CSE221

Lecture 16:
Multiway Search Trees

Acknowledgment: The content of this file is based on the slides of the textbook as well as the slides provided in former lectures at UNIST.

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Outline

- m-way search trees
- B-trees

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Memory Hierarchy

- Von Neumann model limitation
  - Memory is bottleneck
- Memory hierarchy

  CPU ↔ Register ← cache ← memory ← disk

  ← faster but smaller data fetch unit
- Performance implications
  - Performance is closely related to reducing the number of accesses to slow memory
  - Exercising data stored in faster memory is crucial

Improving Performance in Trees

- Number of memory accesses is closely tied to the <u>*height*</u> of the search tree
  - Balanced binary search tree has $\log_2 n$ height
  - Nodes accessed on the path from the root to a leaf are often located in slower memory

Can we break $\log_2 n$ barrier?

- Node data is contiguous and has spatial locality

Can we structure node

to use faster memory more efficiently?

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m-way Search Trees

- Root has **at least two** & **at most m** subtrees
- Node structure (n<m)

Contiguous space: spatial locality

n	A_0	E_1	A_1	E_2	…	E_n	A_n

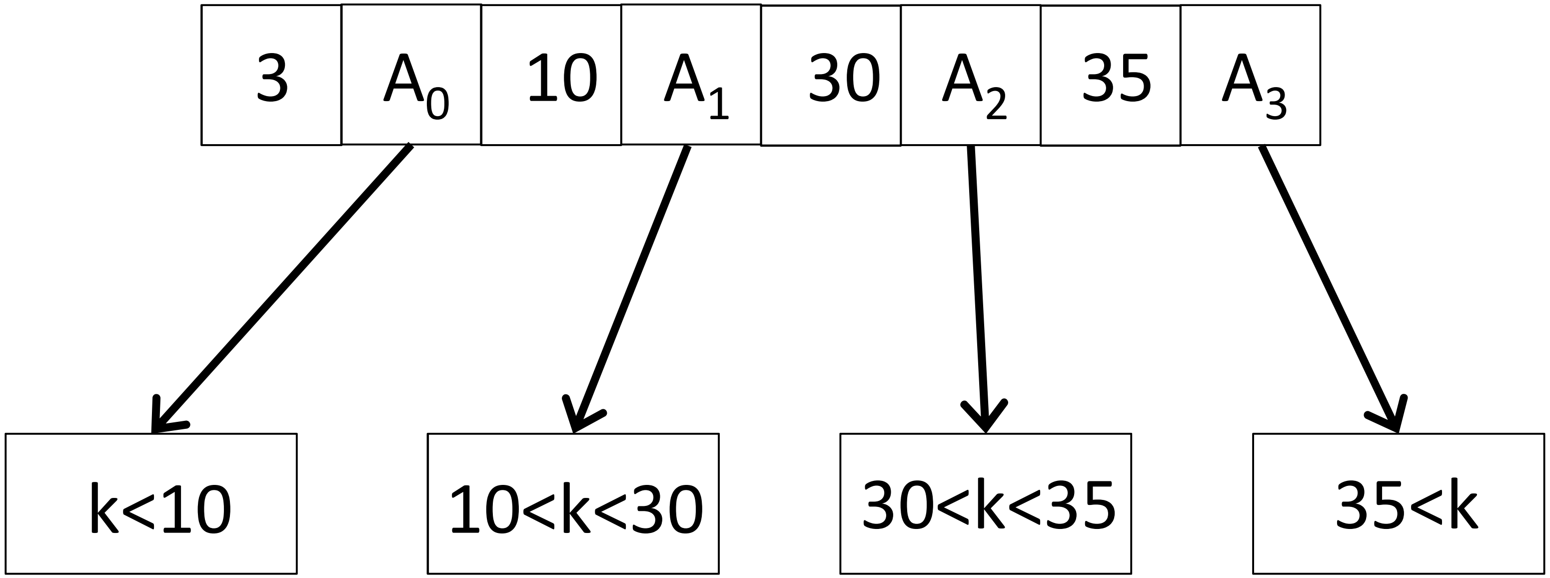
\# of elements　　pointer to subtree　　element

- E_i.key < E_{i+1}.key
- E_i.key < all keys in A_i < E_{i+1}.key

Tree is ordered!

