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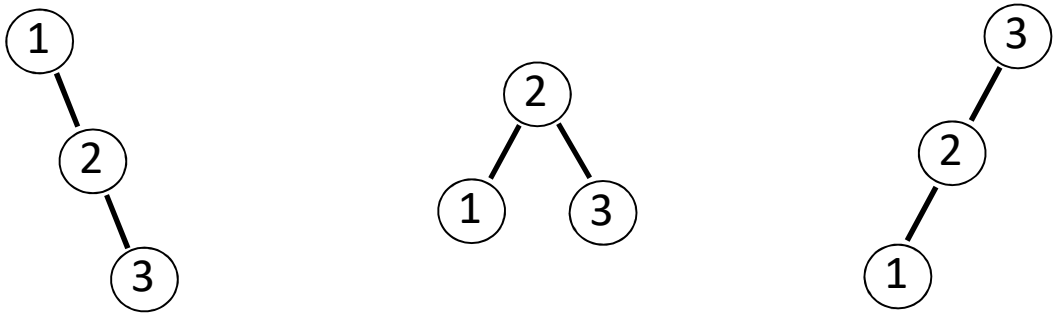

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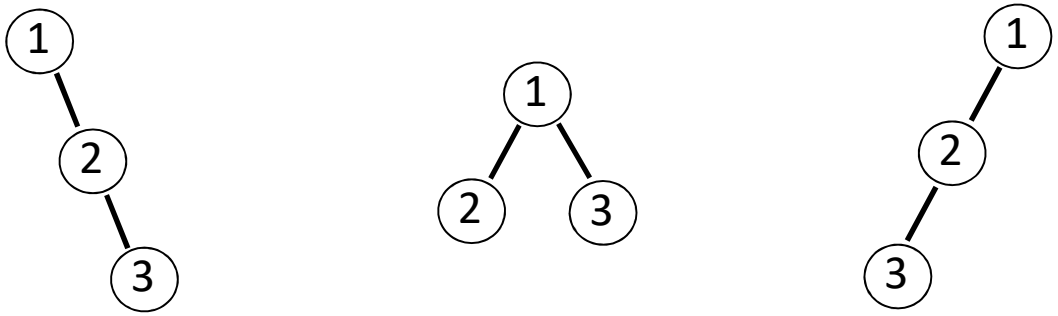

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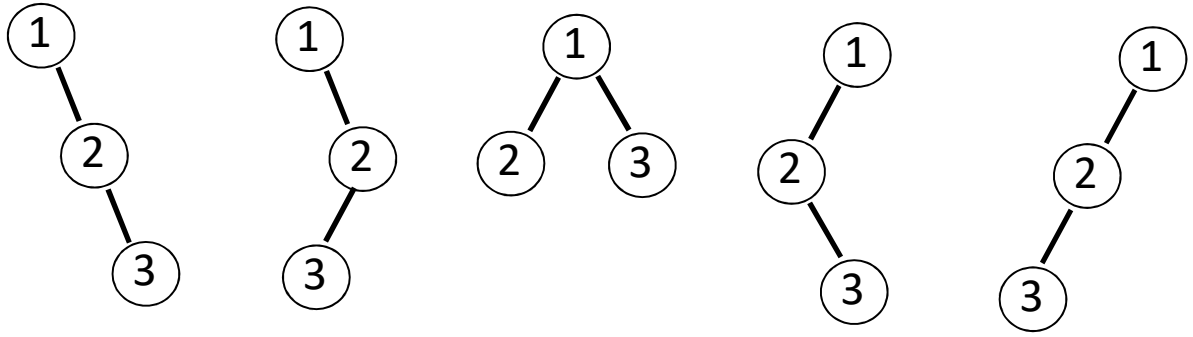

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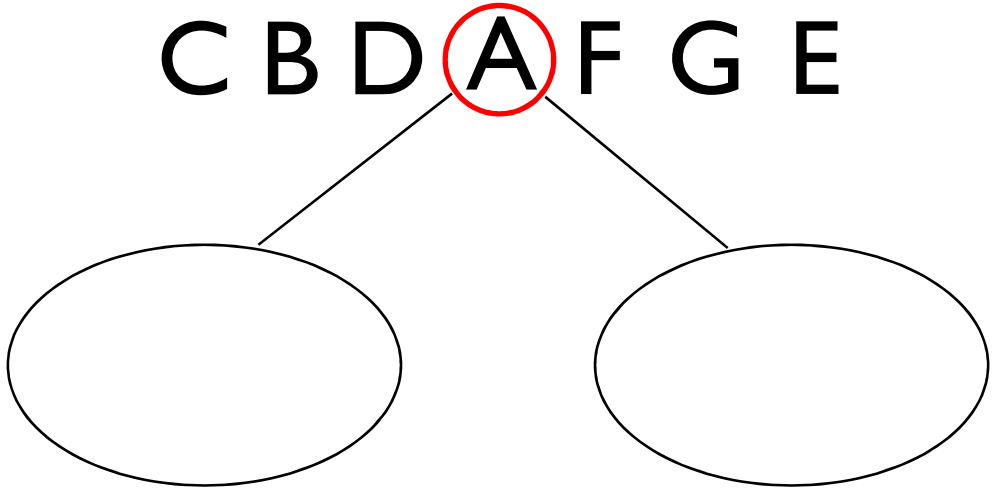

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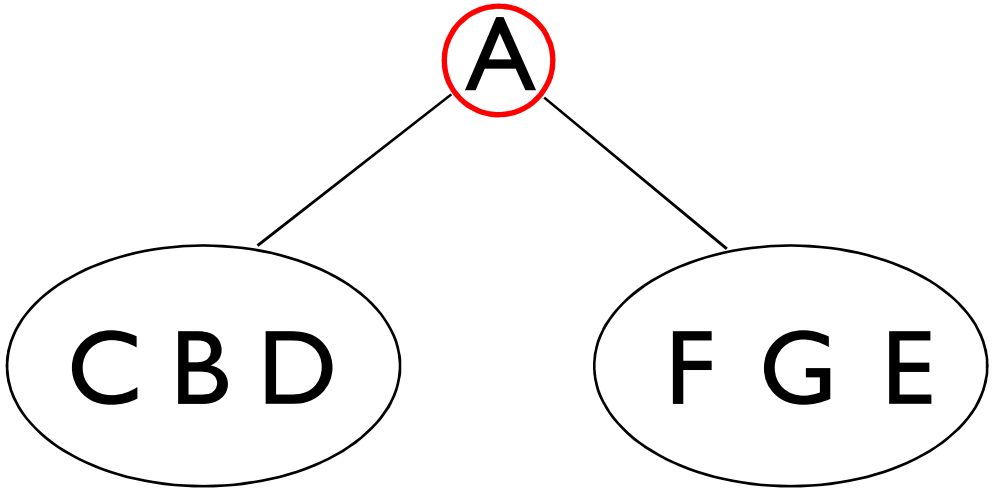

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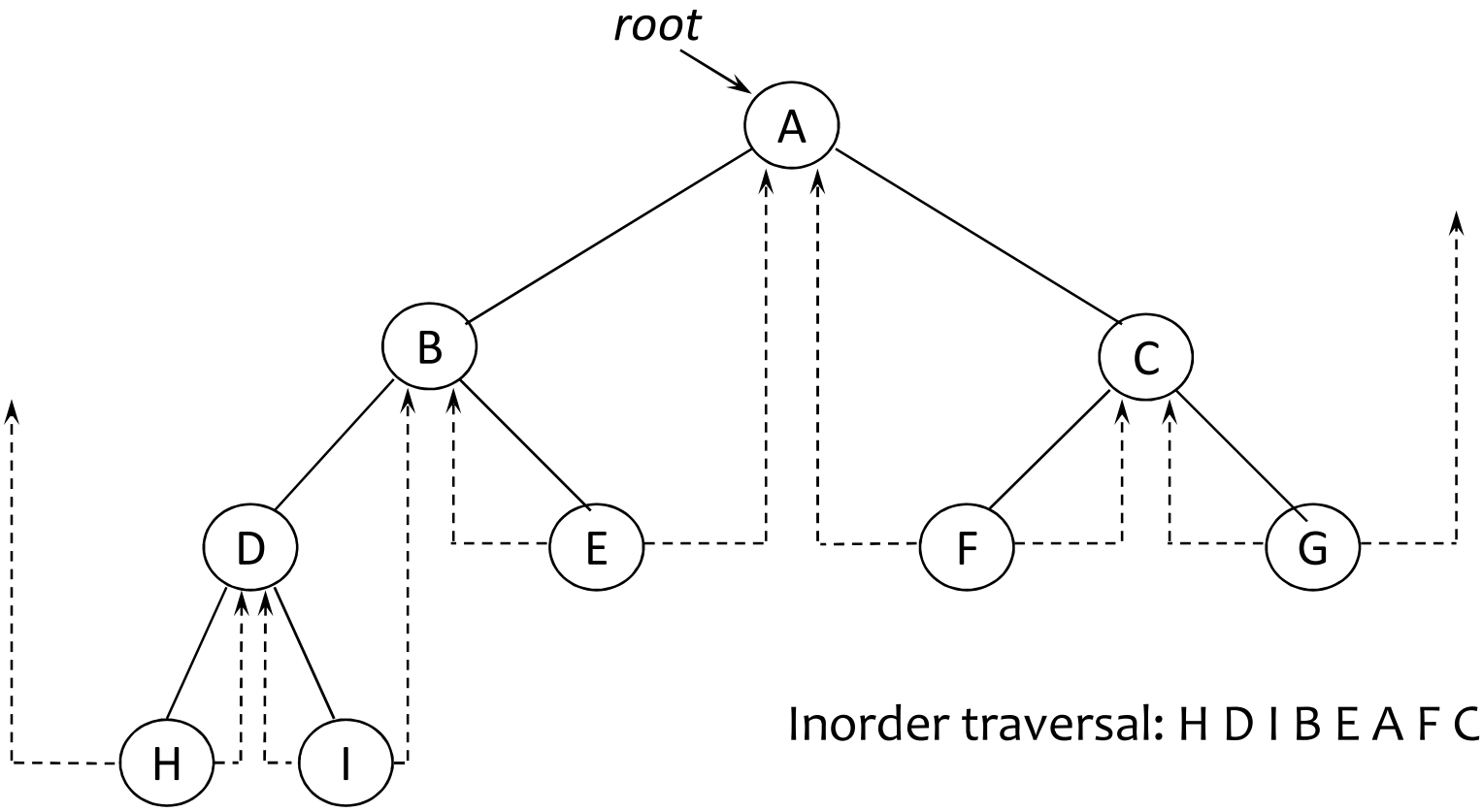

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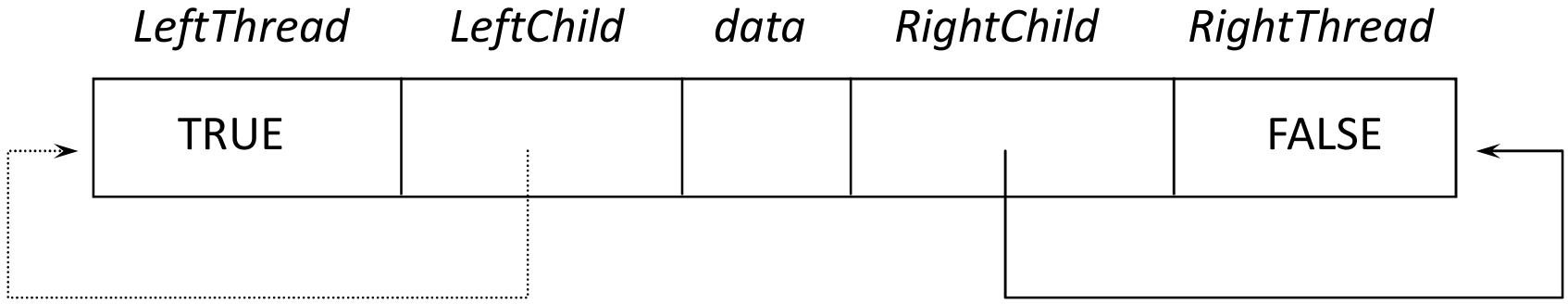

