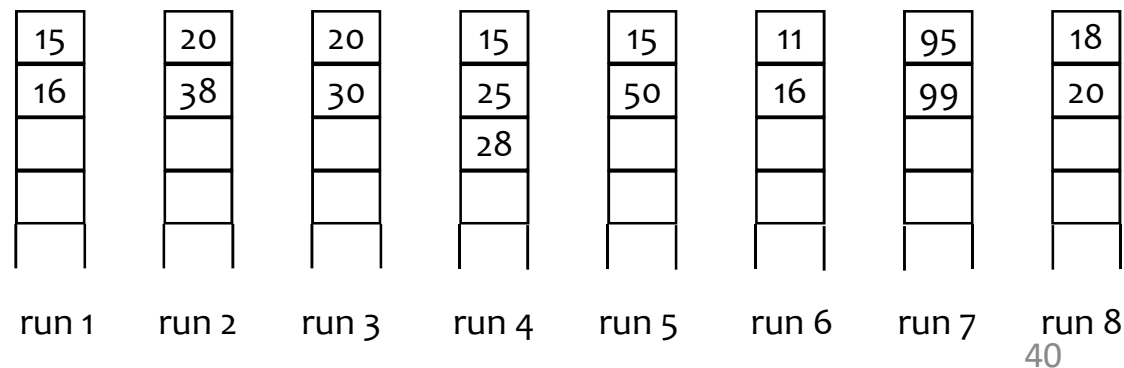


Outline

- Binary search trees
- Selection trees

Merging k ordered list (run)

- Merged list must be ordered
- Need to find the smallest among k runs
 - k-1 comparison per element
 - Expensive if k is large
- Better solution: Selection Trees (winner or loser)

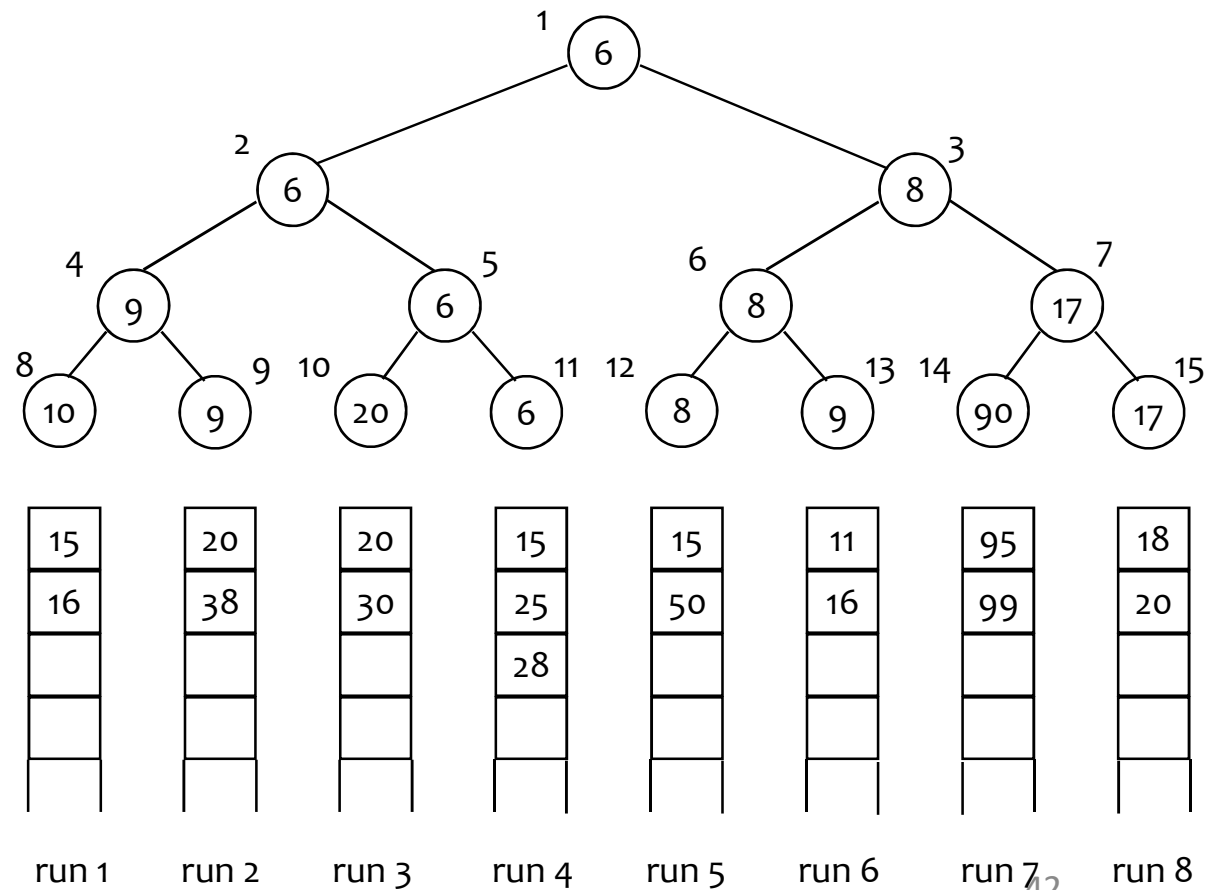


Winner Trees

- Ideas
 - Compare $O(\log k)$ instead $O(k)$
- Complete binary tree
 - Leaf : smallest from each run
 - Non-leaf : winner among two children

