CSE221

Lecture 11:
Binary Search Trees

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Acknowledgment: The content of this file is based on the slides of the textbook as well as the slides provided in former lectures at UNIST.

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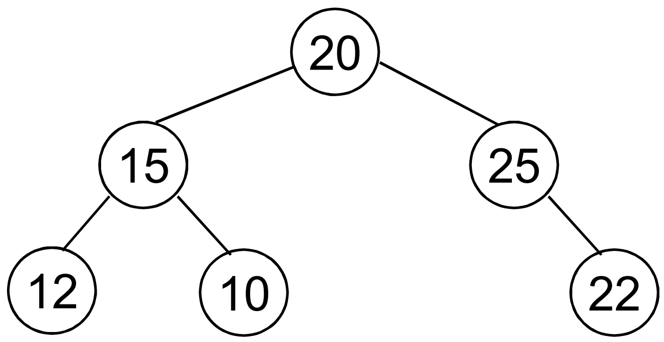
Outline

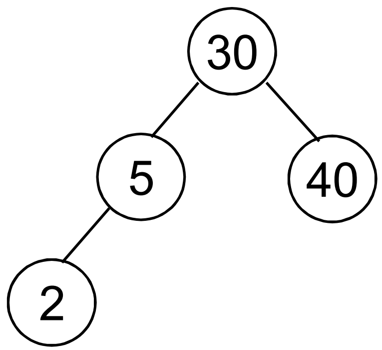
- Binary search trees
- Selection trees

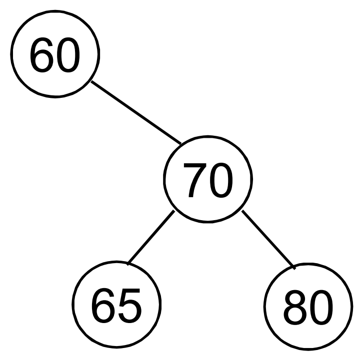
Binary Search Tree

- Definition
  - Every element has a key, and all keys are different
  - Keys in the left subtree are smaller than root
  - Keys in the right subtrees are larger than root
  - Left and right subtrees are also binary search tree

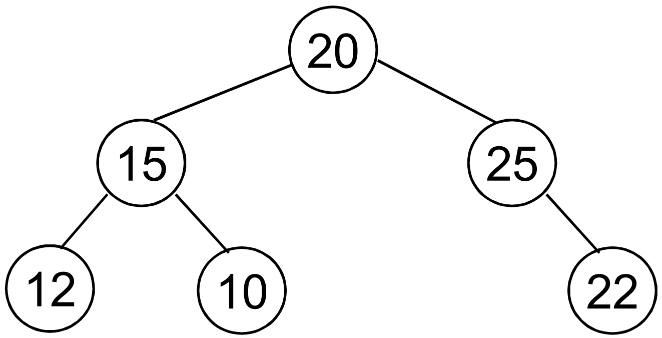
Binary Search Tree?



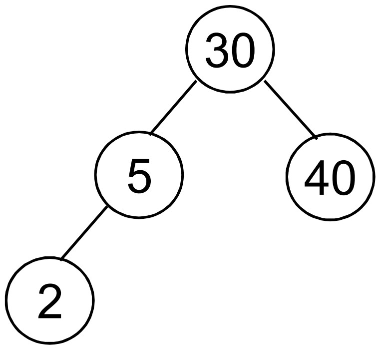




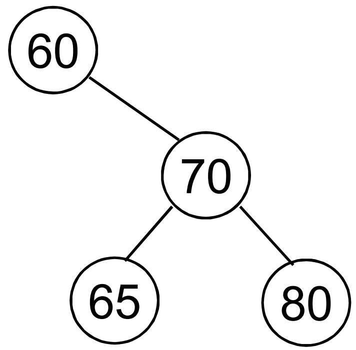
Binary Search Tree?



X



O



O

Searching by the Key Value

- Recursive

```
template<class Type>  // Driver
BstNode<Type> *BST<Type>::Search(const Element<Type> &x)
// Search the binary tree (*this) for a pair with key x
// return 0 if not found
{
    return search(root, x);
}

template<class Type>  // Workhorse
BstNode<Type> *BST<Type>::Search(BstNode<Type> *b,
                                 const Element<Type> &x)
{
   if (!b) return 0;
   if (x.key == b->data.key) return b;
   if (x.key < b->data.key)  return Search(b->LeftChild, x);
   return Search(b->RightChild, x);
}
```

O(h), where h is height of the tree

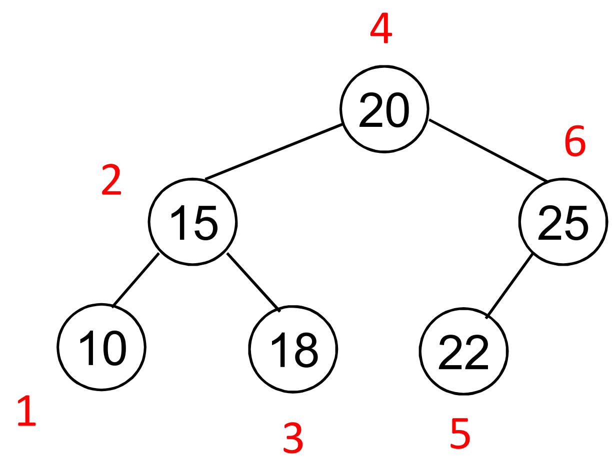
Searching by the Key Value

- Iterative

```
template <class Type>
BstNode<Type> *BST<Type>::IterSearch(const Element<Type> &x)
// Search x in (*this) binary search tree
{
   BstNode<Type> *t = root;
   while(t)
   {
     if (x.key == t->data.key) return t;
     if (x.key < t->data.key) t = t->LeftChild;
     else  t = t->RightChild;
   }
   return 0;
}
```

