CSE221

Lecture 14:
Red-Black Trees

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Acknowledgment: The content of this file is based on the slides of the textbook as well as the slides provided in former lectures at UNIST.

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Red-Black Trees

- Balanced binary search tree
- Guarantee O(log n) insertion, search, delete
- Definition
  - Binary search tree that every node/pointer is colored either red or black
  - Leaf nodes do not contain data
    - Call them "external nodes"

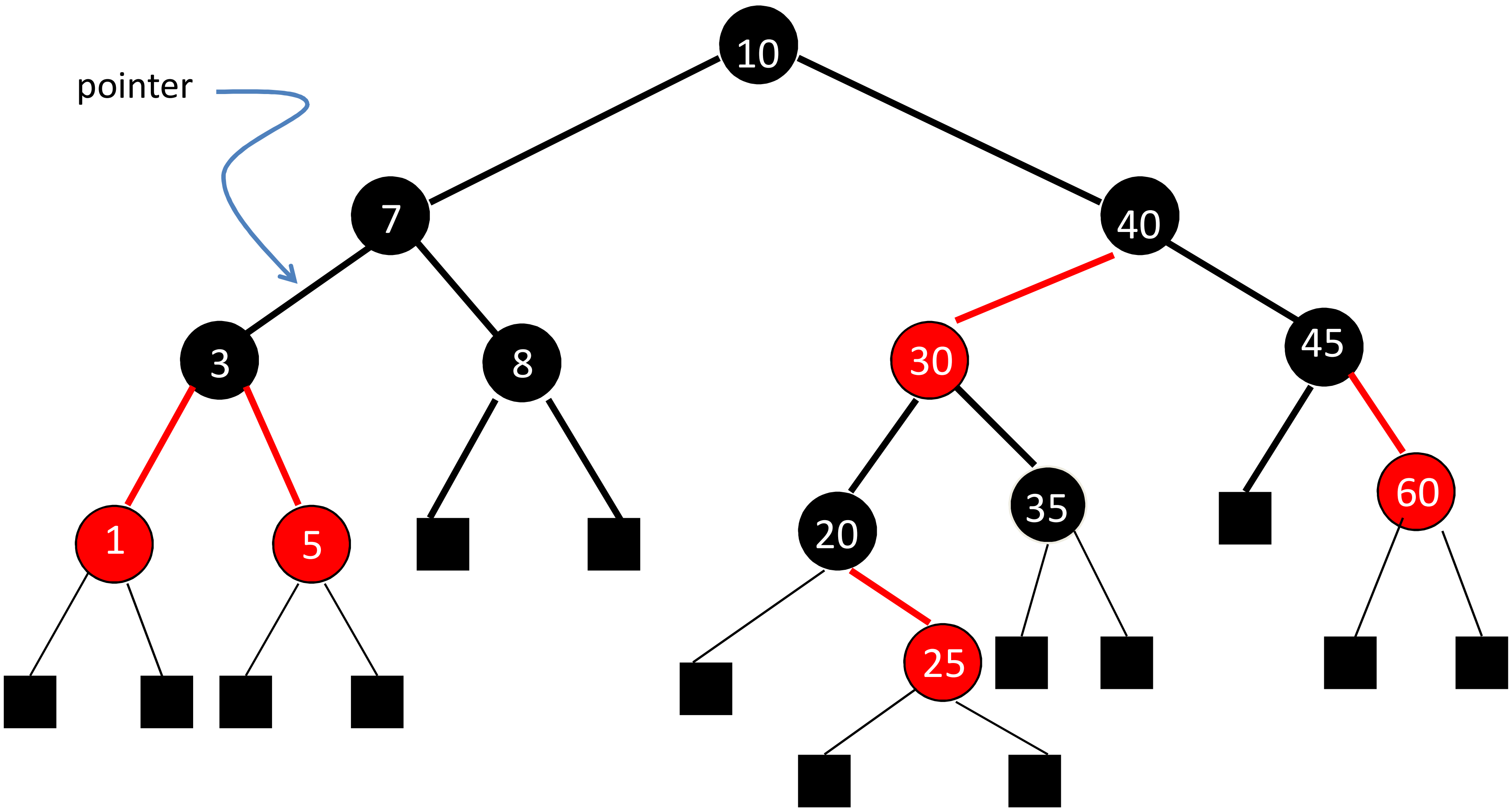
Colored Nodes

- The root and all external nodes are <u>black</u>
- No root-to-external-node path has two consecutive red nodes
  - Red node must have two black children
- All root-to-external-node paths have the same number of black nodes

Colored Pointers

