CSE221

Performance analysis

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Acknowledgment: The content of this file is based on the slides of the textbook as well as the slides provided in former lectures at UNIST.

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Outline

- Performance analysis
- Performance measurement

Performance Evaluation

- Judging programs
- How?
  - Prior estimate
    - Performance analysis
  - Posteriori testing
    - Performance measurement

Performance Analysis

- Theoretical analysis
- Criteria: Complexity
  – Space complexity
  – Time complexity
- Complexity is affected by instance characteristics (size of inputs and outputs)
  – E.g., function f(n) where n is input size

Theoretical Analysis

- Uses a high-level description of the algorithm instead of an implementation
- Characterizes running time as a function of the input size, *n*.
- Takes into account all possible inputs
- Allows us to evaluate the speed of an algorithm independent of the hardware/software environment

Space Complexity

- $S(P) = c + S_p$(instance characteristics)
- Fixed part : c
  - Independent of input/output characteristics
  - Space for instruction, constants, etc.
- Variable part : S_p(instance characteristics)
  - Space for variables whose size is dependent of input/output, recursion stack space, etc.

Since c is constant, we only focus on S_p for space complexity!

Example

```
float Abc(float a, float b, float c)
{
  return a+b+b*c+(a+b-c)/(a+b)+4.0;
}
```

1. What are the instance characteristics?
2. S_p?

Example

```
float Abc(float a, float b, float c)
{
  return a+b+b*c+(a+b-c)/(a+b)+4.0;
}
```

1. What are the instance characteristics?
   : a, b, c
2. $S_p = 0$

Example

```
float Sum(float *a, const int n)
{
   float s = 0;
   for(int i=0; i<n; i++)
     s += a[i];
   return s;
}
```

1. What are the instance characteristics?
2. S_p?

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Example

```
float Sum(float *a, const int n)
{
  float s = 0;
  for(int i=0; i<n; i++)
    s += a[i];
  return s;
}
```

1. What are the instance characteristics?
   : a, n
2. $S_p = 0$

Example

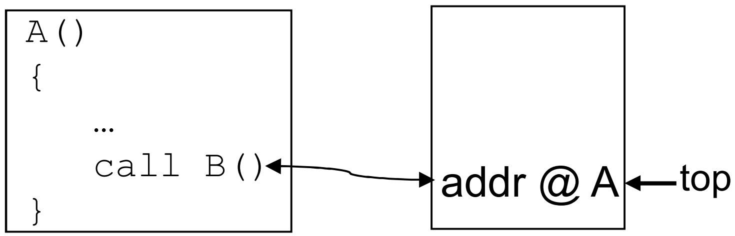
```
float RSum(float *a, const int n)
{
  if(n<=0) return 0;
  else return (Rsum(a, n-1)+a[n-1]);
}
```

1. What are the instance characteristics?
2. S_p ?

Example

```
float RSum(float *a, const int n)
{
  if(n<=0) return 0;
  else return (Rsum(a, n-1)+a[n-1]);
}
```

1. What are the instance characteristics?
   : a, n
2. S_p = A(n+1)
   : A = stack frame size



Time Complexity

- T(P) = compile time + run time
- Compile time: one-time cost (ignore)
- Actual run time: can only be measured
  - Compiler / machine / runtime dependent
  - Difficult to derive exact time for operators
- Use a <u>program step</u> instead

Program Steps (Primitive Operations)

- A segment of program with constant execution time (independent of instance characteristics)

  `e.g., a+b+b*c+(a+b-c)/(a+b)+4.0`

- Set of primitive operations
  - Assigning a value to a variable
  - Calling a function
  - Arithmetic operations (+,-,*,/, ...)
  - Comparing two numbers
  - Indexing into an array
  - Following an object reference
  - Returning from a function...

```
float Sum(float *a, const int n)
{
   float s = 0;
   for (int i = 0; i < n; i++)
   {
       s += a[i];
   }
   return s;
}
```

```
          float Sum(float *a, const int n)
          {
1           float s = 0;
1 * (n + 1) for (int i = 0; i < n; i++)
            {
1 * n           s += a[i];
            }
1           return s;
          }
```

Step count: 2n + 3

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```
float Rsum(float *a, const int n)
{
  if(n <= 0)
  {
      return 0;
  }
  else
  {
      return (Rsum(a, n-1)+a[n-1]);
  }
}
```