Searching by the Rank

- Rank
  - Node position in inorder traversial
- leftsize
  - 1 + # of nodes in left subtree



Rank = numerical order of each node's key

Searching by the Rank

```
template <class Type>
BstNode<Type> *BST<Type>::Search(int k)
// Find k-th smallest pair
{
   BstNode<Type> *t = root;
   while (t) {
     if (k == t->LeftSize) return t;
     if (k < t->LeftSize) t = t->LeftChild;
     else {
        k -= t->LeftSize;
        t = t->RightChild;
     }
   }
   return 0;
}
```

- O(h), where h is height of the tree

- Keep leftsize correct during insert/delete
- If leftsize is not maintained, just perform inorder traversal

Use Case of the Rank

- Percentile: the value below which a given percentage of values are observed
- E.g., 50th-percentile: the value below which 50% of values are observed (i.e., median)
- What is the rank for the 90th-percentile on the binary search tree with N values?
  - Answer: 0.9 x N

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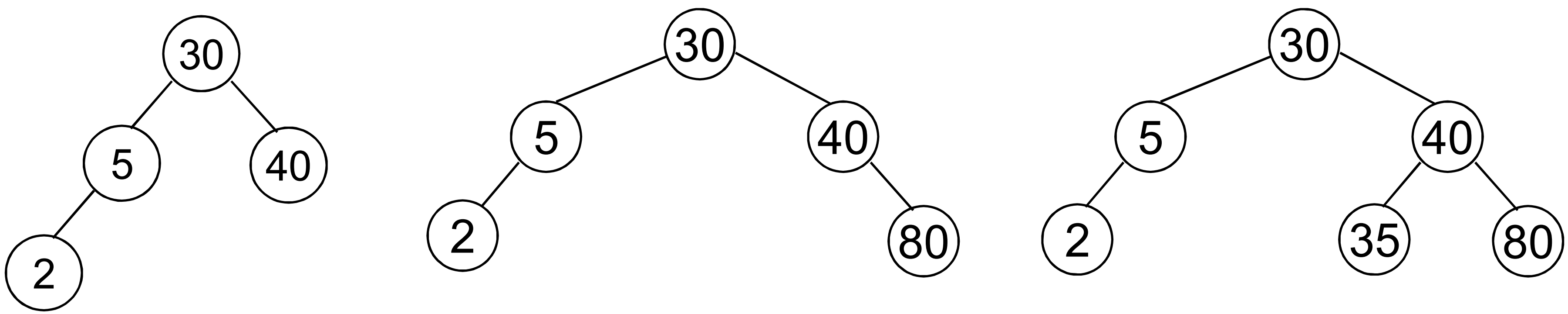
Insertion into a Binary Search Tree

- Note that every key has to be different!
- Search k
  - Failed : insert k where search terminated
  - Success : update element



(a) Insert 80 (b) Insert 35

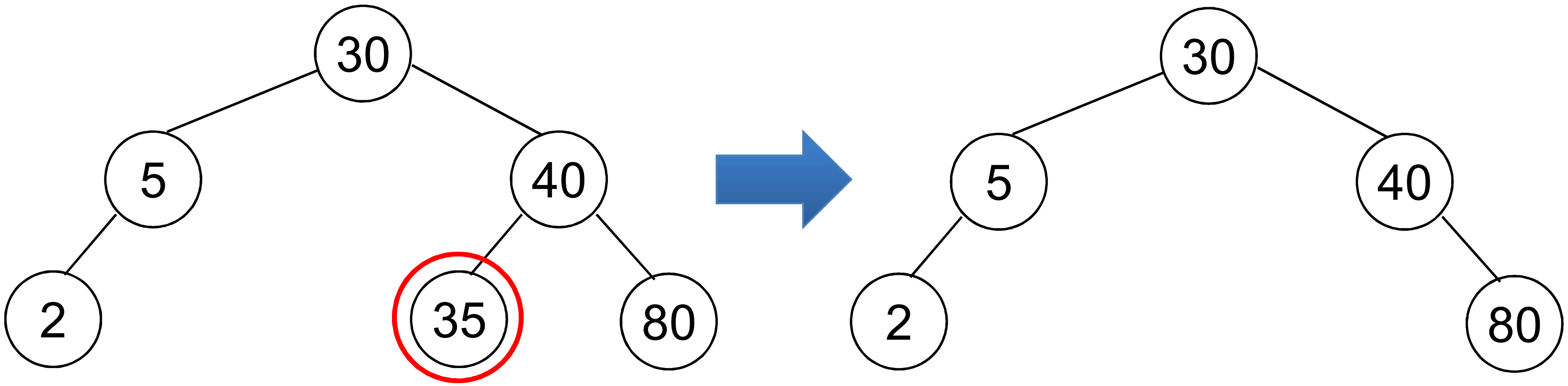
O(h), where h is height of the tree

Deletion from a Binary Search Tree

- Possible status of a node to delete
  - Leaf node
  - One child
  - Two children

