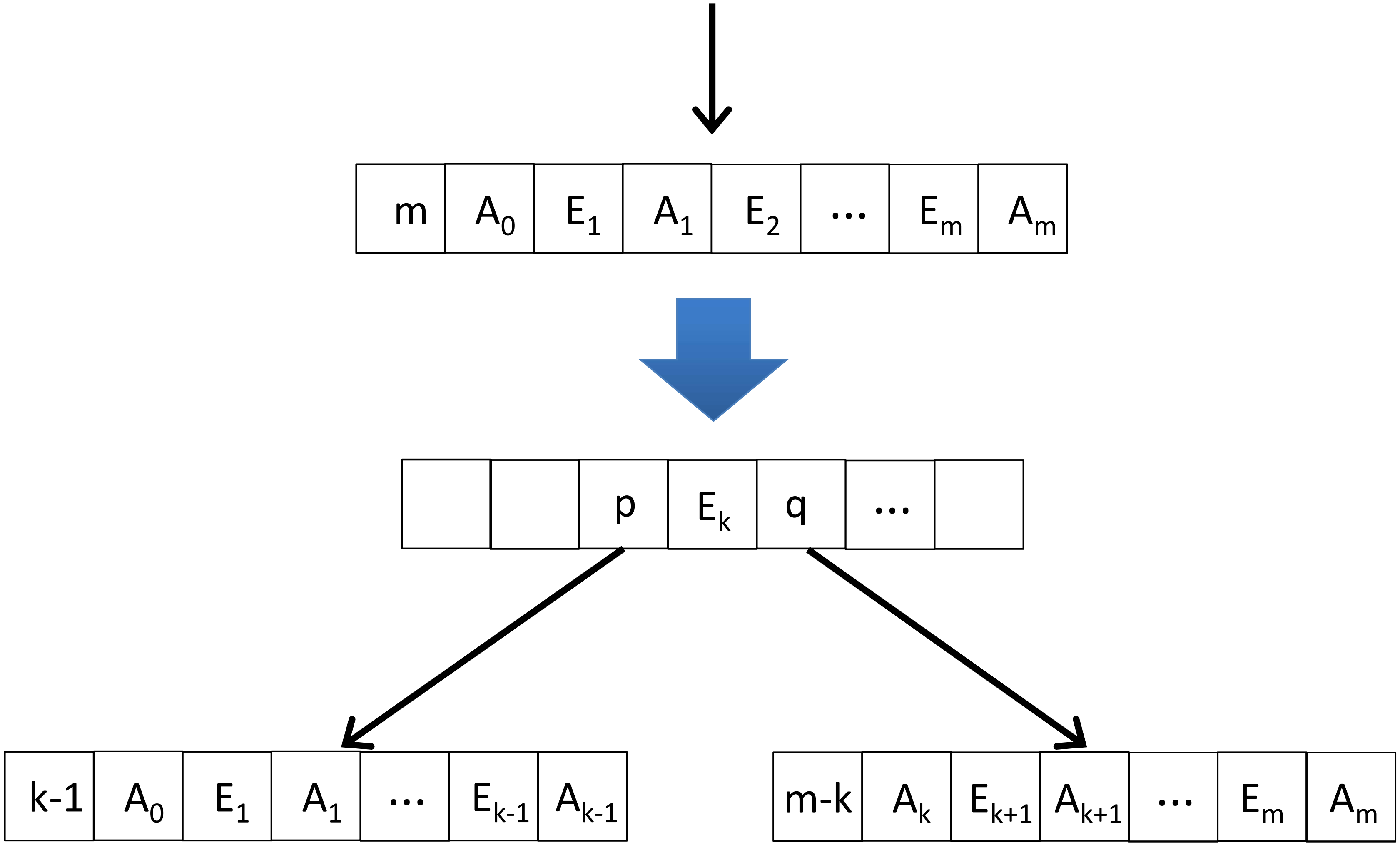
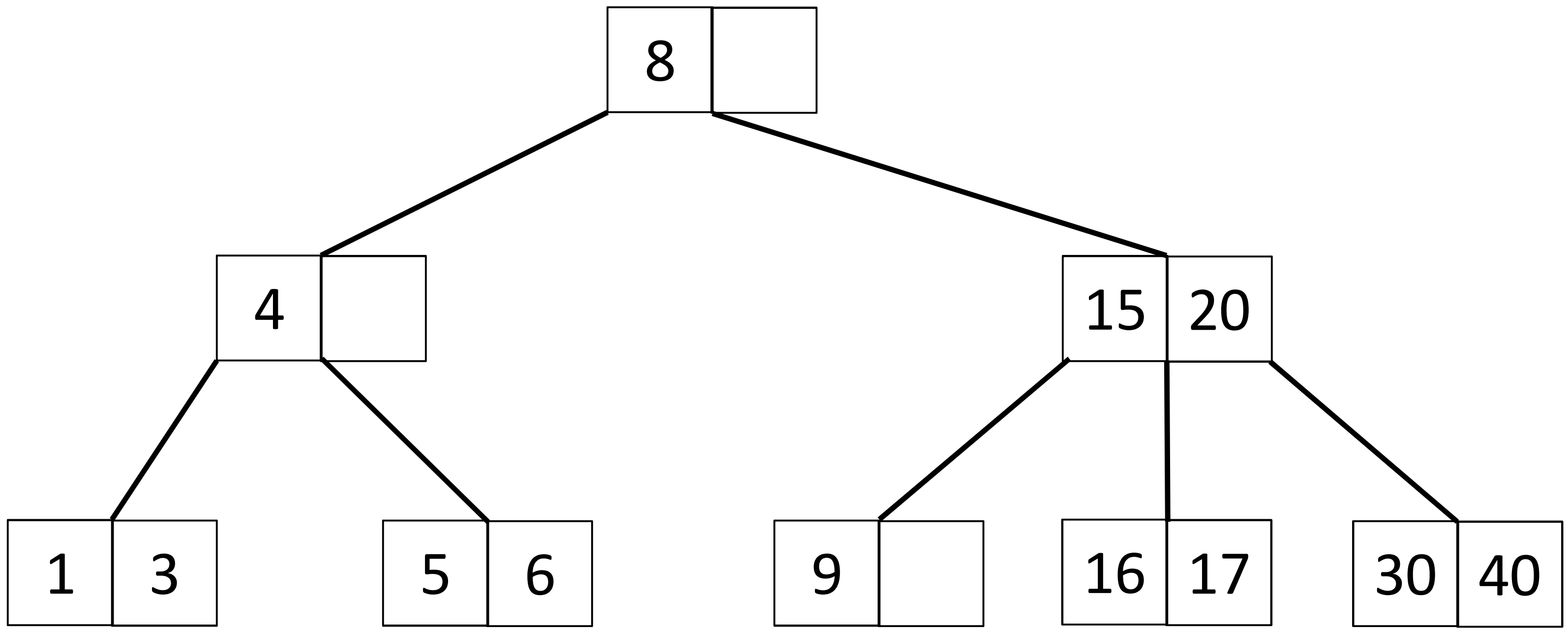
Insert

- If insertion results in overflow (already inserted with m-1 keys), split node
- Let node p have the format after insertion
  - m, A_0, (E_1, A_1), … , (E_m, A_m)
- p is split into two nodes p and q
  - Let k = ceil(m/2)
  - node p: k-1, A_0, (E_1, A_1), …, (E_{k-1}, A_{k-1})
  - node q: m-k, A_k, (E_{k+1},A_{k+1}), …,(E_m, A_m)
  - (E_k,q) is inserted into the parent of p
- Splitting can propagate up to the root

Split Node



Insert (3-way B-tree)



