Outline

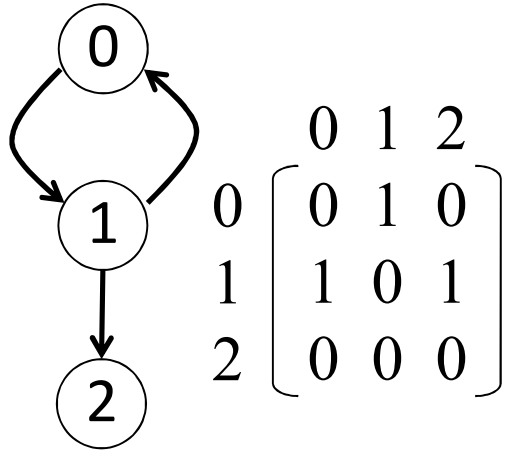
- Graph definitions
- Graph representations
  - Adjacency matrix
  - Adjacency list
  - Adjacency multilist

Adjacency Matrix

- **n x n matrix** for a graph G that has n vertices
  - a[i][j] = 1 if there is an edge (i,j) (or <i,j>)
  - a[i][j] = 0 otherwise

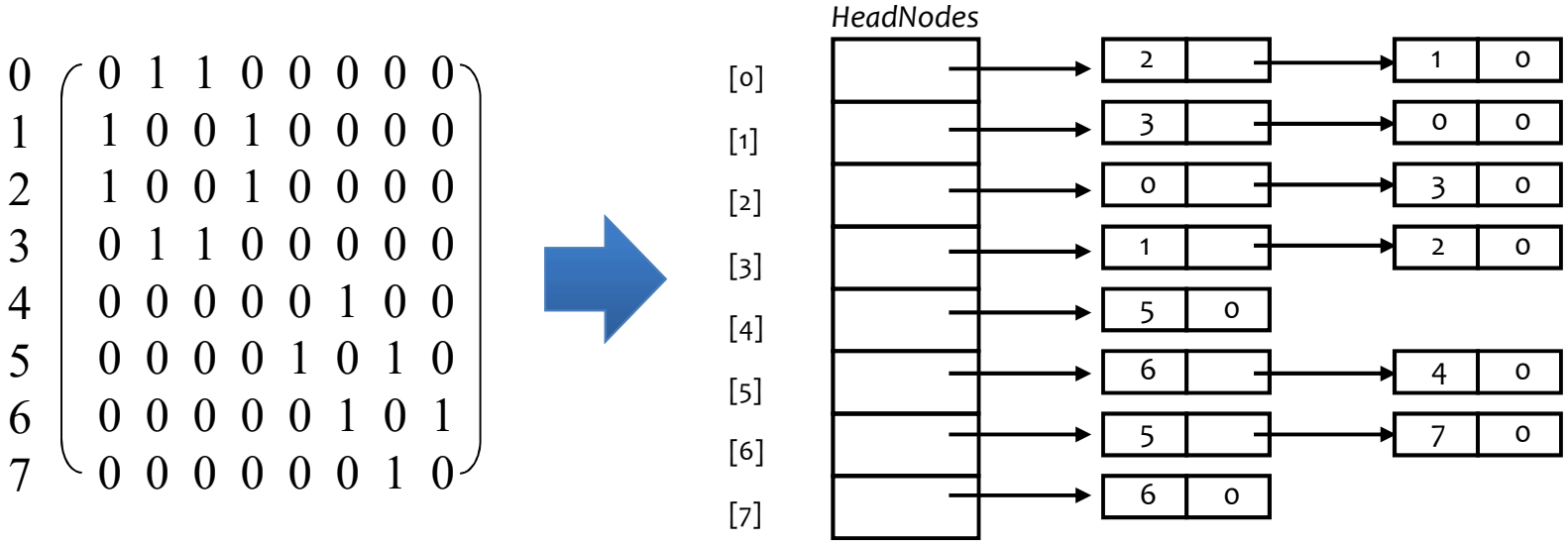
Adjacency Matrix

- Properties
  - Matrix for undirected graph is symmetric
  - Diagonal entries are zero
  - Digraph : row is tail, column is head
    - Sum of row i : out-degree of i
    - Sum of column j : in-degree of j
- Problems
  - Require n^2 bits
  - $O(n^2)$ runtime for several algorithms
    - ex) count the number of edges in a graph