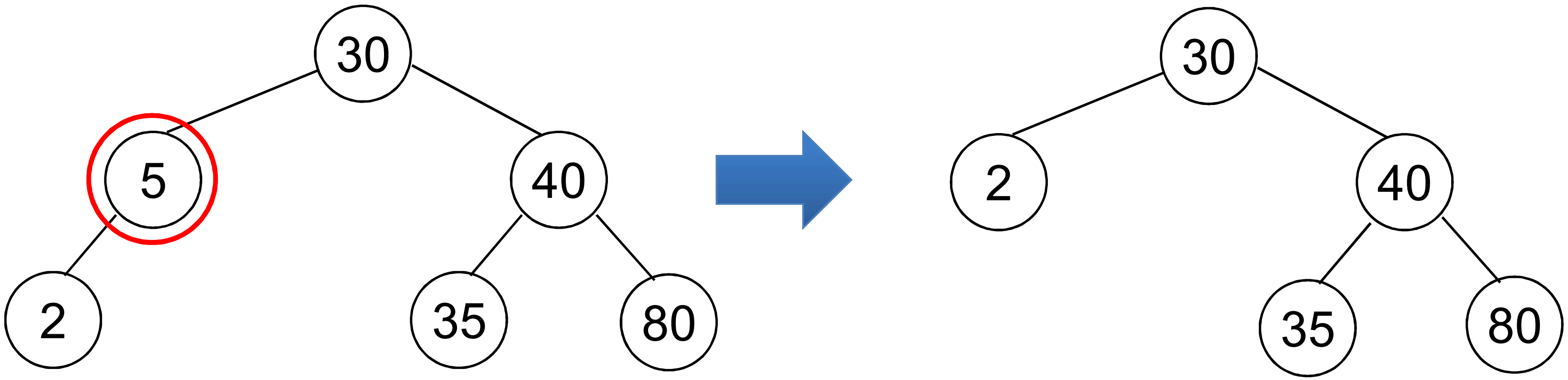
Deletion from a Binary Search Tree

- Leaf node
  - Simply remove the node from its parent
  - e.g., delete 35

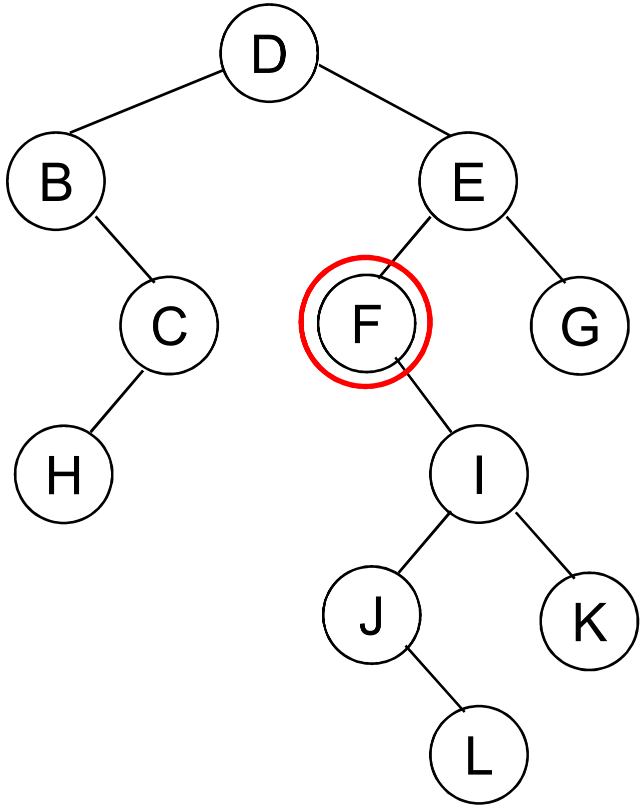


Deletion from a Binary Search Tree

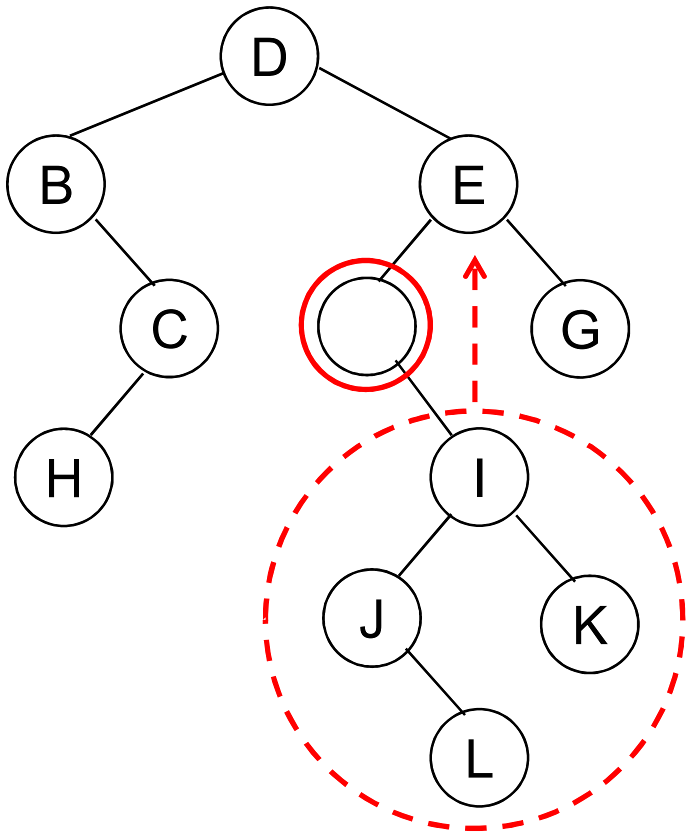
- Non-leaf node with one child
  - Child takes place of the deleted node
  - e.g., delete 5