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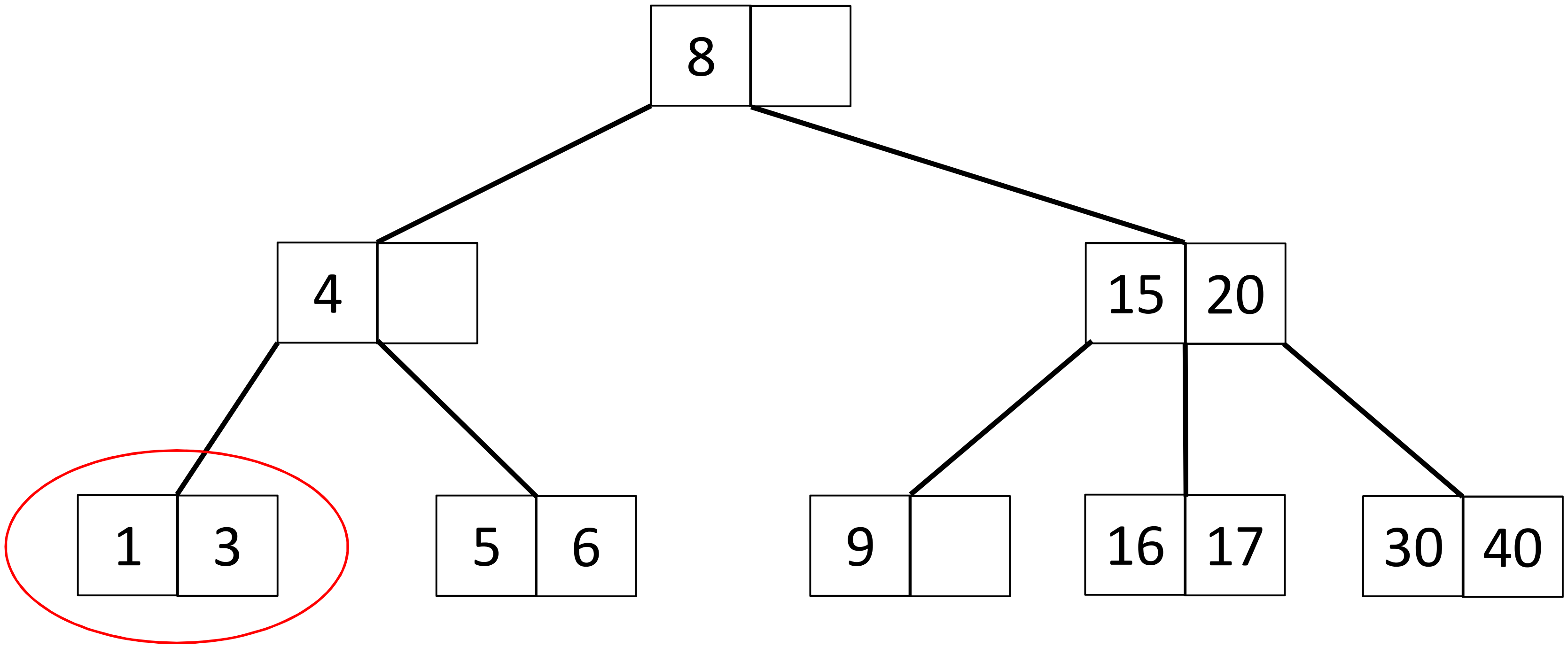
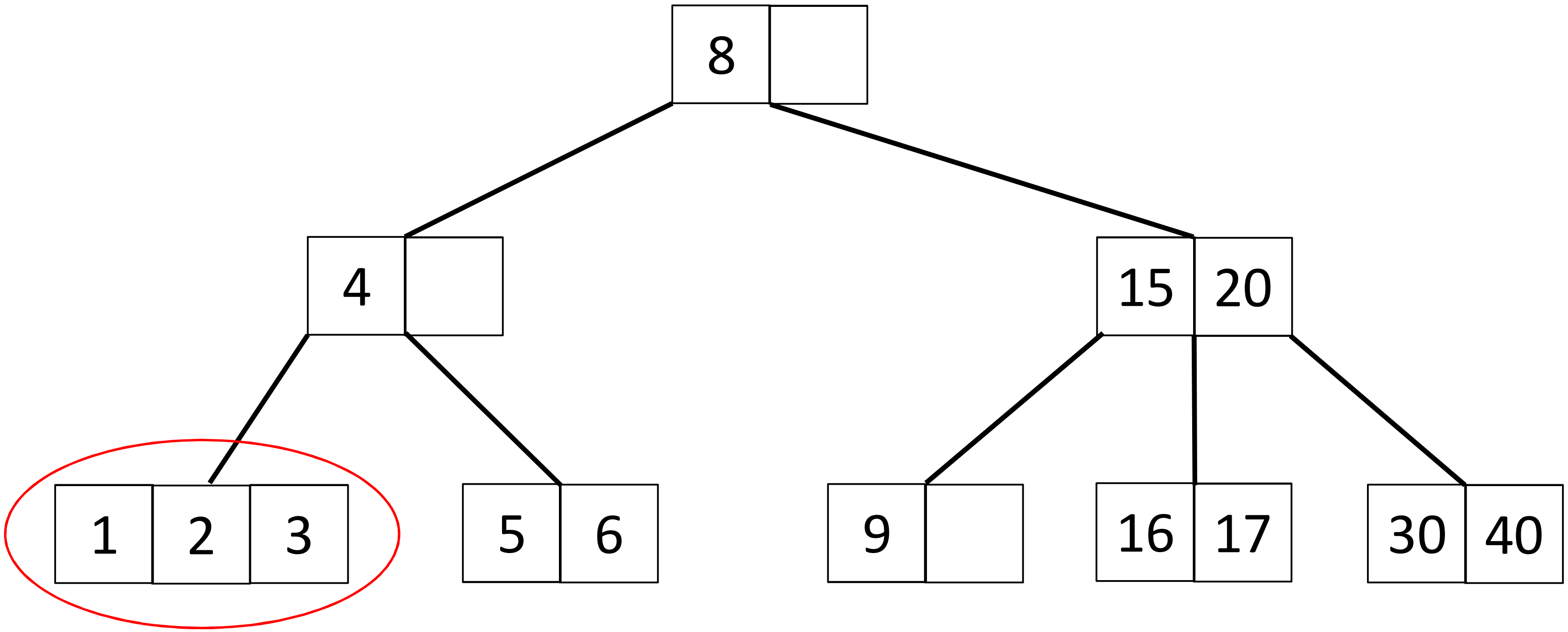
Insert (3-way B-tree)



Insert 2

Insert (3-way B-tree)



need split!

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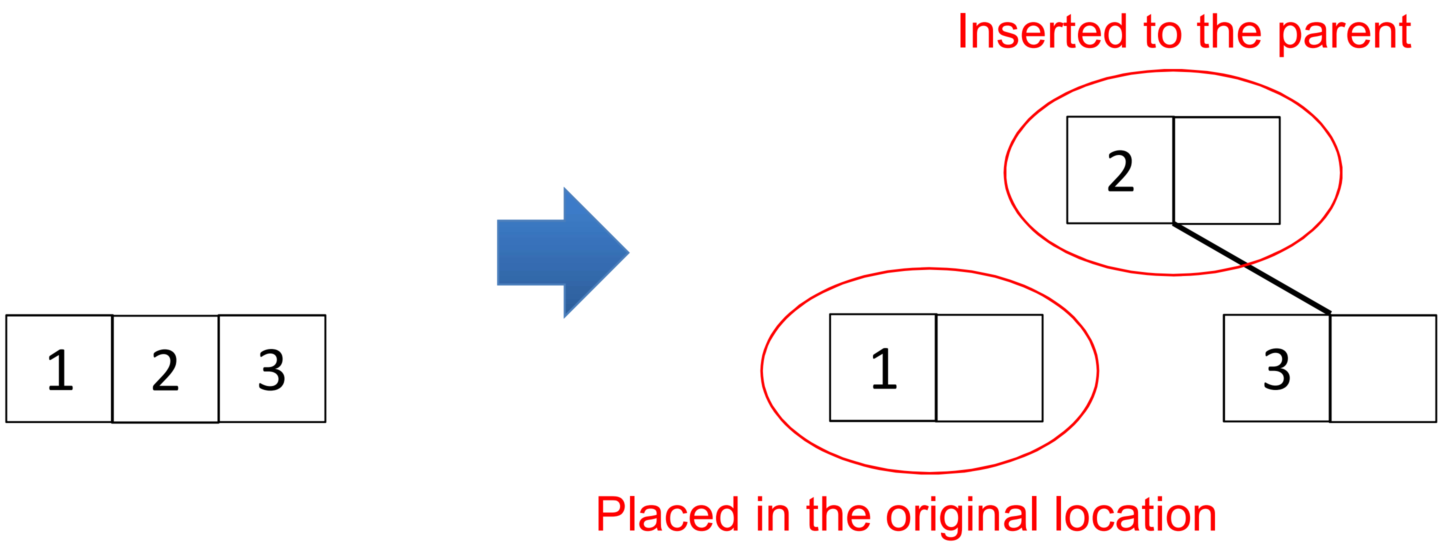
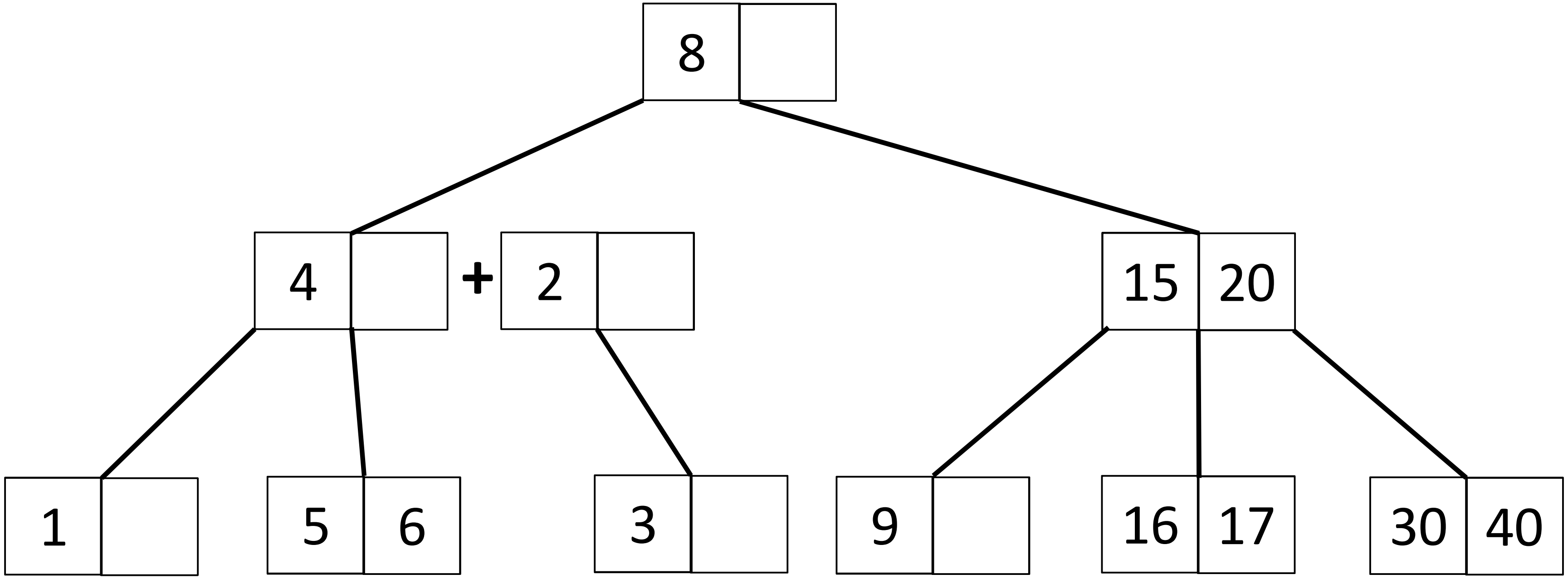
Insert (3-way B-tree)

