CSE221

# Lecture 10:
# Hashing

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Acknowledgment: The content of this file is based on the slides of the textbook as well as the slides provided in former lectures at UNIST.

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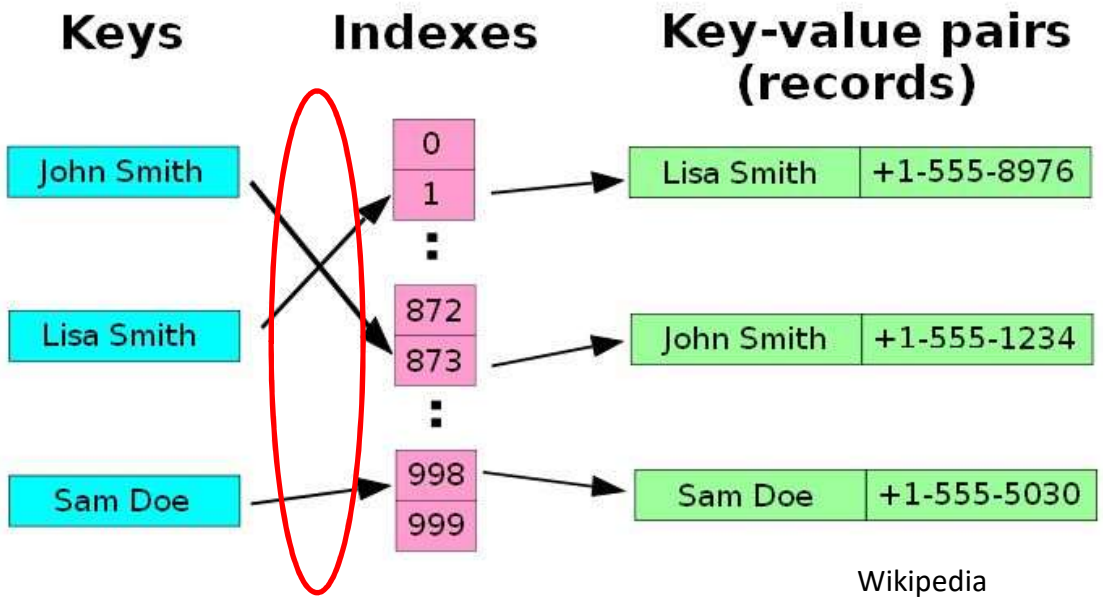
# Outline

- Static hashing
  - Division
  - Mid square
  - Folding
- Overflow handling

# Hashing

- Hash table or hash map is a data structure that associates values with keys
- Example: phone book