```
                 float Rsum(float *a, const int n)
                 {
1                    if(n <= 0)
                     {
1                        return 0;
                     }
                     else
                     {
Rsum + 1                 return (Rsum(a, n-1)+a[n-1]);
                     }
                 }
```

Step count: 2n + 2

T(P) = 2 + Rsum(n-1) = 2 + 2 + Rsum(n-2) = ... = 2n + Rsum(0) = 2n + 2

```
void Add(int **a,int **b,int n,int m)
{
  for(int i=0; i<m; i++)
  {
    for(int j=0; j<n; j++)
    {
      c[i][j] = a[i][j]+b[i][j];
    }
  }
}
```

```
          void Add(int **a,int **b,int n,int m)
          {
1 *(m + 1)  for(int i=0; i<m; i++)
            {
1 *(n + 1) *m for(int j=0; j<n; j++)
              {
1 *n *m         c[i][j] = a[i][j]+b[i][j];
              }
            }
          }
```

Step count: $2nm + 2m + 1$

Best, Worst, Average Step Count

- What if expressions depend on complex characteristics?
- Ex) linear search

```
        int search(float *a, float k, int n)
        {
1*?       for(int i=0; i<n; i++)
          {
1*?           if(a[i] == k)
              {
1                 return i;
              }
          }
        }