- Split overflowed node around middle key
- Insert middle key to its parent



Insert (3-way B-tree)



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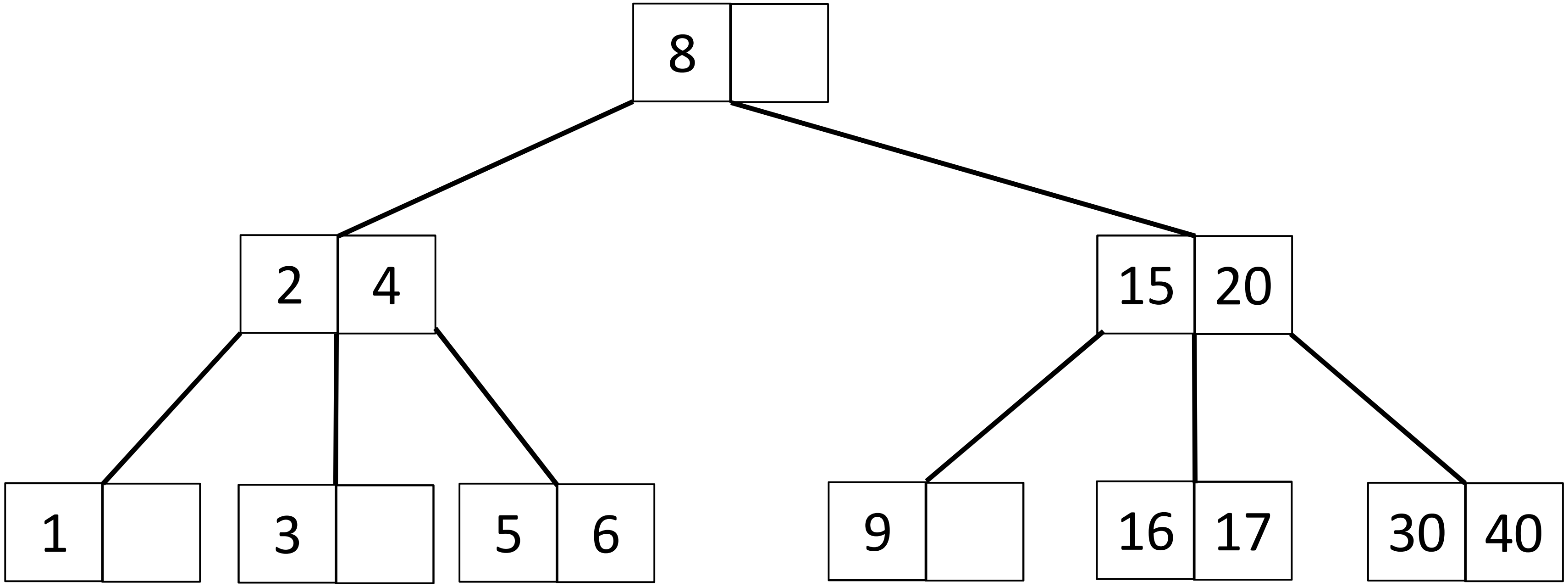
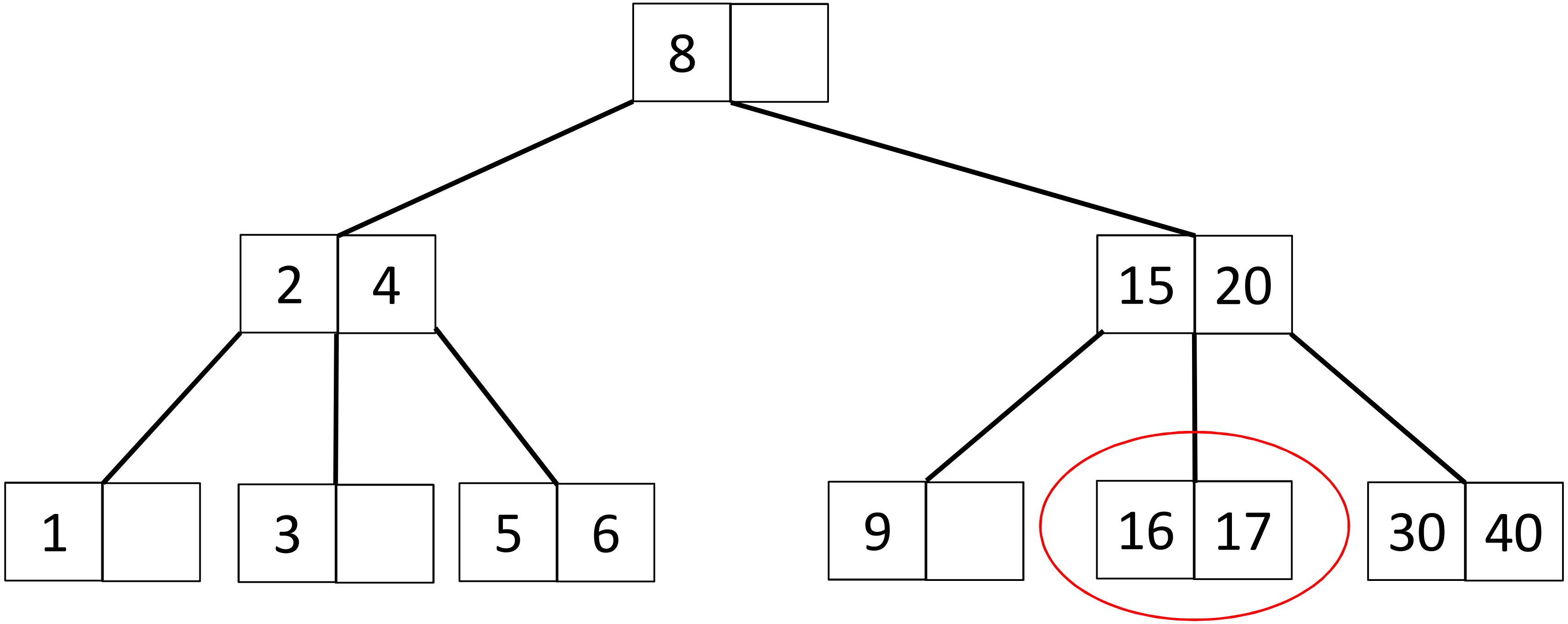
Insert (3-way B-tree)



Insert (3-way B-tree)



