V_t = V_{t-1} + \mu_V + \rho(\bar{P}_t - V_{t-1}),$$

where:

- μ_V: small drift capturing long-term trend.
- ρ: mean-reversion rate toward a moving average of past prices $\bar{P}_t$.

That is, the fundamental value moves gradually and guides long-horizon investors.

3 Agents and Behavioural Rules

The simulation currently contains six primary agent types, sufficient to reproduce several stylised facts of financial markets such as volatility clustering and fat-tailed returns. Each agent class is defined by a small number of parameters controlling its frequency of action, reaction speed, and aggressiveness.

3.1 Noise Trader

Definition: an agent who randomly buys or sells, without informational motive.

$$q_t = s \cdot u_t, \quad s \in \{-1, +1\},\ u_t \sim \text{Uniform}(0, s_{\max})$$

That is, random liquidity providers. They generate baseline fluctuations and ensure constant market activity. Parameters: size = 1.2, frequency = 0.7.

3.2 DCA Investor

Definition: a long-term participant who invests a fixed amount of cash at regular intervals ("Dollar-Cost Averaging").

$$q_t = \frac{C}{P_t} \quad \text{if } t \bmod k = 0.$$

That is, steady demand independent of market conditions. Parameters: cash = 30, period = 28 ticks.

3.3 Momentum Follower

Definition: an agent who buys when prices rise and sells when they fall. The agent computes the slope of a linear regression over a lookback window L:

$$s = \text{sign}\left(\frac{d}{dt} P_t^{(L)}\right), \qquad q_t = k\, s\, |s|.$$

That is, a technical trader exploiting short-term trends. Parameters: lookback = 40 ticks, aggressiveness $k = 1.2$, frequency = 0.8.

3.4 Mean Reverter

Definition: an agent who trades against deviations from a recent moving average.

$$z = \frac{P_t - \bar{P}_t}{\sigma_P}, \quad q_t = \begin{cases} -kz & |z| > \tau, \\ 0 & \text{otherwise.} \end{cases}$$

That is, it assumes the price will revert to the mean. Parameters: window = 80 ticks, threshold $\tau = 1.0$, gain $k = 1.0$, frequency = 0.7.

3.5 Value Investor

Definition: a fundamental agent trading based on mispricing relative to intrinsic value:

$$q_t = k\frac{V_t - P_t}{P_t} \quad \text{if } |V_t - P_t| > \tau P_t.$$

That is, it provides long-term anchoring. Parameters: $k = 2.0$, tolerance $\tau = 0.008$, frequency $= 0.4$.

3.6 Whale (Large Trader)

Definition: an occasional but powerful trader representing institutional or macro shocks. With small probability p, submits a large log-normal order:

$$q_t = s \cdot e^{\mathcal{N}(\mu,\sigma^2)}, \quad s \in \{-1, +1\}.$$

That is, infrequent large events causing fat tails. Parameters: $p = 0.003$, $\sigma = 1.8$, impact_boost = 2.0.

4 Simulation Parameters and Their Interpretation

Parameter	Meaning
T	total number of ticks (time steps)
P_0	initial price (set to 100)
κ	price impact coefficient; higher = more sensitivity to flow
L_0	base liquidity; higher = deeper market, smaller moves
α	persistence of volatility EWMA
μ_V	drift of fundamental value (slow trend)
ρ	rate of mean reversion of value to recent price mean
w_i	population weight of agent type i
k	aggressiveness factor for trading response
τ	threshold triggering trades (z-score or mispricing level)
p	probability of whale event per tick
σ	dispersion of whale order magnitude (fat-tail control)

The baseline configuration used in experiments:

```
cfg = Config(T=6000, kappa=0.022, L0=140.0)
agents = {
  "Noise":      (NoiseTrader(size=1.2, freq=0.7), 0.30),
  "DCA":        (DCAInvestor(period=28, cash=30.0), 0.10),
  "Momentum":   (MomentumFollower(lookback=40, k=1.2, freq=0.8), 0.22),
  "MeanRevert":(MeanReverter(L=80, thresh=1.0, k=1.0, freq=0.7), 0.18),
  "Value":      (ValueInvestor(k=2.0, tol=0.008, freq=0.4), 0.10),
  "Whale":      (Whale(p=0.003, sigma=1.8, impact_boost=2.0), 0.10),
}
```

5 Implementation Summary

The Python implementation is structured as follows:

- `Agent`: base class defining the interface `decide(t, price, state)`.
- Subclasses implement individual trading logics.
- `Simulator`: maintains arrays for price, value, volatility, liquidity; applies updates per tick.
- `LiveRunner`: uses `matplotlib.animation.FuncAnimation` for live rendering.