- Subtrees A_i are also m-way search trees (recursive definition)

Example: 4-way Search Tree



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m-way Search Trees

- Maximum # of nodes: when all internal nodes are m-nodes (having m subtrees)
  - A tree of degree m and height h

$$1 + m + m^2 + m^3 + \ldots + m^h = (m^{h+1} - 1)/(m - 1)$$

- Each m-node has up to m – 1 elements
- Maximum # of elements = $m^{h+1} - 1$

Searching

```
// Search m-way search tree for an element with key x
E0.key=-MAXKEY;
for(p=root; p; p=Ai)
{
        Let p is a node (n, A0, E1, A1, .. , En, An)
        En+1.key = MAXKEY
        Determine i such that Ei.key <= x < Ei+1.key;
        if(x == Ei.key) return Ei; // x is found
}
// x is not found
return NULL;
```


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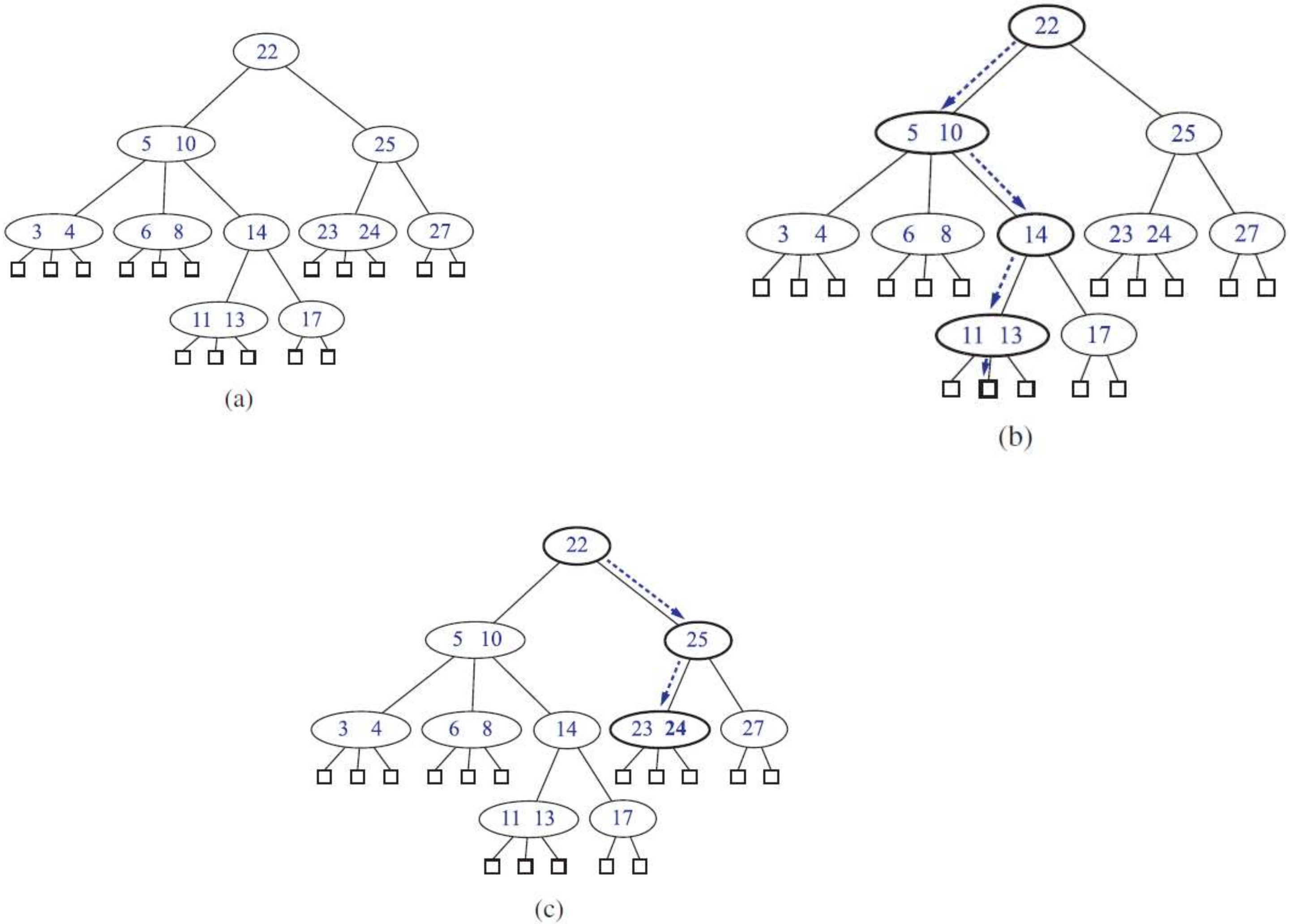

Figure 10.20: (a) A multi-way search tree T; (b) search path in T for key 12 (unsuccessful search); (c) search path in T for key 24 (successful search).