- Pointers from an internal node to an external node are black
- No root-to-external-node path has two consecutive red pointers
- All root-to-external-node paths have the same number of black pointers

Example Red-Black Tree



- The pointer from a parent to a black child is black and to a red child is red.

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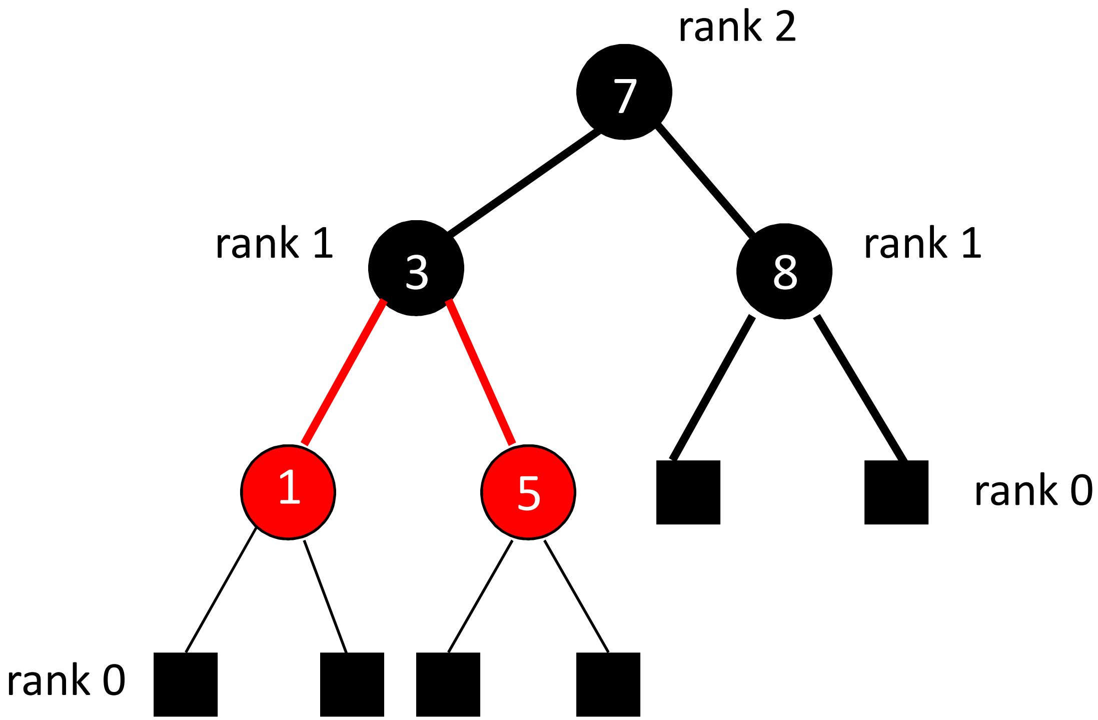
Properties

- Rank : # of black pointers on any path from a node to any external node



Properties

- If P and Q are two root-to-external-node paths

  length(P) <= 2*length(Q)

  length= the number of pointers on the path

- Longest path length is bounded by 2*shortest path length
  - Shortest path : B-B-B-....-B
  - Longest path : B-R-B-R...-B
  - Number of B must be same for all paths by definition, so the above statement is true

