CSE221

Lecture 2: Arrays

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Acknowledgment: The content of this file is based on the slides of the textbook as well as the slides used in former lectures at UNIST.

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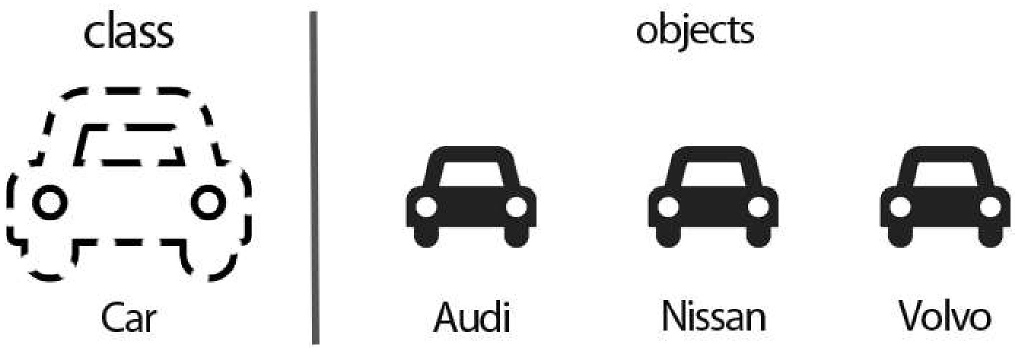
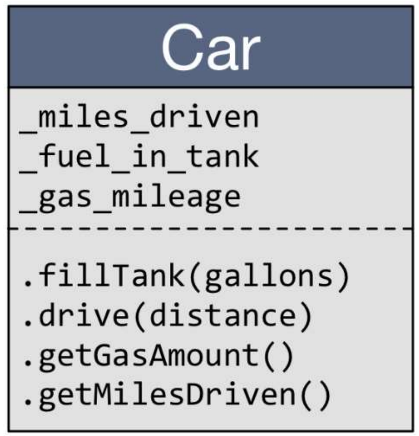
Outline

- Basic concepts
- Arrays
- Examples
  - Sparse matrices
  - Selection sort

Object

- Entity that performs *computations* and has a *local state*
  - –Procedural elements + variables
- An instance of a class





Credits: https://javatutorial.net/java-oop

Object-oriented Design

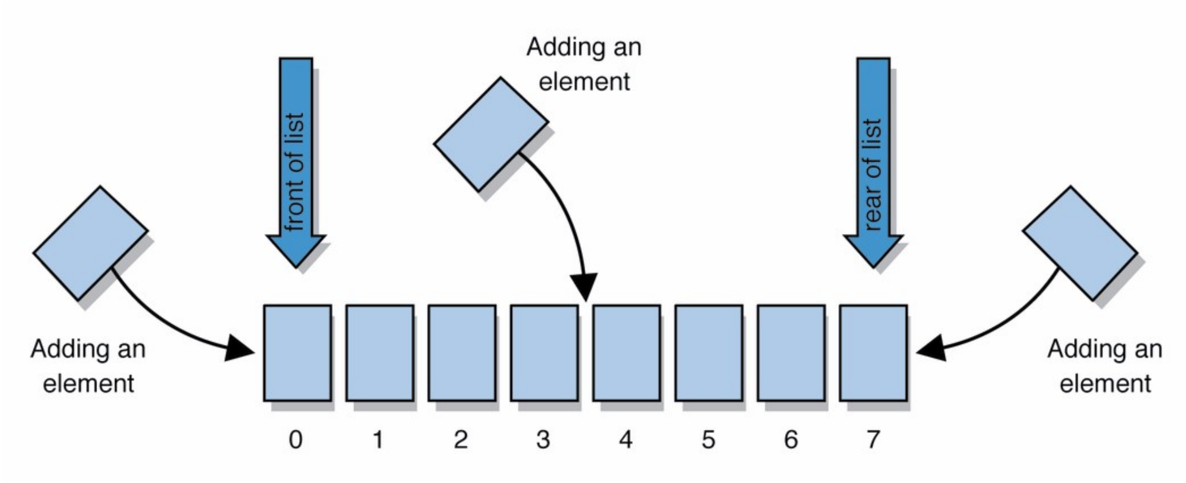
- Object as fundamental building block
  - View software as a set of interacting objects that model entities
  - Use *divide and conquer*
- Benefits
  - Encourage code reusability and flexibility
  - Localize changes: related objects *only*