Numerical choices:

- The multiplicative update ensures $P_t > 0$.
- EWMA volatility smooths sudden jumps, mimicking persistent risk.
- Liquidity function introduces reflexivity: volatility up $\Rightarrow$ liquidity down.
- Random number generators (`numpy.random.default_rng`) provide reproducibility via a fixed seed.

6 Results

Below are the result of 3 test runs with random seeds at $T = 5000$.

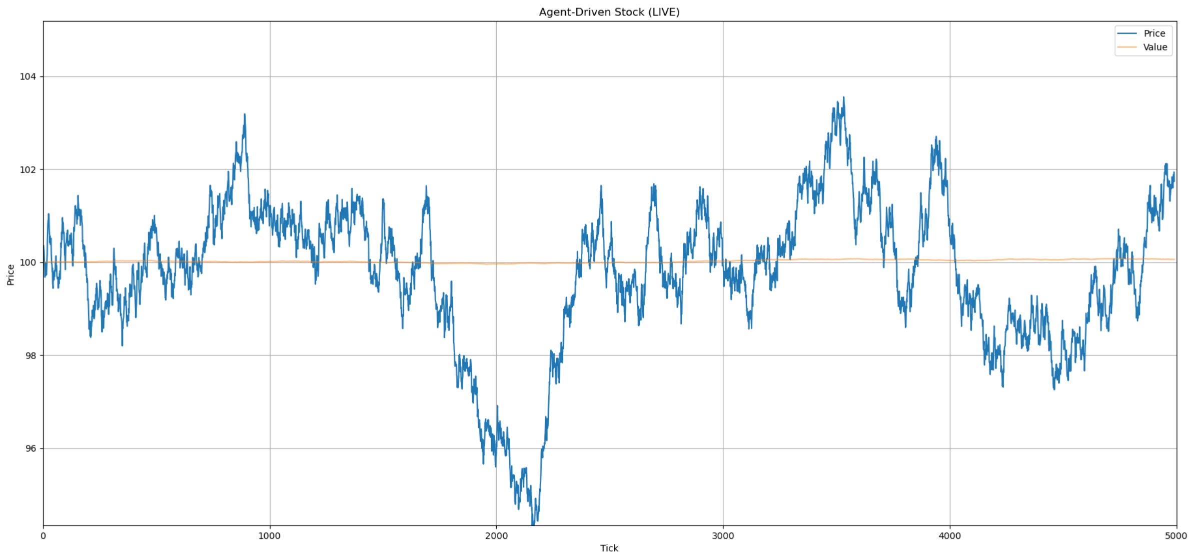

Figure 1: Test run 1

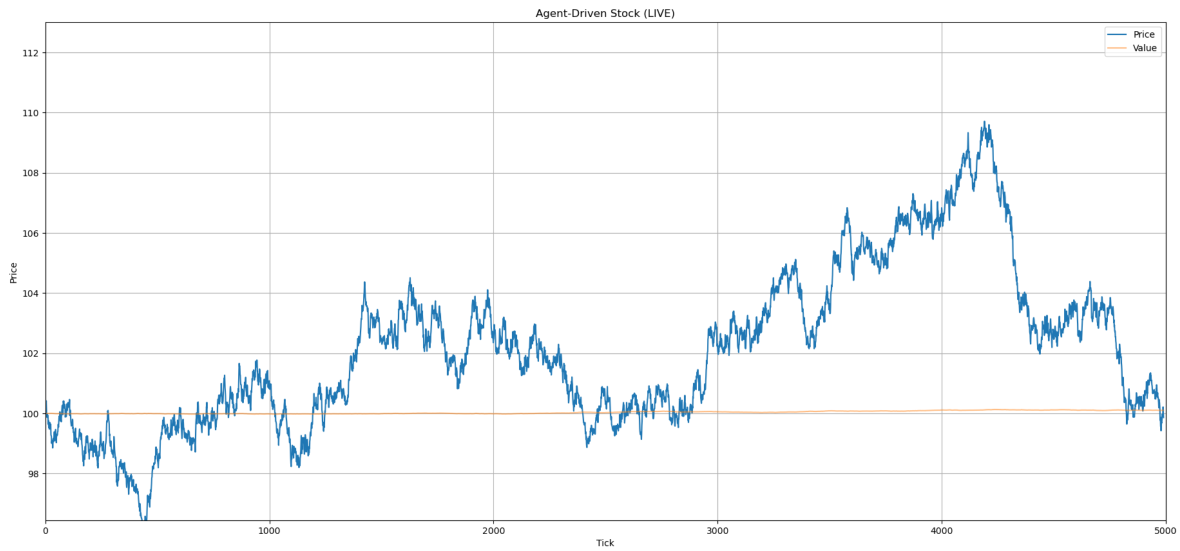

Figure 2: Test run 2

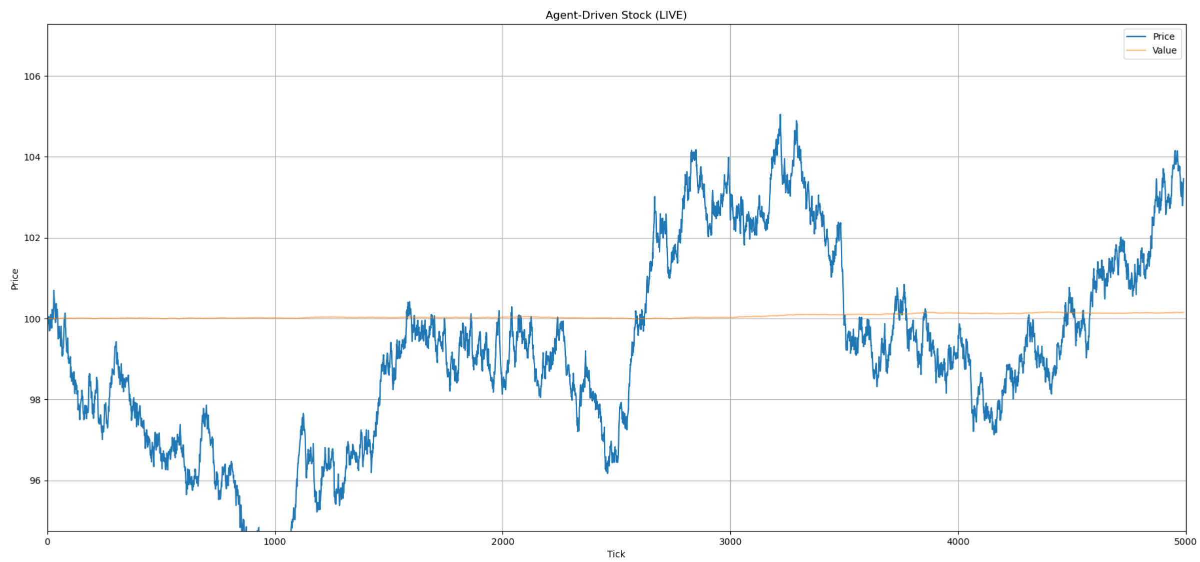

Figure 3: Enter Caption

7 Discussion

This simulator is a minimal agent-based market. Despite its simplicity, it reproduces hallmark features of real financial data. These include:

- **Volatility clustering:** feedback between volatility and liquidity produces bursts of activity.
- **Fat-tailed returns:** rare but large "whale" events generate heavy-tailed distributions.

- **Mean reversion and momentum:** the coexistence of opposite strategies yields alternating trends and stabilisations.
- **Endogenous fluctuations:** no external random process drives price; the chaos comes from agent interaction.