Properties

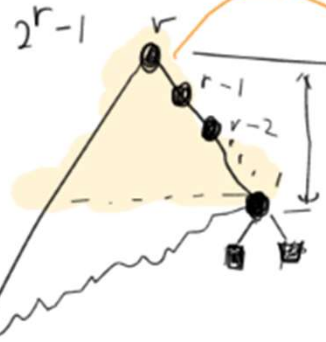
- h : height, r : rank of the root, n : # of nodes

① $h \le 2r$

longest path length $\le$ 2·shortest path length

$h \le$ longest path length $\le 2 \cdot r$

② $n \ge 2^r - 1$



$r \Rightarrow 2^r - 1$ nodes

$\therefore n \ge 2^r - 1$

$\therefore \log_2(n+1) \ge r \ge \frac{1}{2}h$ (by ① & ②)

$\therefore h \le 2\log_2(n+1)$

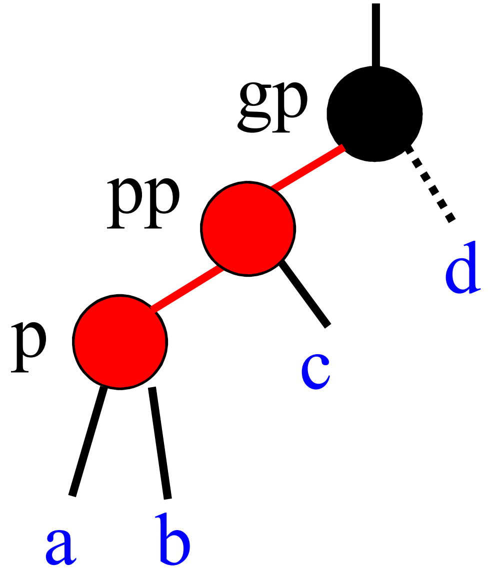
$\Rightarrow$ search, Insert, delete $\simeq O(h) = O(\log n)$

Insert

- Same as regular binary search tree insertion
- Must assign color
  - If the tree is empty, new node is root so assign black
  - If the tree was not empty, assign black node in the path?
    - NO! Why?
    - Will violate same # of black nodes for all paths
      - difficult to resolve
  - Then, assign red?
    - May (or may not) lead to two consecutive red nodes in the path?
    - Can be resolved by rotation and color flips