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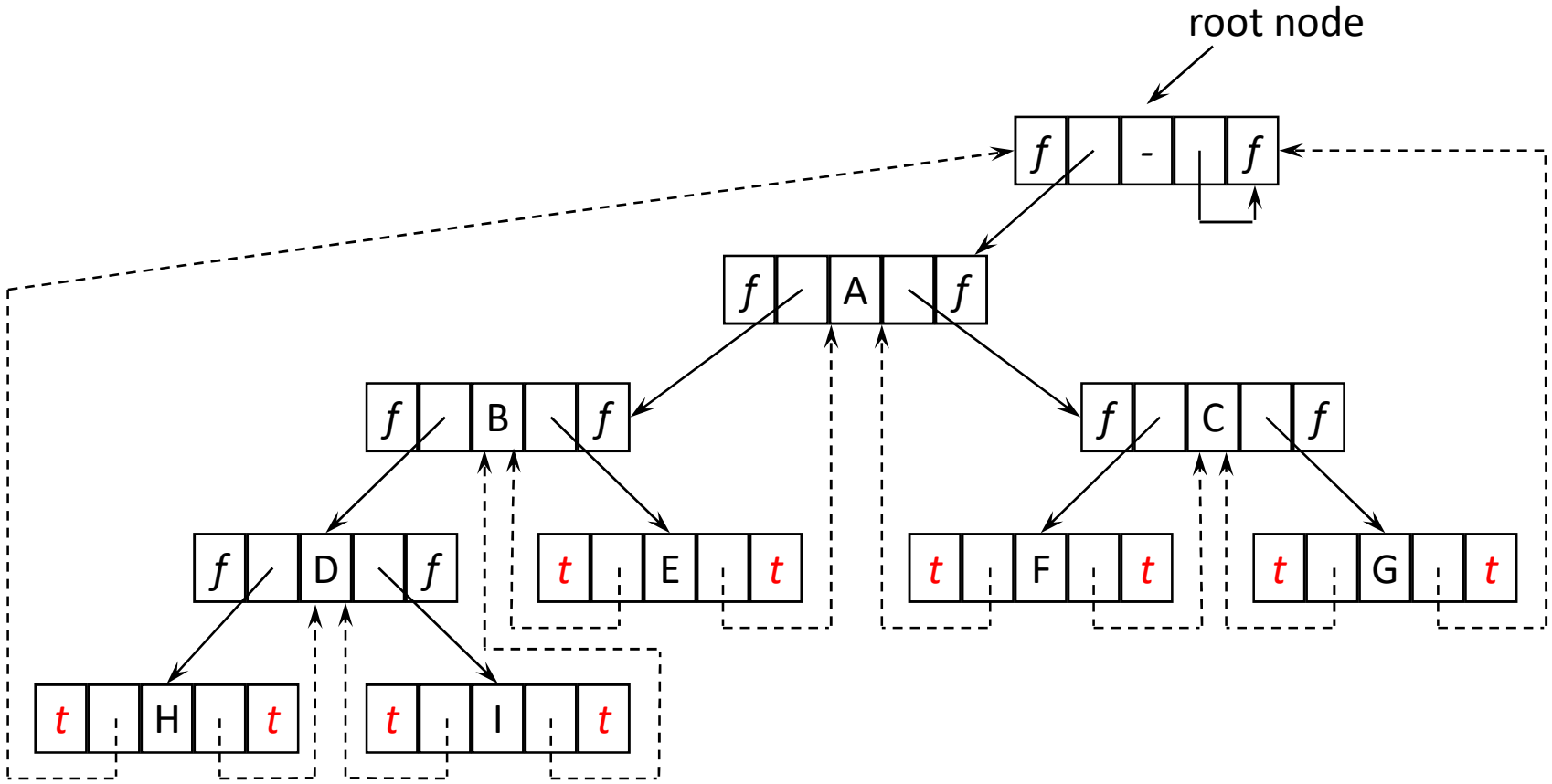

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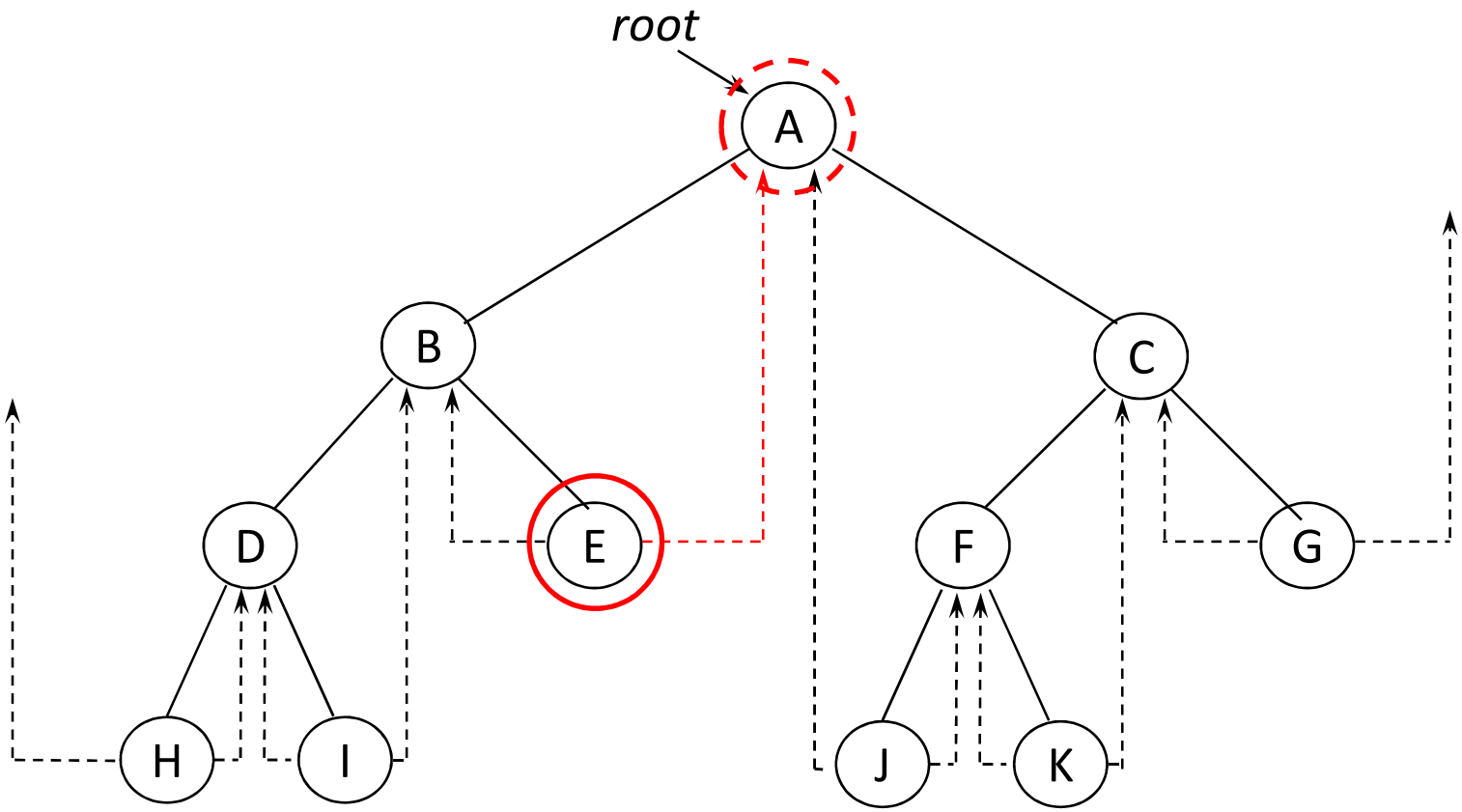

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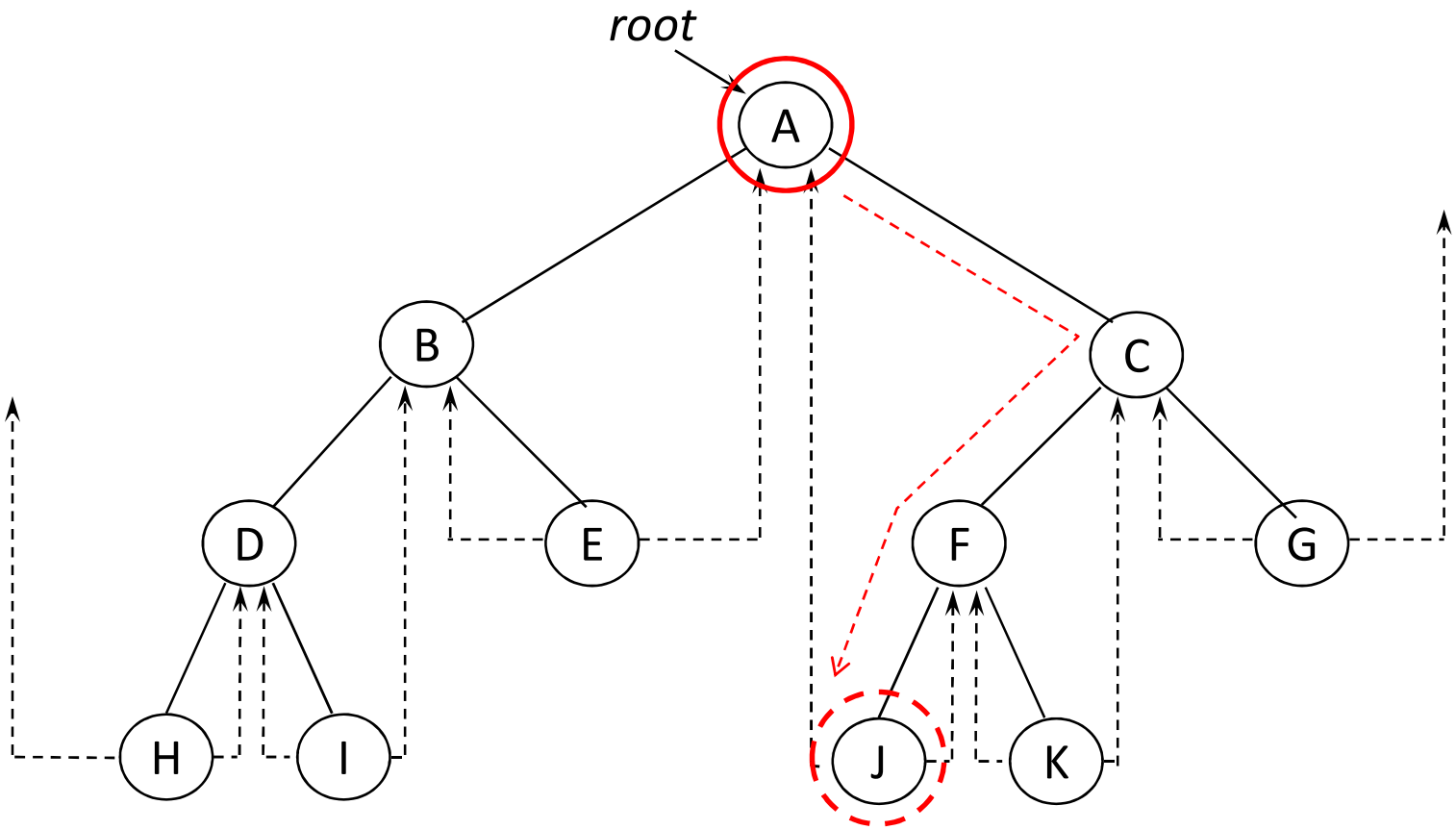