```

? = n(worst case), 1(best case), n/2(average)

average = ∑(all cases) / # cases = (1+2+3+4+…+n) / n

Comparing Step Count

- What step count means?
  - How the run time changes when the instance characteristics changes
  - e.g., how much the program becomes slower if the input size grows

- P_A : $2n + 1$
- P_B : $3n^2$
  - When n is doubled, P_A becomes 2x slower but P_B becomes 4x slower

Comparing Step Count

- P_A : $n^2 + 200$
- P_B : $n^2 + 100$
  - When n = 1, running time of P_A and P_B become 201 and 101, respectively (1.99x difference)
  - When n = 100, running time of P_A and P_B become 10200 and 10100, respectively (1.009x difference)
  - If n becomes very large, P_A and P_B converges (n^2 term dominates)
- P_A runs same as P_B *asymptotically!*

Asymptotic Analysis

- Asymptotic behaviour
  - behaviour as input n approaches infinity
    - input: instance characteristics
  - dominant factor prevails
    - constants become "insignificant"

Asymptotic Analysis

- We need relatively easy ways to compare functions as problem size n vary:
  - A is 'at most' as fast/big as B
  - A is 'at least' as fast/big as B
  - A is 'equal' in performance/size to B
- Asymptotic behaviour
  - The behaviour as some key parameter n increases towards infinity

Big-O

- Formally, given functions *f(x), g(x),*

$$f(x) = O(g(x))$$

  if there exist positive constants c and x_0 such that $f(x) \leq cg(x)$ for all $x \geq x_0$

- Meaning of Big-O
  - For sufficiently large x, *f* is bounded by *g* with a scalar factor
  - *f* is "at most" *g* beyond some value x_0
    - *g* is an upper bound of *f*

Examples

- $3n+2 = O(n)$?
  - *Yes.* $3n + 2 \le 4n$ for all $n \ge 2$
- $1000n^2+100n-6 = O(n^2)$?
  - *Yes.* $1000n^2 + 100n - 6 \le 1001n^2$ for all $n \ge 100$
- $6*2^n+n^2 = O(2^n)$?
  - *Yes.* $6*2^n + n^2 \le 7*2^n$ for all $n \ge 4$

Asymptotic Complexity

- Order of magnitude – higher one dominates
  - O(1) : constant
  - O(log n) : logarithmic
  - O(n) : linear
  - O(nlog n) : log linear
  - O(n^2) : quadratic
  - O(n^3) : cubic
  - O(2^n) : exponential
  - O(n!) : factorial

Higher

Informative Big-O

- How good the bound is?
  - $3n+2 = O(n) = O(n^2)$?
  - Both are correct statement
  - $O(n)$ is tighter bound and more informative
  - $O(n^2)$ is less informative and not used

Big-Oh Rules

- If is $f(n)$ a polynomial of degree d, then $f(n)$ is $O(n^d)$, i.e.,