Classification of 2 Red Nodes/Pointers

pp: parent
gp: grand parent

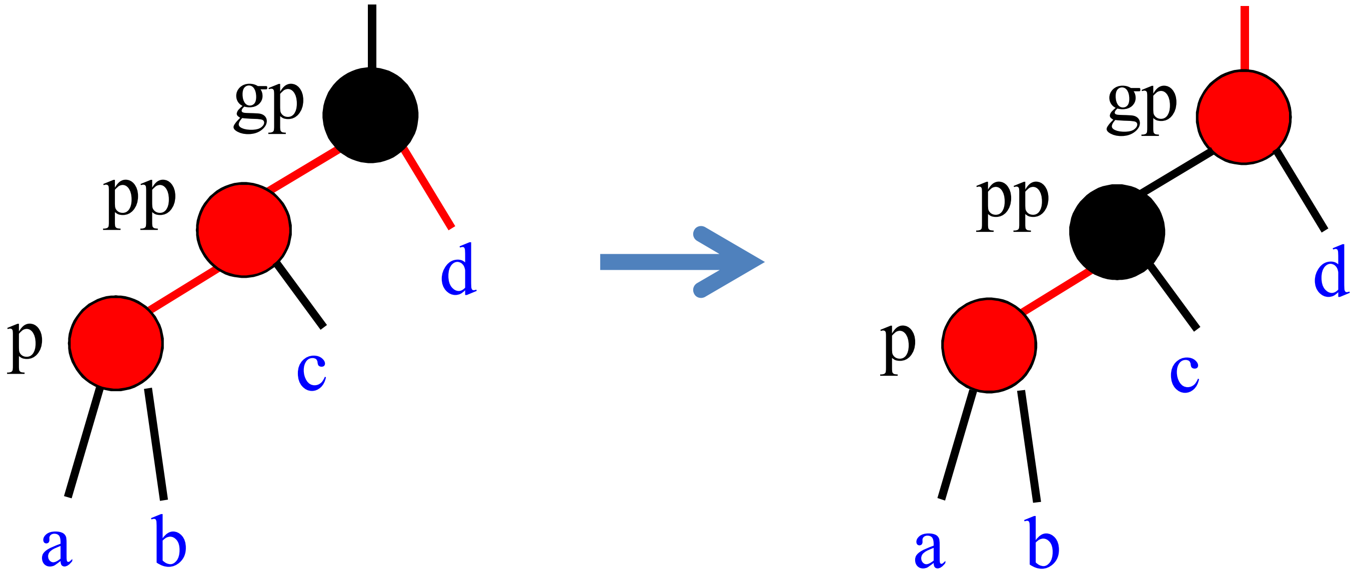


LLb example

- XYz
  - X: relationship (Left or Right) between gp and pp
  - Y: relationship (Left or Right) between pp and p
  - z: color (red or black) of the other-side node of gp

XYr

- Perform color flipping

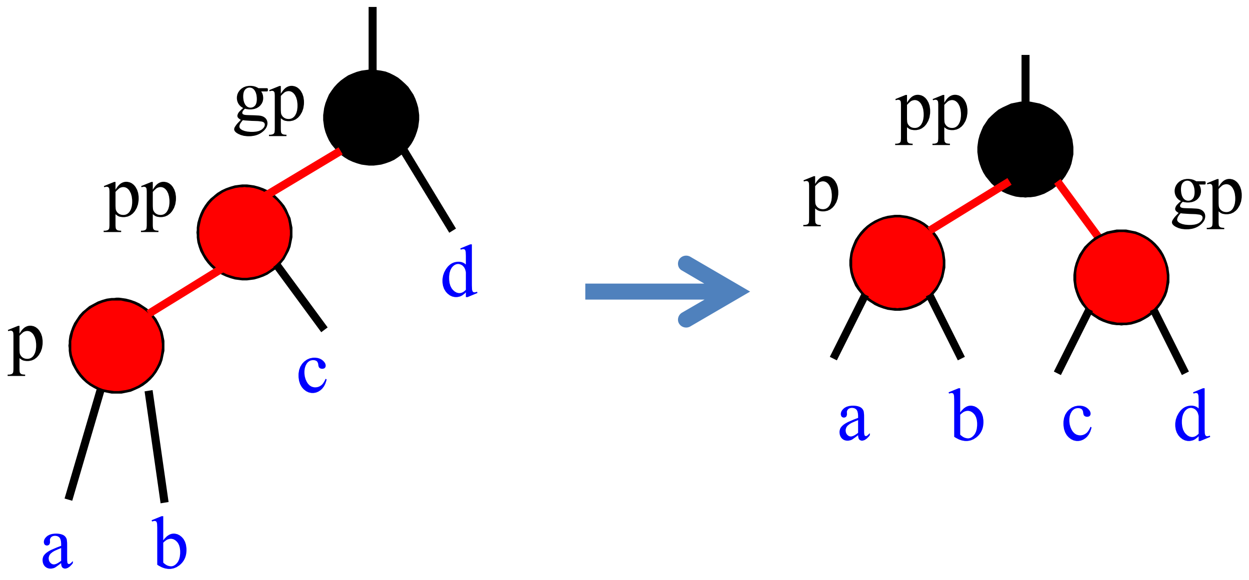


- Flip color of pp, gp, d and pointers of gp
  - Does not increase # of black
- Reapply transformation to gp by p ← gp
  - Because gp’s parent might be red