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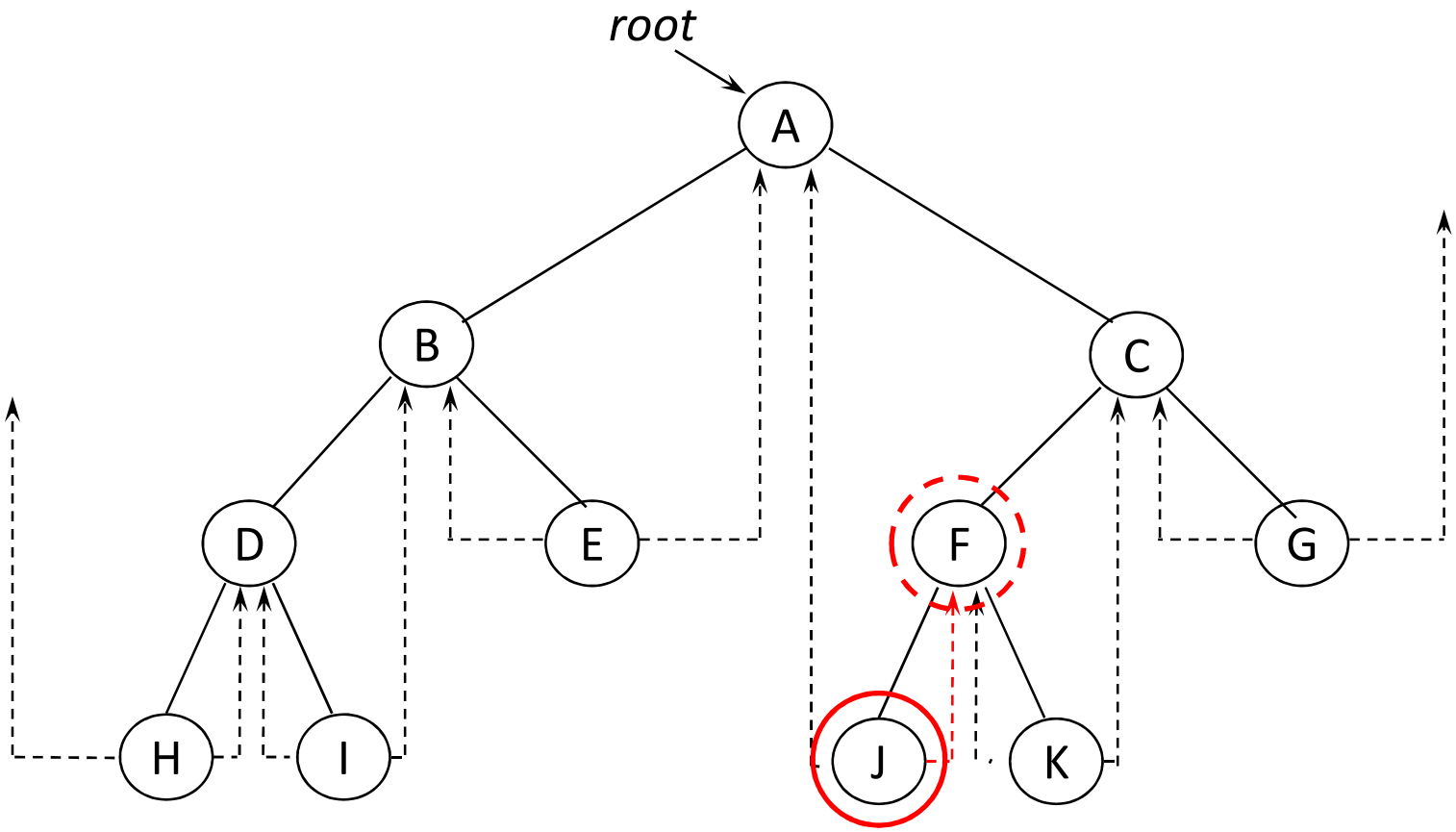

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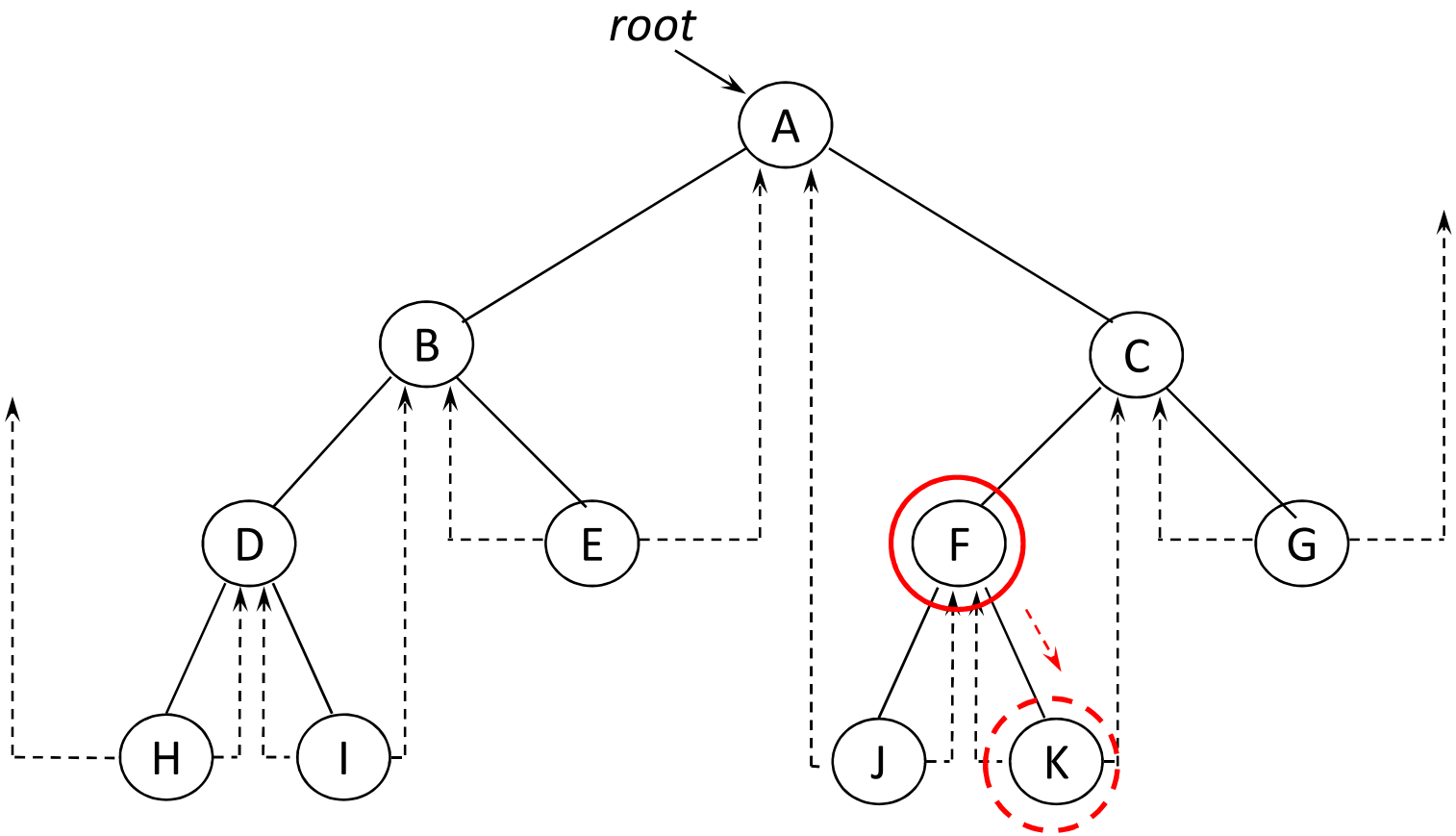

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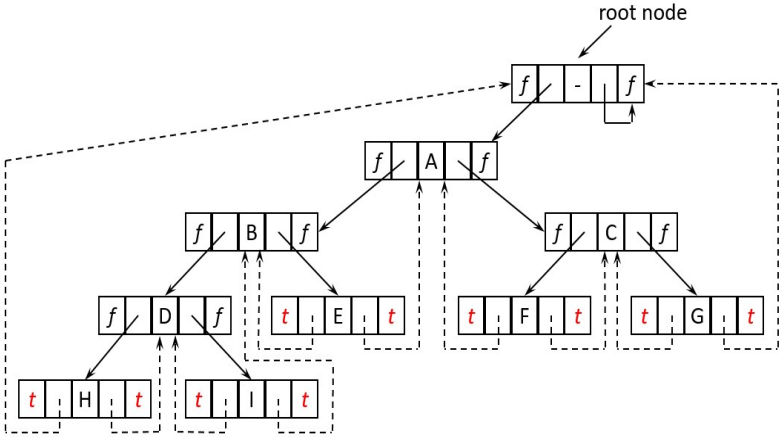

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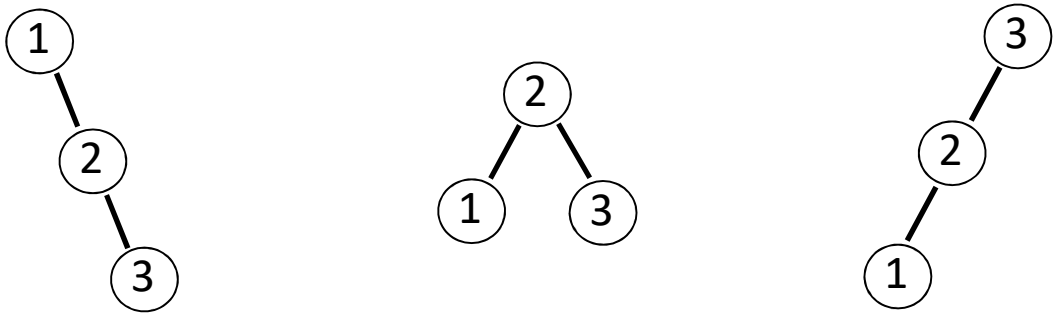

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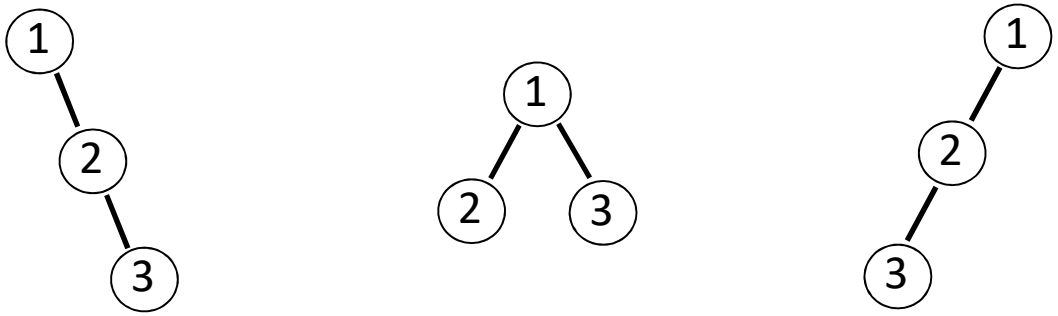

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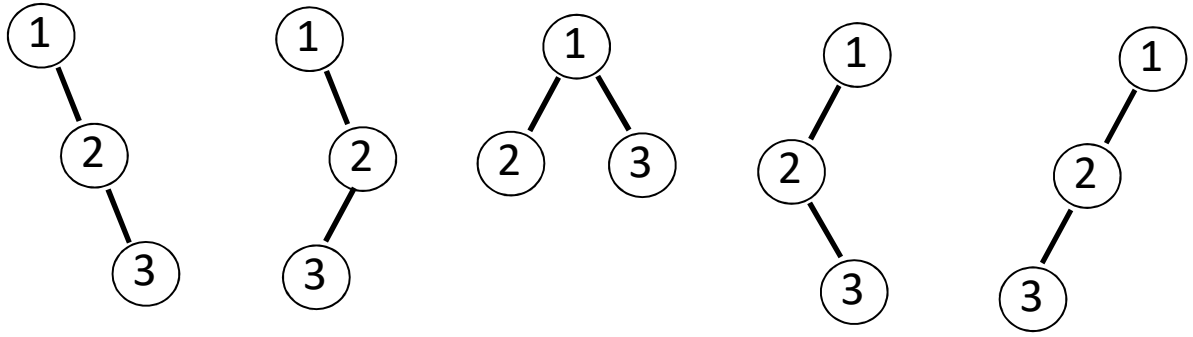

