CSE221

Lecture 17:
Graphs

Acknowledgment: The content of this file is based on the slides of the textbook as well as the slides provided in former lectures at UNIST.

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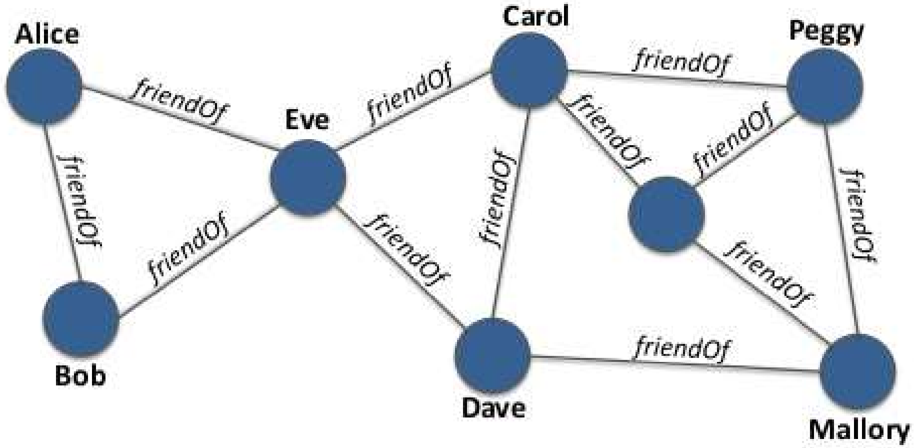
Powerful Representation of Social Network

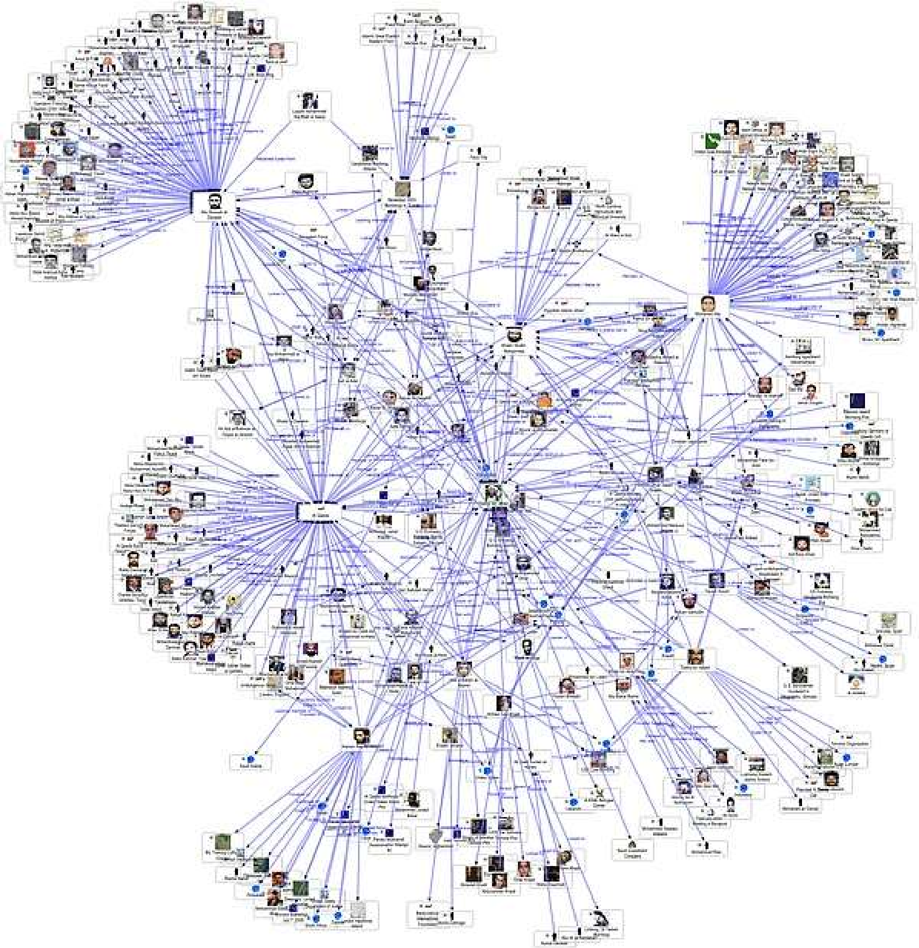
"Graphs are everywhere"



$$Graph = (\mathbf{Users}, Friendships)$$

Picture credit: https://www.slideshare.net/FlinkForward/martin-junghans-gradoop-scalable-graph-analytics-with-apache-flink

Can You Identify Key People?