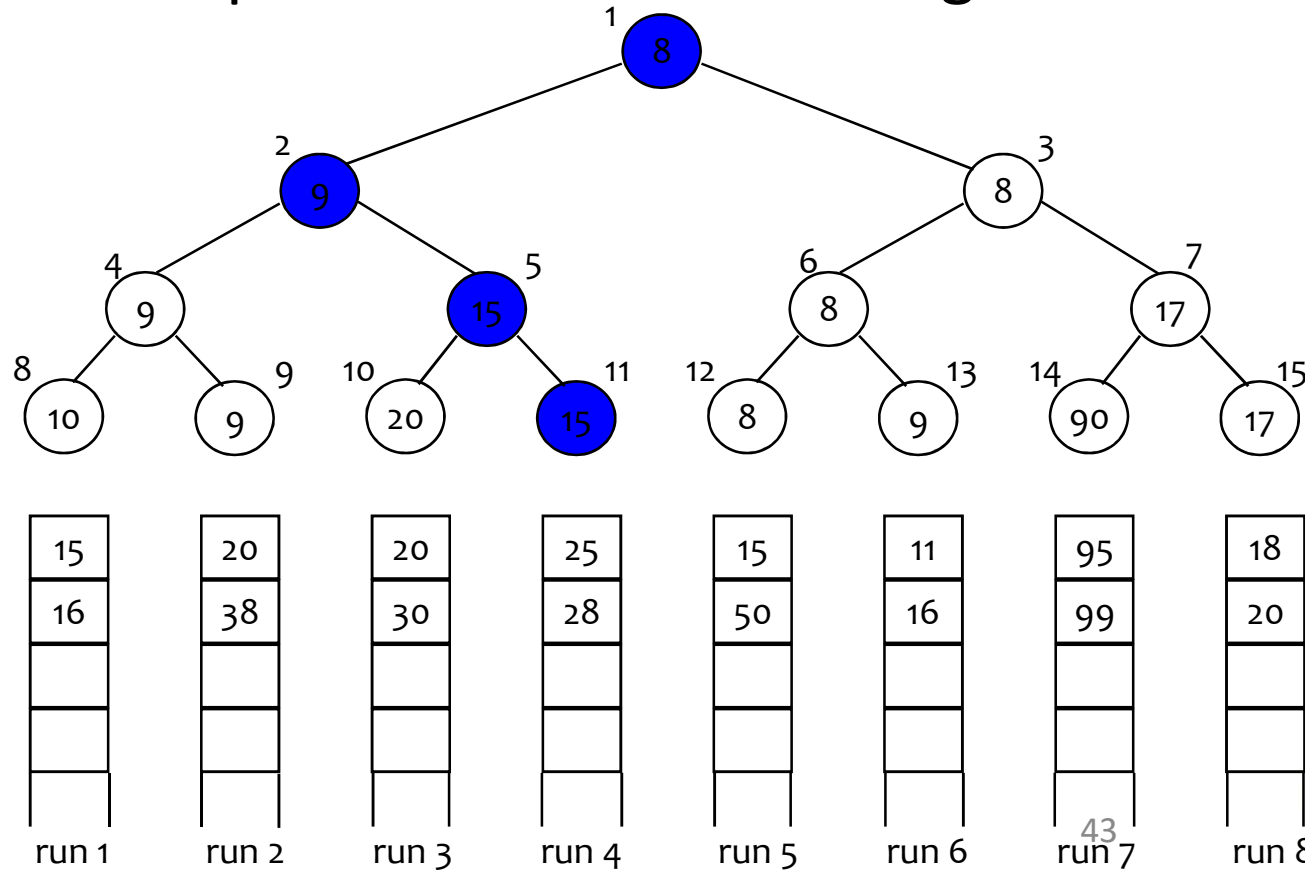
Example of Winner Tree

- 6 is the smallest among 8 values



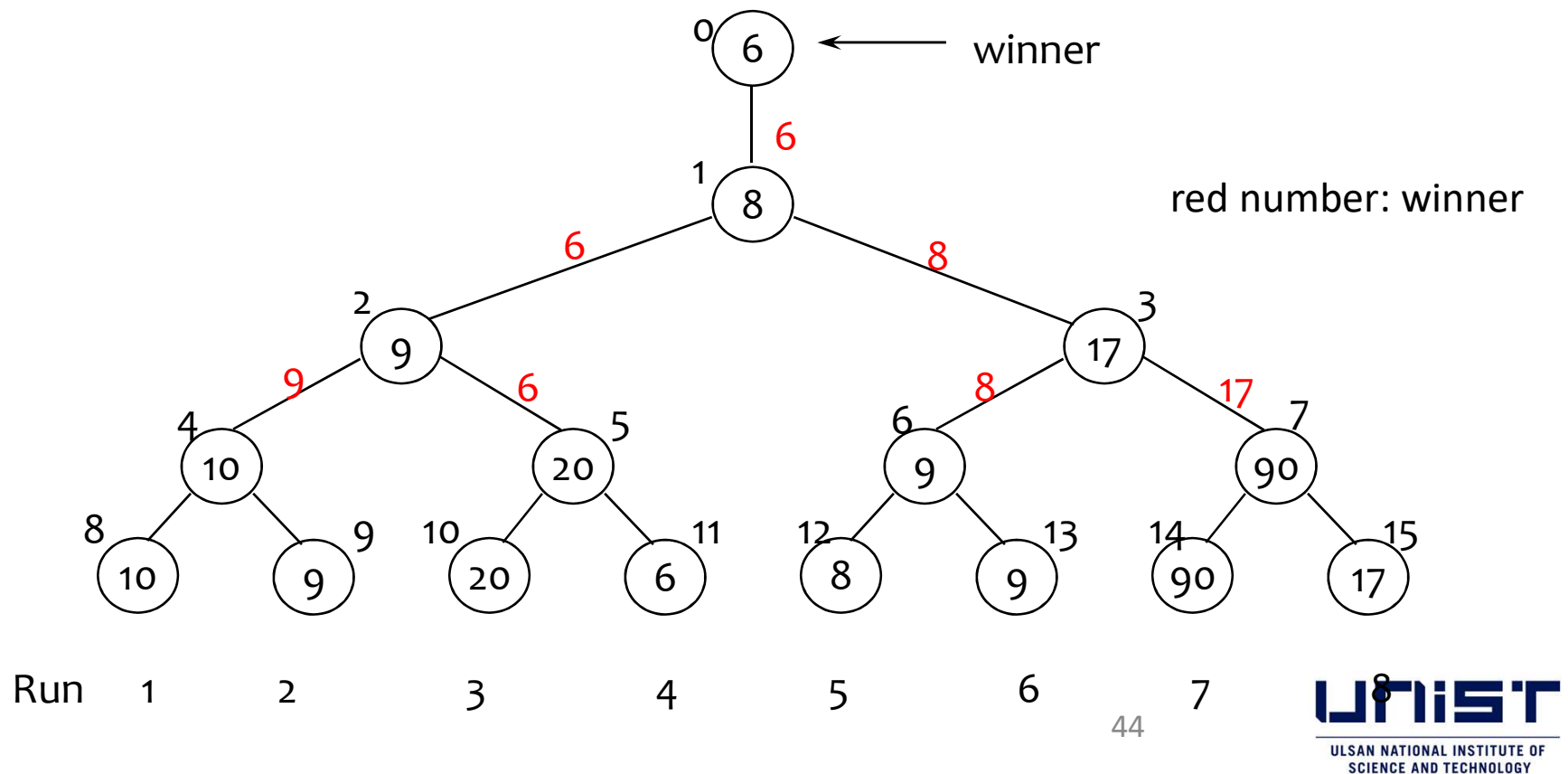