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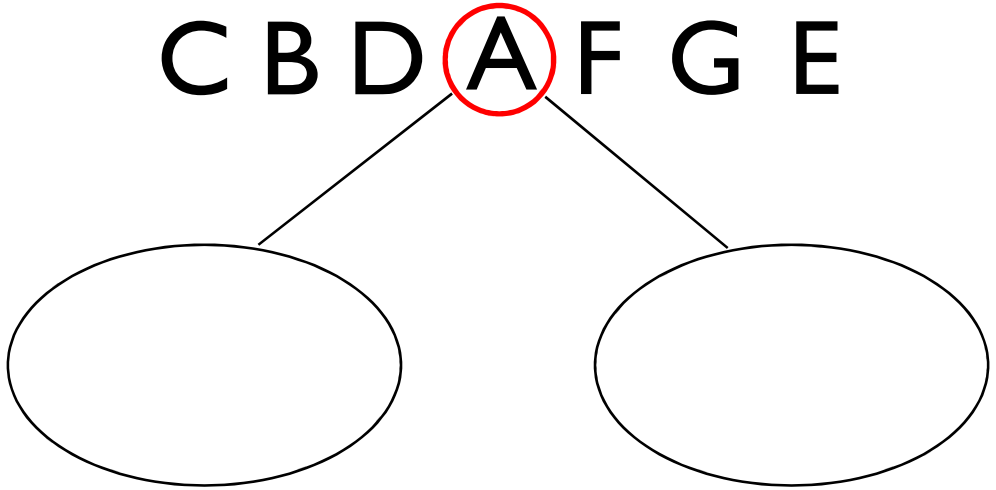

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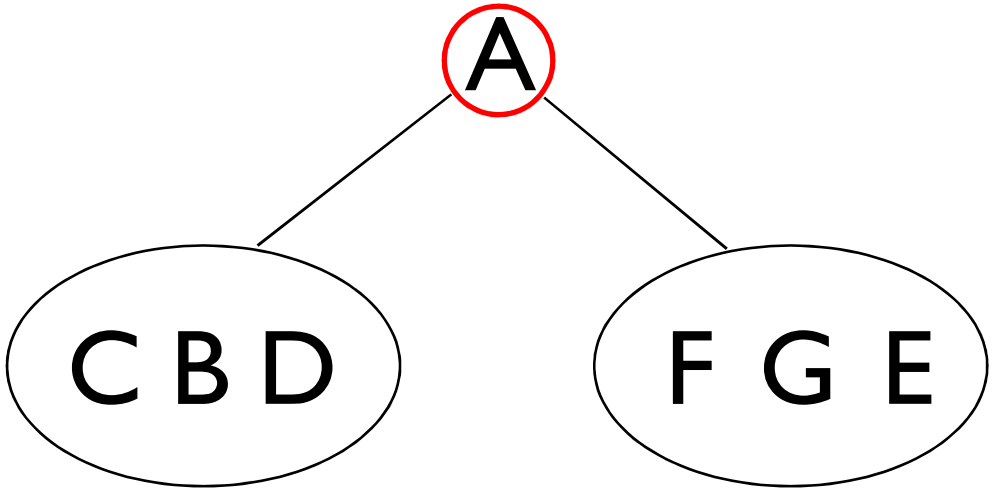

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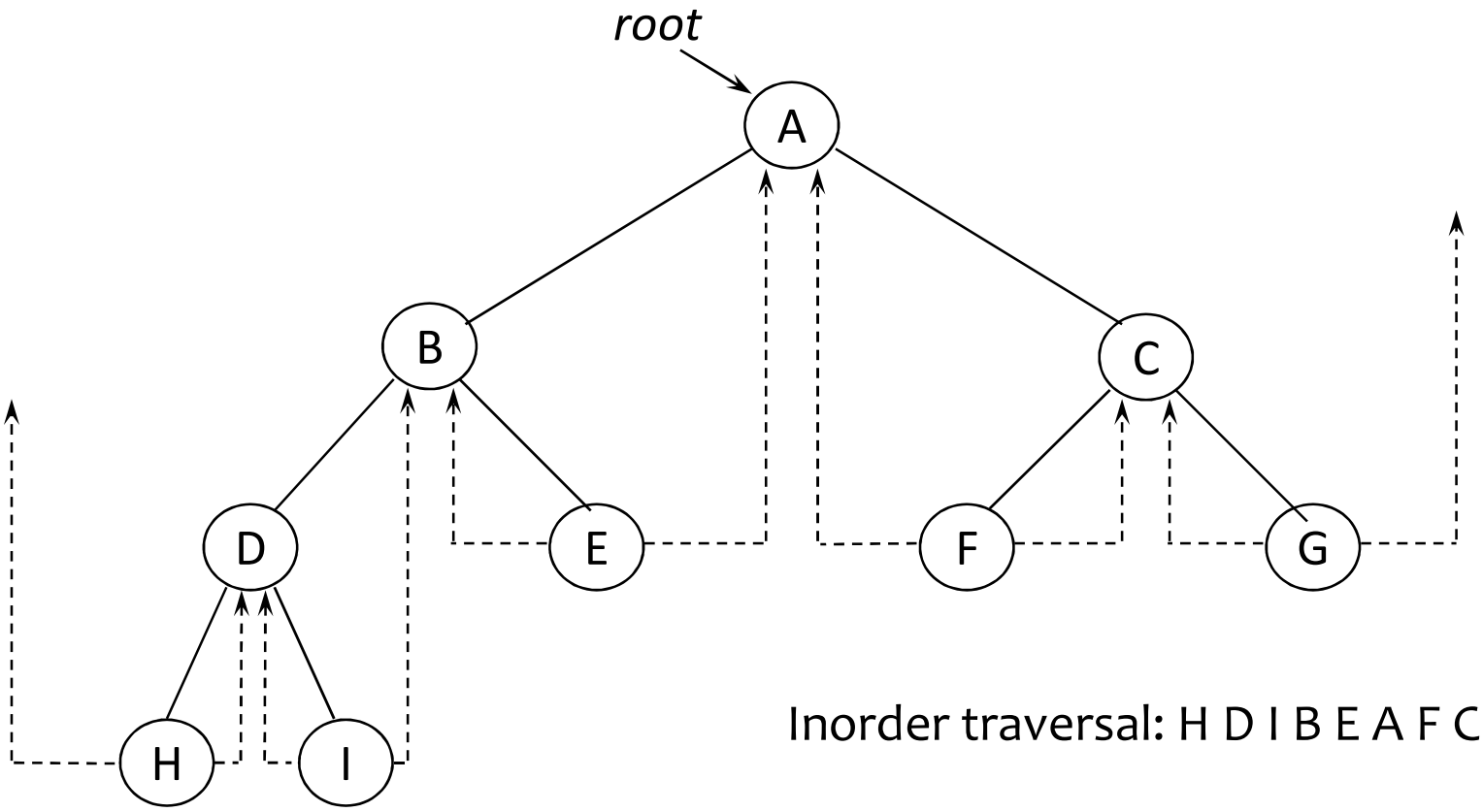

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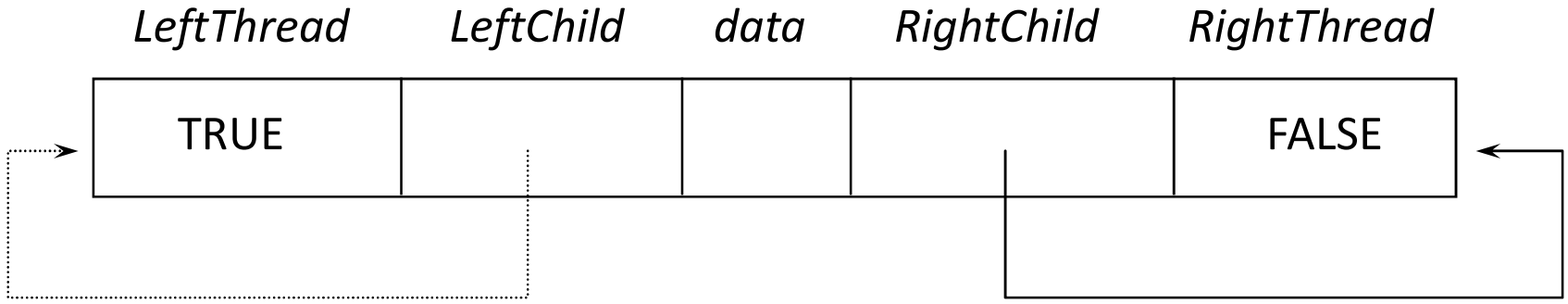

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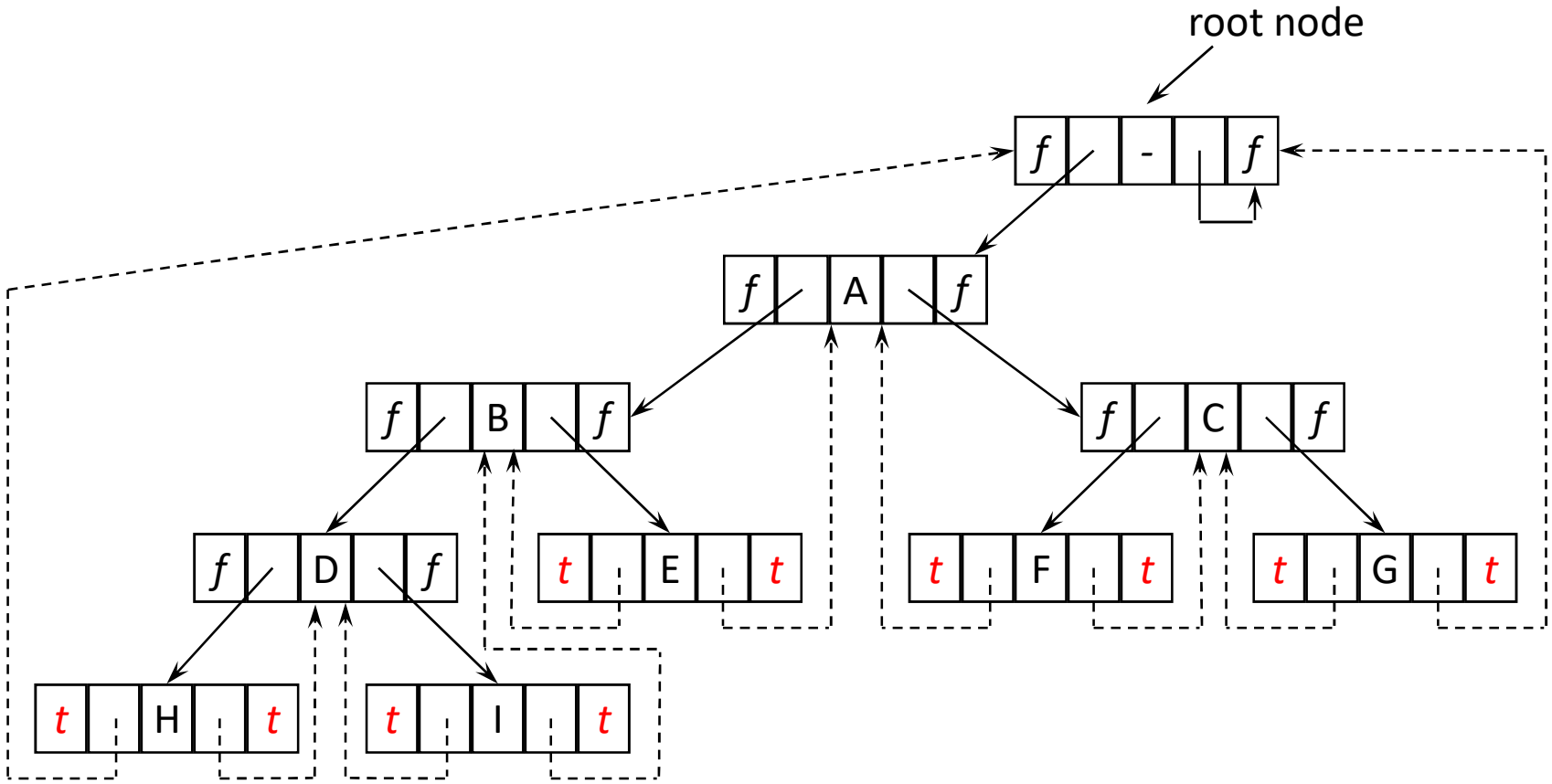