LLb

- Perform rotation

RRb is symetric

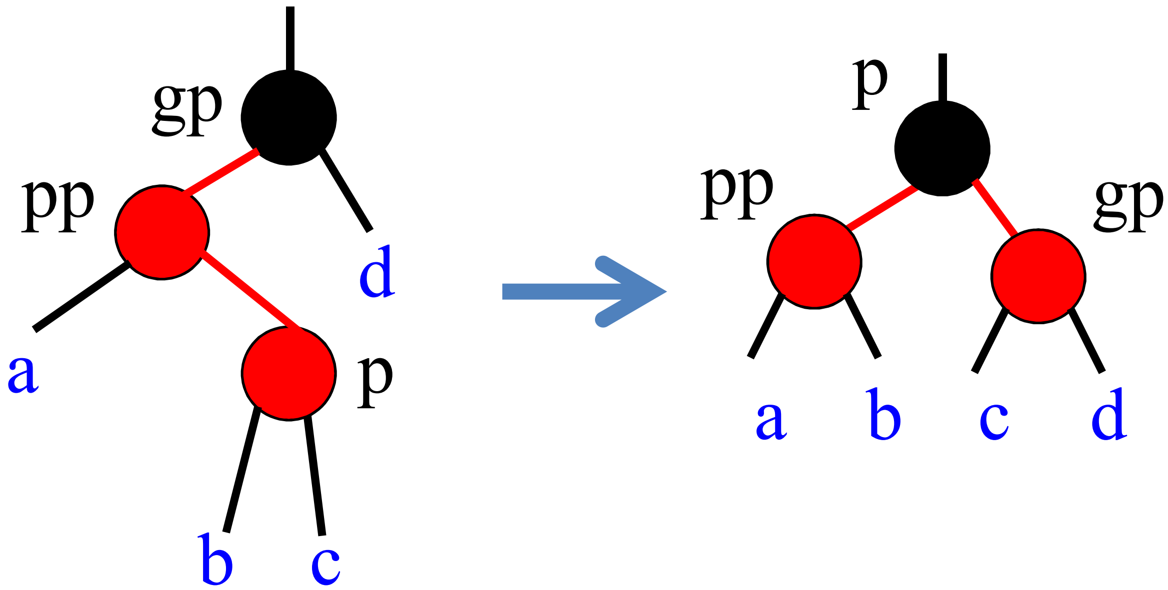


- Same as LL rotation of AVL tree (same inorder)
- Flip color of pp and gp after rotation
- No reapply: root of this subtree is still black

LRb

- Perform rotation

RLb is symetric

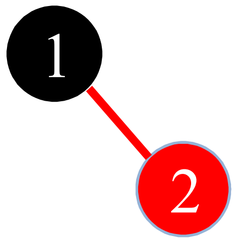


- Same as LR rotation of AVL tree (same inorder)
- Flip color of p and gp
- No reapply