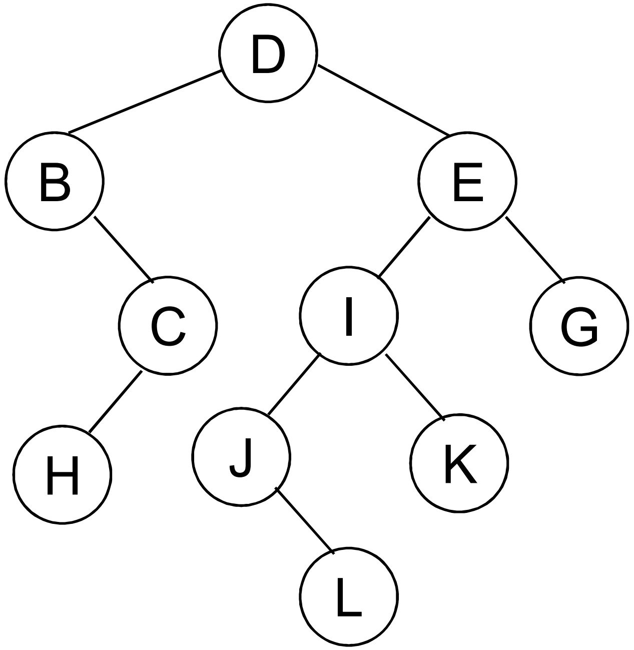
Deletion from a Binary Search Tree



Deletion from a Binary Search Tree

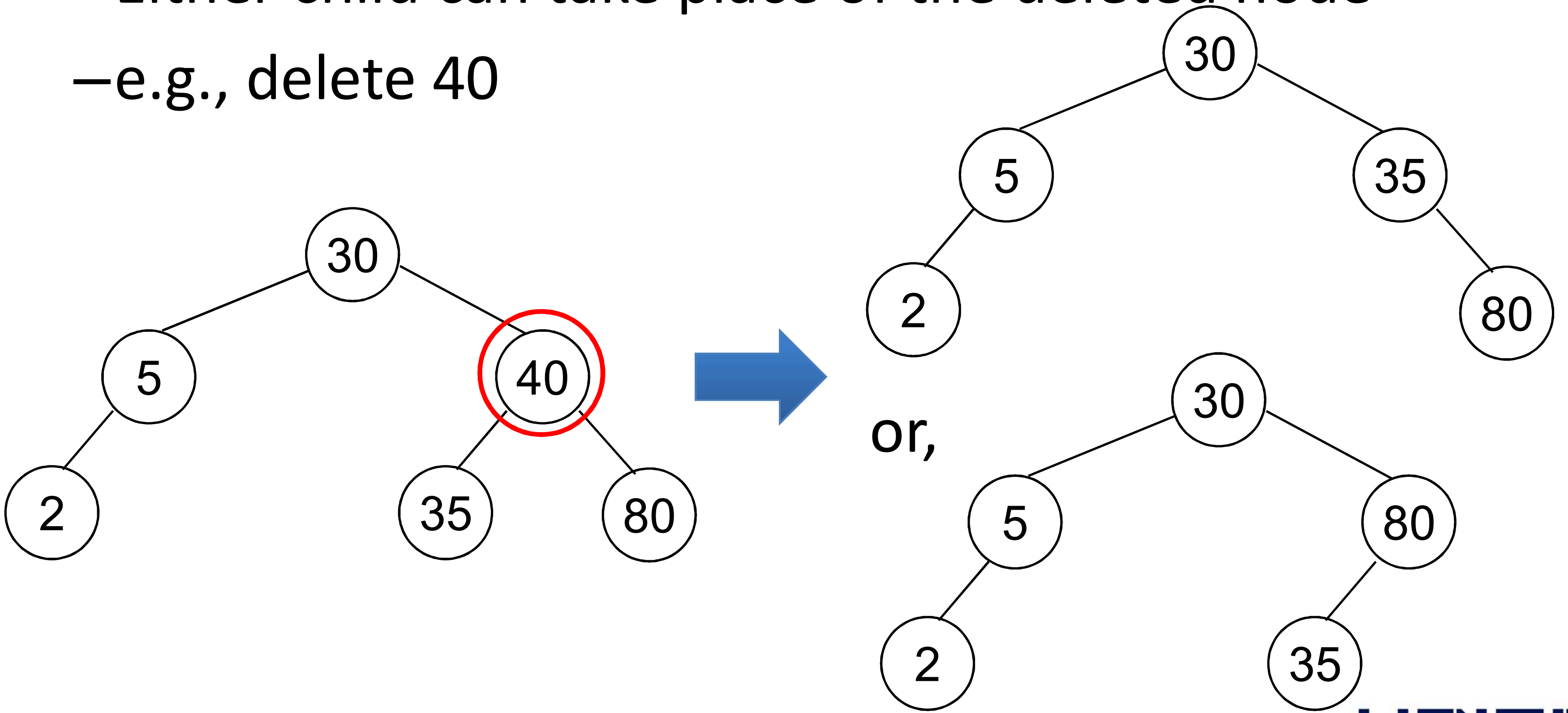


Deletion from a Binary Search Tree



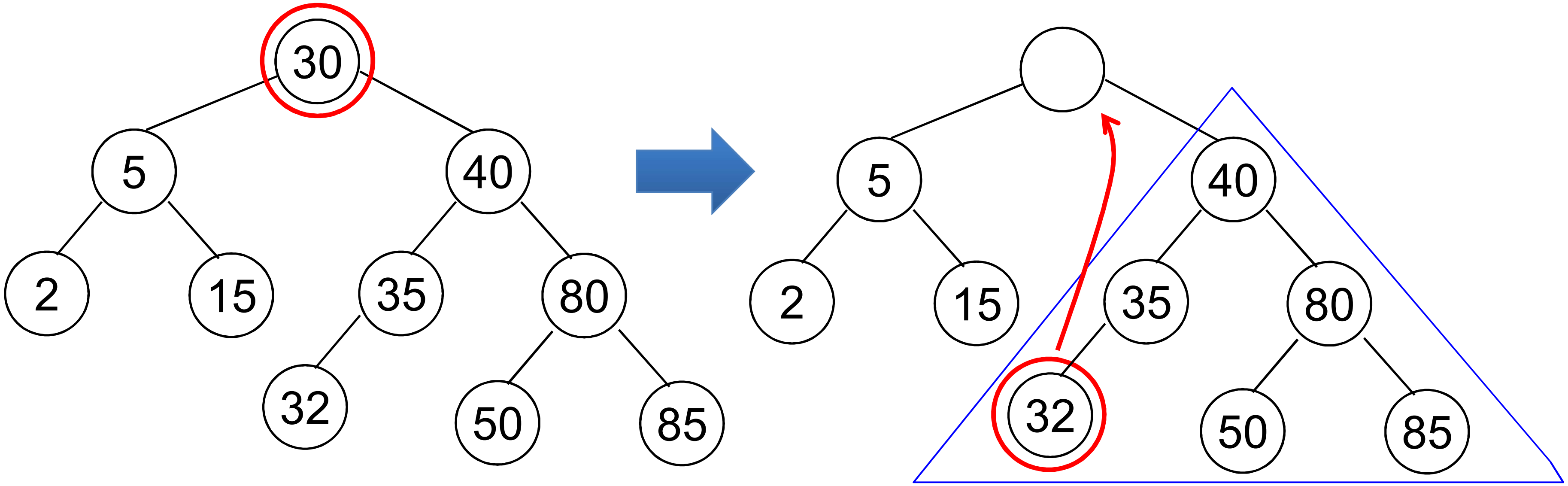
Deletion from a Binary Search Tree

- Non-leaf node with two children
  - Either child can take place of the deleted node