Insert 18

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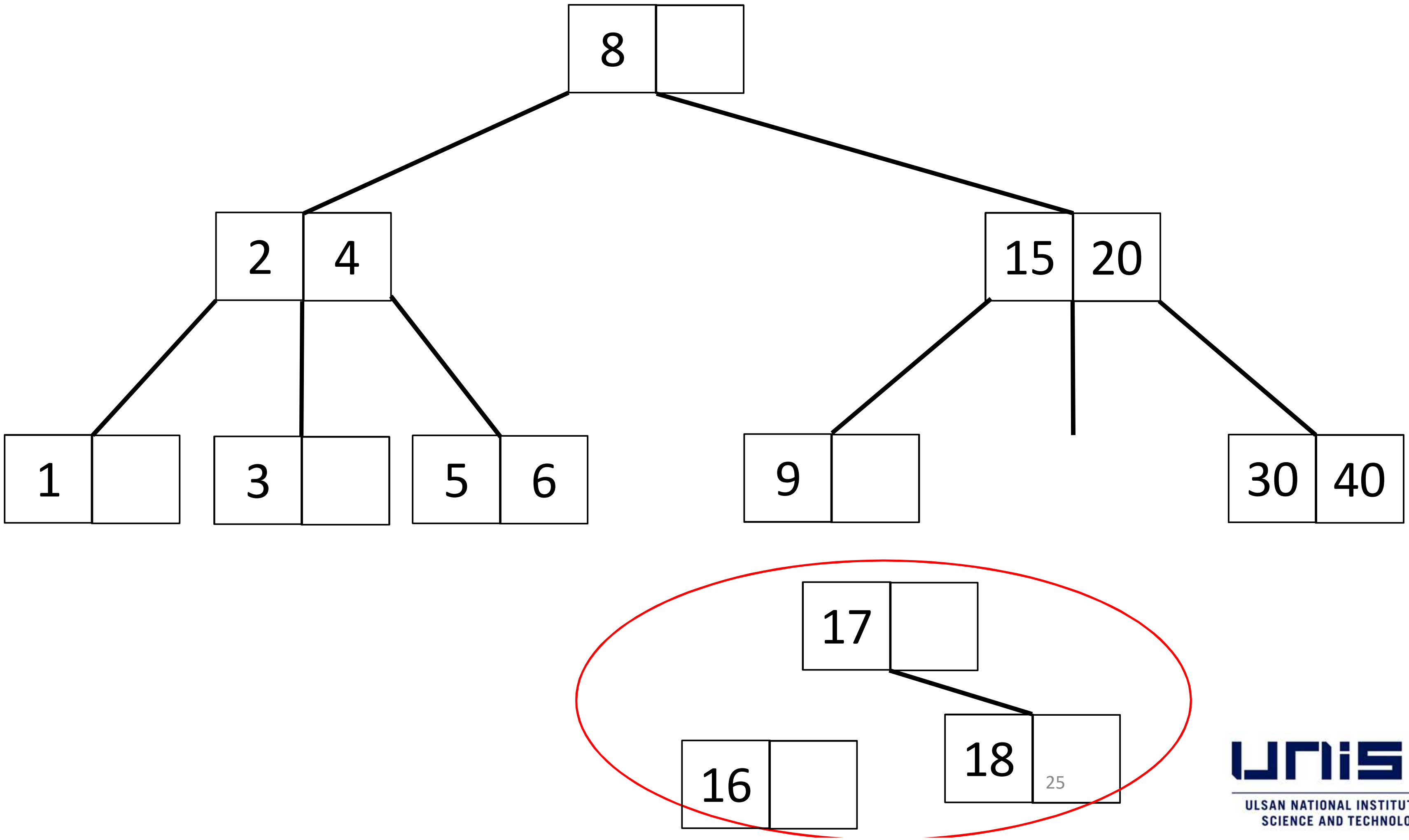
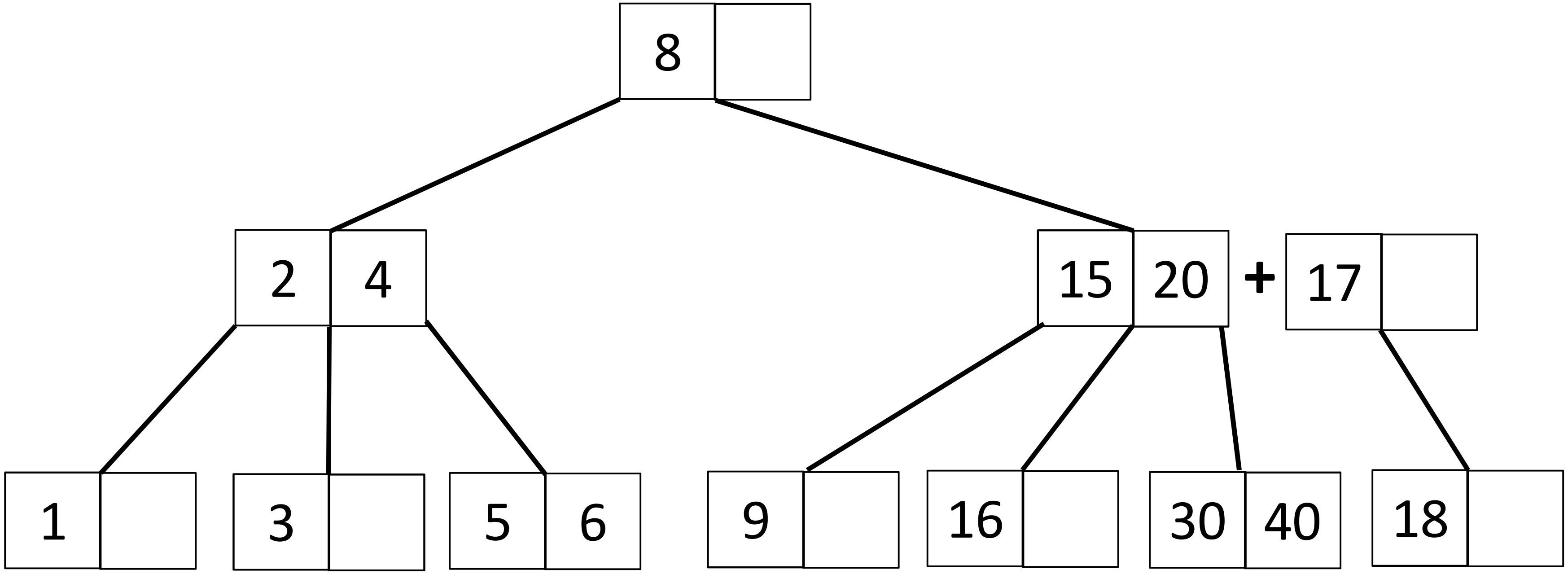
Insert (3-way B-tree)



Insert (3-way B-tree)