Wikipedia

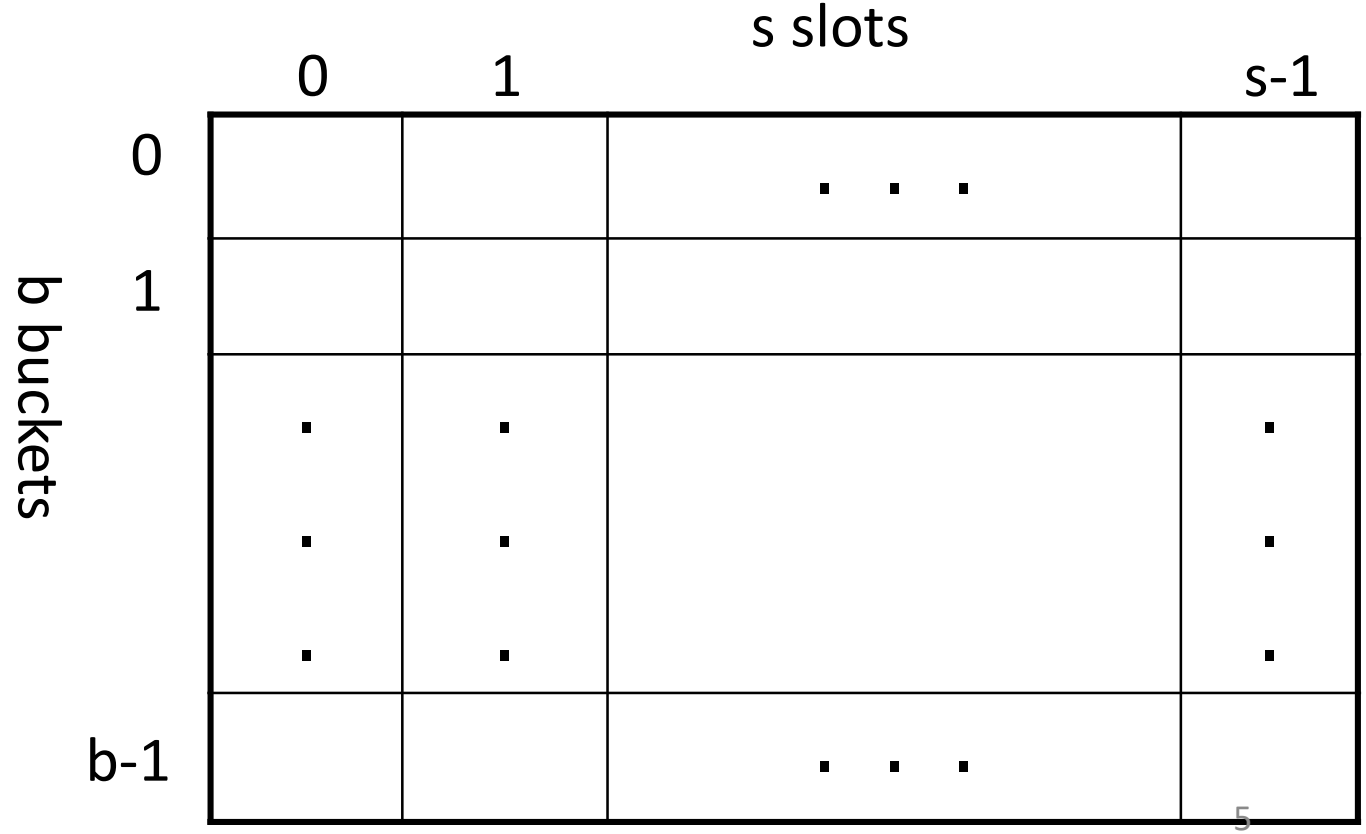
Hash function

# Hashing

- Hash function: hashing method
  - Calculate indexes from keys
  - Expected time: O(1)
  - Input data is not known in advance
    - Should consider all possible key values
- Types
  - Static hashing vs. Dynamic hashing

# Static Hashing

- Key-value pairs are stored in a fixed size hash table



# Static Hashing

- Key-value pairs are stored in a fixed size hash table
  - 2-D structure:
    - A hash table is partitioned into *b buckets*
    - Each bucket has *s slots*
    - Each slot holds one record
  - Hash function *h(k)* transforms *key k* into an address (or index) to a bucket in the hash table
    - Each key appears only once in the table

# Static Hashing

- n: current number of key-value pairs in the table
- T: all possible keys
- Key density : n/T
  - Fraction of keys in the table
  - Usually very low because not all keys are used
- Loading density(factor) : $\alpha = n/(sb)$
  - How much the hash table is used
  - 1 : table is full, 0 : table is empty

# Hashing

- Hash function *h* computes hash table address of a key without any other information
  - *h*(*k*) is a function of *k*
- Hash function may generate identical addresses for different keys
  - Why? Key density is usually very low
    - There is a chance for two keys map to a same location
  - A bucket may store multiple keys in slots

# Example

- b = 26, s = 2, n = 10
- Key: a string starting with a character
- Hash function: A~Z to 0~25, from the first character of key
  - A -> 0
  - A2 -> 0
  - D -> 3
  - G -> 6
  - GA -> 6

| | Slot 1 | Slot 2 |
|---|---|---|
| 0 | A | A2 |
| 1 | | |
| 2 | | |
| 3 | D | |
| 4 | | |
| 5 | | |
| 6 | GA | G |
| . | . | . |
| . | . | . |
| . | . | . |
| 25 | | |