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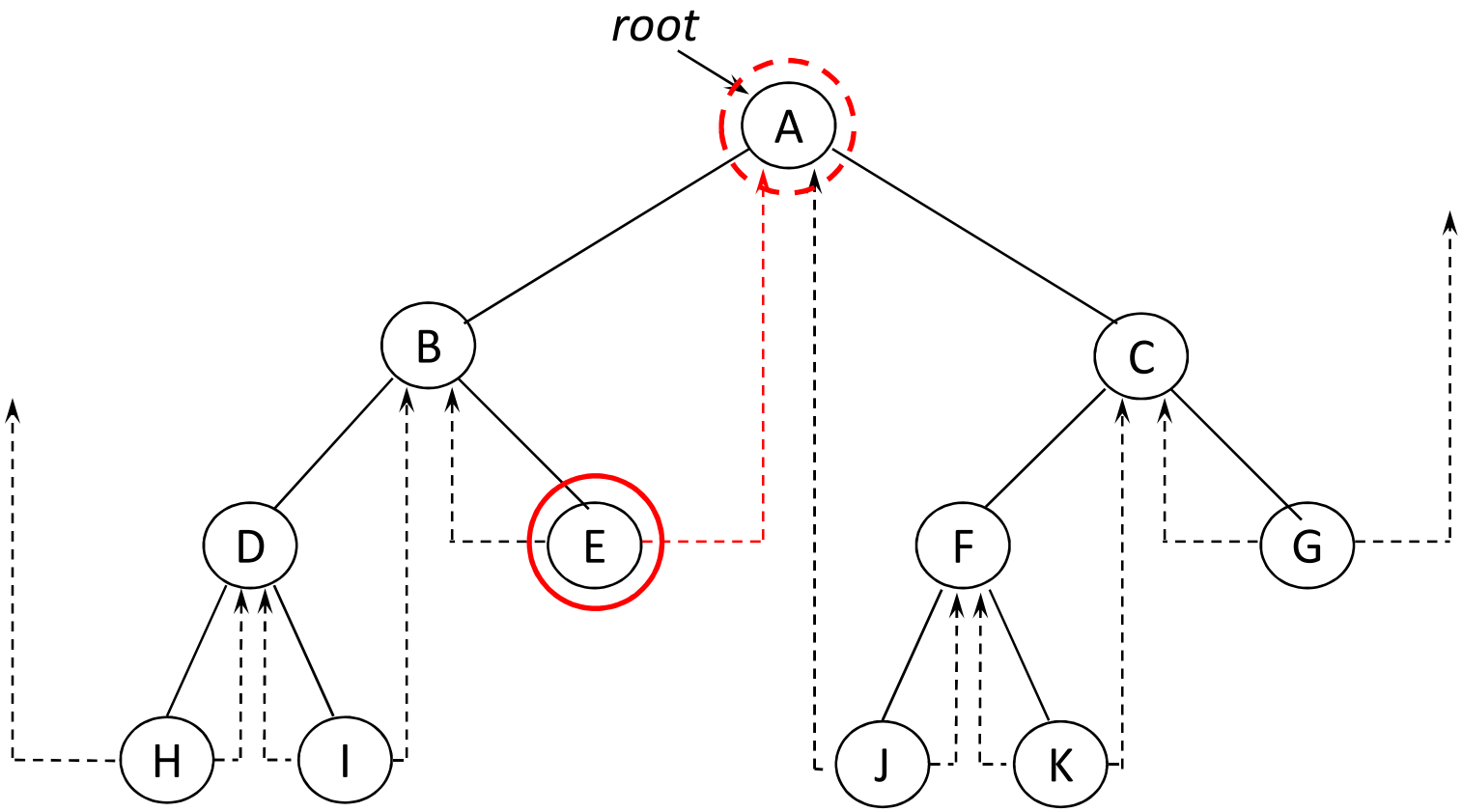

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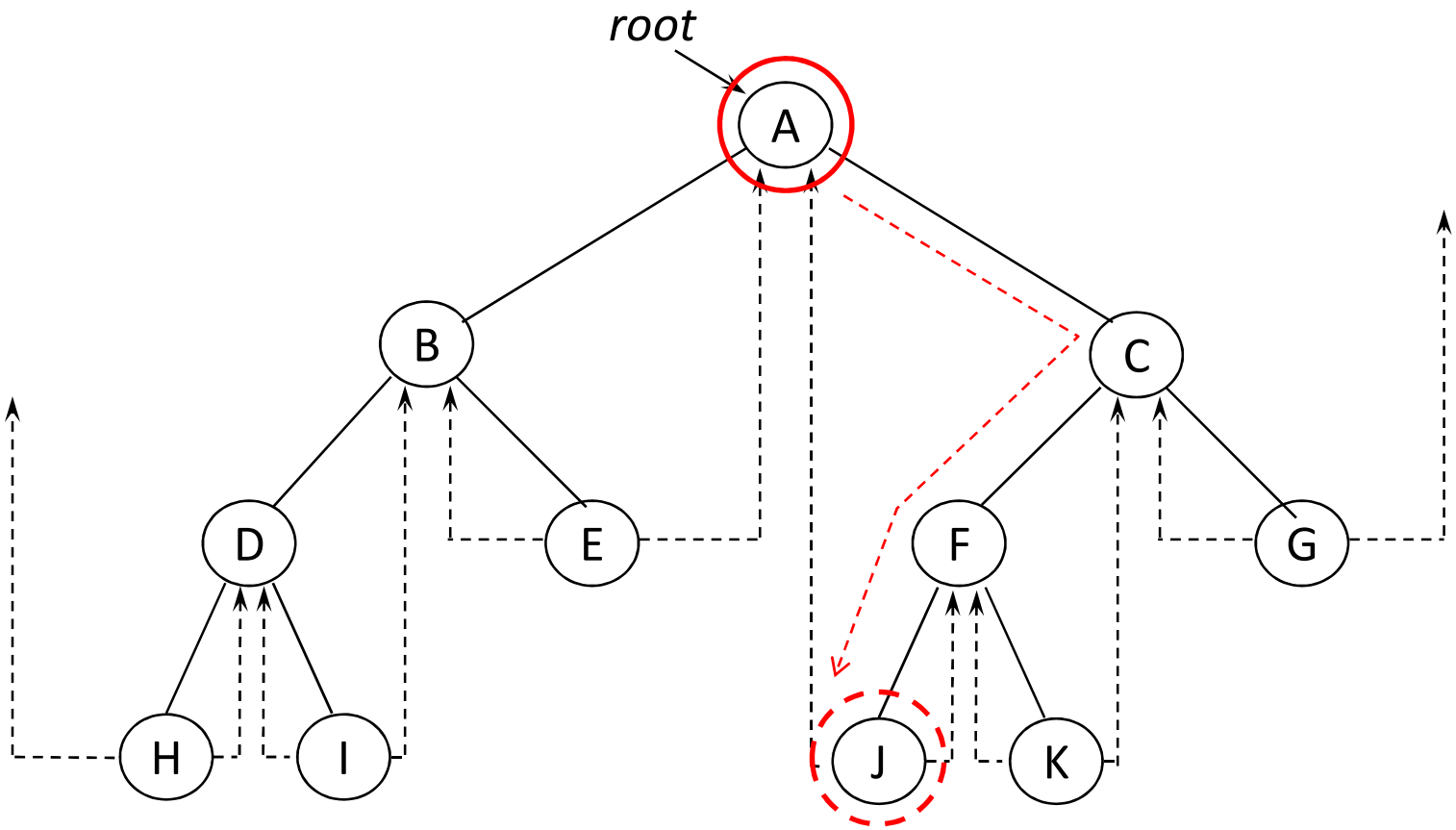

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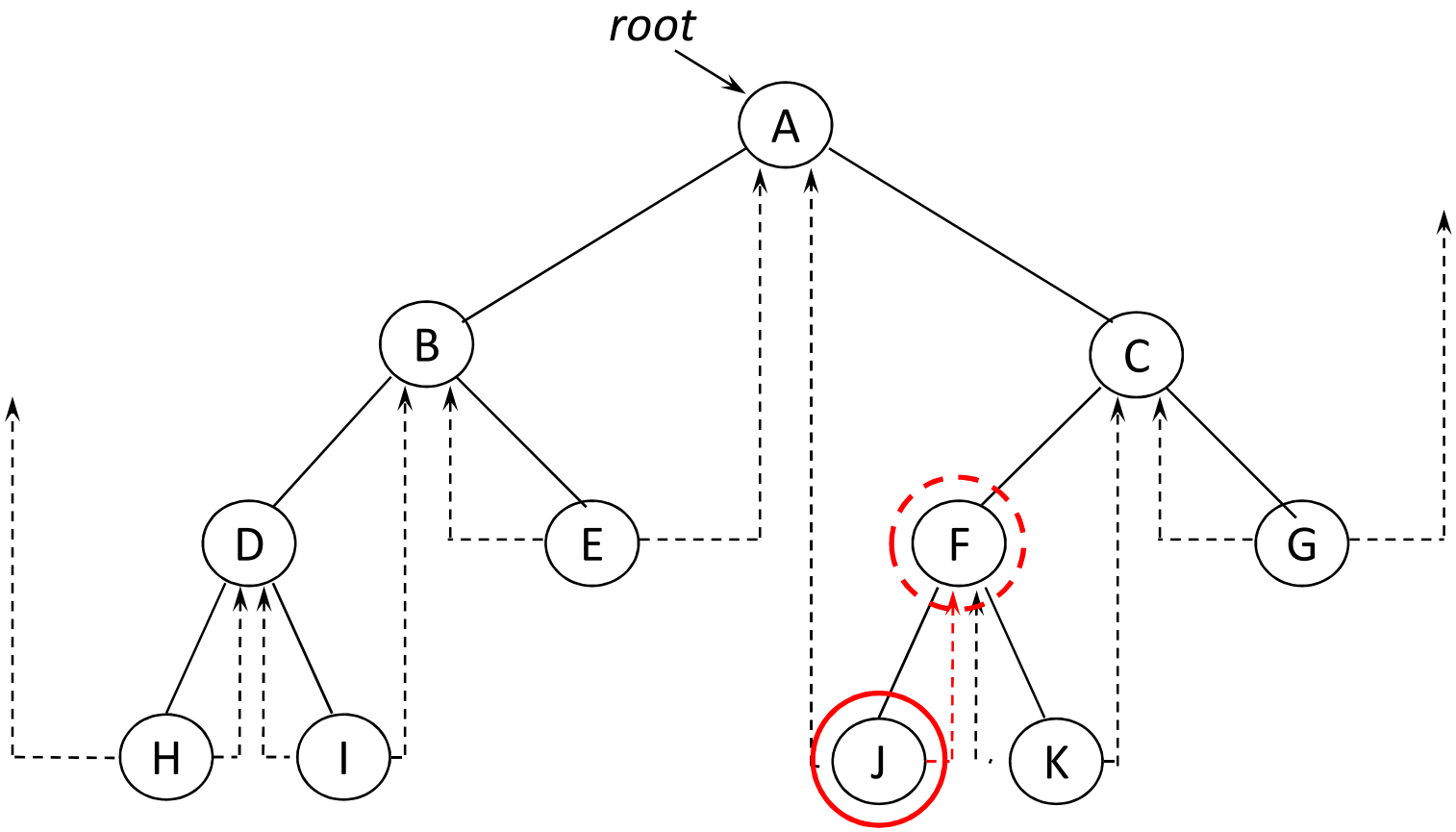