  1. Drop lower-order terms
  2. Drop constant factors
- Use the smallest possible class of functions
  - Say "$2n$ is $O(n)$" instead of "$2n$ is $O(n^2)$"
- Use the simplest expression of the class
  - Say "$3n + 5$ is $O(n)$" instead of "$3n + 5$ is $O(3n)$"

Theta and Omega

- Omega (Ω) meaning "at least" (lower bound):

$$f(x) = \Omega(g(x))$$

  - If there exist positive constants c and x_0 such that $cg(x) \leq f(x)$ for all $x \geq x_0$

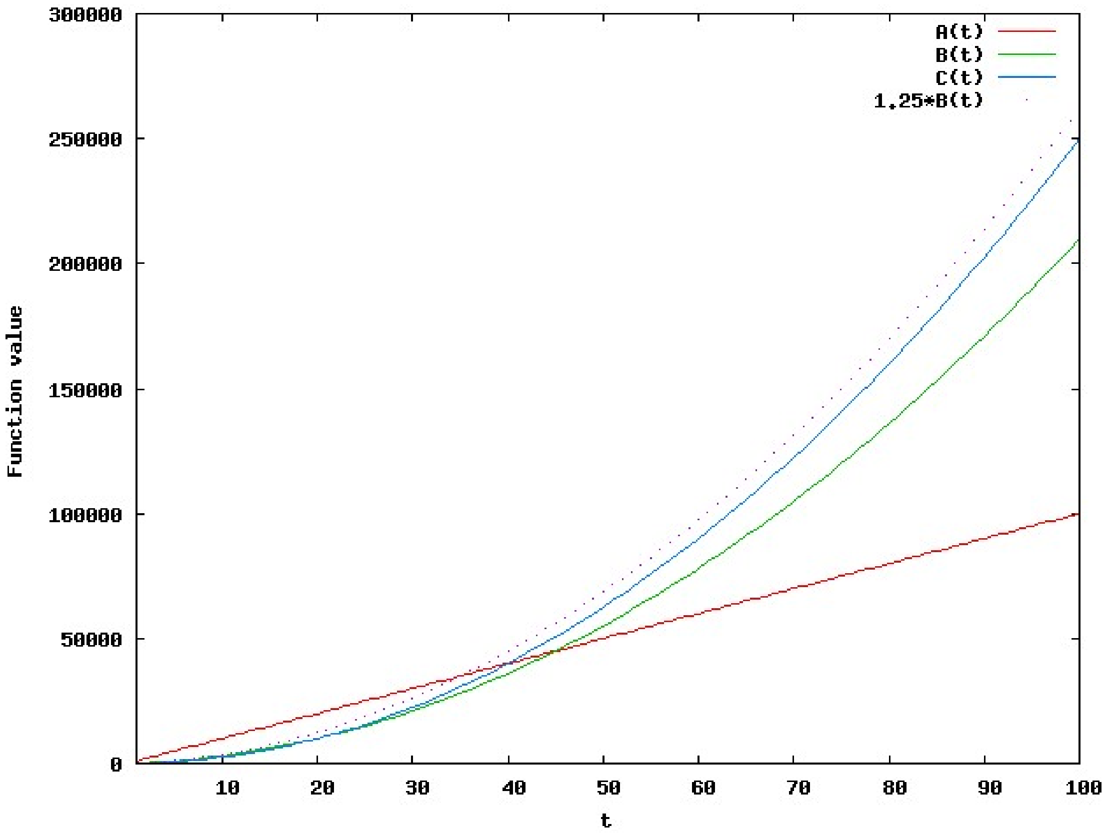
- Theta (Θ) "equals" or "goes as":

$$f(x) = \Theta(g(x))$$

  - If there exist positive constants c_1, c_2, and x_0 such th at $c_1 g(x) \leq f(x) \leq c_2 g(x)$ for all $x \geq x_0$

Graph

- For *t > 45*, *B(t)* is always greater than *A(t) : A=O(B)*



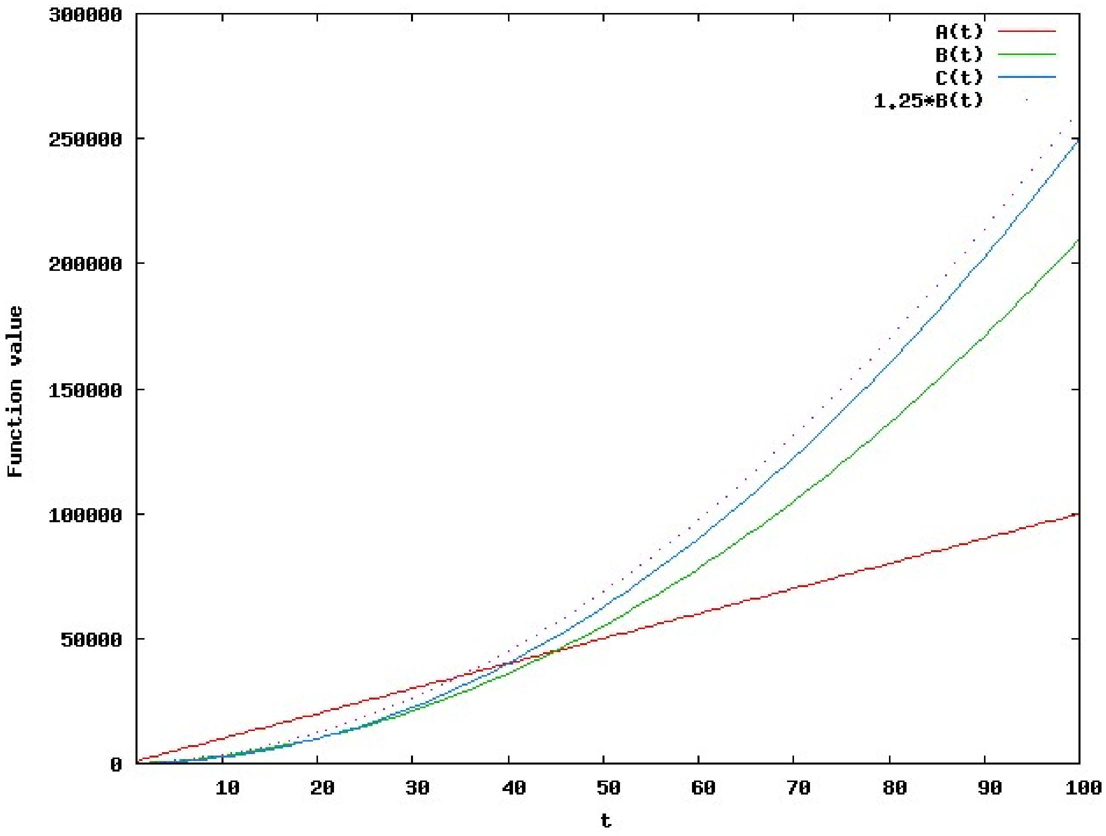
$$A(t) = 1000t$$

$$B(t) = 100t + 20t^2$$

$$C(t) = 25t^2$$

Graph

- Can we say B=*O(A)*?



$$A(t) = 1000t$$

$$B(t) = 100t + 20t^2$$

$$C(t) = 25t^2$$

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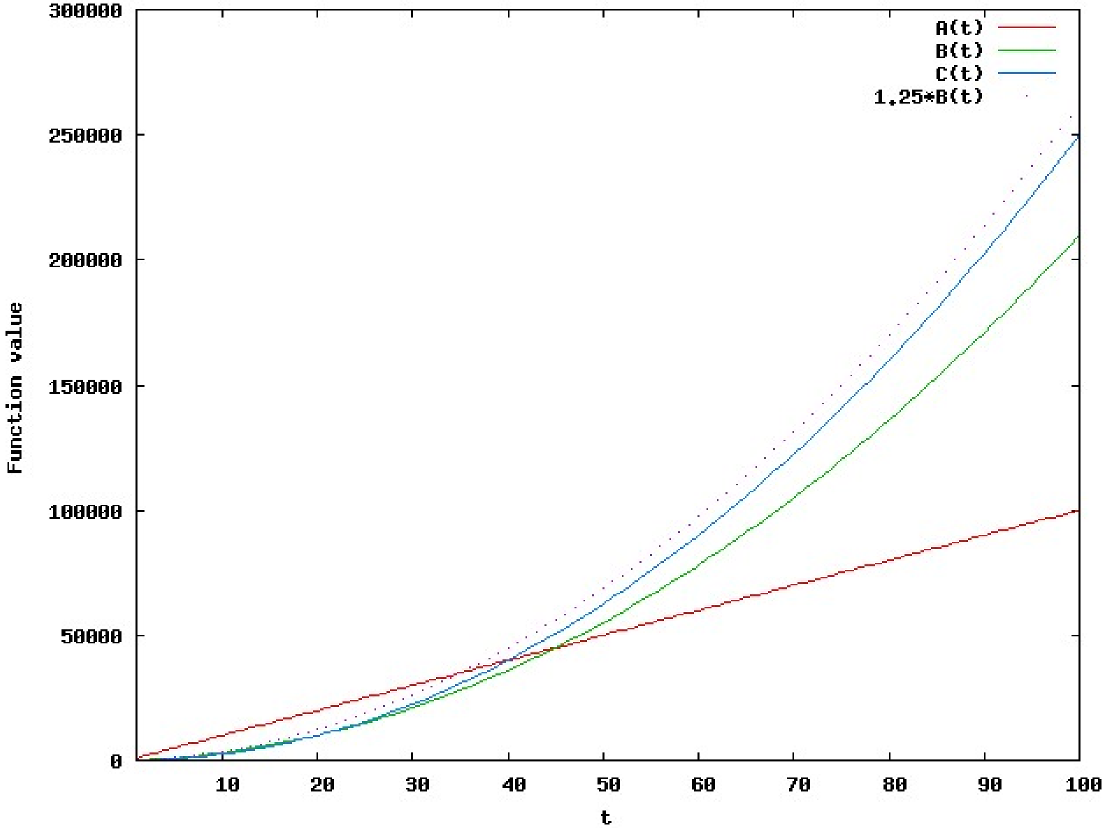
Graph

- For *t > 20, B(t)* is always less than C*(t) : ?*
- For *t > 0, 1.25*B(t)* is always greater than *C(t) : ?*



$$A(t) = 1000t$$