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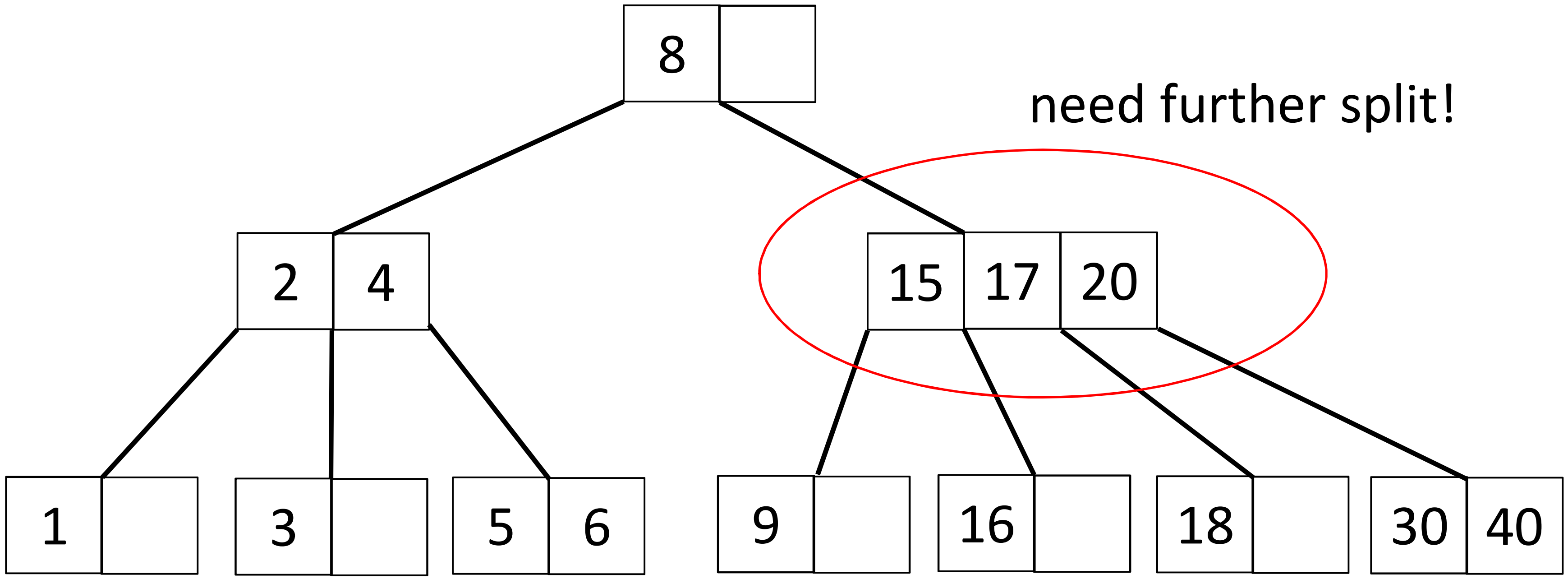
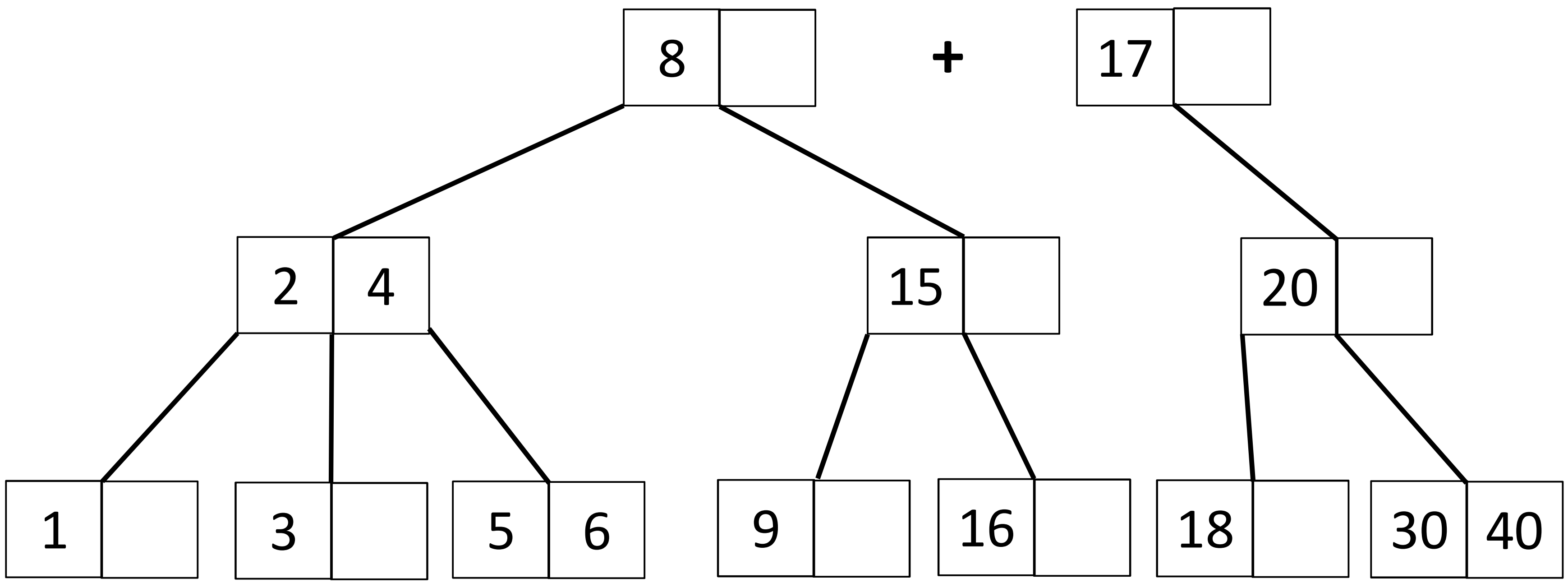
Insert (3-way B-tree)



Insert (3-way B-tree)



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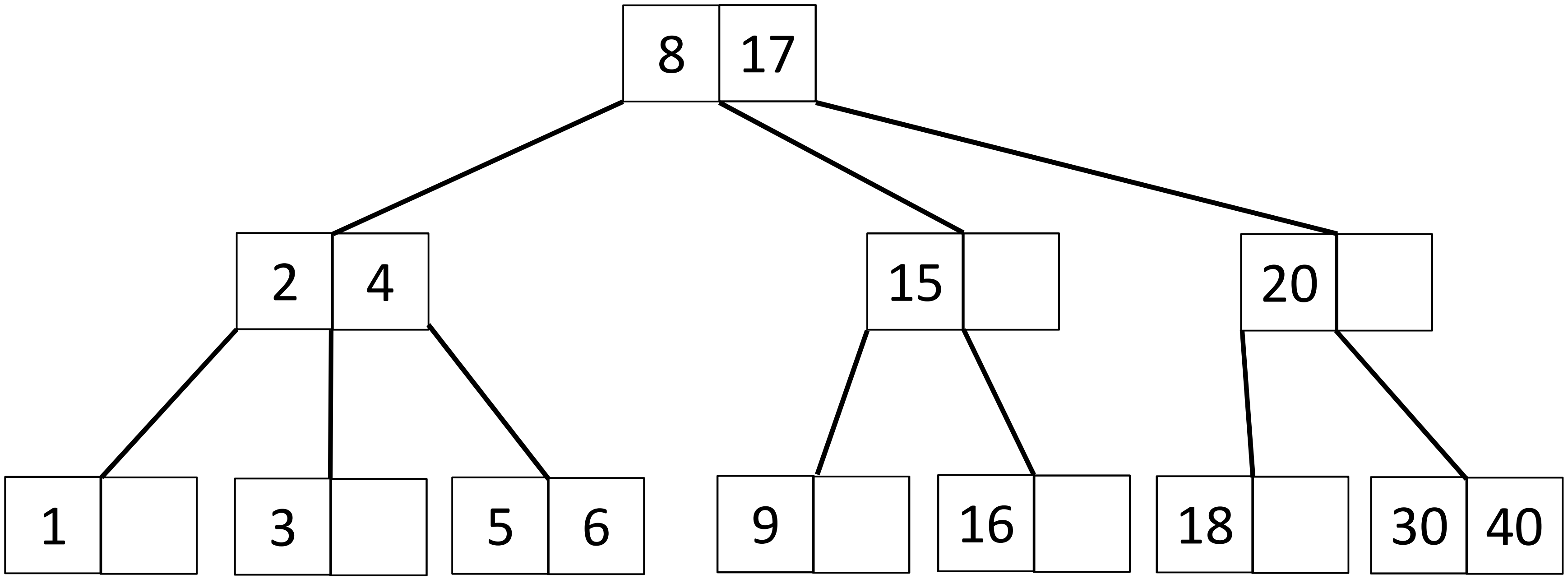
Insert (3-way B-tree)