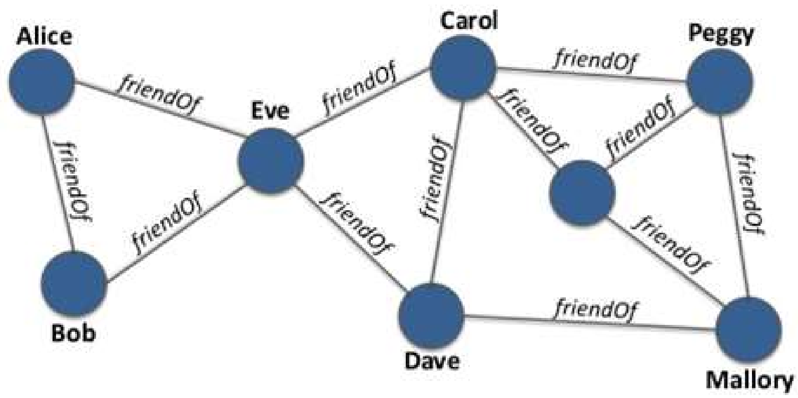
Picture credit: http://www.fmsasg.com/SocialNetworkAnalysis/

Outline

- Graph definitions
- Graph representations
  - Adjacency matrix
  - Adjacency list
  - Adjacency multilist

Definitions

- Graph G consists of two sets, V and E
  - G=(V,E)
  - V (vertices) : finite, nonempty set of nodes
  - E (edges) : set of pairs of (different) vertices



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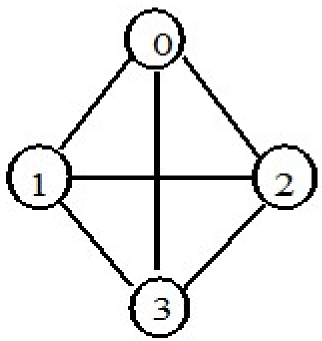
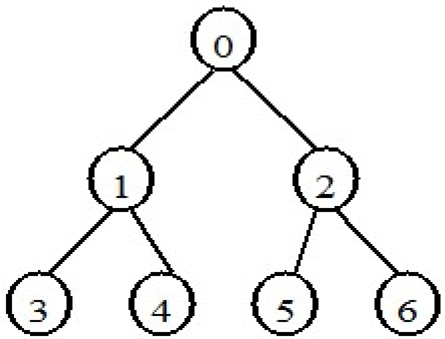
Definitions

- Undirected graph
  - All edges (i.e., all pairs of vertices) are **undirected**
  - (u, v) : u and v are *adjacent*
  - (u, v) = (v, u)

V: {0, 1, 2, 3}
E: {(0,1), (0,2), (0,3), (1,2), (1,3), (2,3)}





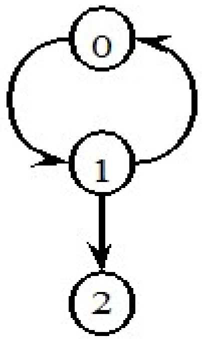
V: {0, 1, 2, 3, 4, 5, 6}
E: {(0,1), (0,2), (1,3), (1,4), (2,5), (2,6)}

Tree: undirected graph in which any two vertices are connected by exactly one path

Definitions

- Directed graph
  - Each edge represents **directed pair** <u, v>
  - <u, v> - u : tail, v : head
  - <u, v> != <v, u>

V: {0, 1, 2}
E: {<0,1>, <1,0>, <1,2>}



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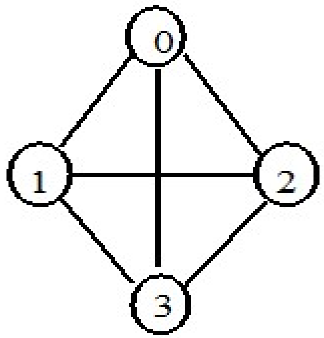
Definitions

- # unordered edges in a graph with n vertices
  - $(n-1) + (n-2) + .. + 2 + 1 = {}_{n}C_{2} = \mathbf{n(n-1)/2}$
- Complete graph
  - Graph that has maximum number of edges
  - For n vertices, **n(n-1)/2** edges