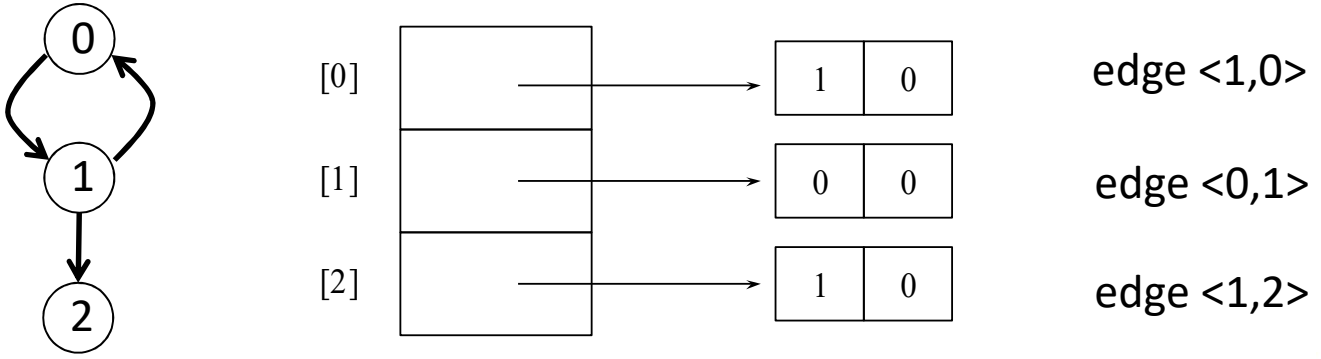
Adjacency List

- *n* rows are represented as *n* linked lists
  - Keep head vertices for each row in linked list
- Vertices in each row can be unordered



Inverse Adjacency List

- Adjacency list can be used for calculating out-degree of a vertex
  - In-degree vertex is not easy
- A row of inverse adjacency list stores incoming vertex list