Data Encapsulation and Abstraction

- Data encapsulation
  - Concealing of the implementation details of a data object from the outside
- Data abstraction
  - Separation between the specification (what) of a data object and its implementation (how)

Abstract Data Type (ADT)

- Definition for expected operations and behavior
- Example: list
  - A collection storing an ordered sequence of elements
  - Operations:



Data Structure and ADT

- Data structure
  - How we organize/store data points
  - Determines how each operation would behave
- One ADT can be implemented in many ways
- One data structure can be used for multiple ADTs

ADT First, Then Data Structure

- Wrong way:
  - I will represent my data A as an array
- Right way:
  - I will be frequently accessing a random data point of my data A, so I need operation []
  - I will therefore use an array

Benefits of Abstraction and Encapsulation

- Simplification of software development
  - Enables parallel development of subtasks
- Testing and debugging
  - Easier to identify problems by testing subtasks
- Reusability
  - Easier to take out a part and use elsewhere
- Modification of data type
  - Implementation can be modified without changing i nterfaces

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- Basic concepts
- Arrays
- Examples
  - –Sparse matrices
  - –Selection sort

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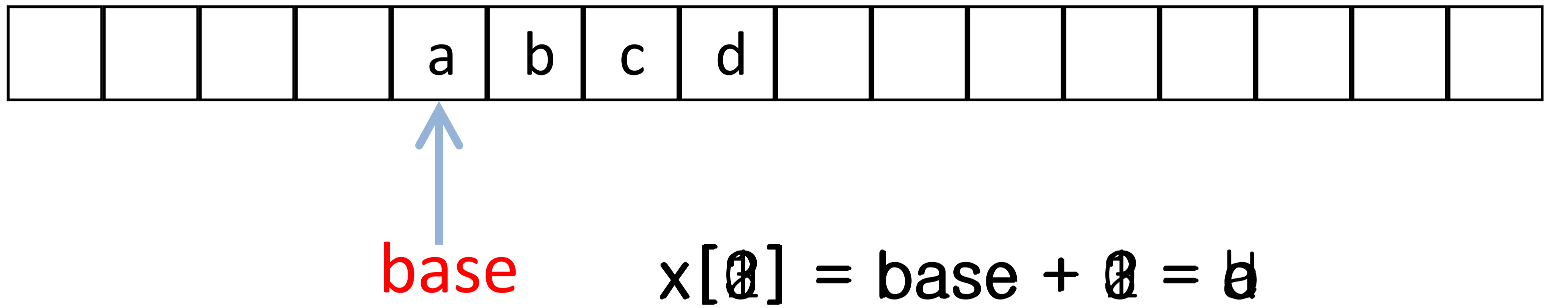
Arrays in C/C++

- Continuous memory space with the same data type
- `type name [elements];`
- Example:
  - int a[100];
  - float b[10][20];
  - mytype c[5]; // user-defined data type
- Each element accessed by index