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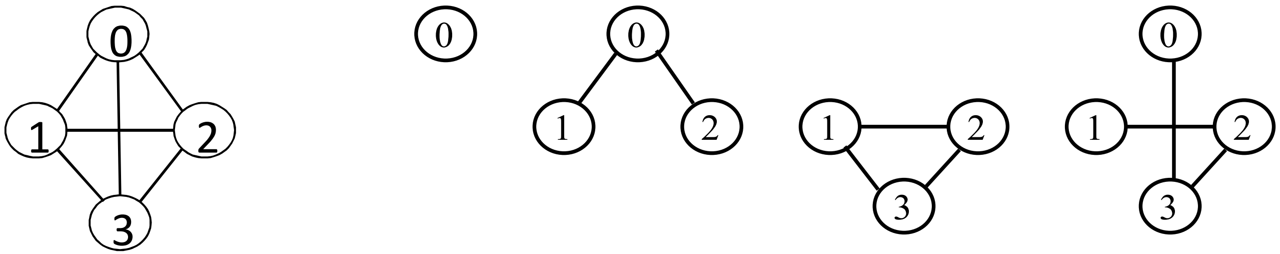
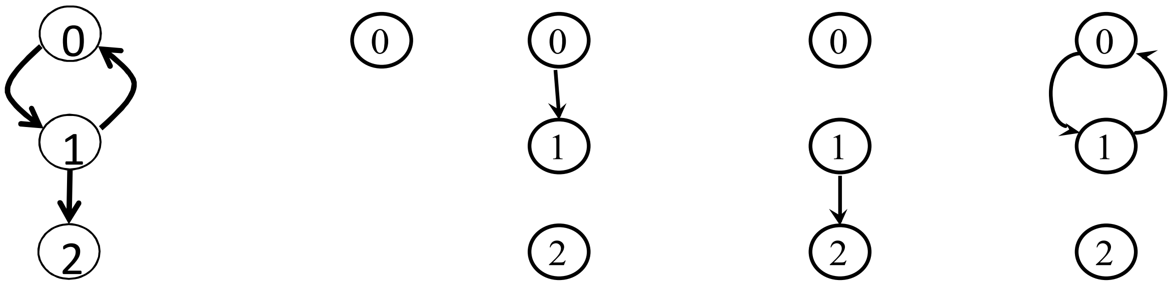
Definitions

- Subgraph of G=(V,E)
  - G'=(V',E'), V' ⊆ V and E' ⊆ E



(i) (ii) (iii) (iv)



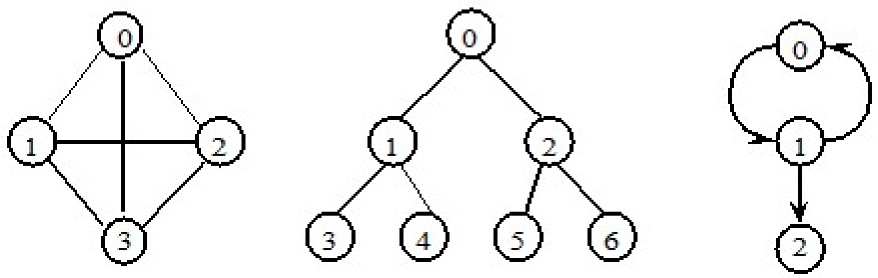
(i) (ii) (iii) (iv)

Definitions

- Path from u to v in G
  - Sequence of vertices, u, i_1, i_2, ..., i_k, v such that (u,i_1), (i_1,i_2), ..., (i_k,v) are edges in E of G
- Length
  - Number of <u>edges</u> in the path
- Simple path
  - Path that has **all distinct vertices** except first and last vertices (i.e., first and last vertices can be same)
    - Path 0,1,3,2,4 : length 4, simple
    - Path 0,1,3,1,4 : length 4, not simple

Definitions

- Cycle
  - –Simple path that has the same first and last vertices



any cycle?

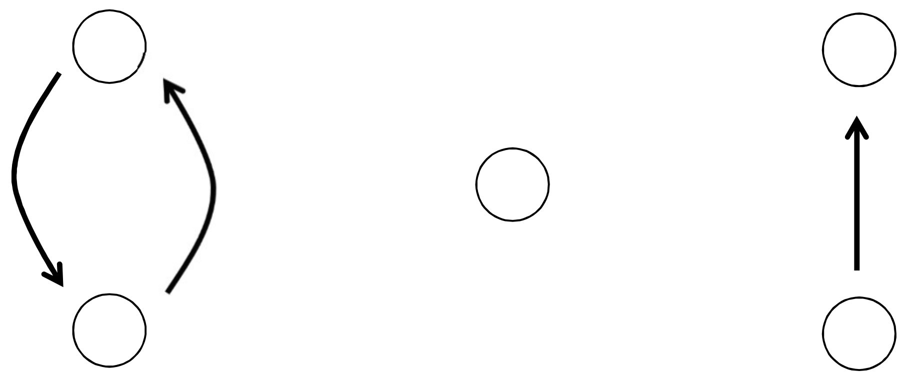
Definitions

- Vertices u and v are *connected* iff
  - There is a path from u to v

- Undirected graph G is *connected* iff
  - There is a path from u to v for every pair (u, v) in G

Definitions

- Directed graph is *strongly connected* iff
  - For every pair of (distinct) u and v, there is a directed path from u to v and v to u



Which one is strongly connected?

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Definitions

- Degree of a vertex
  - Number of edges directly connected to that vertex
- In-degree of a vertex in a directed graph
  - Number of incoming edges
- Out-degree of a vertex in a directed graph
  - Number of outgoing edges
- Digraph = Directed graph