8 Related Work

As mentioned in the introduction, such models are broadly known as **Agent-Based Models (ABMs)** or **Heterogeneous Agent Models (HAMs)** in computational economics and econophysics. Similar frameworks include:

- Lux & Marchesi (1999): "Scaling and criticality in a stochastic multi-agent model of a financial market."
- LeBaron (2000): "Agent-based computational finance: Suggested readings and early research."
- Farmer & Foley (2009): "The economy needs agent-based modelling," *Nature*.

9 Conclusion

The presented simulation illustrates how simple deterministic and stochastic behavioural rules can collectively produce complex market-like dynamics. By employing heterogeneous agents instead of an exogenous random walk, the model generates price series that fluctuate, trend, crash, and recover purely from internal interactions. The framework can be eaasily extended with additional agent types (for example, market makers, stop-loss cascades, volatility-targeters, etc) or multi-asset coupling to explore further systemic phenomena.

Code: implemented in Python 3.11 using `numpy` and `matplotlib`.

References

[1] T. Lux and M. Marchesi, *Scaling and criticality in a stochastic multi-agent model of a financial market*, Nature, **397** (1999), pp. 498–500.

[2] B. LeBaron, *Agent-based computational finance: Suggested readings and early research*, Journal of Economic Dynamics and Control, **24** (2000), pp. 679–702.

[3] J.D. Farmer and D. Foley, *The economy needs agent-based modelling*, Nature, **460** (2009), pp. 685–686.

[4] L. Tesfatsion and K.L. Judd (eds.), *Handbook of Computational Economics, Vol. 2: Agent-Based Computational Economics*, Elsevier, Amsterdam, 2006.

[5] A. Kirman, *Whom or what does the representative individual represent?*, Journal of Economic Perspectives, **6**(2) (1992), pp. 117–136.

[6] C. Hommes, *Heterogeneous agent models in economics and finance*, in *Handbook of Computational Economics*, Vol. 2, Elsevier, 2006, pp. 1109–1186.

[7] R. Cont, *Empirical properties of asset returns: stylized facts and statistical issues*, Quantitative Finance, **1** (2001), pp. 223–236.

[8] B. Mandelbrot, *The variation of certain speculative prices*, Journal of Business, **36**(4) (1963), pp. 394–419.

[9] W.B. Arthur, *Complexity and the economy*, Science, **284** (1999), pp. 107–109.

[10] C. Chiarella, G. Iori, and J. Perelló, *The impact of heterogeneous trading rules on stylized facts of prices*, Quantitative Finance, **9**(3) (2009), pp. 397–409.

[11] T. Lux, *Applications of statistical physics in finance and economics*, in *Complexity and Evolution*, eds. D.S. Sornette et al., Springer, 2018, pp. 123–148.

[12] G. Ehrenstein, F. Westerhoff, and D. Stauffer, *Fat tails and volatility clustering in a simple artificial stock market*, Physica A, **350** (2005), pp. 277–290.

[13] J.-P. Bouchaud and R. Cont, *A Langevin approach to stock market fluctuations and crashes*, European Physical Journal B, **6** (1998), pp. 543–550.

[14] D. Lamper, S. Howison, and N.F. Johnson, *Predictability of large future changes in a competitive evolving population*, Physical Review Letters, **88**(1) (2002), 017902.

[15] E. Samanidou, E. Zschischang, D. Stauffer, and T. Lux, *Agent-based models of financial markets*, Reports on Progress in Physics, **70**(3) (2007), pp. 409–450.

[16] J. Hellthaler, C. Hommes, and F. Wagener, *Dynamic complexity in speculative markets with heterogeneous agents: A numerical investigation*, Structural Change and Economic Dynamics, **16**(2) (2005), pp. 179–201.

[17] A. Kirman, *The Economic Crisis is a Crisis for Economic Theory*, CESifo Economic Studies, **56**(4) (2010), pp. 498–535.

[18] L. Tesfatsion, *Agent-based computational economics: A constructive approach to economic theory*, in *Handbook of Computational Economics*, Vol. 2, Elsevier, 2017, pp. 831–880.