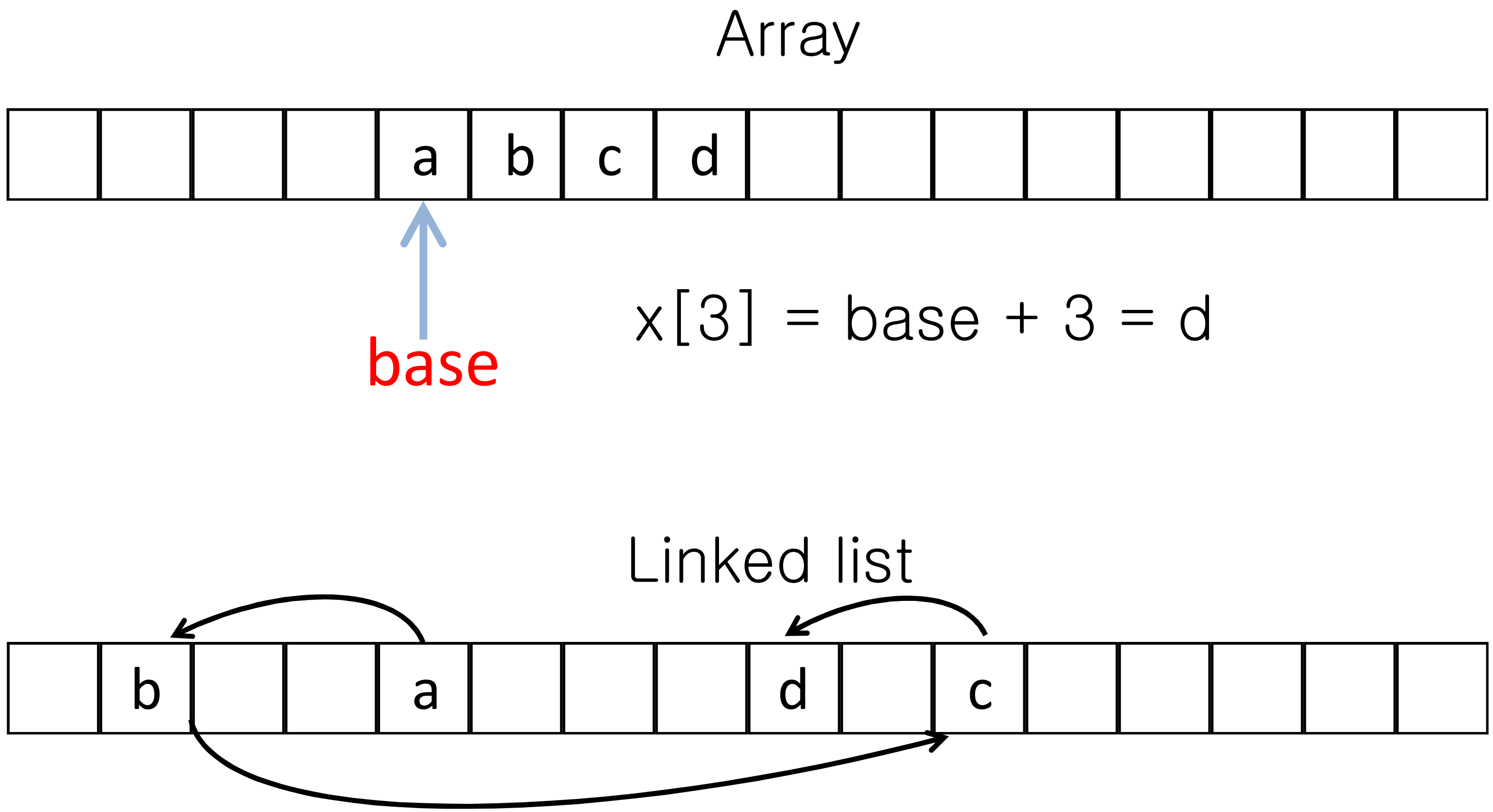
1D Array Representation In C

1D Memory Space



- 1-dimensional array x = [a, b, c, d]
- map into contiguous memory locations
- location(x[i]) = base + i

Array v.s. Pointer-based Linked List



Array v.s. Pointer-based Linked List

- Arrays
  - Good for random and sequential access
    - Indexing
    - Modern architecture reads data at a chunk
  - Good when the size is quite fixed
- Linked list
  - Good for frequent inserting and deleting
  - Resizing is easier

2D Arrays

- The elements of a 2-dimensional array x declared as:
- int x[3][4];
- may be shown as a table

x[0][0]	x[0][1]	x[0][2]	x[0][3]
x[1][0]	x[1][1]	x[1][2]	x[1][3]
x[2][0]	x[2][1]	x[2][2]	x[2][3]