Example of Winner Tree

- Add next element from run 4 to the tree
 - Need comparisons between siblings



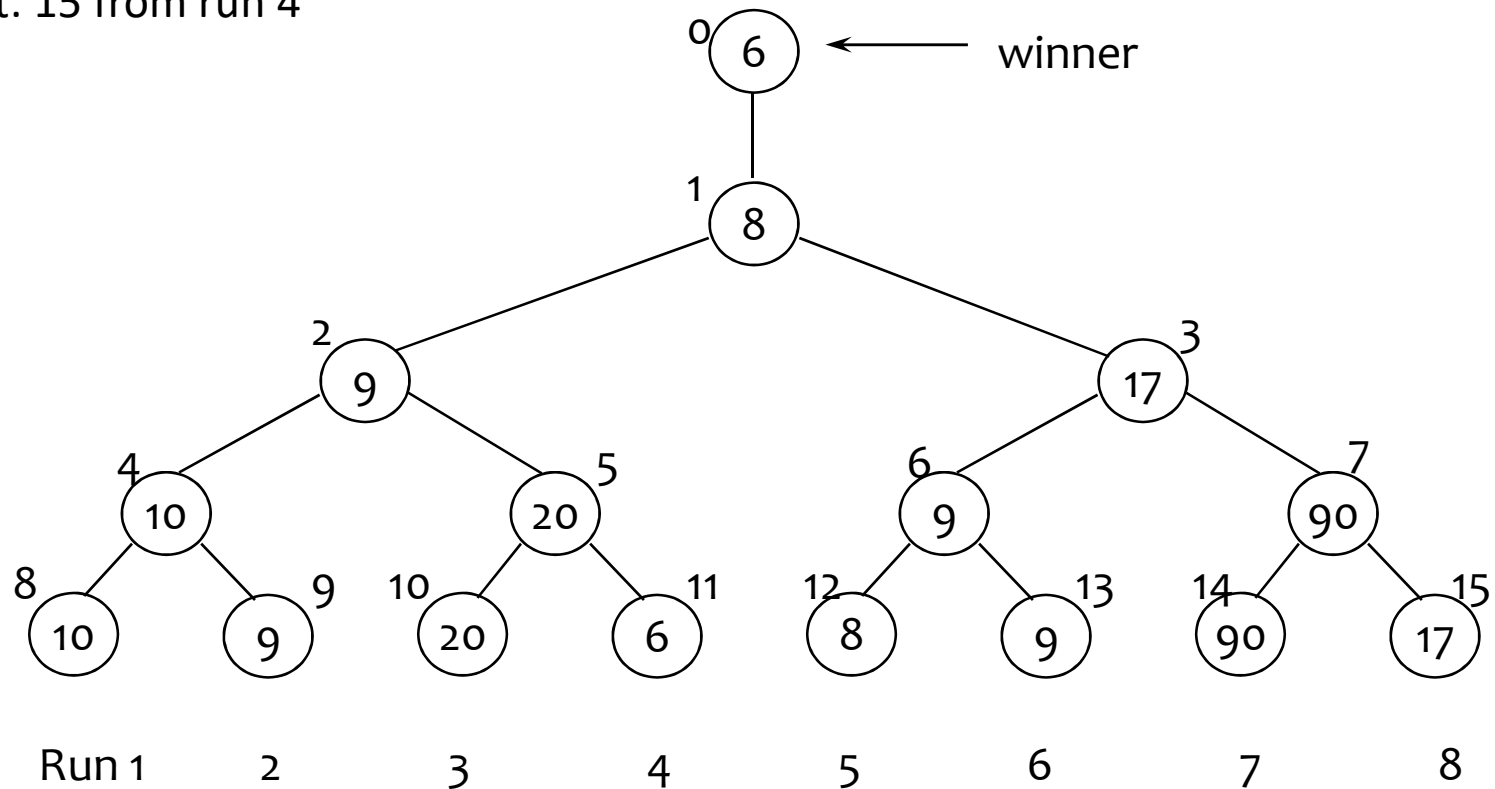
Loser Tree

- Non-leaf is loser, push winner to the top
- No need to compare with siblings