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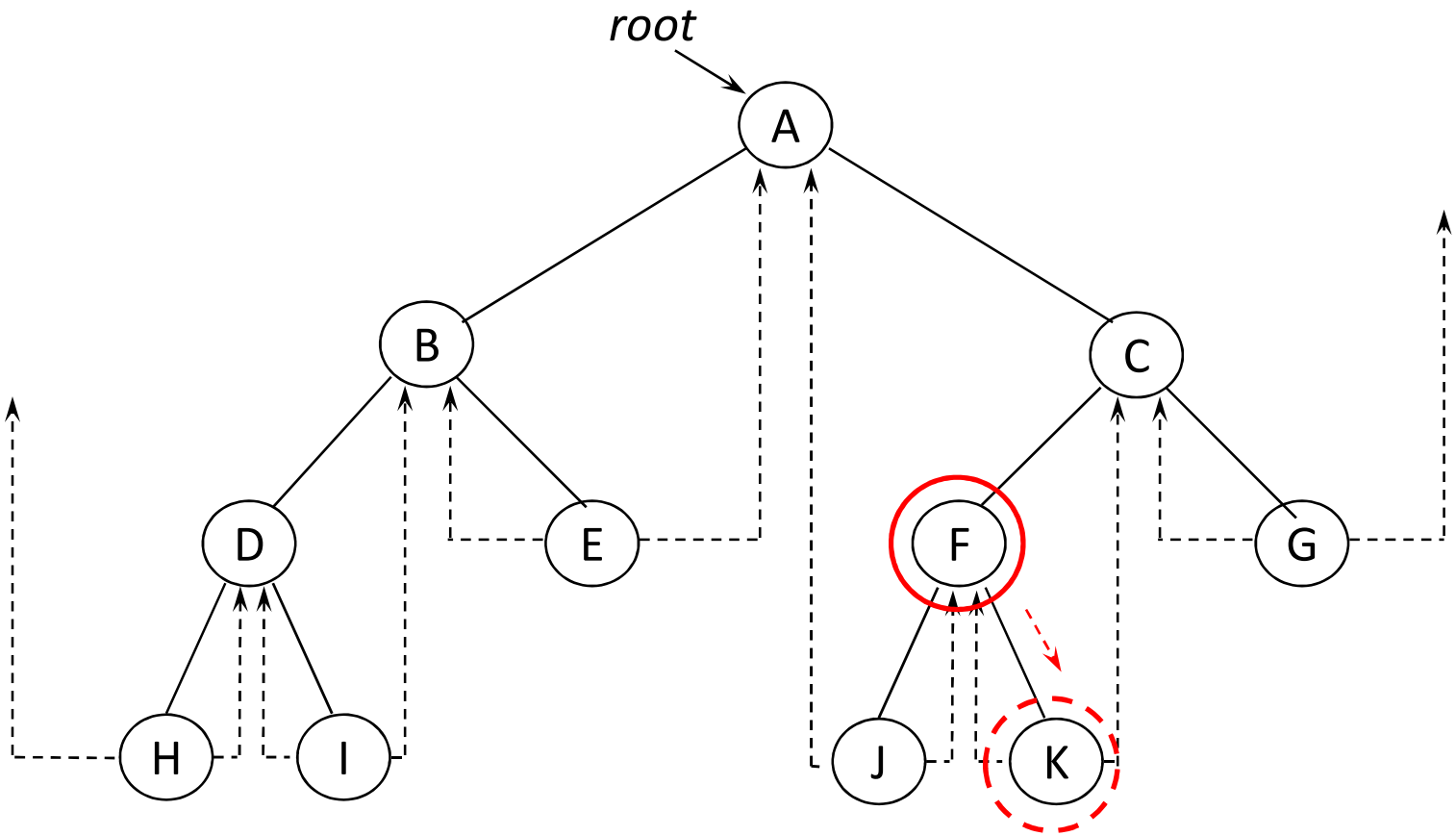

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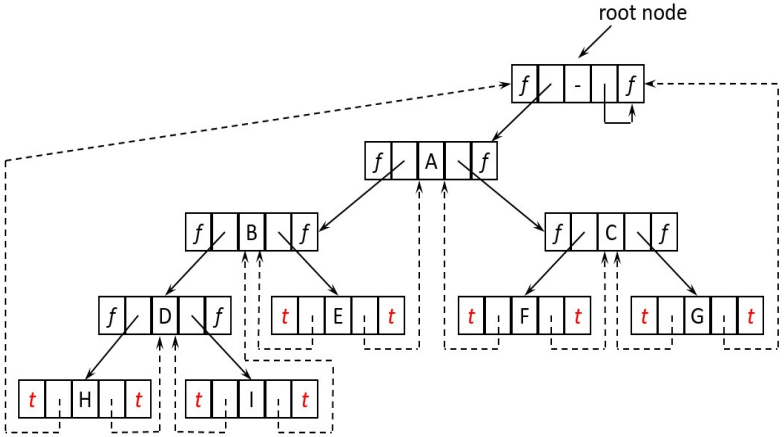

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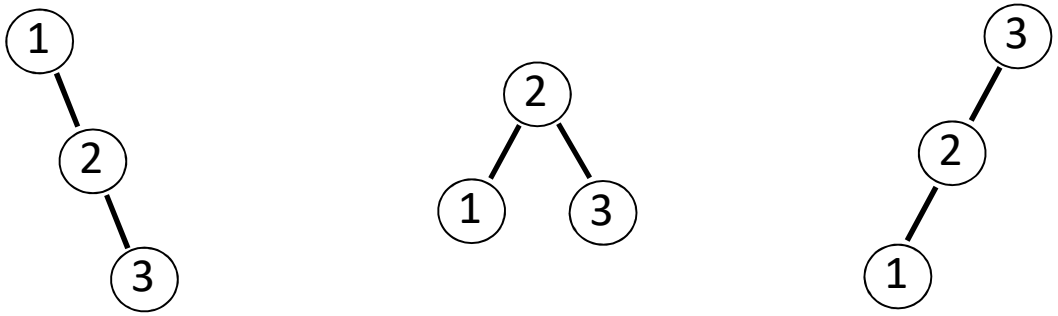

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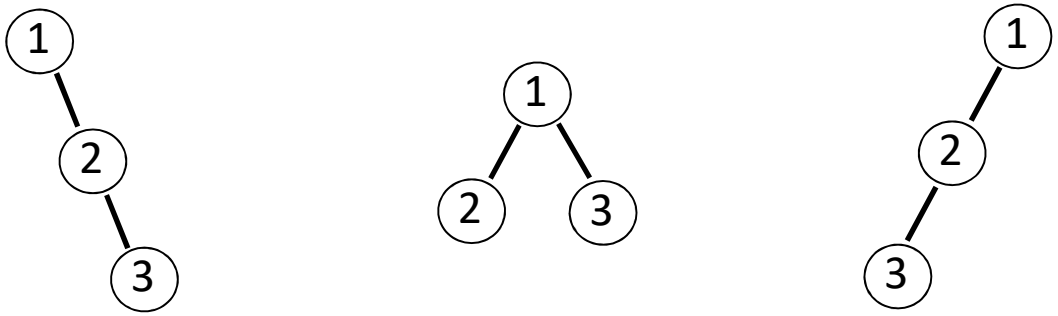

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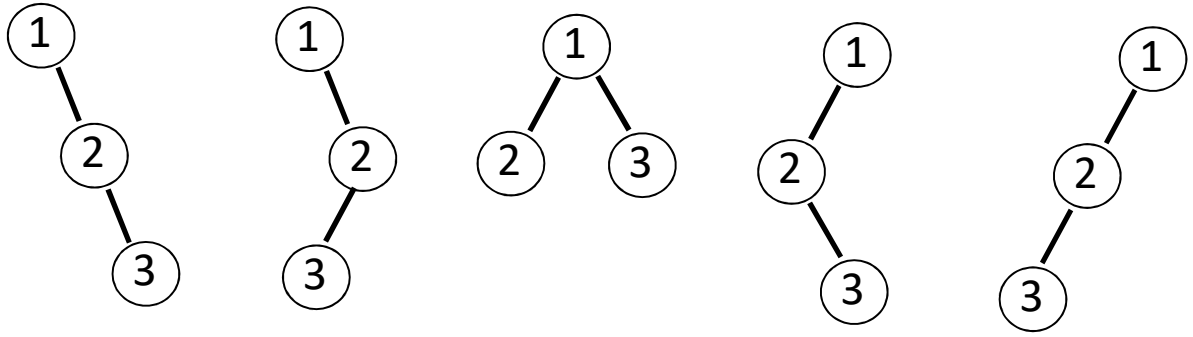

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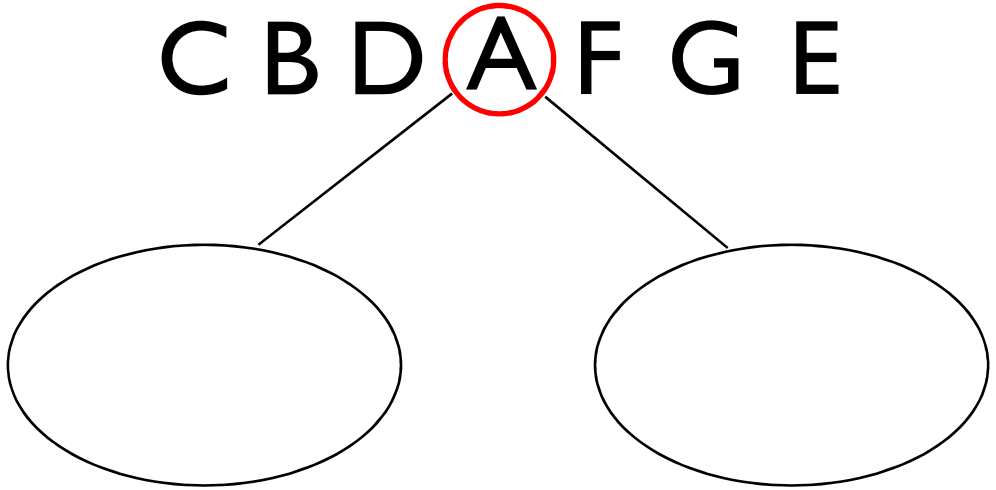

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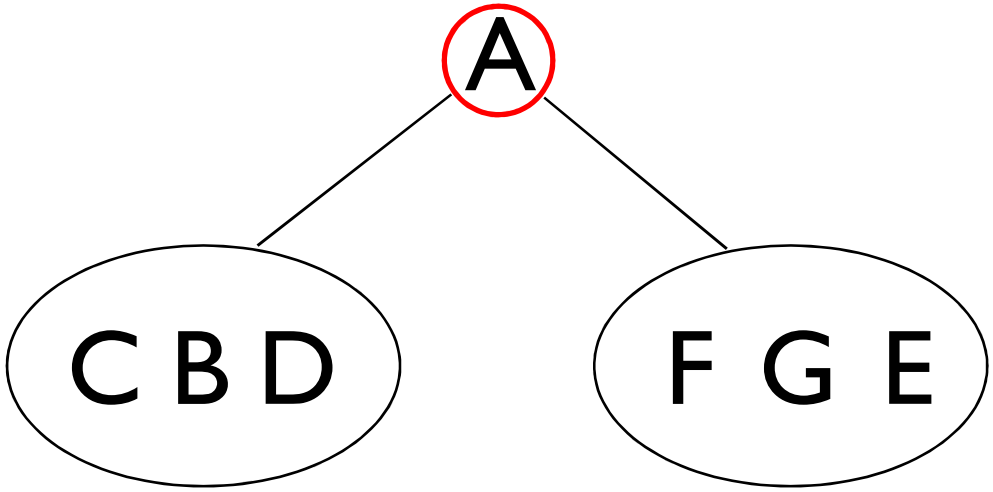

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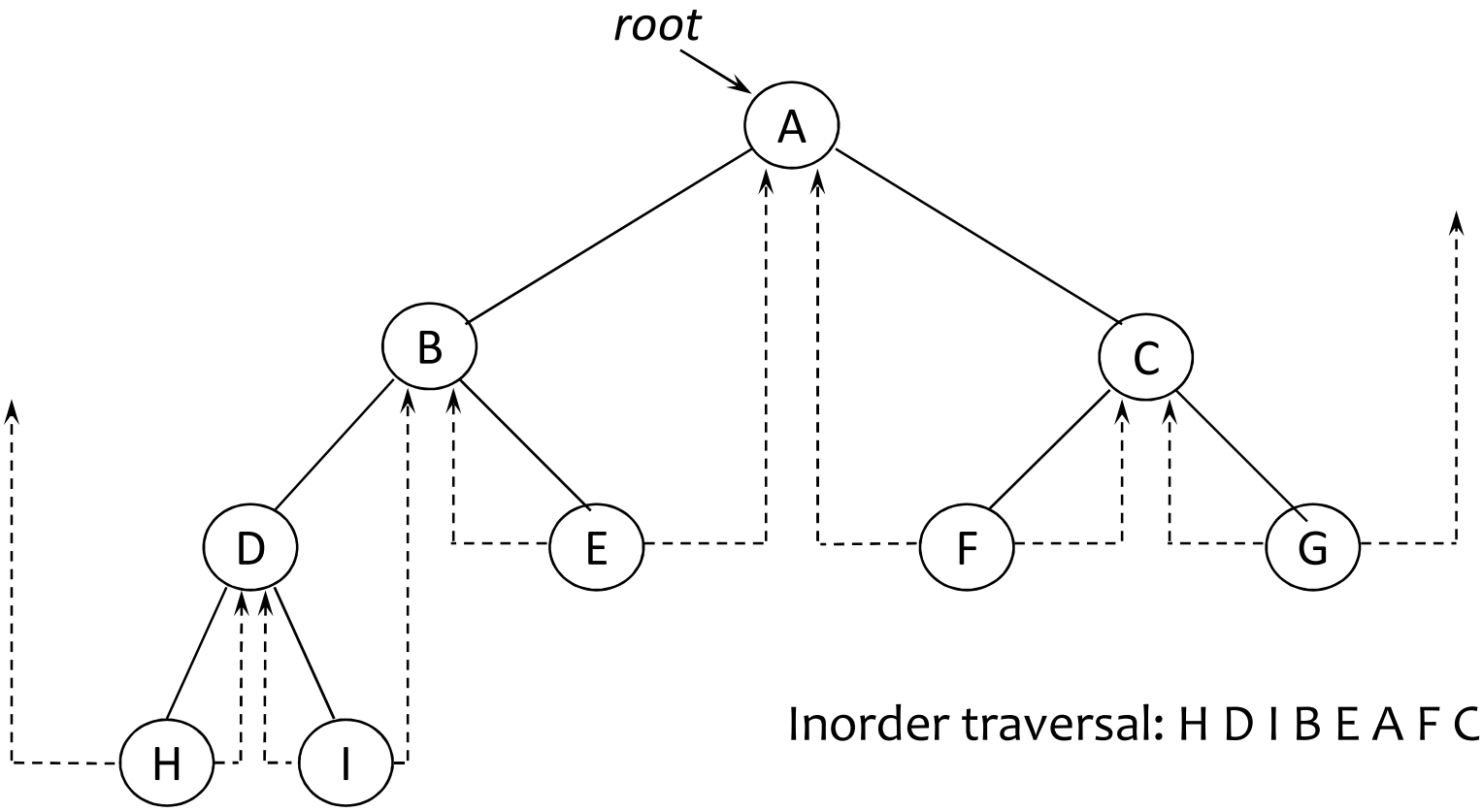