  - e.g., delete 40



Deletion from a Binary Search Tree

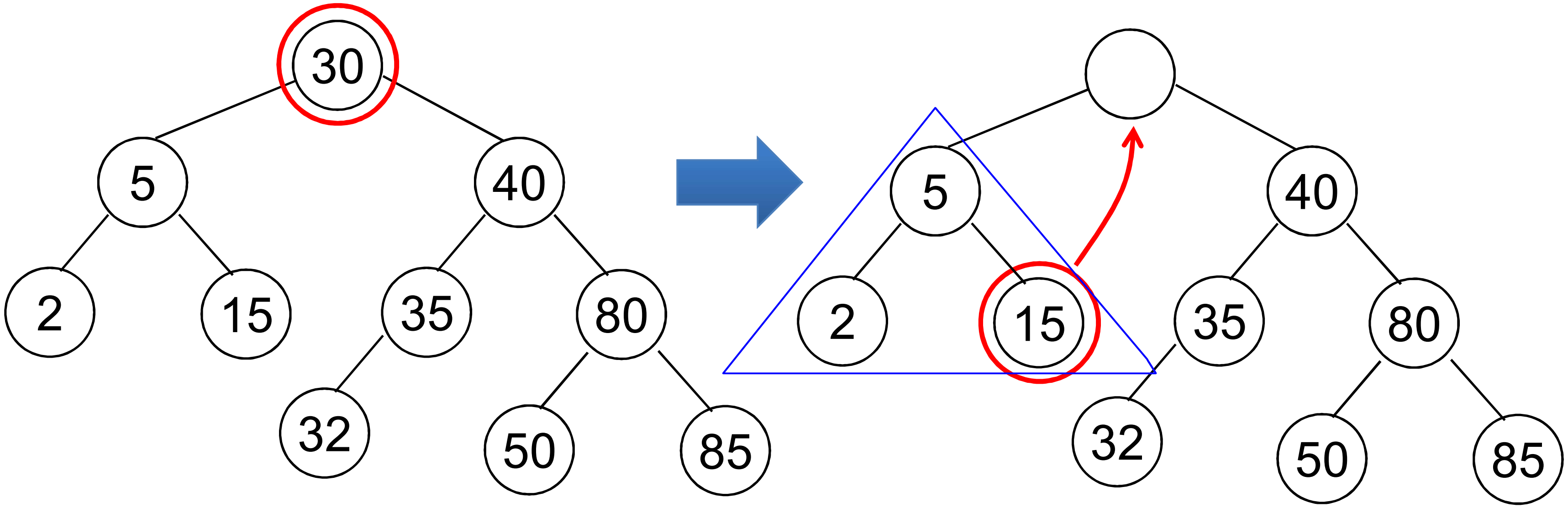
- Non-leaf node with two subtrees
  - Replace with max/min node



delete min from right subtree

Deletion from a Binary Search Tree

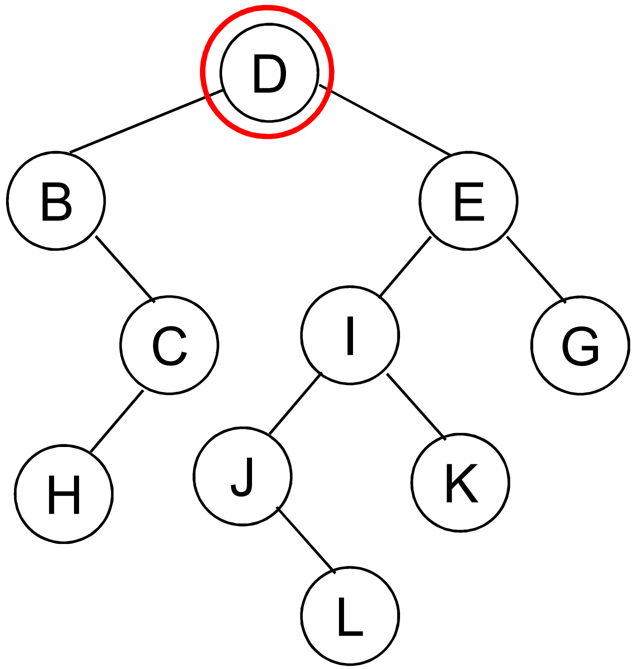
- Non-leaf node with two subtrees
  - Replace with max/min node