Rows of a 2D Array

x[0][0] x[0][1] x[0][2] x[0][3] → row 0

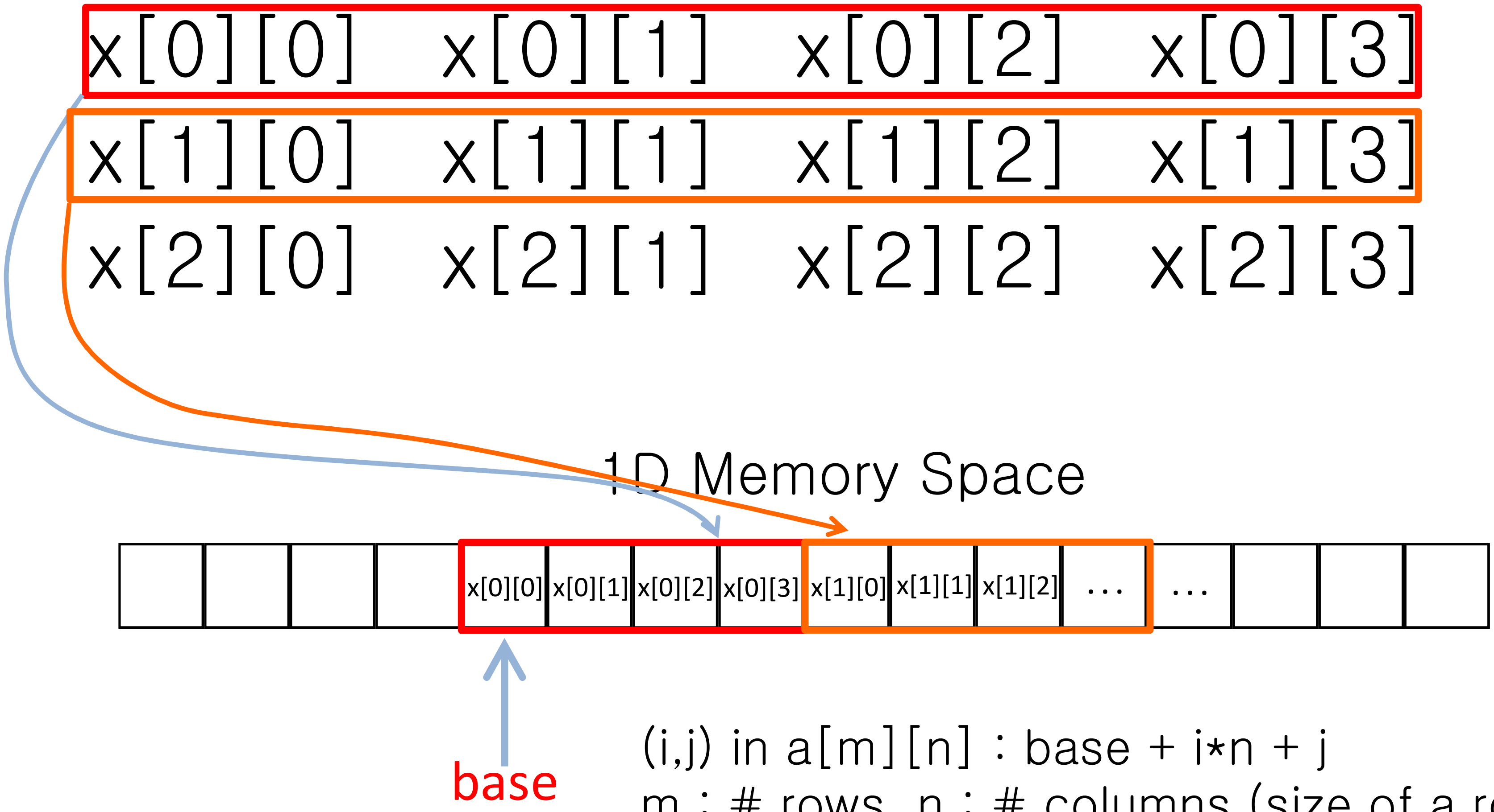
x[1][0] x[1][1] x[1][2] x[1][3] → row 1

x[2][0] x[2][1] x[2][2] x[2][3] → row 2

Columns of a 2D Array

x[0][0]	x[0][1]	x[0][2]	x[0][3]
x[1][0]	x[1][1]	x[1][2]	x[1][3]
x[2][0]	x[2][1]	x[2][2]	x[2][3]
col 0	col 1	col 2	col 3