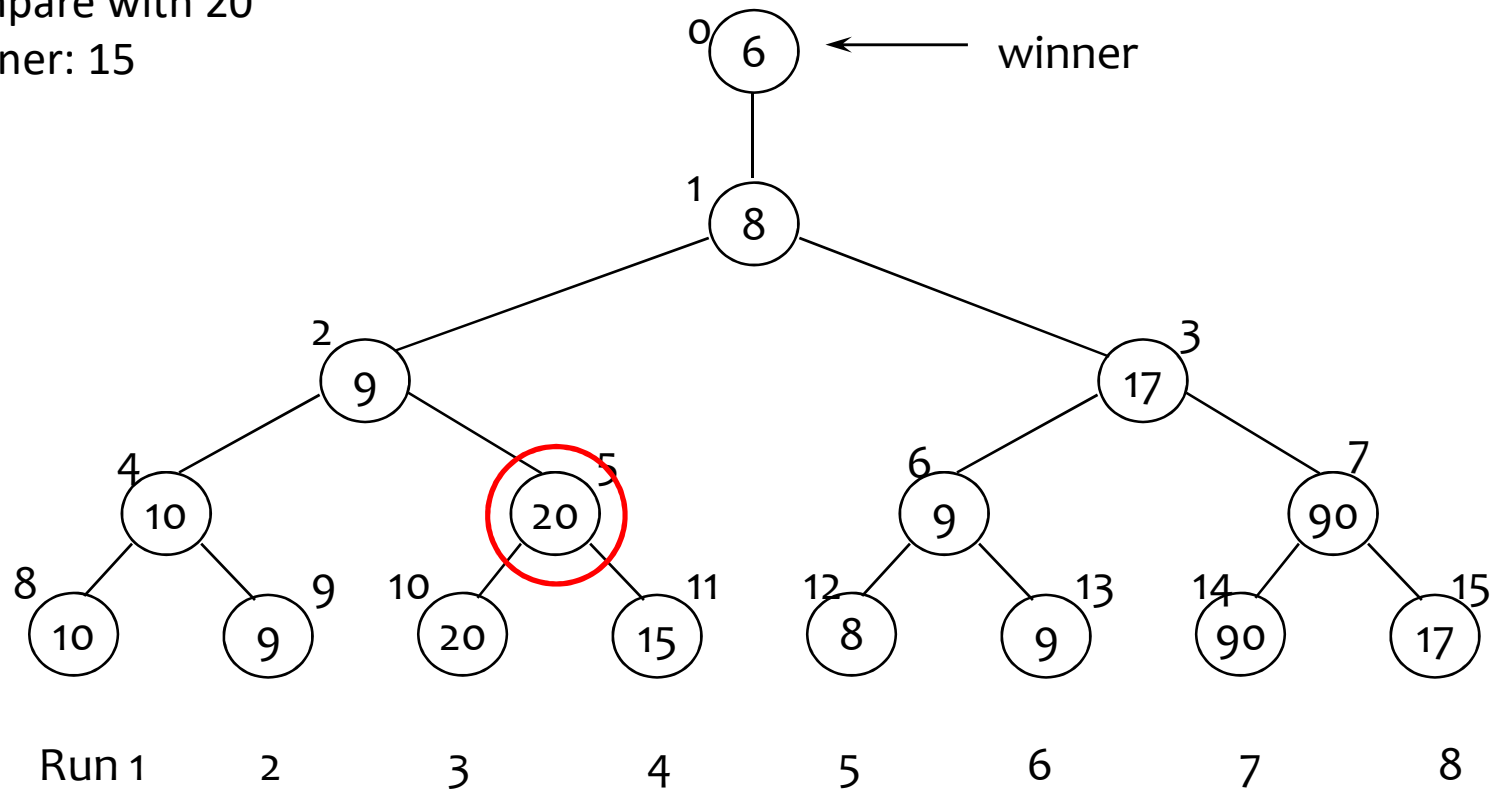
Loser Tree

Next: 15 from run 4



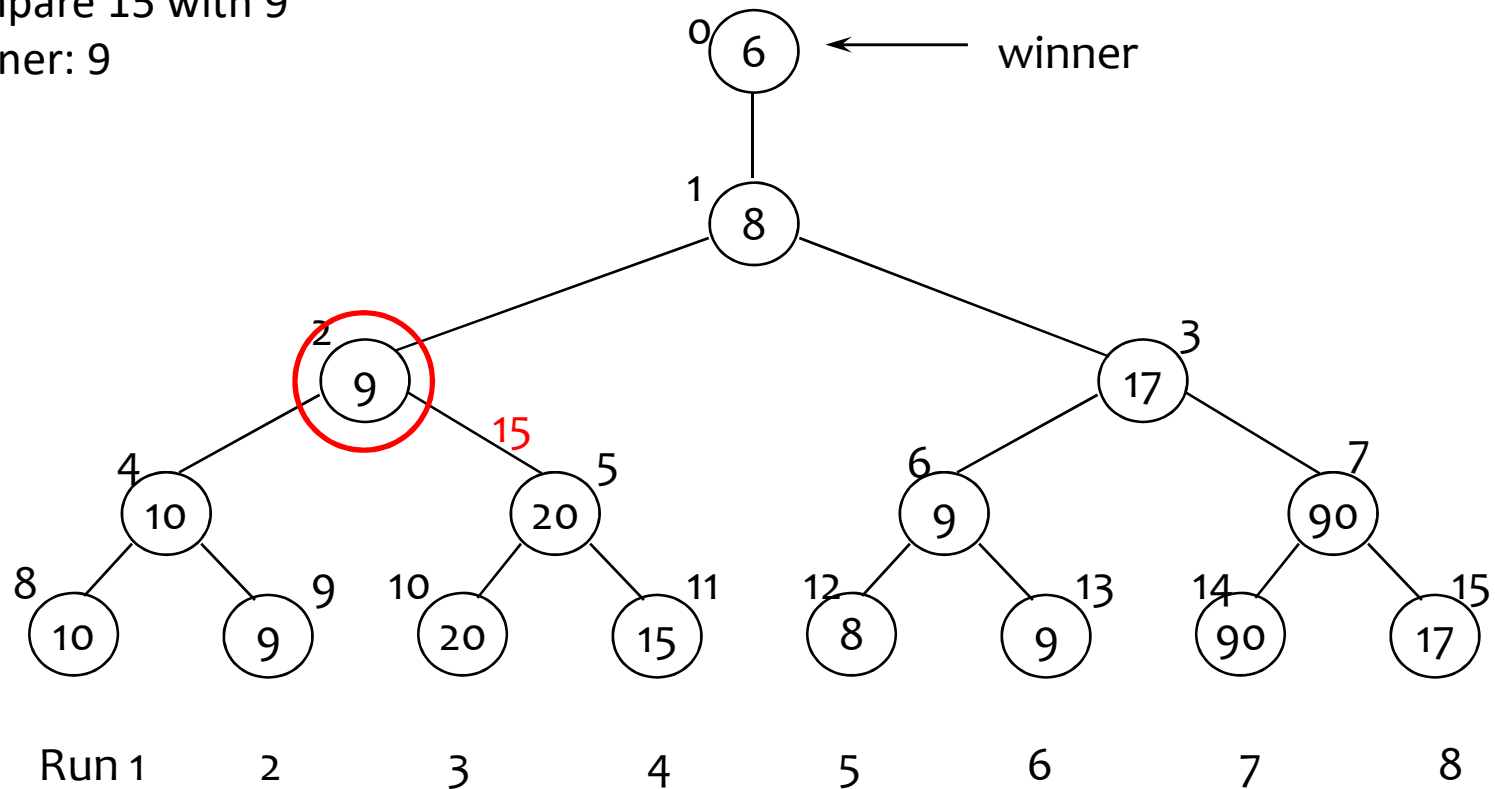
Loser Tree

Compare with 20