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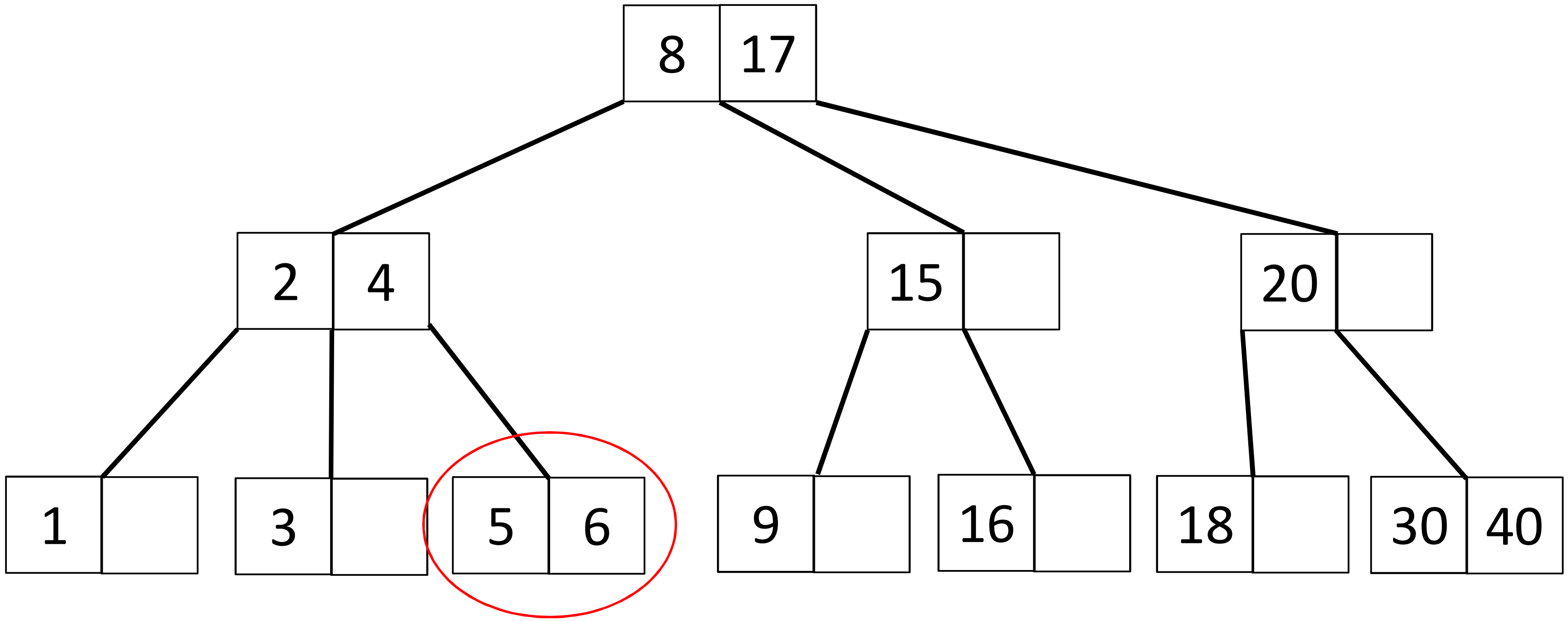
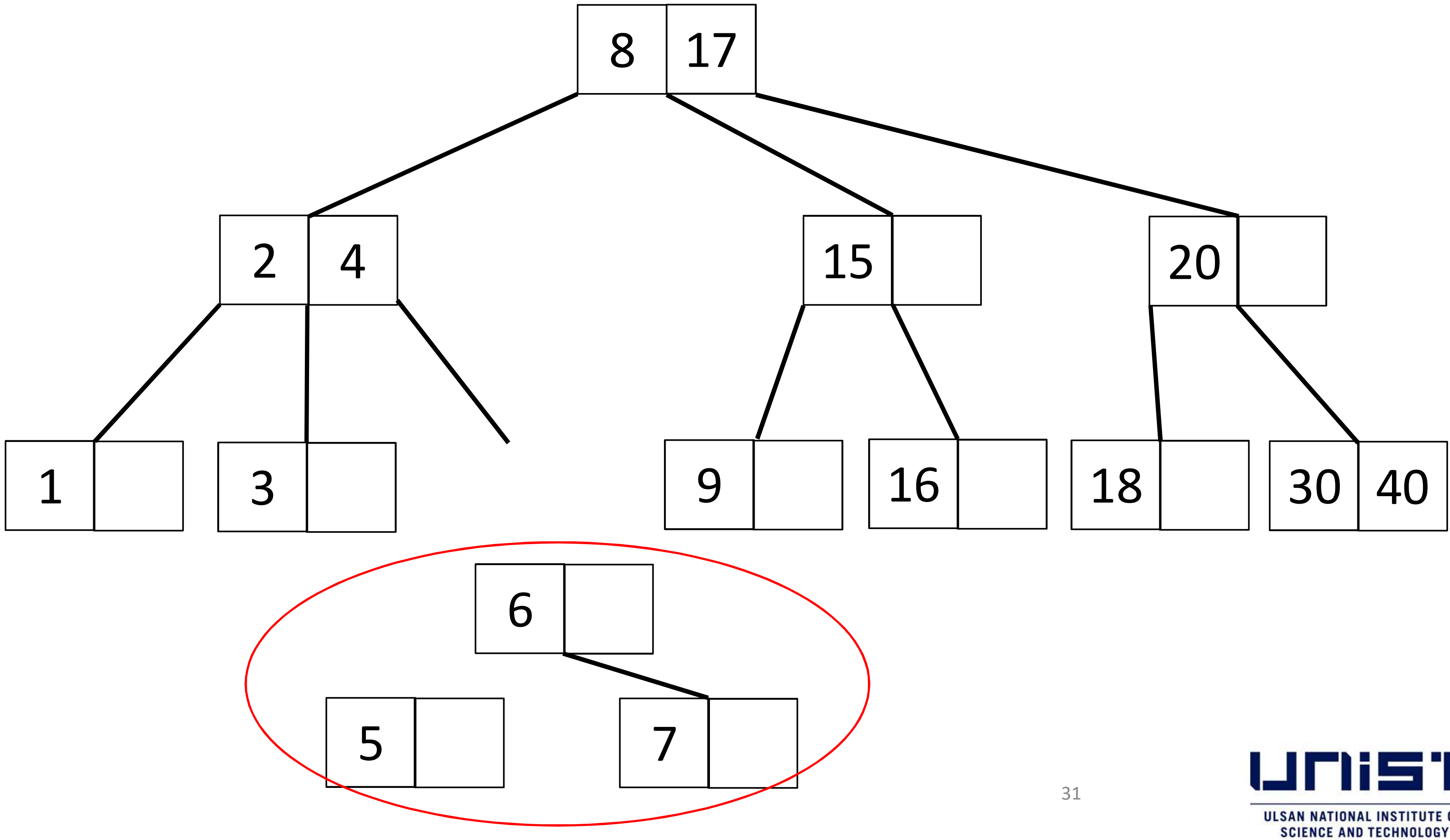
Insert (3-way B-tree)



Insert 7

Insert (3-way B-tree)



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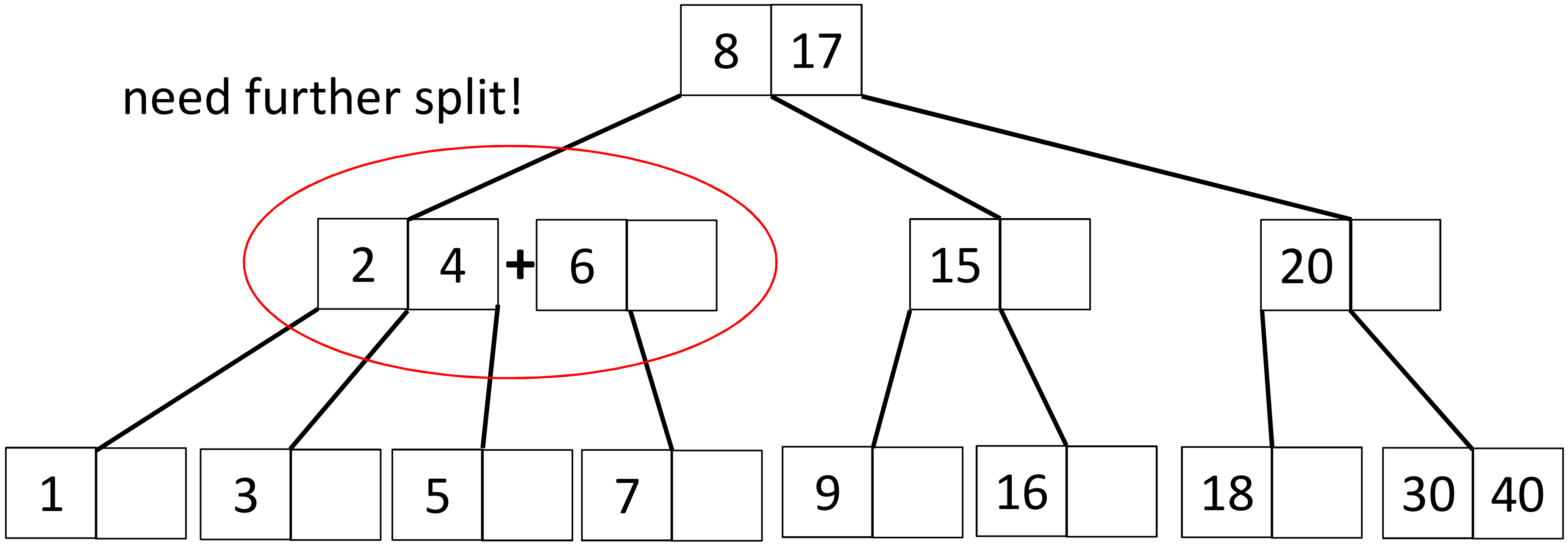
Insert (3-way B-tree)



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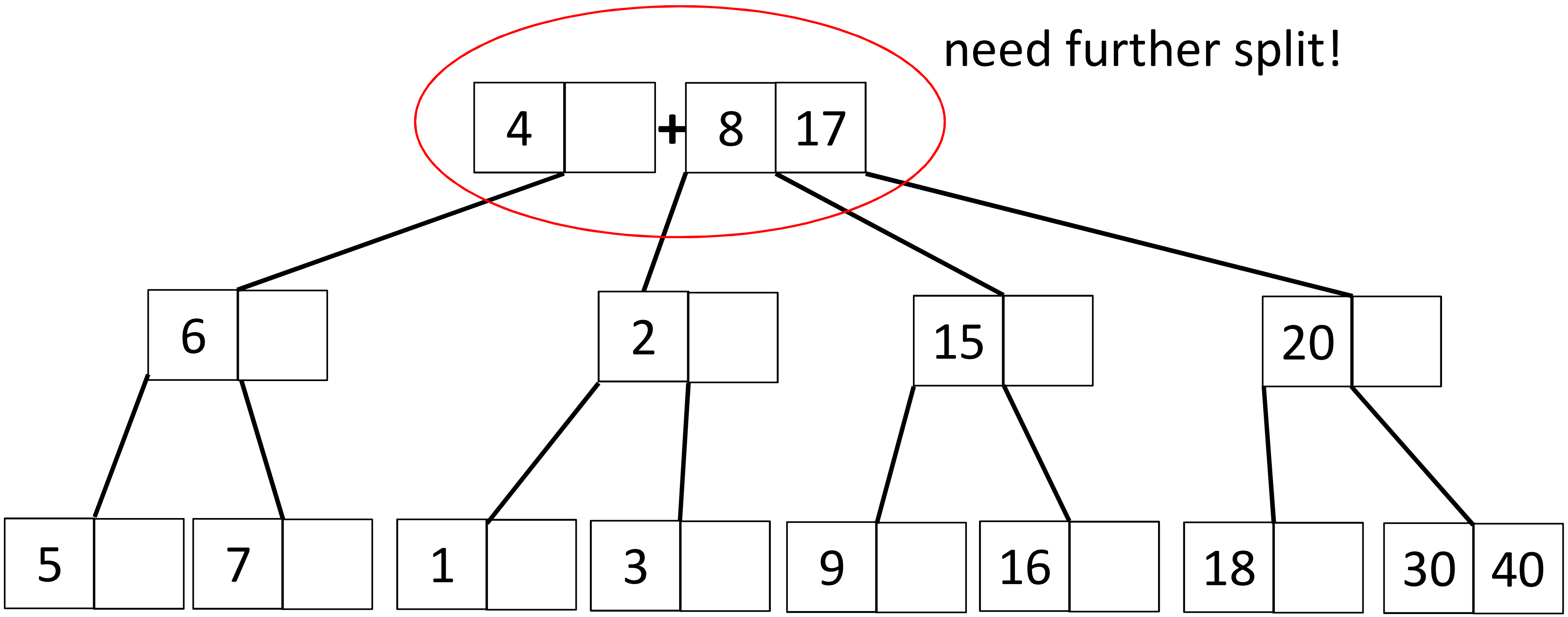
Insert (3-way B-tree)