Insert Example

- Insert 1
  - –Root

- Insert 2
  - –Red



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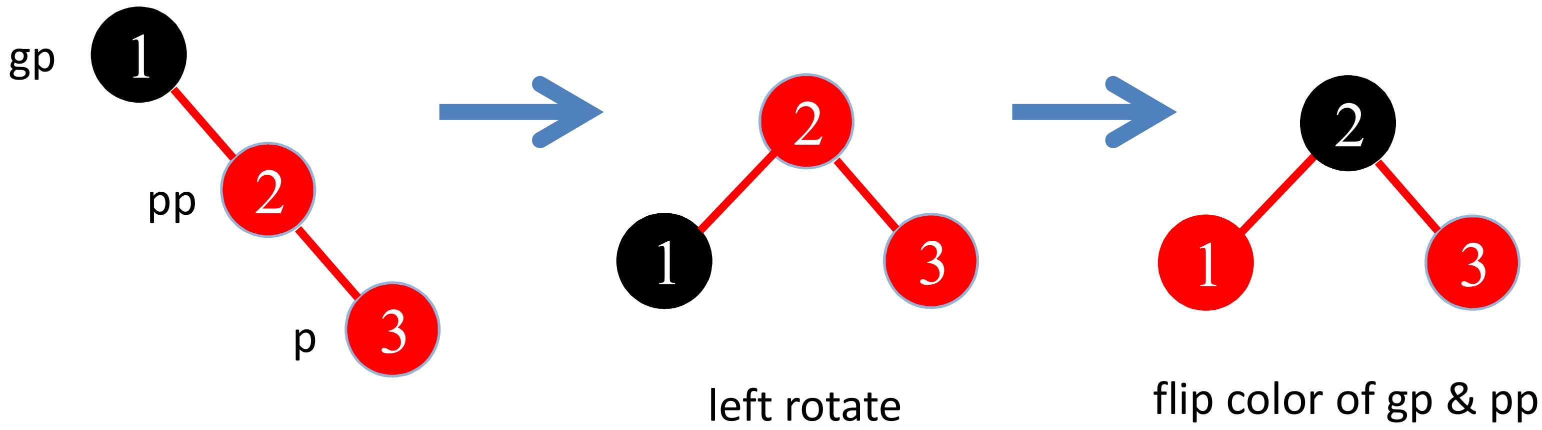
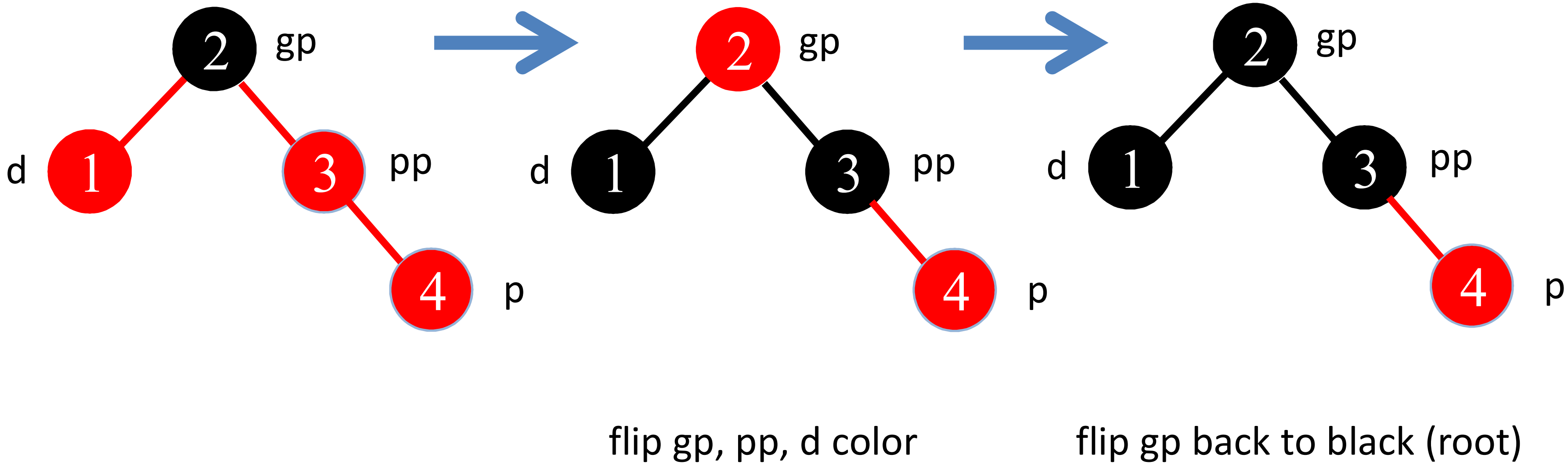
Insert Example

- Insert 3
  - –RRb: rotate (external node at left child of gp: black)



Insert Example

- Insert 4
  - RRr: flip color



- If gp is not root
  - Move up and reapply transformation because gp's parent might be red
  - p ← gp, pp ← gp's pp, gp ← gp's gp

Insert Example

- Insert 5
  - RRb: rotate