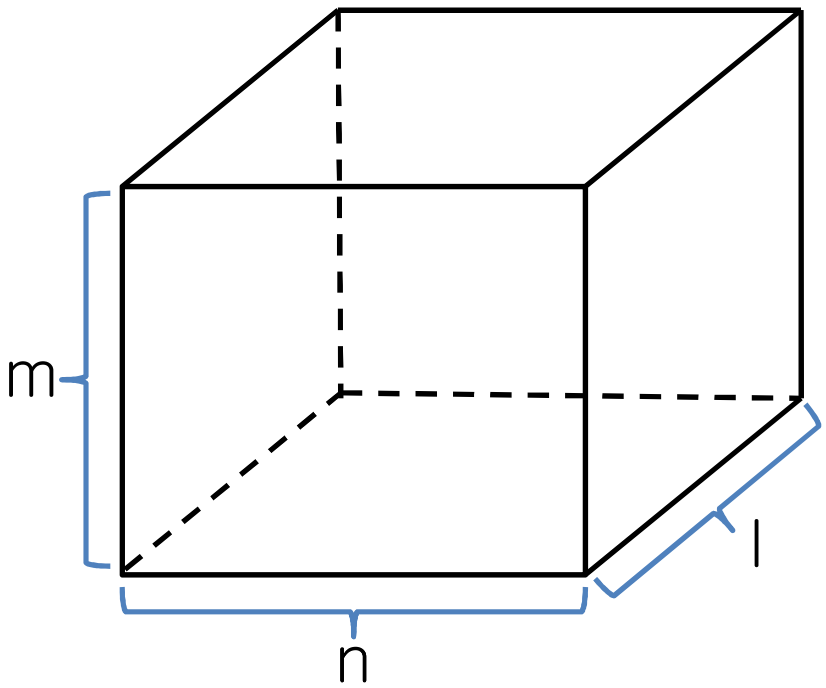
1D Indexing of 2D Array



(i,j) in a[m][n] : base + i*n + j
m : # rows, n : # columns (size of a row)
i : row index, j : column index

1D Indexing of 3D Array

- (i,j,k) in x[l][m][n] : base + i*m*n + j*n + k



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Array Abstract Data Type

- More general than array in C/C++
  - Contiguous memory location is more of a implementation issue
- Properties:
  - Mapping between index and value
    - Do not need to be implemented with pure array!
  - Operators to retrieve and store value

```
class GeneralArray {
// A set of pairs <index, value> where for each
// value of index in index set, there is a value of
// type float
public:
      // Constructor.
      // j : dimension, list : range of each dimension.
      // initValue : initial value
      GeneralArray(int j,
                   RangeList list,
                   float initValue);

      // Return the float associate with i if i is in
      // the index set. Otherwise throw an exception
      float Retrieve(index i);

      // Replace the old value associate with i if i is
      // in the index set. Otherwise throw an exception
      void Store(index i, float x)
}
```

Outline

- Recap OOP: programming concepts
- Arrays
- Examples
  - Sparse matrices
  - Selection sort

Matrices

- A[m][n]
  - m x n matrix A
  - m : # of rows
  - n : # of column
  - m*n : # of elements
- Accessing an element at i-th row and j-th col
  - A[i][j]
- 2D array is often used to represent matrices → Dense matrix

Sparse Matrices

- Mostly zero

$$
\begin{array}{c c}
 & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\
\begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array} &
\begin{bmatrix} -27 & 3 & 4 \\ 6 & 82 & -2 \\ 109 & -64 & 11 \\ 12 & 8 & 9 \\ 48 & 27 & 47 \end{bmatrix}
\end{array}
$$

(a) Dense Matrix

$$
\begin{array}{c c}
 & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} &
\begin{bmatrix} 15 & 0 & 0 & 22 & 0 & -15 \\ 0 & 11 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & -6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 91 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 28 & 0 & 0 & 0 \end{bmatrix}
\end{array}
$$

(b) Sparse Matrix

Sparse Matrix Representation

- Array of triples <row, col, value> for nonzero elements
- # elements to store (space requirement) for nxn matrix
  - –Dense : n^2
  - –Sparse diagonal matrix : 3*n

Meaningful when n is very large!