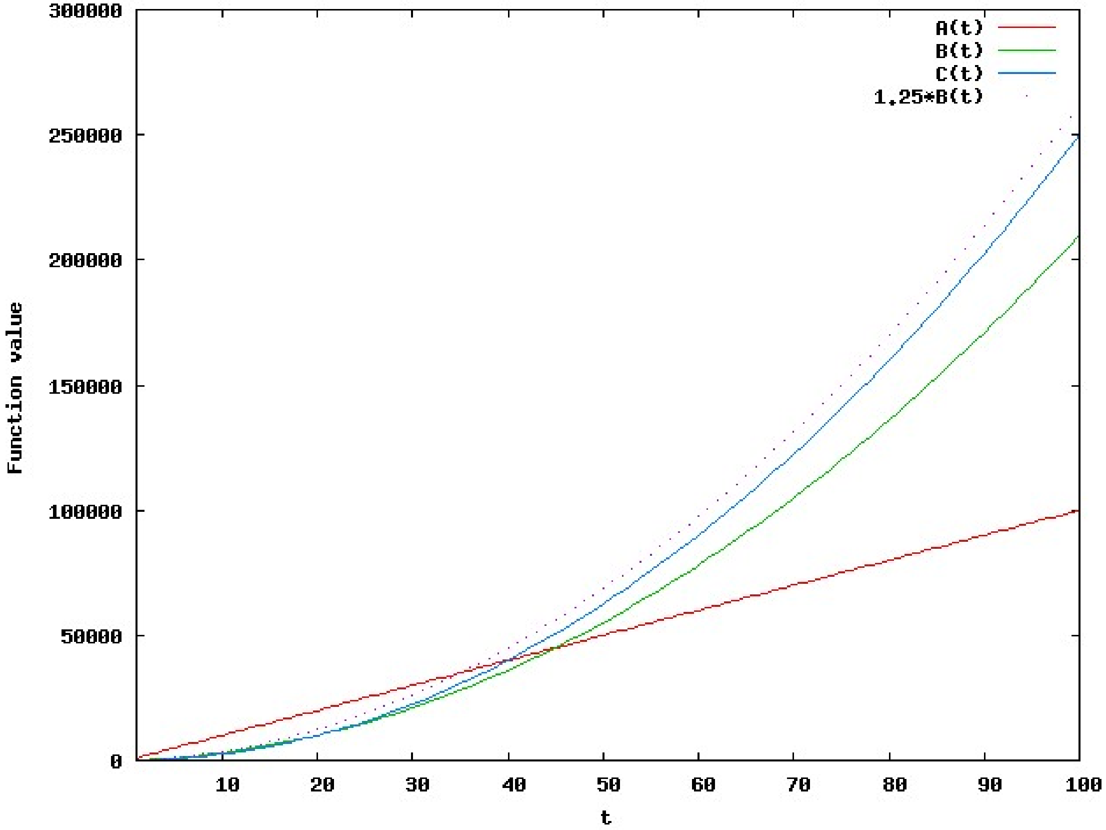
$$B(t) = 100t + 20t^2$$

$$C(t) = 25t^2$$

Graph

C=Θ(B)!

- For $t > 20$, $B(t)$ is always less than $C(t)$: $C=\Omega(B)$
- For $t > 0$, $1.25*B(t)$ is always greater than $C(t)$: $C=O(B)$

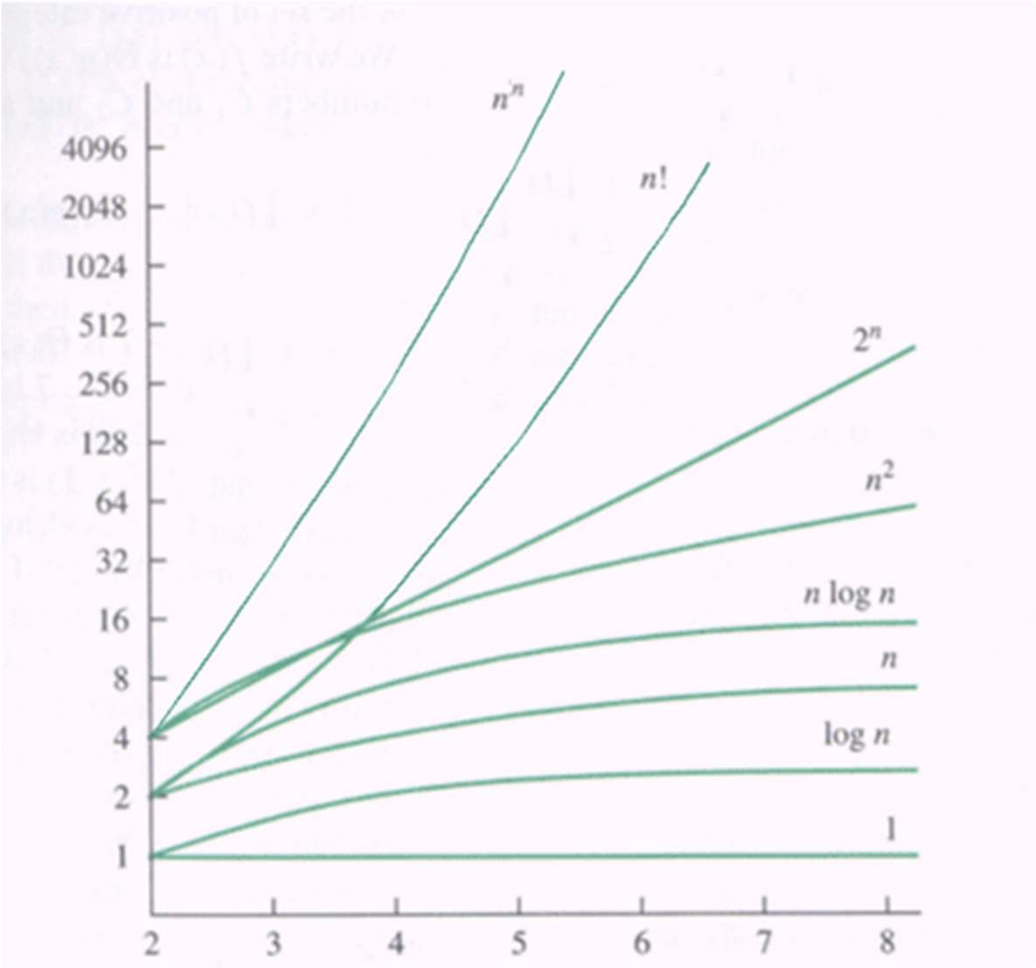


$$A(t) = 1000t$$

$$B(t) = 100t + 20t^2$$

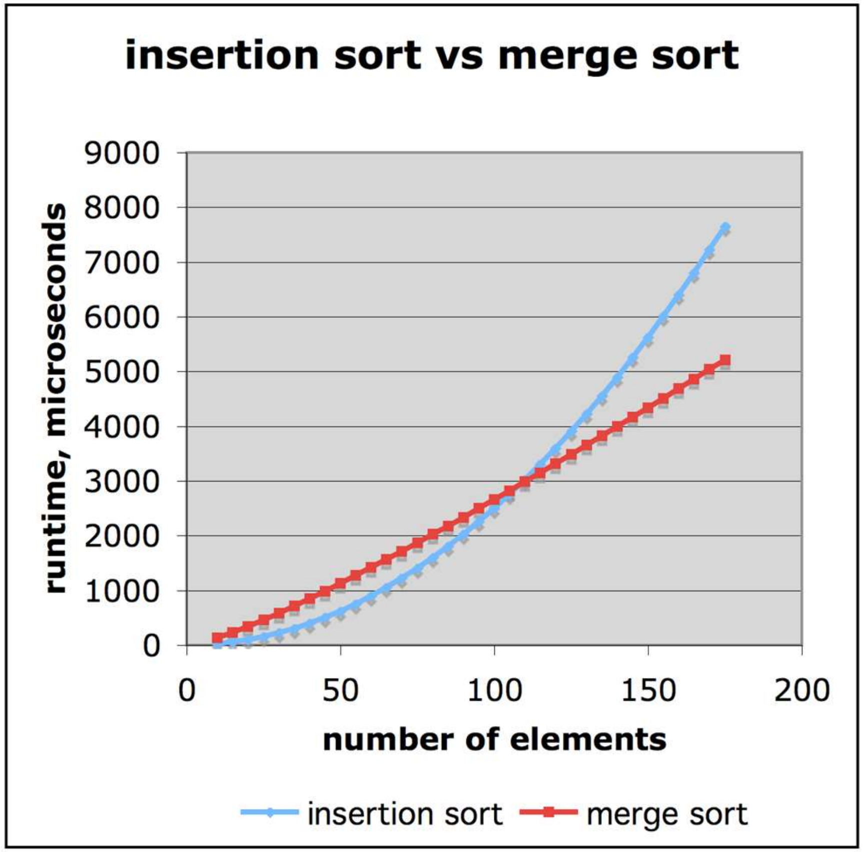
$$C(t) = 25t^2$$

Plot of Functions



Picture credit: https://www.cs.odu.edu/~cs381/cs381content/function/growth.html

Comparison of Two Algorithms



insertion sort is

$n^2 / 4$

merge sort is

$2 \ n \lg n$

sort a million items?

insertion sort takes roughly 70 hours

while

merge sort takes roughly 40 seconds

This is a slow machine, but if 100x as fast then it's 40 minutes versus less than 0.5 seconds

Stallman

Run time for 1billion steps / sec computer

n	n	$n\log_2 n$	n^2	n^3	n^4	n^{10}	2^n
10	.01 μs	.03 μs	.1 μs	1 μs	10 μs	10 s	1 μs
20	.02 μs	.09 μs	.4 μs	8 μs	160 μs	2.84 h	1 ms
30	.03 μ	.15 μ	.9 μ	27 μ	810 μ	6.83 d	1 s
40	.04 μs	.21 μs	1.6 μs	64 μs	2.56 ms	121 d	18 m
50	.05 μs	.28 μs	2.5 μs	125 μs	6.25 ms	3.1 y	13 d
100	.10 μs	.66 μs	10 μs	1 ms	100 ms	3171 y	$4*10^{13}$ y
10^3	1 μs	9.96 μs	1 ms	1 s	16.67 m	$3.17*10^{13}$ y	$32*10^{283}$ y
10^4	10 μs	130 μs	100 ms	16.67 m	115.7 d	$3.17*10^{23}$ y	
10^5	100 μs	1.66 ms	10 s	11.57 d	3171 y	$3.17*10^{33}$ y	
10^6	1 ms	19.92 ms	16.67 m	31.71 y	$3.17*10^7$ y	$3.17*10^{43}$ y	

μs = microsecond = 10^{-6} seconds; ms = milliseconds = 10^{-3} seconds
s = seconds; m = minutes; h = hours; d = days; y = years

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Performance Measurement

- Use timer function
  - CPU time only for serial program
  - Wall-clock time for parallel program
- total = start – stop
- Timer has accuracy limitation
  - 1/100 sec, etc
- Multiple runs and average gives accurate time

Timing in C

```
clock_t start, stop;

start = clock(); /* set start to current time in
                    hundredths of a second */

/* code to be timed comes here */

stop = clock(); /* set stop to current time */

runTime = (double) (stop – start)/
                        CLOCKS_PER_SEC;
```

Multiple Runs

```
do {
      counter++;
      start = clock();
      doSomething();
      stop = clock();
      elapsedTime += stop – start;
   } while (elapsedTime < 1000)
elapsedTime /= counter;
```

Problem?

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Questions?