Outline

- m-way search trees
- B-trees

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B-trees

- m-way search trees with special properties:
  - Replace a NULL pointer to an external node
- Definition
  - If not empty, root node has **at least two** children
  - All internal nodes (except root) have **at least ceil(m/2) children**
  - All external nodes are at the same level
- Balanced m-way search trees

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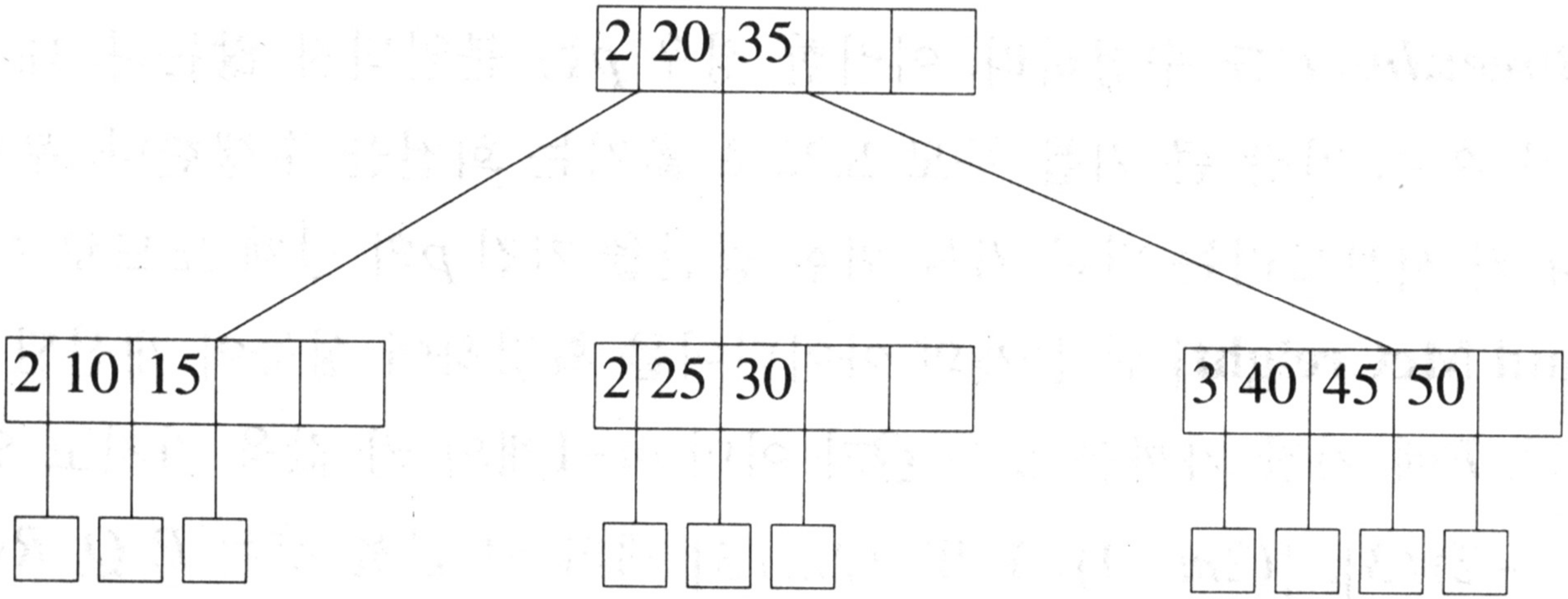
Example