Winner: 15



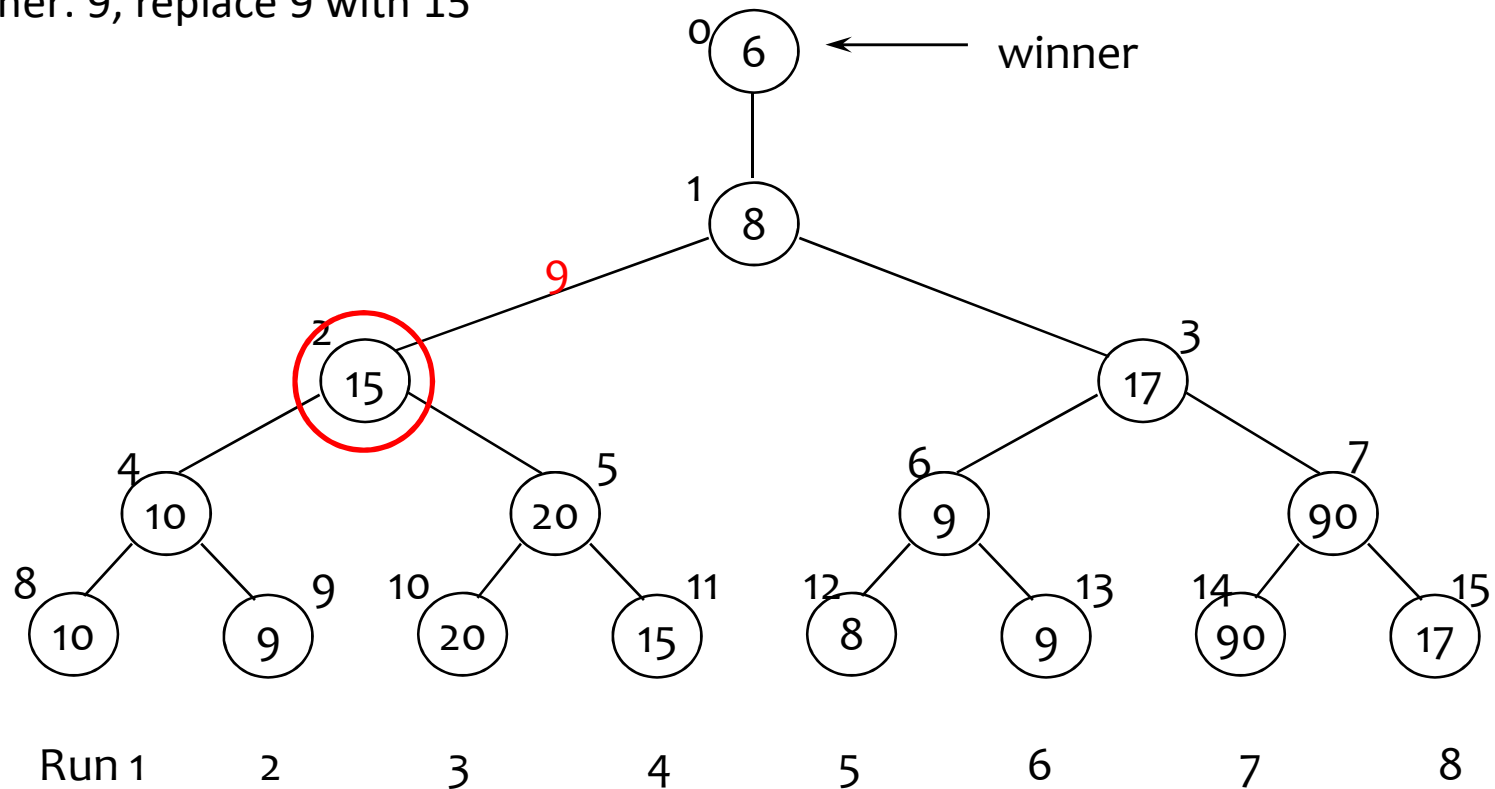
Loser Tree

Compare 15 with 9
Winner: 9



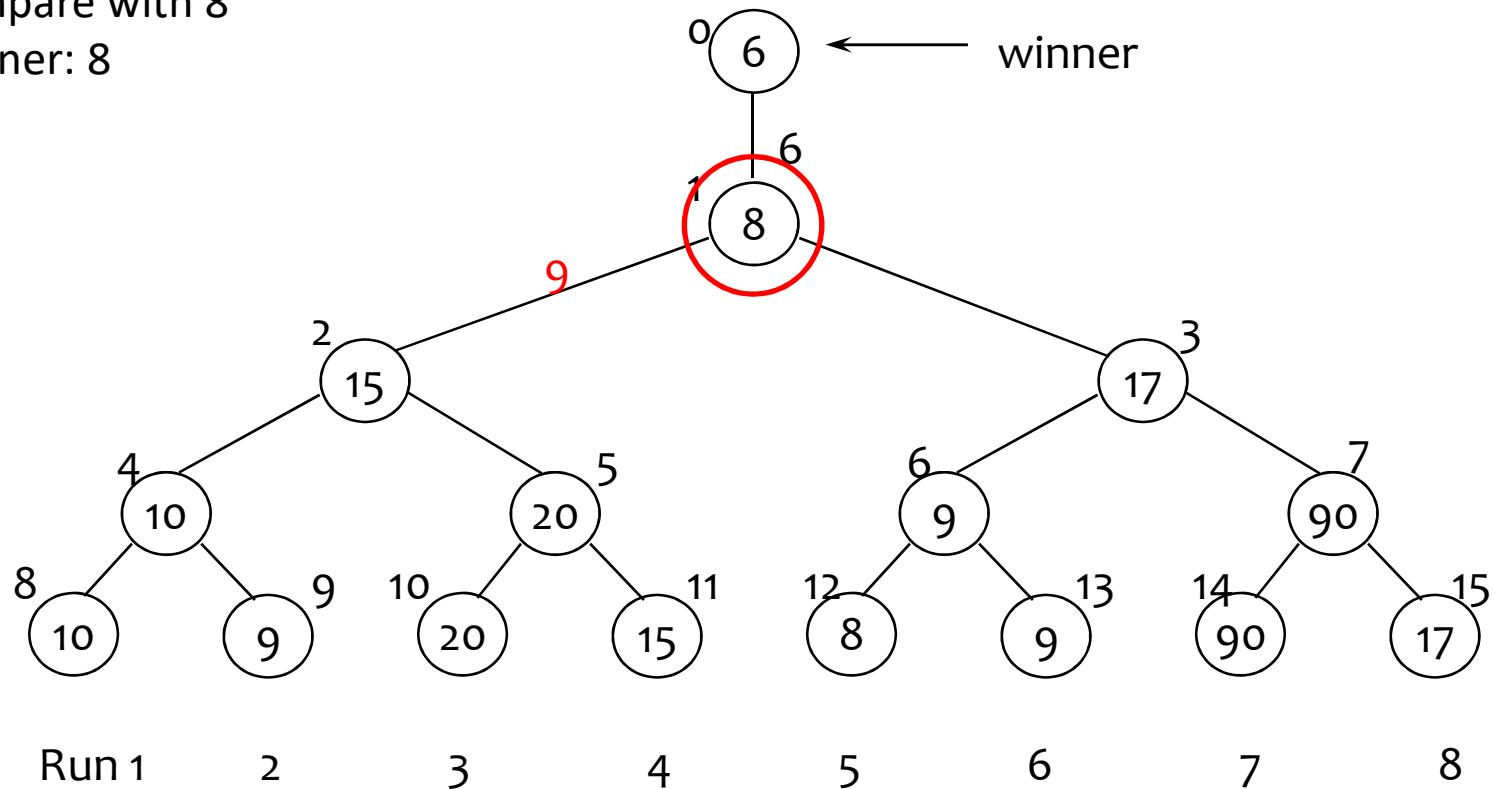