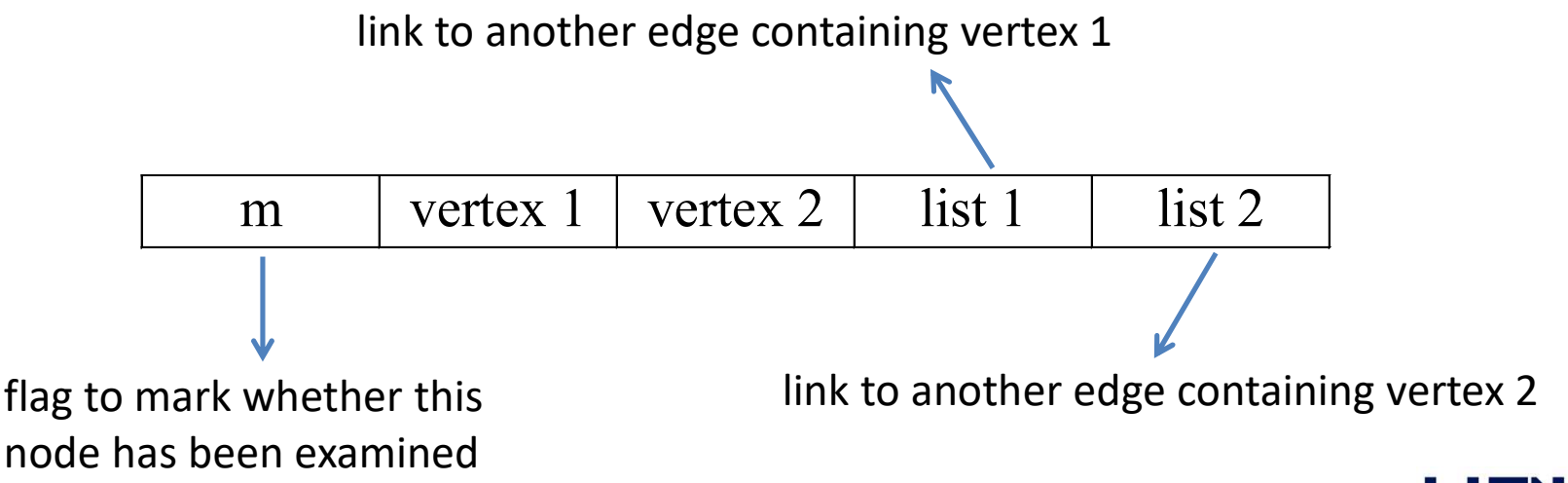


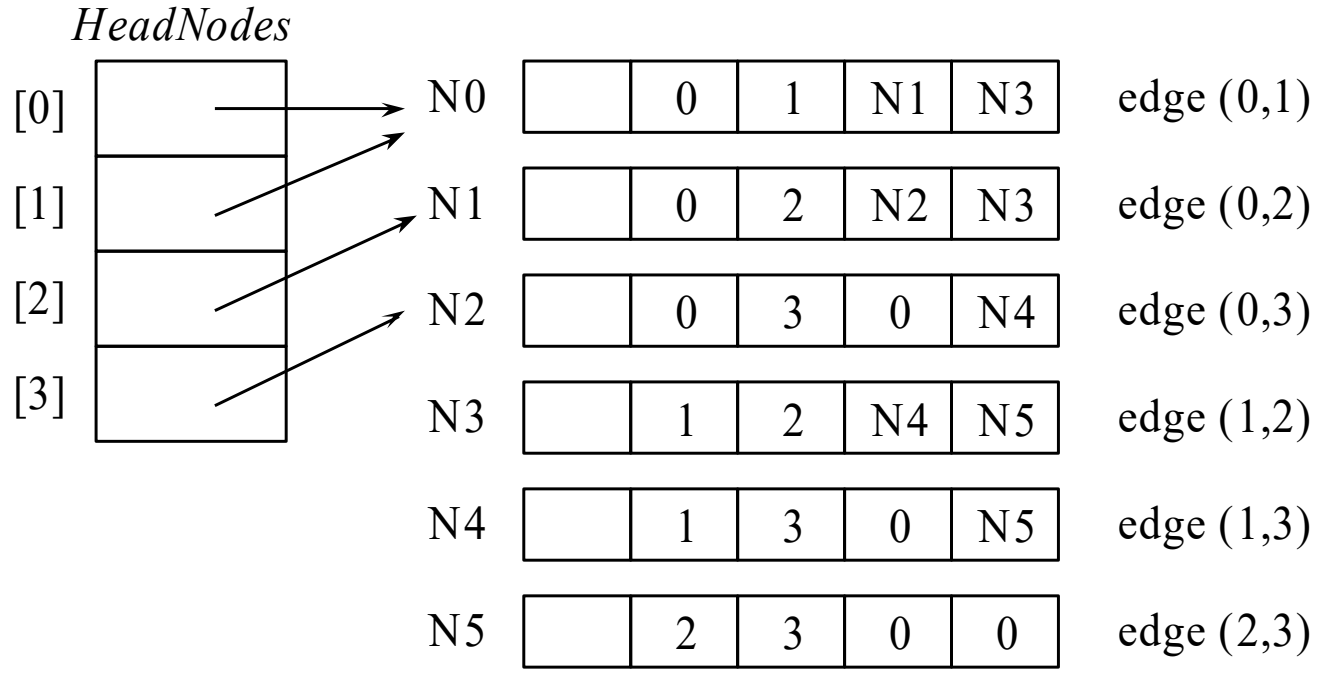
Adjacency Multilist

- Each edge in an undirected graph shows twice in adjacency list
  - Costly to manage
  - What if we should examine an edge only once?

Adjacency Multilist

- Proposal
  - Each edge is represented by exactly one node
  - This node appears in two adjacency lists
- New node structure:





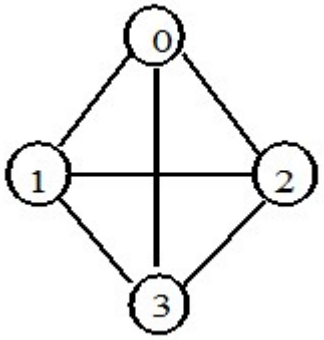
The lists are

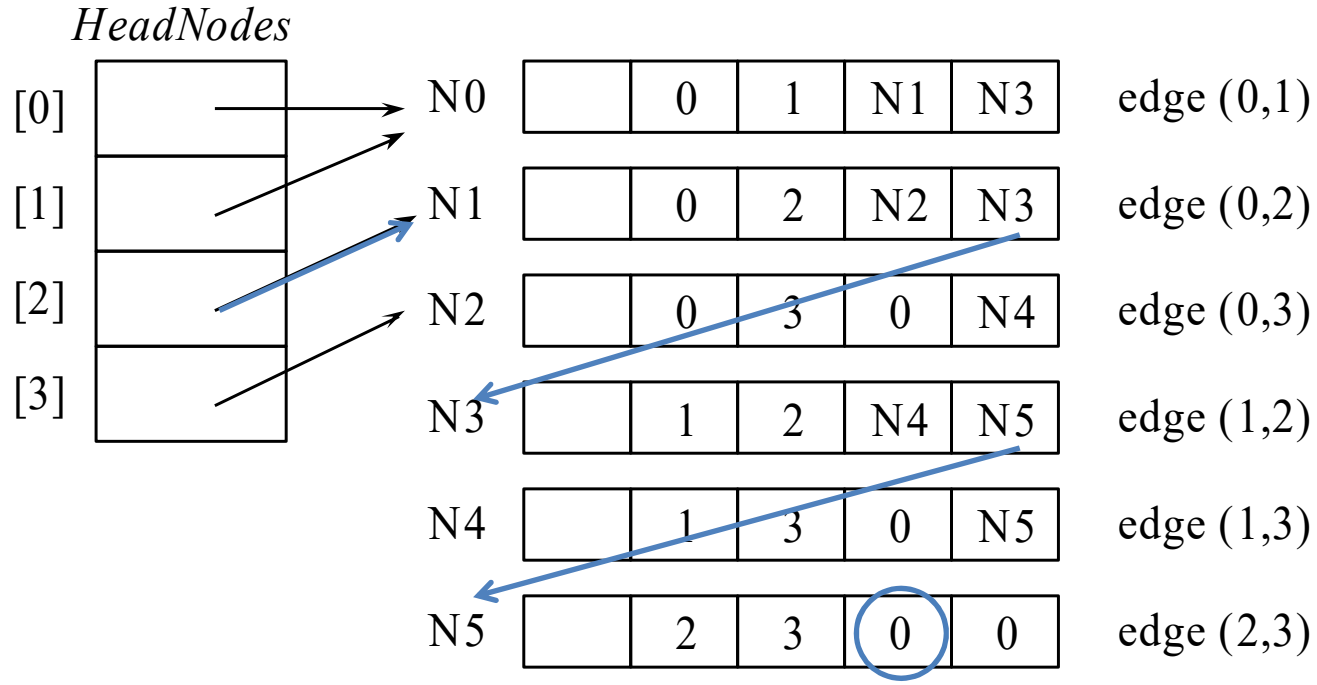
vertex 0: N0 -> N1 -> N2

vertex 1: N0 -> N3 -> N4

vertex 2: N1 -> N3 -> N5

vertex 3: N2 -> N4 -> N5





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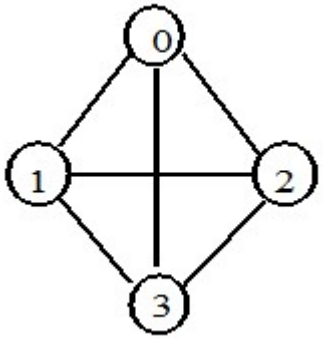
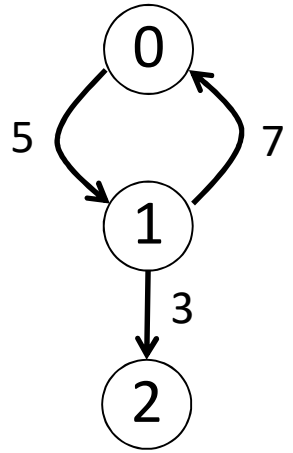


Weighted Edges

- Edges of a graph may have weights assigned
- Adjacency matrix
  - Each entry is weight
- Adjacency list
  - Extra field required
- Network
  - A graph with weighted edges

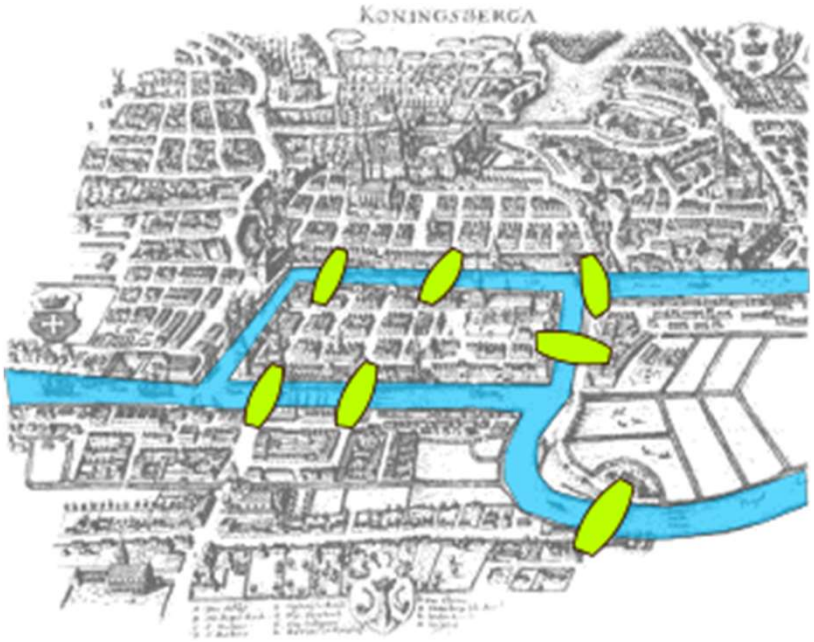
$$\begin{array}{c|ccc} & 0 & 1 & 2 \\ 0 & 0 & 5 & 0 \\ 1 & 7 & 0 & 3 \\ 2 & 0 & 0 & 0 \end{array}$$



Questions?

Historically Notable Graph Problem: The Seven Bridges of Königsberg

- Is it possible to cross each of the bridges only and only once?



Wikipedia