delete max from left subtree

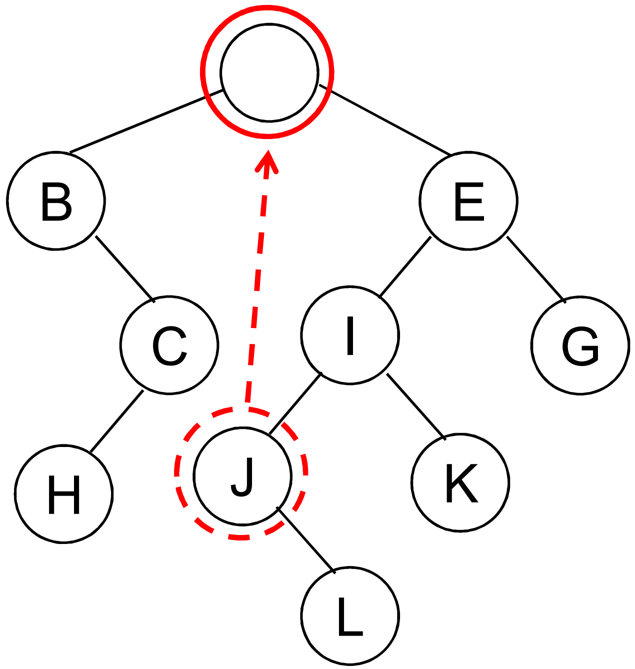
Deletion from a Binary Search Tree

- Non-leaf node with two subtrees
  - If min/max node is not a leaf, apply general rule again



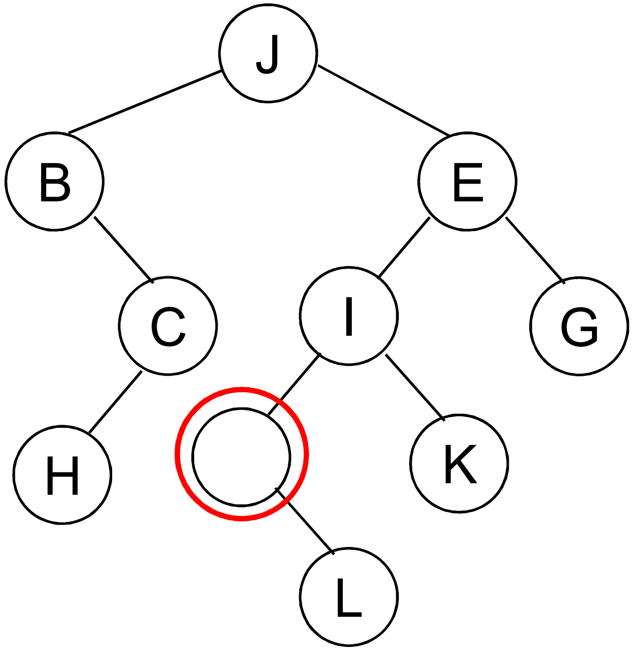
Deletion from a Binary Search Tree

- Non-leaf node with two subtrees
  - If min/max node is not a leaf, apply general rule again



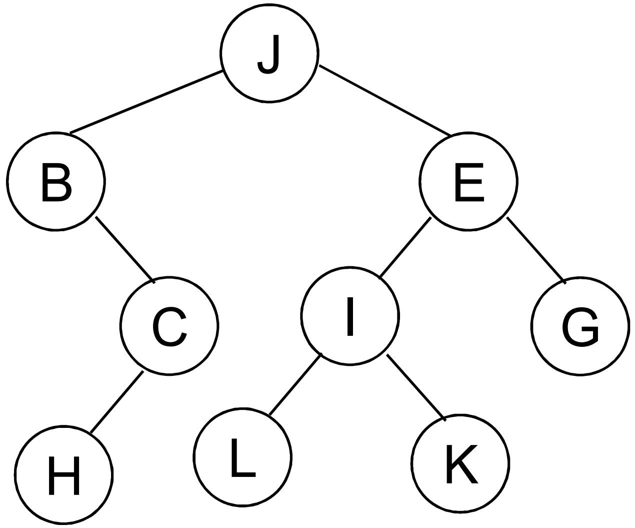
Deletion from a Binary Search Tree

- Non-leaf node with two subtrees
  - If min/max node is not a leaf, apply general rule again



Deletion from a Binary Search Tree

- Non-leaf node with two subtrees
  - –If min/max node is not a leaf, apply general rule again

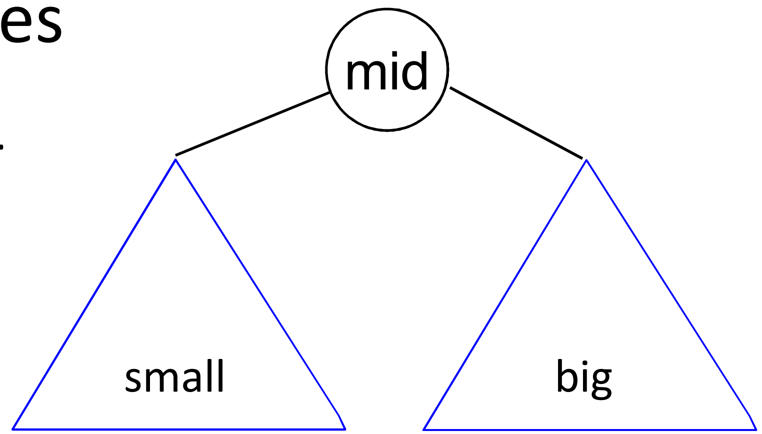


Joining and Splitting Binary Trees

- Three-way join
  - Two binary search trees and a mid value are merged as a single binary search tree
- Two-way join
  - Two binary search trees are merged to a single binary search tree
- Split
  - A single binary search tree is split into two binary search trees with respect to a given mid value