5-way B-tree example, ceil(5/2) = 3

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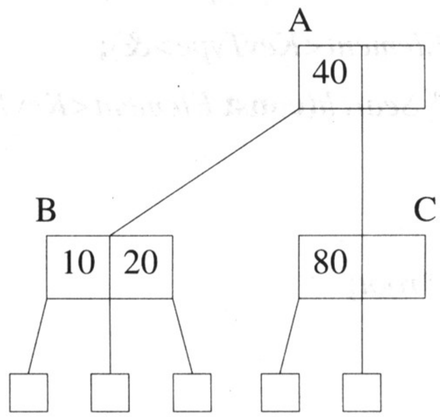
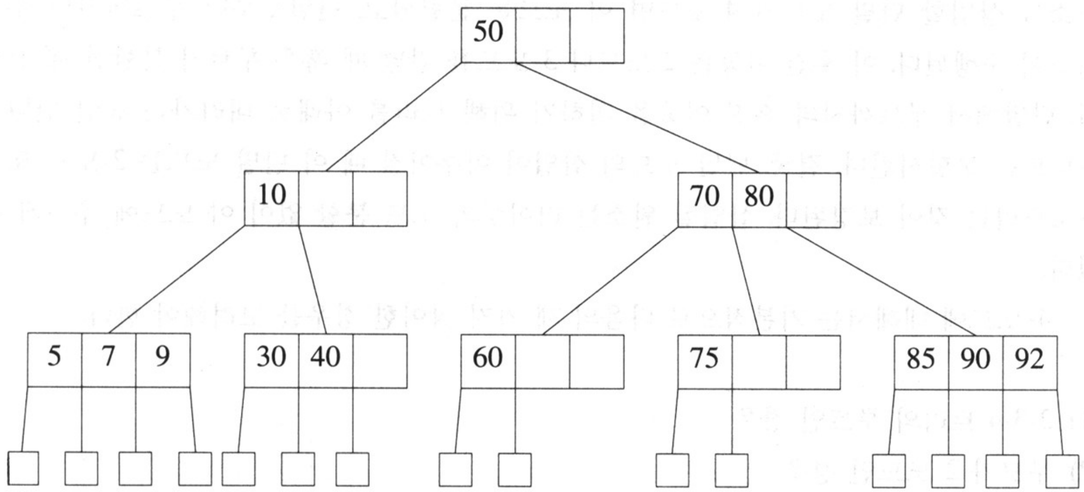
2-3 and 2-3-4 Trees

- 2-3 tree is B-tree of order 3
- 2-3-4 tree is B-tree of order 4
  - Also called (2,4) tree or 2-4 tree



2-3 tree



2-3-4 tree

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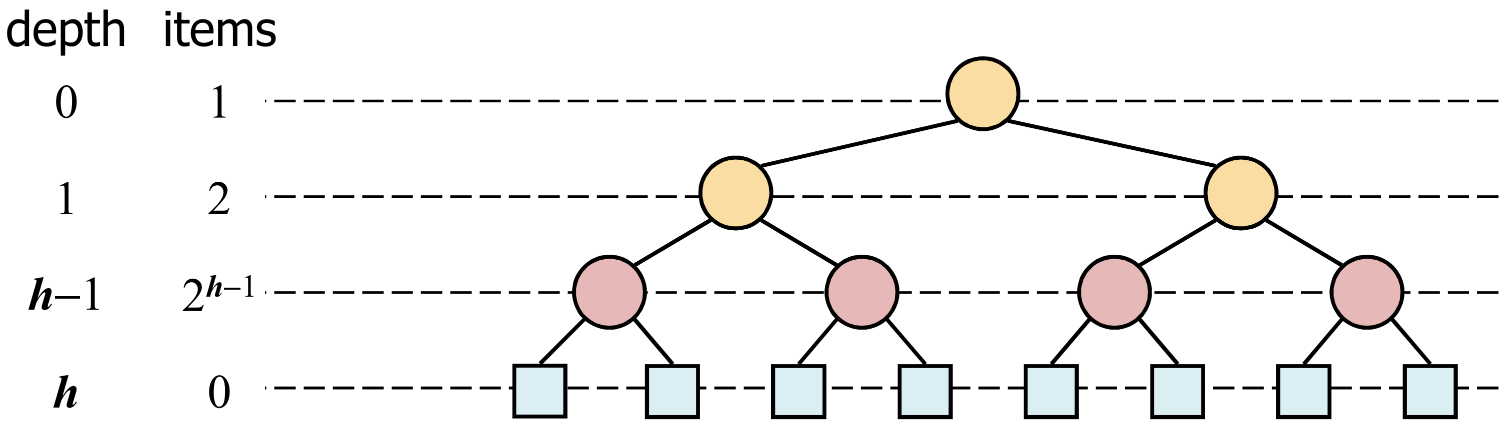
Height of 2-3-4 Tree

- 2-3-4 tree storing $\boldsymbol{n}$ items has height $\boldsymbol{O}(\log \boldsymbol{n})$
  - Let $\boldsymbol{h}$ be the height
  - Since there are at least 2^i items at depth $\boldsymbol{i} = 0, \ldots, \boldsymbol{h}$ and no items at depth $\boldsymbol{h}$, we have

$$\boldsymbol{n} \geq 1 + 2 + 4 + \ldots + 2^{\boldsymbol{h}-1} = 2^{\boldsymbol{h}} - 1$$

  - Thus, $\boldsymbol{h} \leq \log(\boldsymbol{n} + 1)$



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Choice of m

- Worst-case search time
  - (time to fetch a node + time to search node) x height
- Search time ↑ if m is too small or too large
- Pick m so that single node fits to a single memory fetch unit (e.g., cache line)
  - Time to fetch a node is constant
  - Time to search node is fast with spatial locality
  - Height of the tree is smaller (fewer non-spatial accesses)