Tridiagonal Matrix

$$f_n = \begin{vmatrix} a_1 & b_1 & & & \\ c_1 & a_2 & b_2 & & \\ & c_2 & \ddots & \ddots & \\ & & \ddots & \ddots & b_{n-1} \\ & & & c_{n-1} & a_n \end{vmatrix}.$$

Row Major vs. Column Major

- 2D->1D layout
- Row major
  - Store each row in contiguous memory
- Column major
  - Store each column in contiguous memory

Sparse Matrix

- Array of triples <row, col, value> for nonzero elements

```
class SparseMatrix;
    class MatrixTerm {
    friend class SparseMatrix;
    private:
        int row, col, value;
    };

private:
    int rows, cols, terms, capacity;
    MatrixTerm * smArray;
```

Example

$$
\begin{array}{c|cccccc}
 & 0 & 1 & 2 & 3 & 4 & 5 \\
\hline
0 & 15 & 0 & 0 & 22 & 0 & -15 \\
1 & 0 & 11 & 3 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & -6 & 0 & 0 \\
3 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 91 & 0 & 0 & 0 & 0 & 0 \\
5 & 0 & 0 & 28 & 0 & 0 & 0
\end{array}
$$

		row	col	val
smArray	[0]	0	0	15
	[1]	0	3	22
	[2]	0	5	-15
	[3]	1	1	11
	[4]	1	2	3
	[5]	2	3	-6
	[6]	4	0	91
	[7]	5	2	28

Row-major ordering

Space Cost Analysis

1 integer

$$\begin{array}{c|c}
 & \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 \end{matrix} \\
\begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} &
\begin{bmatrix}
15 & 0 & 0 & 22 & 0 & -15 \\
0 & 11 & 3 & 0 & 0 & 0 \\
0 & 0 & 0 & -6 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
91 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 28 & 0 & 0 & 0
\end{bmatrix}
\end{array}$$

3 integers
(3x more space)

	row	col	val
[0]	0	0	15
[1]	0	3	22
[2]	0	5	-15
[3]	1	1	11
[4]	1	2	3
[5]	2	3	-6
[6]	4	0	91
[7]	5	2	28

space saving = 12
(36/24 integers in dense/sparse matrix)

- Sparse representation is more effective when there are many empty elements

Sparse Matrix Transpose

- Exchange (i,j) to (j,i)
- Pseudo code:
  - for each row i, store (i,j) to (j,i)
  - Problem

    (0, 0, 15) -> (0, 0, 15)

    (0, 3, 22) -> (3, 0, 22)

    (0, 5, -15) -> (5, 0, -15)

    (1, 1, 11) -> (1, 1, 11) **X** Violate row-major order

Sparse Matrix Transpose

- New approach
  - Use column major order traversal
    - After transpose, matrix is row-major order

```
for (all element in column j)
  store (i,j,value) of the original matrix as (j,i,value) of transposed matrix
```

Example

		row	col	val
smArray	[0]	0	0	15
	[1]	0	3	22
	[2]	0	5	-15
	[3]	1	1	11
	[4]	1	2	3
	[5]	2	3	-6
	[6]	4	0	91
	[7]	5	2	28

		row	col	val
smArray	[0]	0	0	15
	[1]	0	4	91
	[2]			
	[3]			
	[4]			
	[5]			
	[6]			
	[7]			

for Column 0

Example

		row	col	val
smArray	[0]	0	0	15
	[1]	0	3	22
	[2]	0	5	-15
	[3]	1	1	11
	[4]	1	2	3
	[5]	2	3	-6
	[6]	4	0	91
	[7]	5	2	28

		row	col	val
smArray	[0]	0	0	15
	[1]	0	4	91
	[2]	1	1	11
	[3]			
	[4]			
	[5]			
	[6]			
	[7]			

for Column 1

Example

		row	col	val
smArray	[0]	0	0	15
	[1]	0	3	22
	[2]	0	5	-15
	[3]	1	1	11
	[4]	1	2	3
	[5]	2	3	-6
	[6]	4	0	91
	[7]	5	2	28