Three-way Join

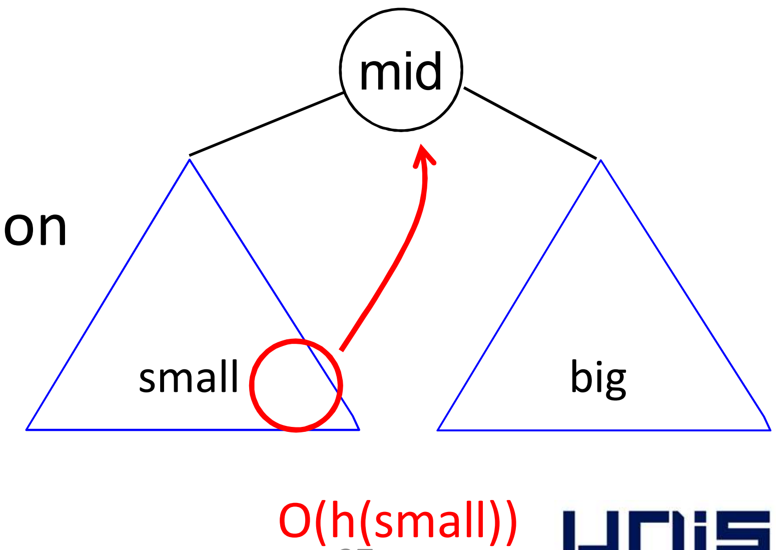
- Input
  - Two binary search trees (small, big) and mid value
- Algorithm
  - Create a mid node and attach small / big binary trees as left / right subtrees
  - h= max(h(small),h(big))+1



O(1)

Two-way join

- Input
  - Two binary search tree (small, big)
- Algorithm
  - Find mid by searching largest (smallest) key in small (big) and three way join
  - h= max(h'(small),h(big))+1
    - h': height of small after deletion

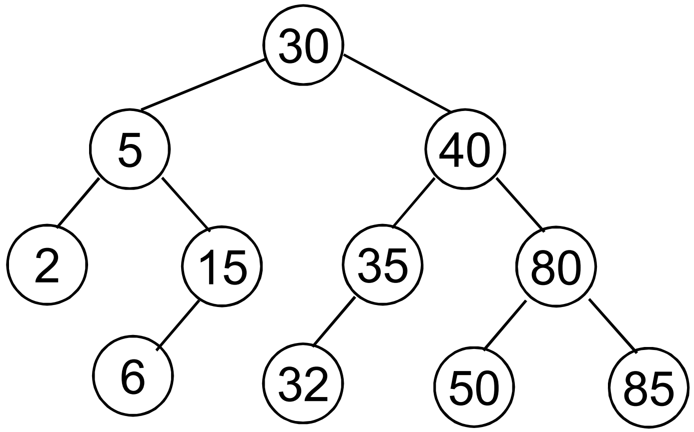


Split

- Input
  - A binary search tree and the mid value k
- Output
  - Two binary search trees (small and big), mid node if k exists

Split

k = 35



small

big

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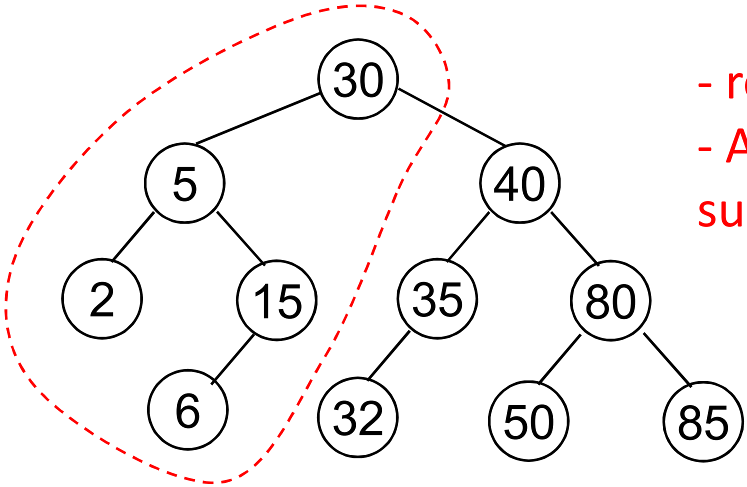
Split

k = 35



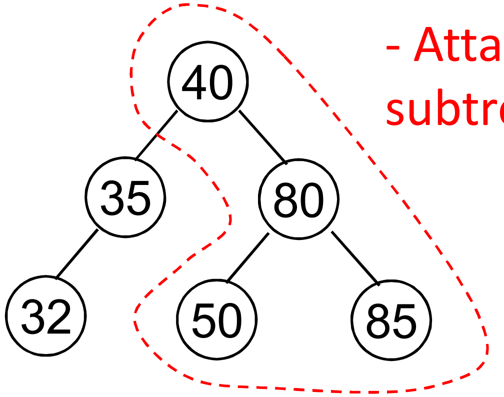
- root = 30 < k
- Attach root & left subtree to small