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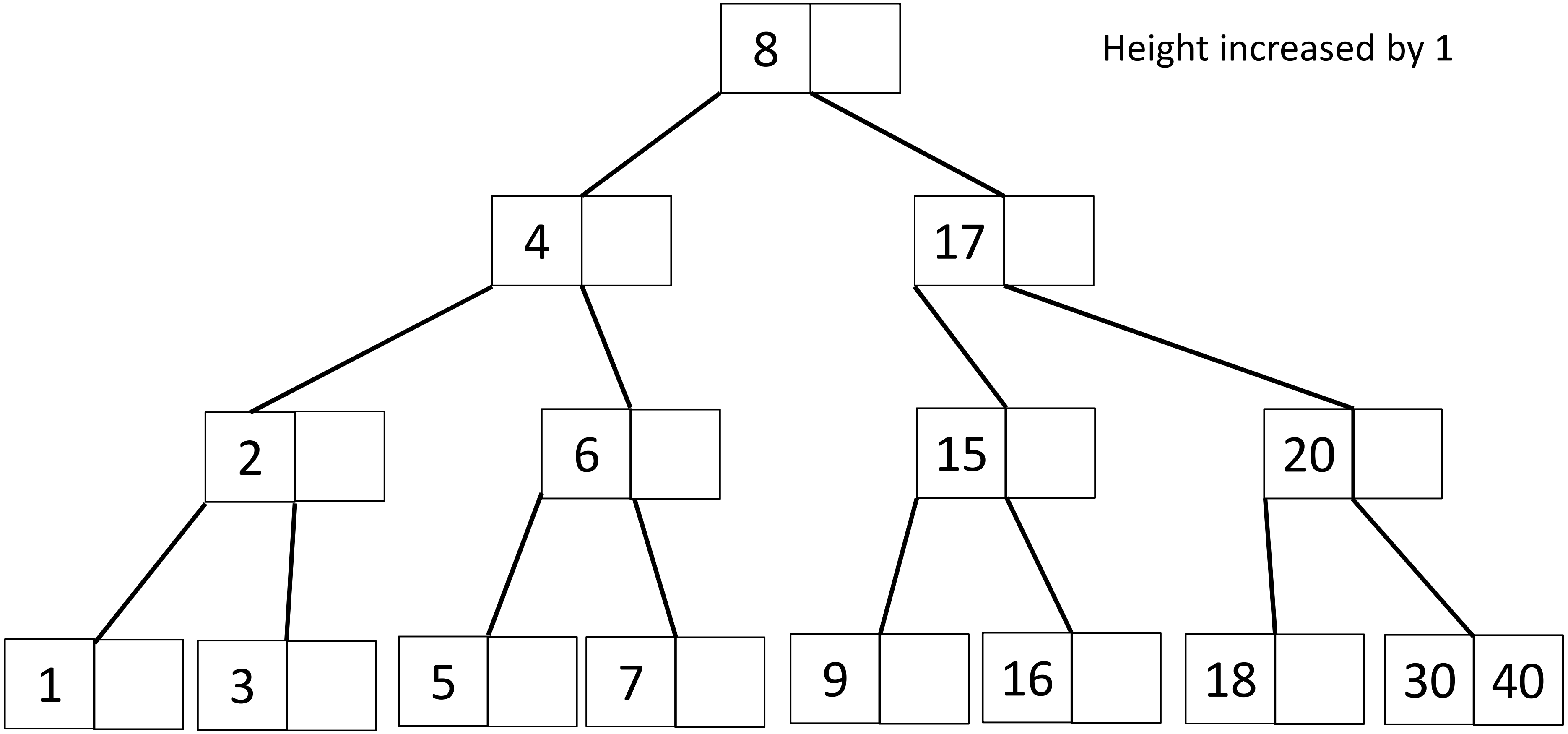
Insert (3-way B-tree)



Height increased by 1

Deletion

- Delete from interior node
  - Replace with the largest in left subtree or the smallest in right subtree
    - The smallest/largest is in the leaf node
    - Deletion from an interior node is transformed into a **deletion from a leaf node**
  - If deletion results in less than ceil(m/2) children, rotation or combine must be done

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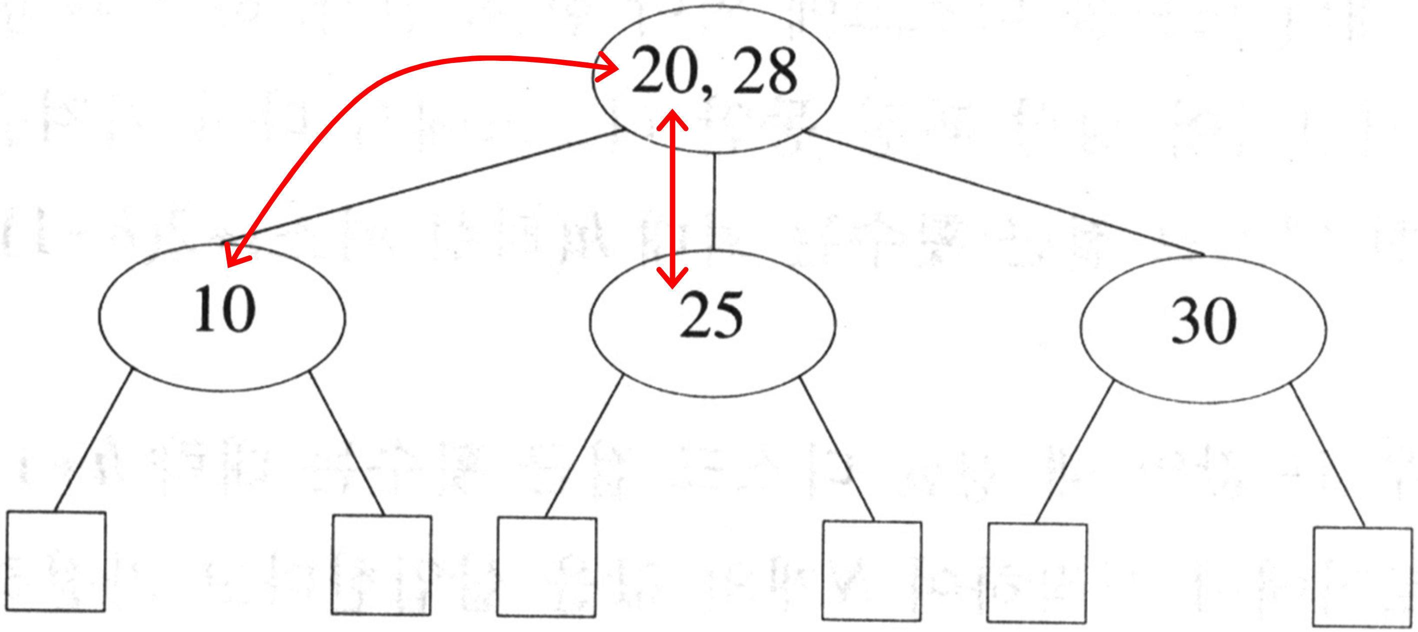
Example

- Delete 20
  - Replace with 10 or 25



Deletion

Four cases when deleting an element from a leaf node p

1. p is root and left with at least one element after delete
   – OK: root is not empty = **at least two** children
2. p is internal and left with at least ceil(m/2)-1 elements after delete
   – OK: **ceil(m/2)-1 elements = ceil(m/2) children**