Loser Tree

Winner: 9, replace 9 with 15



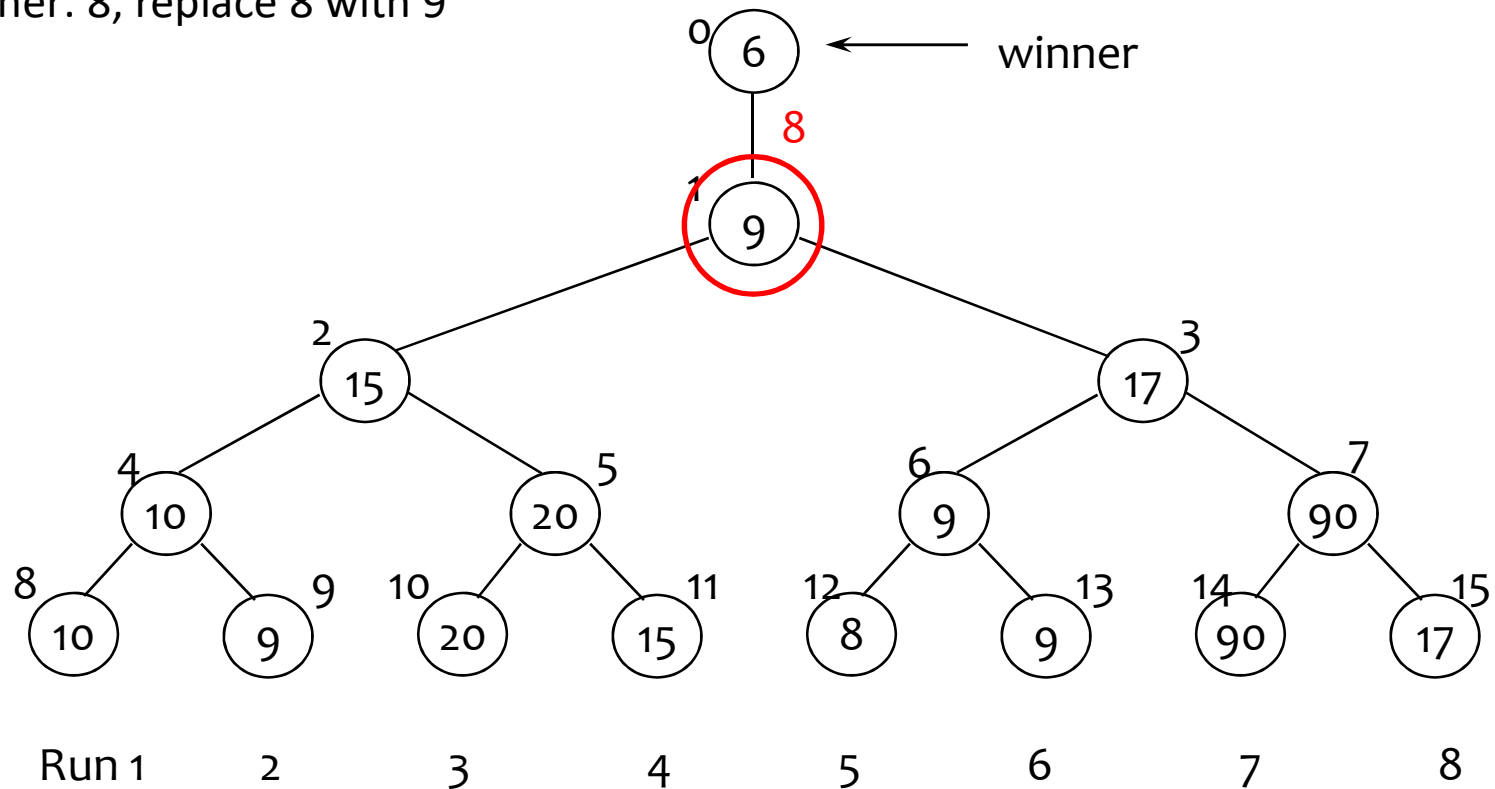
Loser Tree

Compare with 8
Winner: 8



