

# CSE221

## Lecture 21: Sorting

Acknowledgment: The content of this file is based on the slides of the textbook as well as the slides provided in former lectures at UNIST.

# Search Records using a Key

- Sequential search in  $a[1:n]$ 
  - Search successful at  $a[i]$  :  $i$  comparisons
  - Search unsuccessful :  $n$  comparisons
  - Average comparison :  $(\sum_{1 \leq i \leq n} i) / n = (n + 1) / 2$
  - $O(n)$
- Searching in an ordered list
  - Binary search :  $O(\log n)$

we need sorting!

# Insertion Sort

- Assume  $a[1:i]$  is ordered
- Insert a new record into the list to get a new ordered list  $a[1:i+1]$

j	[1]	[2]	[3]	[4]	[5]
-	5	4	3	2	1
2	4	5	3	2	1
3	3	4	5	2	1
4	2	3	4	5	1
5	1	2	3	4	5

Elements in red are already sorted

# Insertion Sort

```
void InsertionSort(*a, n)
{
    for(j=2; j<=n; j++)
    {
        temp = a[j];
        Insert(temp, a, j-1);
    }
}
```

```
void Insert(e, *a, i)
{
    a[0]=e;
    while(e<a[i])
    {
        a[i+1]=a[i];
        i--;
    }
    a[i+1]=e;
}
```

insert e to sorted list a[1:i] to make a[1:i+1]

		<i>List L</i>	<i>Priority Queue P</i>
Input		(7, 4, 8, 2, 5, 3, 9)	()
Phase 1	(a)	(4, 8, 2, 5, 3, 9)	(7)
	(b)	(8, 2, 5, 3, 9)	(4, 7)
	(c)	(2, 5, 3, 9)	(4, 7, 8)
	(d)	(5, 3, 9)	(2, 4, 7, 8)
	(e)	(3, 9)	(2, 4, 5, 7, 8)
	(f)	(9)	(2, 3, 4, 5, 7, 8)
	(g)	()	(2, 3, 4, 5, 7, 8, 9)

# Insertion Sort

- Analysis

- Worst case:  $i+1$  comparison for  $\text{Insert}(e,a,i)$

$$O\left(\sum_{i=1}^{n-1} (i+1)\right) = O(n^2)$$

- Best case :  $O(n)$

- Average case :  $O(n^2)$

- Additional space requirement:  $O(1)$

- In-place swapping a pair of records

# Quick Sort

- Partition the list into three sublists using pivot
  - Left, middle, right
  - Middle = pivot
  - Left  $\leq$  pivot
  - Right  $\geq$  pivot
- Recursively sort left and right sublists

# Quick Sort

pivot  $\downarrow$   $i$   $\downarrow$   $j$   
26 5 37 1 61 11 59 15 48 19

26 5 37 1 61 11 59 15 48 19  
 $\uparrow$   $\uparrow$

$i++$ , swap

26 5 19 1 61 11 59 15 48 37  
 $\uparrow$   $\uparrow$

$i++$ ,  $j--$ , swap

26 5 19 1 15 11 59 61 48 37  
 $\uparrow$   $\uparrow$

$i++$ ,  $j--$ ,  $i > j$ , swap(pivot,  $a[j]$ )

11 5 19 1 15 26 59 61 48 37

# Quick Sort

$R_1$	$R_2$	$R_3$	$R_4$	$R_5$	$R_6$	$R_7$	$R_8$	$R_9$	$R_{10}$
[26	5	37	1	61	11	59	15	48	19]
[11	5	19	1	15]	26	[59	61	48	37]
[1	5]	11	[19	15]	26	[59	61	48	37
1	5	11	[19	15]	26	[59	61	48	37]
1	5	11	15	19	26	[59	61	48	37]
1	5	11	15	19	26	[48	37]	59	[61]
1	5	11	15	19	26	37	48	59	[61]
1	5	11	15	19	26	37	48	59	61



# Quick Sort

```
void QuickSort(*a, left, right)
{
    if(left<right) {
        int i = left, j=right+1, pivot=a[left];
        do {
            do i++; while(a[i]<pivot); // move i to right
            do j--; while(a[j]>pivot); // move j to left
            if(i<j) swap(a[i],a[j]);
        } while(i<j)
        swap(a[left],a[j]);
        QuickSort(a,left,j-1);
        QuickSort(a,j+1,right);
    }
}
```

# Quick Sort

- Analysis
  - $O(n)$  to partition a list with  $n$  records
  - Worst :  $O(n^2)$ 
    - Either left or right sublist is empty
  - Best :  $O(n \log n)$ 
    - Left and right sublists are roughly same size
  - Average :  $O(n \log n)$
  - Additional space requirement :  $O(n)$  for stack

# Questions?