# Example

- How about additionally inserting A1 and A3?
  - *Collision*: h(A1) = h(A3) = h(A) = h(A2) = 0
  - *Overflow*: no slot left

| | Slot 1 | Slot 2 |
|---|---|---|
| 0 | A | A2 |
| 1 | | |
| 2 | | |
| 3 | D | |
| 4 | | |
| 5 | | |
| 6 | GA | G |
| . | . | . |
| . | . | . |
| . | . | . |
| 25 | | |

A, A2, A1, A3 : collisions

A1, A3 : overflows

# Hash Table Issues

- Choice of hash function
  - Easy to compute
  - Avoid collision as much as possible
- Overflow handling method
  - Should handle when there is no space in the bucket for the new pair
- Size of hash table
  - If too small, the collision occurs often

# Two Steps for Hashing with Strings

- In many cases, keys are given as strings
- We first transform a string to an integer
- We then produce an address from the integer

# String To Integer

- `char` : 1 byte, `int` : 4 bytes
- A character has a unique value (i.e., ASCII code)

- We can pack and convert multiple characters in a string into a single integer
  - E.g. two-character string key[0:1] can be converted into a unique 4 byte non-negative `int`

```
number = key[0];
number += ((int) key[1]) << 8;
```

# String To Integer

- Example
  - Key: SA
  - S: ASCII code 83 = 01010011
  - A: ASCII code 65 = 01000001
  - S + A<<8
    = 0000000001010011 + 0100000100000000
    = 0100000101010011 (Binary)
    = 16723 (Decimal)

# Generalization

```
unsigned int stringToInt(char *key)
{
    int number = 0;
    while(*key)
    {
        number += *key++;
        if (*key)
            number += ((int) *key++) <<8;
    }
    return number;
}
```

This code generates 16bit integer for a string of arbitrary length (every two characters are converted into a 16bit integer and added together)

# Uniform Hash Function

- If *k* is a key chosen randomly, then we want probability of *h*(*k*)=*i* to be *1/b* for all buckets
  - Distribute key values uniformly over the range
- Uniform hash function minimizes collision / overflow when keys are selected randomly
- E.g., division, mid-square, folding, etc

# Hash Function: Division

- Domain is all nonnegative integers
- h(k)=k % D  (D is usually b)
- Generated address: 0 ~ D-1
- For *b buckets*, the number of integers that get hashed into bucket *i* is approximately $2^{31}/b$
- The division maps approximately the same number of keys into each bucket
  - Uniform hashing function

# Hash Function: Division

- Issue: keys tend to be biased
- If divisor (D) is an even number
  - Odd integers hash into odd buckets
  - Even integers into even buckets
  - Example
    - 20%14 = 6, 30%14 = 2, 8%14 = 8
    - 15%14 = 1, 3%14 = 3, 23%14 = 9
- If divisor is an odd number
  - Odd (even) integers may hash into any bucket
  - Example
    - 20%15 = 5, 30%15 = 0, 8%15 = 8
    - 15%15 = 0, 3%15 = 3, 23%15 = 8

# Hash Function: Mid-square

- Squaring the key and using r middle bits
- Example: r=2 digits in 10-base
  - k = 4567, $k^2$=20857489, h(k) = 57
- Avoid division operation, but expensive
- All bits of the key contribute the result

# Hash Function: Folding

- Partition the key x into several parts, and add the parts together to obtain the hash address
  - Part's size matches the size of the required address
- ex) x=12320324111220, 1000 addresses
  - partition x into 123,203,241,112,20
  - Shift folding
    - return the address 123+203+241+112+20=699
  - Folding at the boundaries
    - return the address 123+302+241+211+20=897