small

big

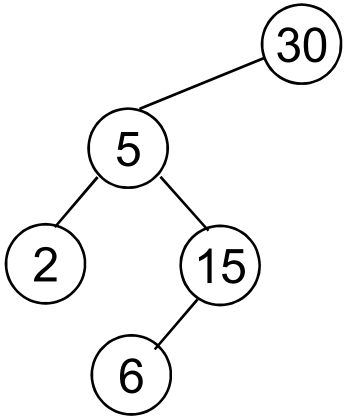
Split

k = 35



- root = 40 > k
- Attach root & right subtree to big

small

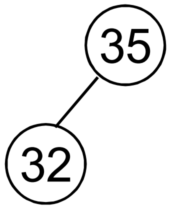


big

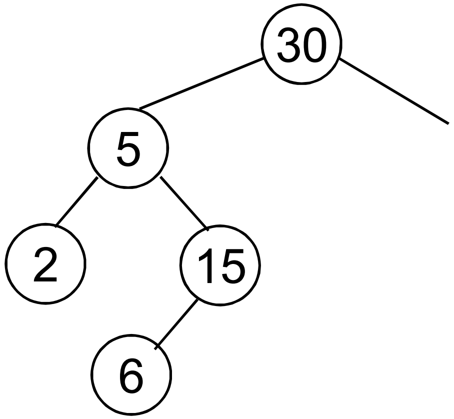
Split

k = 35

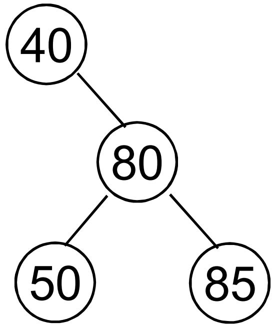
- root = 35 = k
- Attach left subtree to small and right subtree to big



small



big

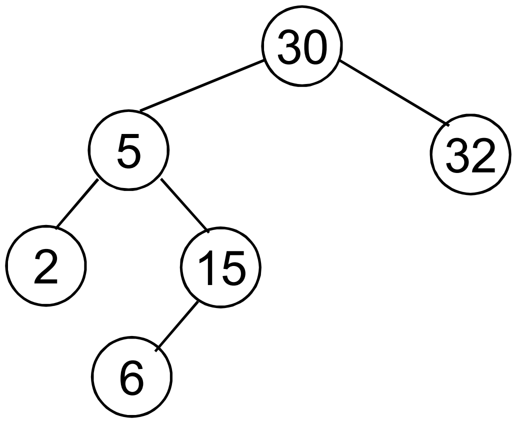


Split

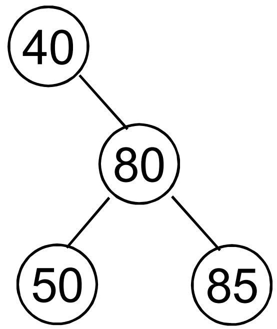
k = 35

- root = 35 = k
- Return 35 as mid

small



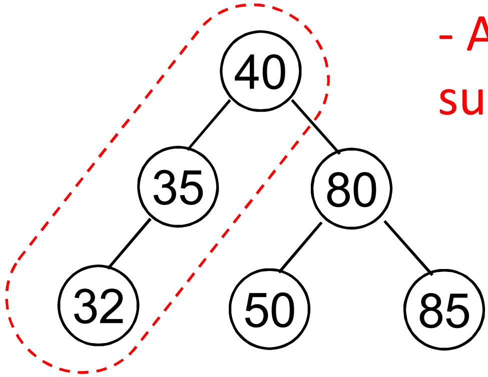
big



Split

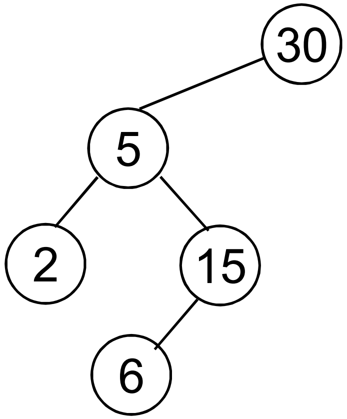
What if k = 80?

- root = 40 < k
- Attach root & left subtree to small



small

big



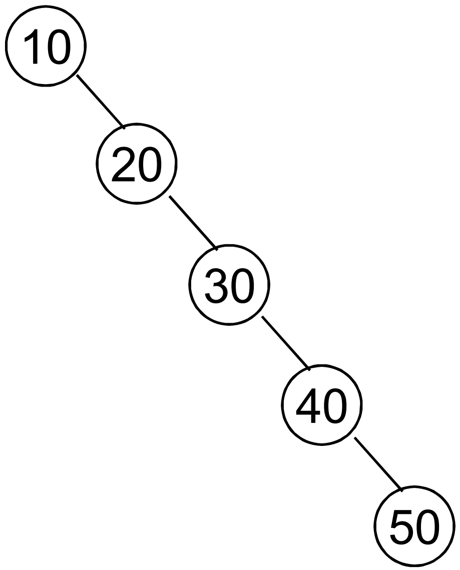
Split Algorithm

- Start with empty small and big binary search trees
- Repeat the following process:
  - If k > root, attach root + left subtree to small
    - Perform two-way join with current small: incoming small is bigger
  - If k < root, attach root + right subtree to big
    - Perform two-way join with current big: incoming big is smaller
  - If k = root, attach left subtree to small and right subtree to big, and return a node containing k as mid

Split

- Worst case
  - Each split, only one node (root) will be attached to small

k = 50



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Split

- Complexity
  - O(h(input))
  - h(small), h(big) <= h(input)

Height of a Binary Search Tree

- Worst case
  - O(n)
  - e.g., 1,2,3,4,5,..,n (right skewed tree)
- Best case
  - O(log n)
  - e.g., complete/full binary tree
- Balanced search tree
  - Search tree with worst-case height O(log n)