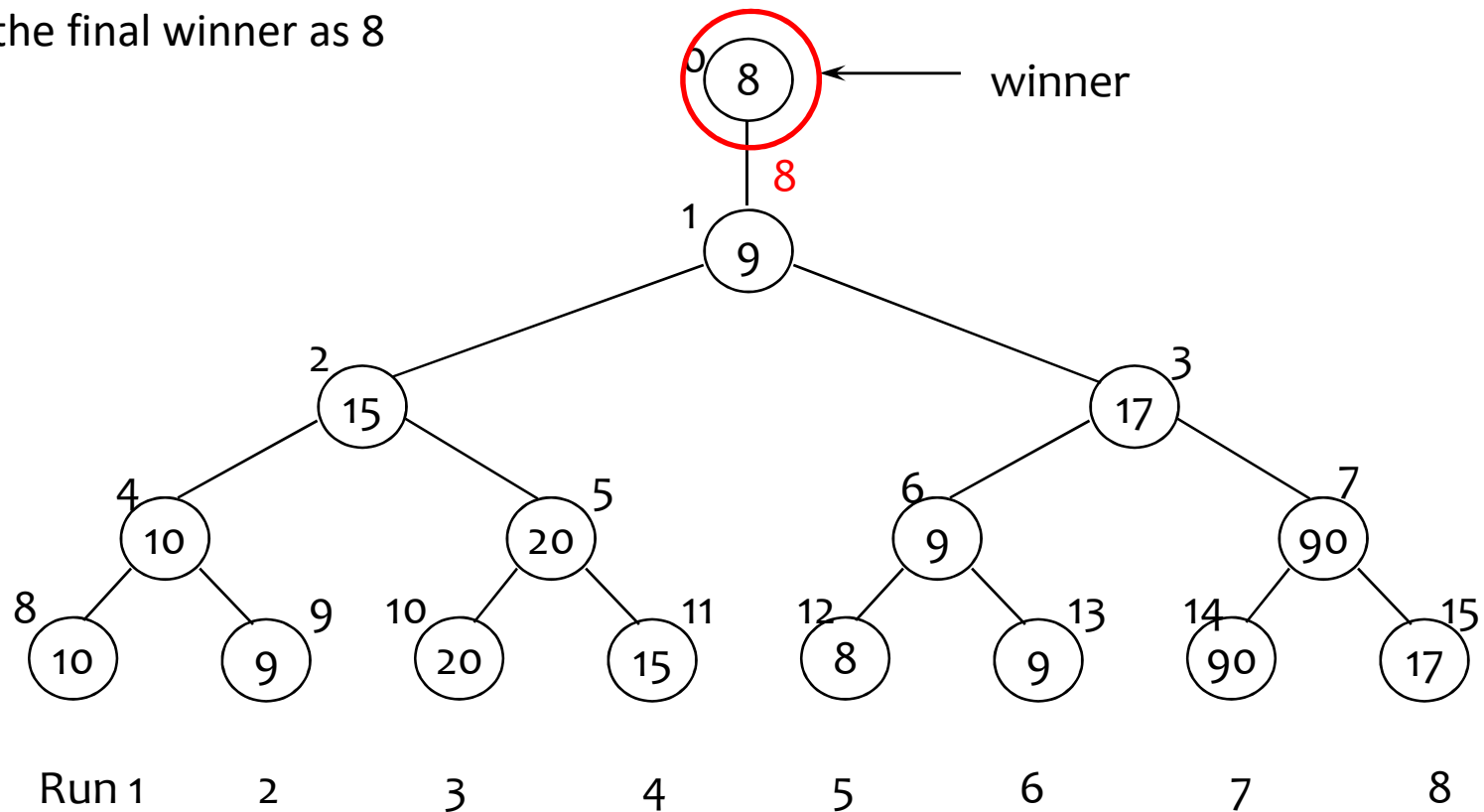
Loser Tree

Winner: 8, replace 8 with 9



Loser Tree

Set the final winner as 8



Questions?