		row	col	val
smArray	[0]	0	0	15
	[1]	0	4	91
	[2]	1	1	11
	[3]	2	1	3
	[4]	2	5	28
	[5]			
	[6]			
	[7]			

for Column 2

Example

		row	col	val
smArray	[0]	0	0	15
	[1]	0	3	22
	[2]	0	5	-15
	[3]	1	1	11
	[4]	1	2	3
	[5]	2	3	-6
	[6]	4	0	91
	[7]	5	2	28

		row	col	val
smArray	[0]	0	0	15
	[1]	0	4	91
	[2]	1	1	11
	[3]	2	1	3
	[4]	2	5	28
	[5]	3	0	22
	[6]	3	2	-6
	[7]			

for Column 3

Example

		row	col	val
smArray	[0]	0	0	15
	[1]	0	3	22
	[2]	0	5	-15
	[3]	1	1	11
	[4]	1	2	3
	[5]	2	3	-6
	[6]	4	0	91
	[7]	5	2	28

		row	col	val
smArray	[0]	0	0	15
	[1]	0	4	91
	[2]	1	1	11
	[3]	2	1	3
	[4]	2	5	28
	[5]	3	0	22
	[6]	3	2	-6
	[7]			

for Column 4 : No Match

Example

		row	col	val
smArray	[0]	0	0	15
	[1]	0	3	22
	[2]	0	5	-15
	[3]	1	1	11
	[4]	1	2	3
	[5]	2	3	-6
	[6]	4	0	91
	[7]	5	2	28

		row	col	val
smArray	[0]	0	0	15
	[1]	0	4	91
	[2]	1	1	11
	[3]	2	1	3
	[4]	2	5	28
	[5]	3	0	22
	[6]	3	2	-6
	[7]	5	0	-15

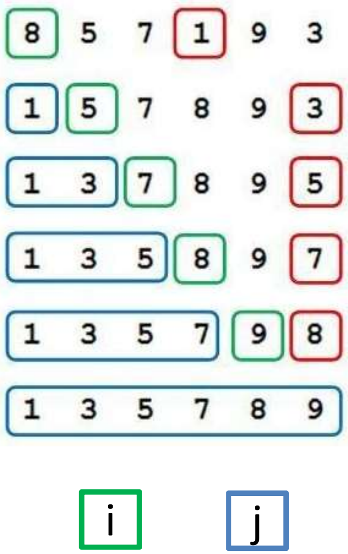
for Column 5

```
SparseMatrix SparseMatrix::Transpose(){
    SparseMatrix b;             // b : output (transposed matrix)
    b.Rows = Cols;              // b rows = a cols
    b.Cols = Rows;              // b cols = a rows
    b.Terms = Terms;            // # of elements is same

    if (Terms > 0){             // non empty matrix
        int CurrentB = 0;
        for (int c = 0; c < Cols; c++)          // per each column
            for (int i = 0; i < Terms; i++)     // for all nonzero elements in sparse
                if (smArray[i].col ==c) {       // matrix
                    b.smArray[CurrentB].row = c;
                    b.smArray[CurrentB].col = smArray[i].row;
                    b.smArray[CurrentB].value = smArray[i].value;
                    CurrentB++;
                }
    }
    return b;
}
```

Selection Sort

- *From those integers that are currently unsorted, find the smallest and place it next in the sorted list*




```
for (int i = 0; i < n; i++) [
   examine a[i] to a[n-1] and suppose the smallest is at a[j];
   interchange a[i] and a[j];
}
```

This is algorithm!

Selection Sort

```
void sort(int *a, const int n)
// sort n integers a[0] to a[n-1] into nondecreasing order.
{
    for (int i = 0; i < n; i++)
    {
        int j = i;
        // find smallest integer in a[i] and a[n-1]
        for (int k = i + 1; k < n; k++)
            if (a[k] < a[j]) j = k;
        // swap
        int temp = a[i]; a[i] = a[j]; a[j] = temp;
     }
 }
```

This is C++ program!

Questions?