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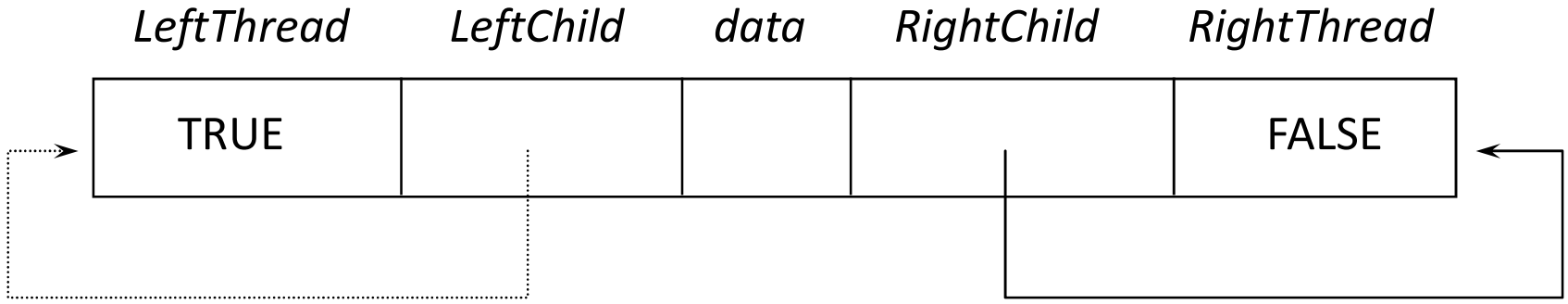

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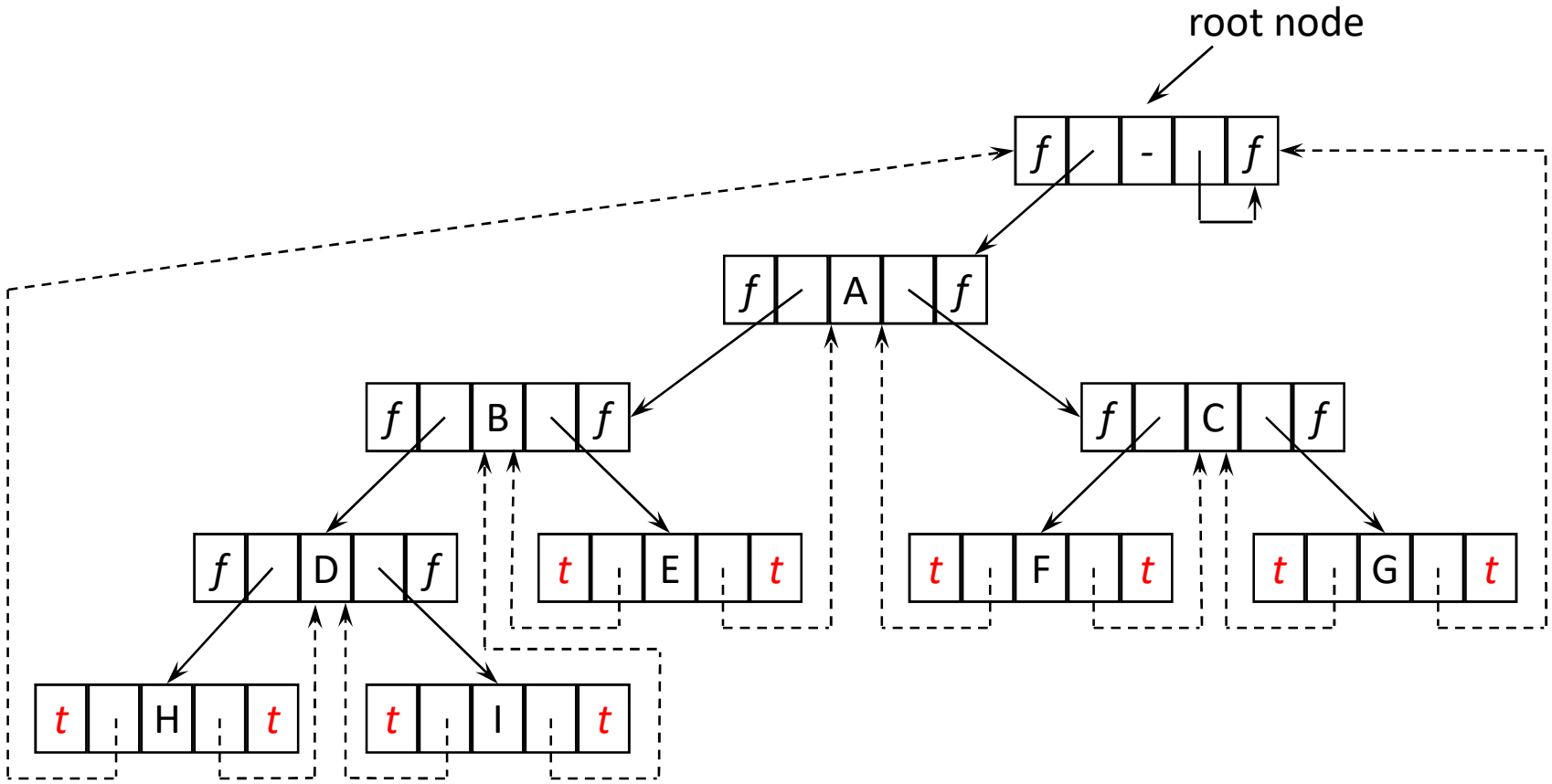

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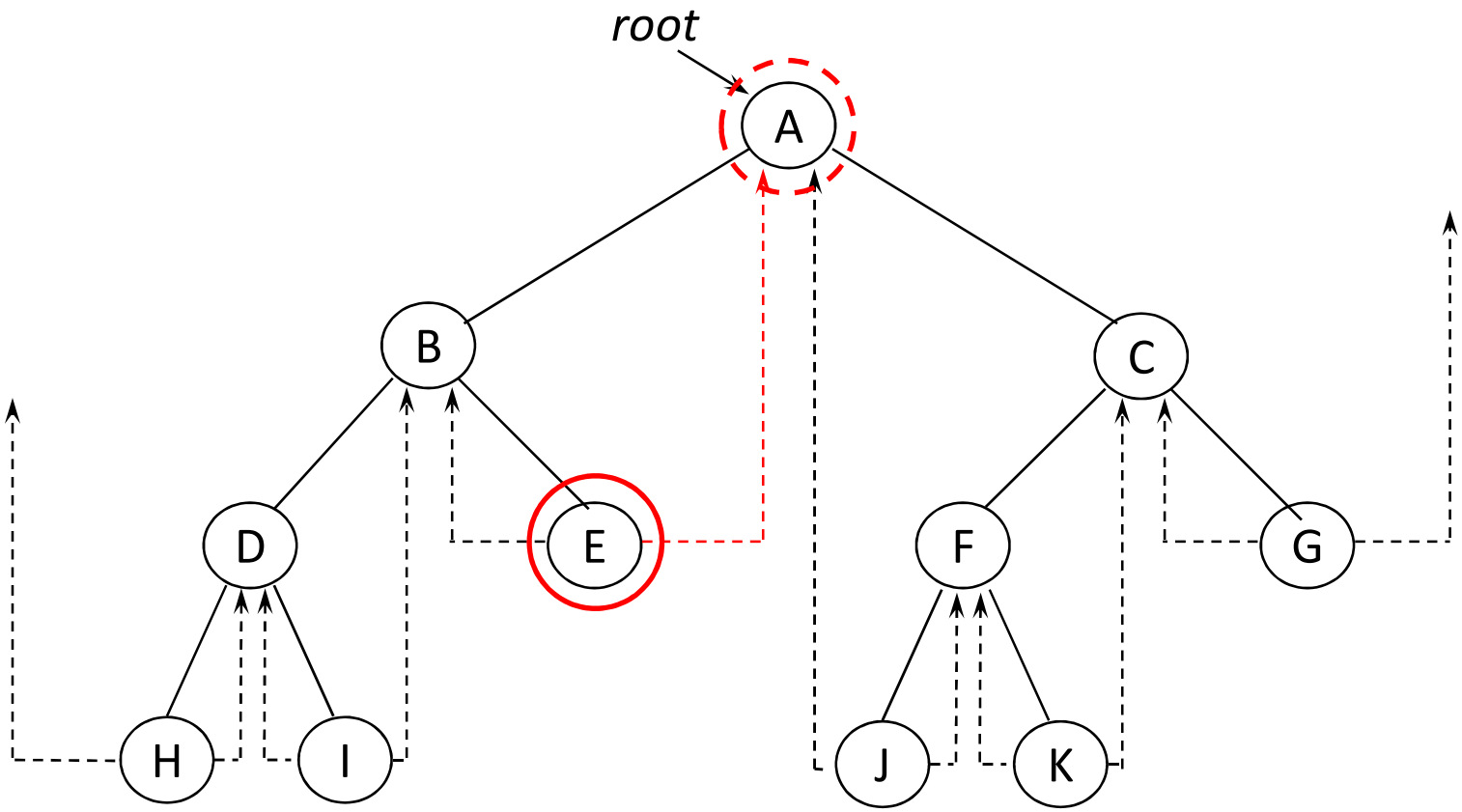
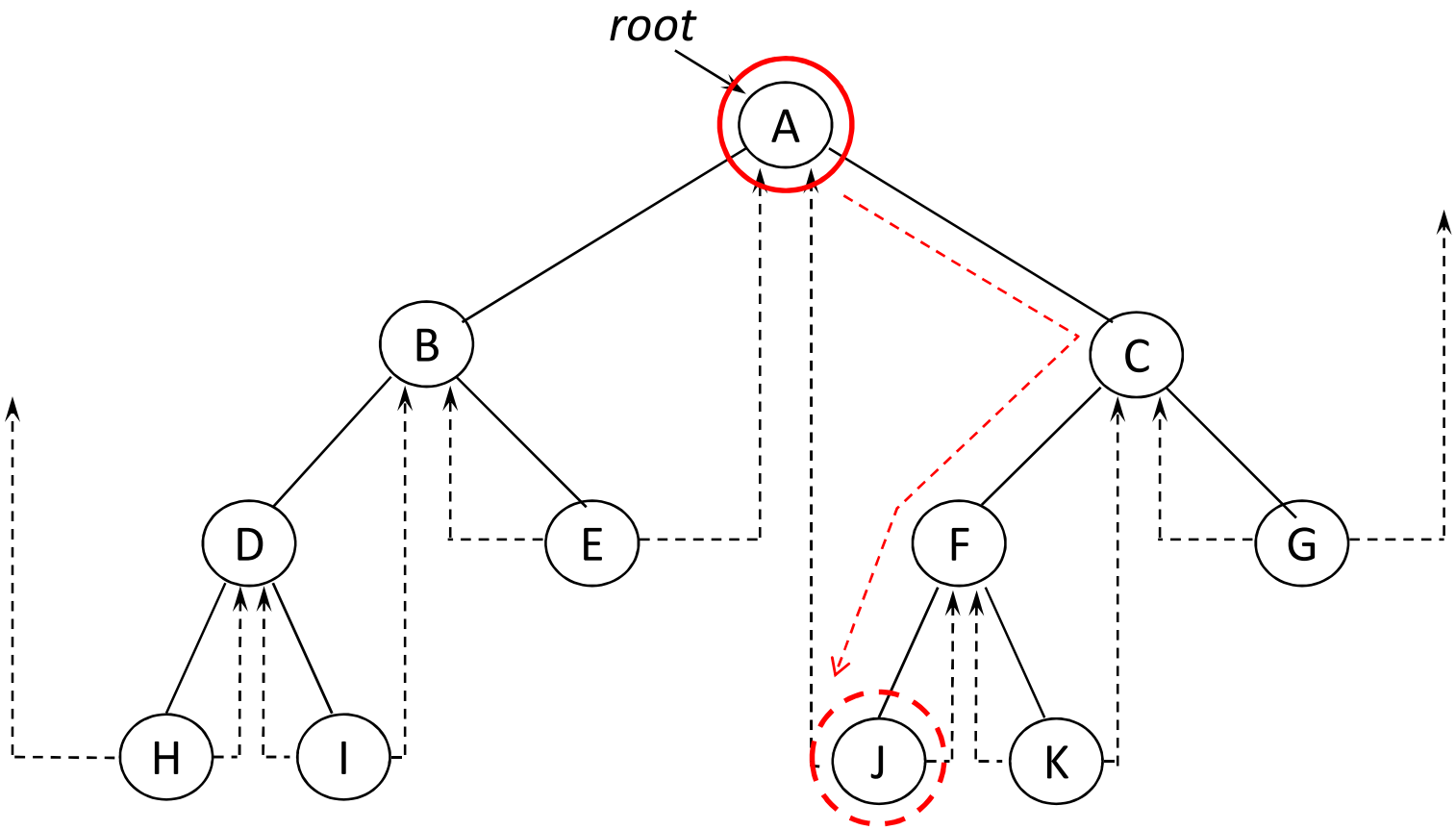
Current node : E

RightThread is True, so the next node for inorder traversal is E->RightChild = A



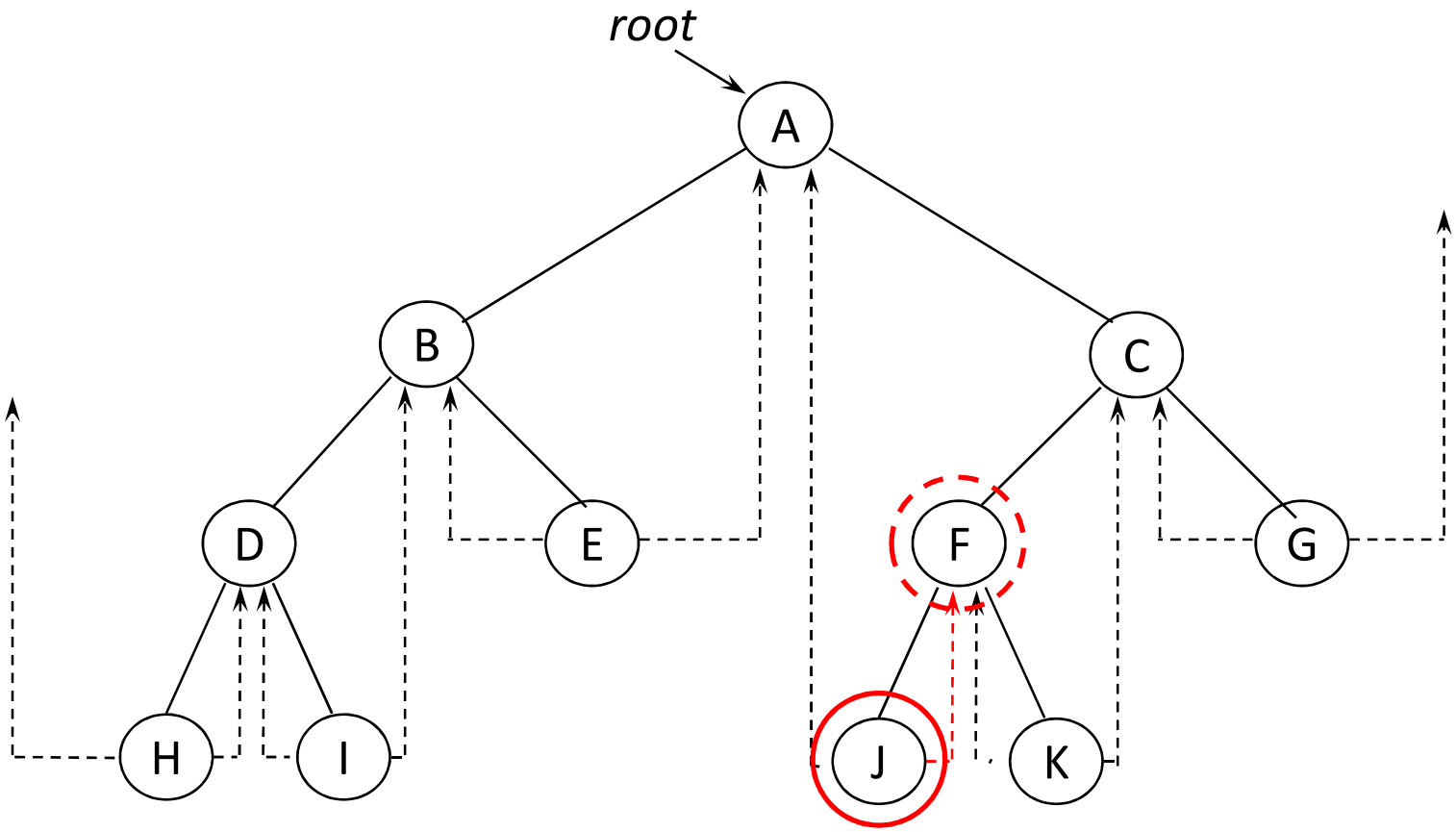
Current node : A

RightThread is False, so you follow the LeftChild link of RightChild node (C) until you reach the node with the LeftThread is True (which is J)



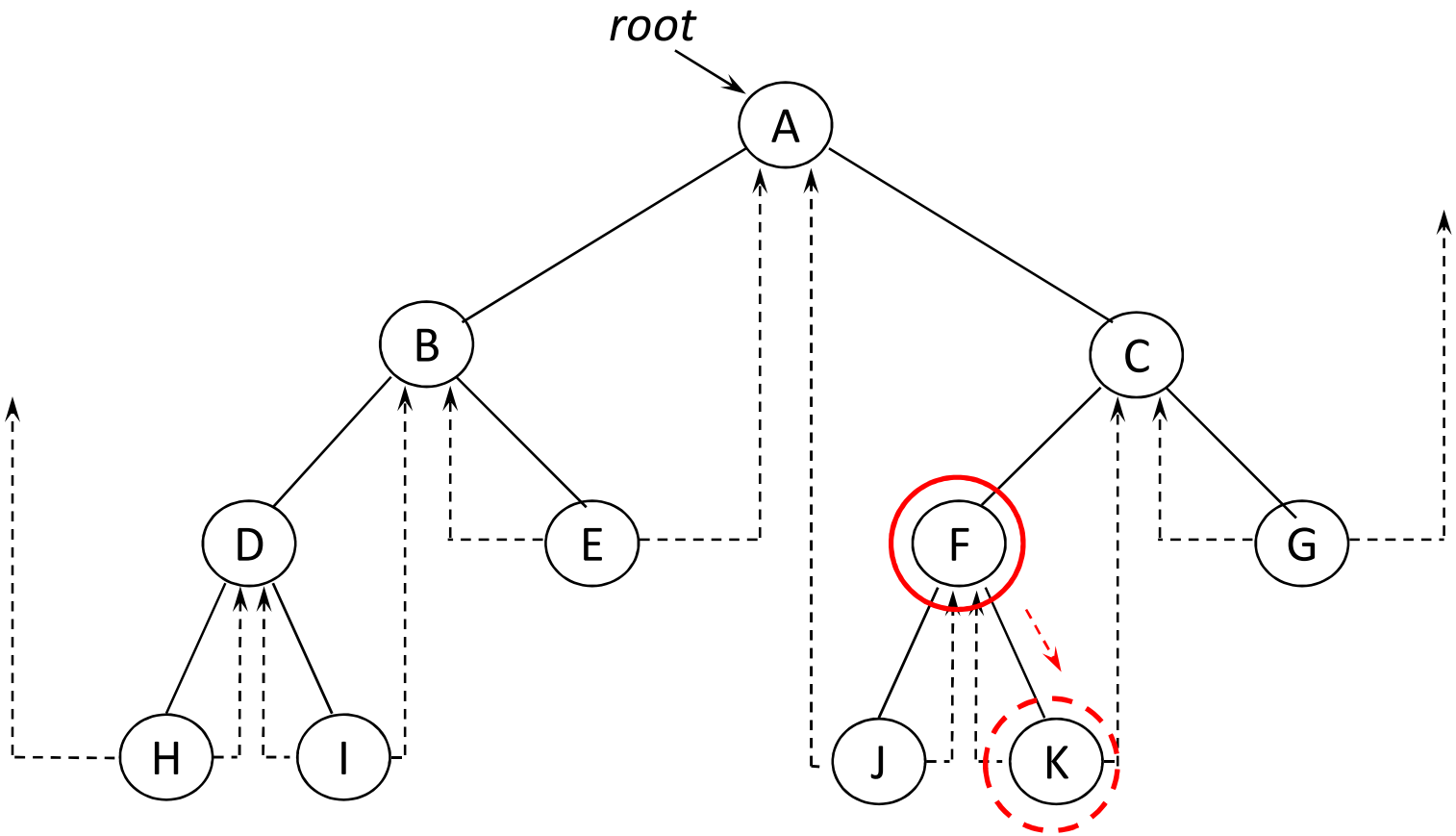
Current node : J

RightThread is True, so the next node for inorder traversal is J->RightChild = F



Current node : F

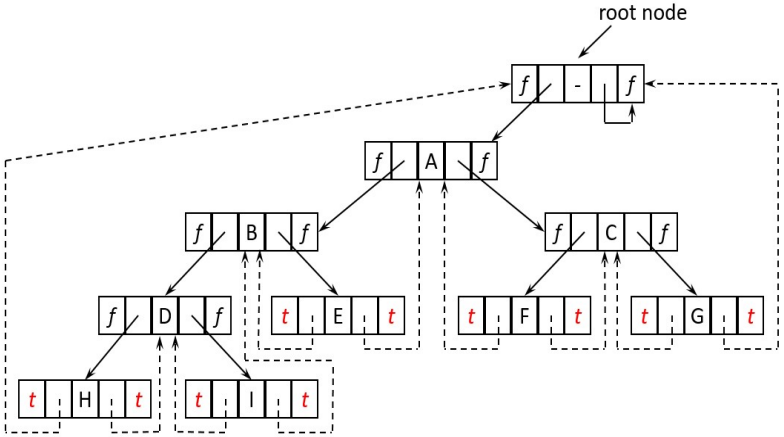
RightThread is False, so you follow the LeftChild link of RightChild node (K) until you reach the node with the LeftThread is True (which is K itself)



Traverse Threaded Binary Tree

- Traverse inorder without using a stack

```
Next() // return next inorder
{
     ThreadedNode *temp = CurrentNode->RightChild;
     if (!CurrentNode->RightThread)
          while (!temp->LeftThread) temp = temp->LeftChild;
     CurrentNode = temp;
}
```



1) Start from root node: RightThread is false, so all the way down to the left
2) If CurrentNode is root node, done with inorder traversal

Questions?