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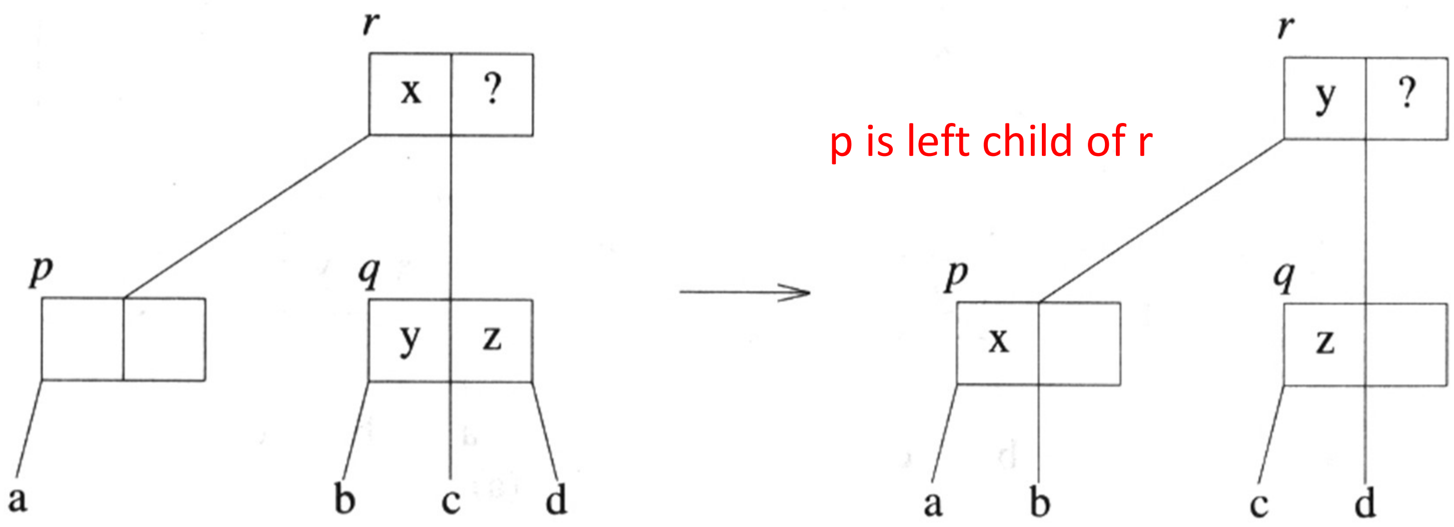
Deletion

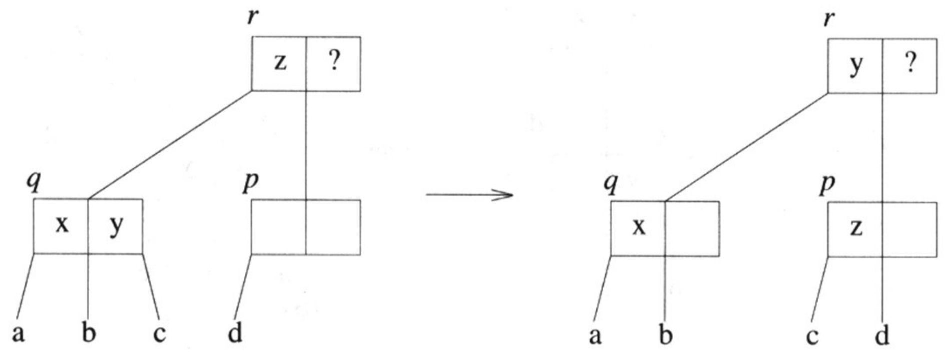
3. p has **ceil(m/2)-2** elements and its sibling q has **at least ceil(m/2)** elements
   - Rotation: **ceil(m/2)-1** elements in p



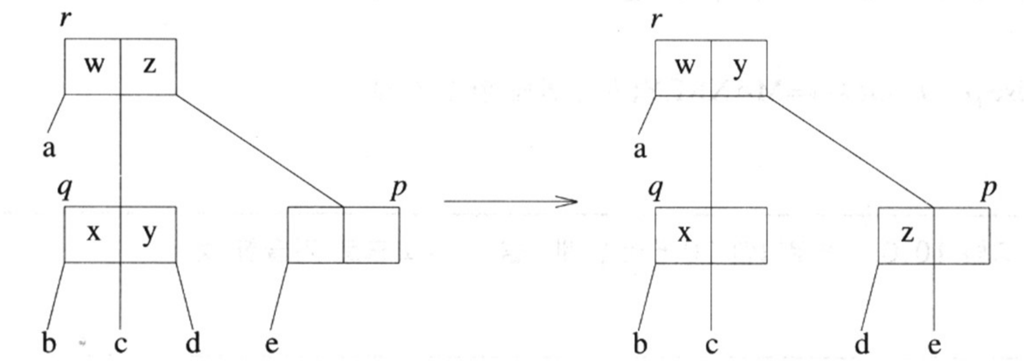
Order kept: a < x < b < y < c < z < d

Deletion

3. More rotation examples



p is middle child of r



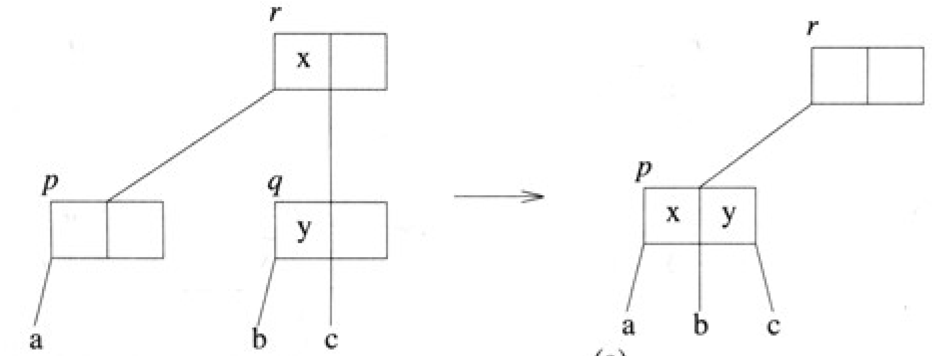
p is right child of r

Deletion

4. p has **ceil(m/2)-2** elements and its sibling q has **ceil(m/2)-1** elements
   - q has the minimum number of elements
     - Cannot rotate as we cannot reduce q's element
   - p and q are combined while borrowing a key (in-between element) from parent r
   - If r has ceil(m/2)-2 elements, rotation or combine is applied upward to the root

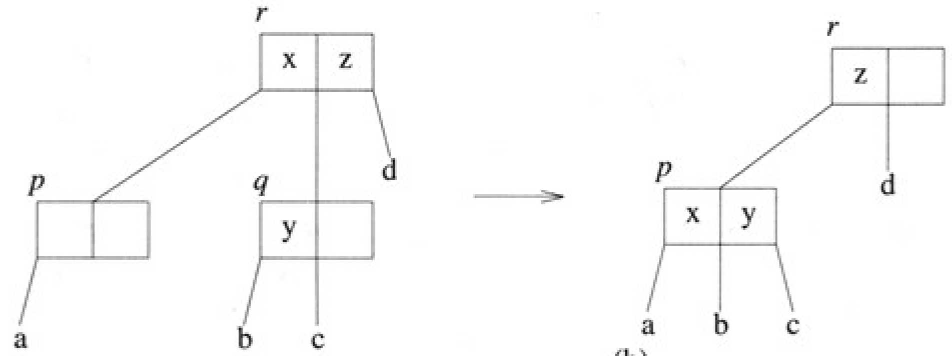
Deletion

4. p has **ceil(m/2)-2** elements and its sibling q has **ceil(m/2)-1** elements



(a)

r has insufficient element
→ rotation/combine is applied upward



(b)

push x down
→ p is left child of r

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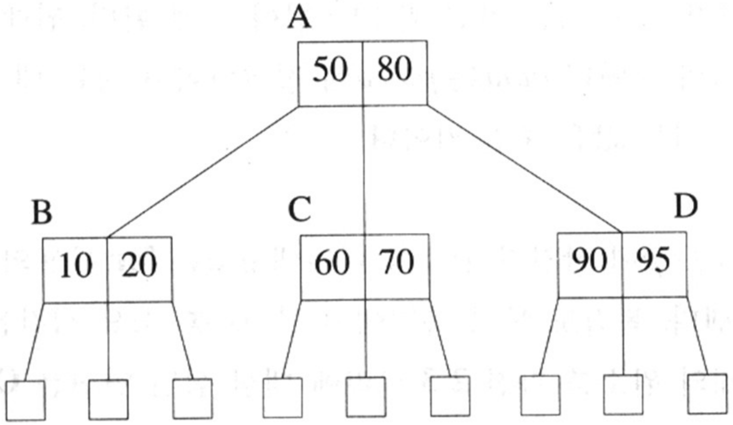
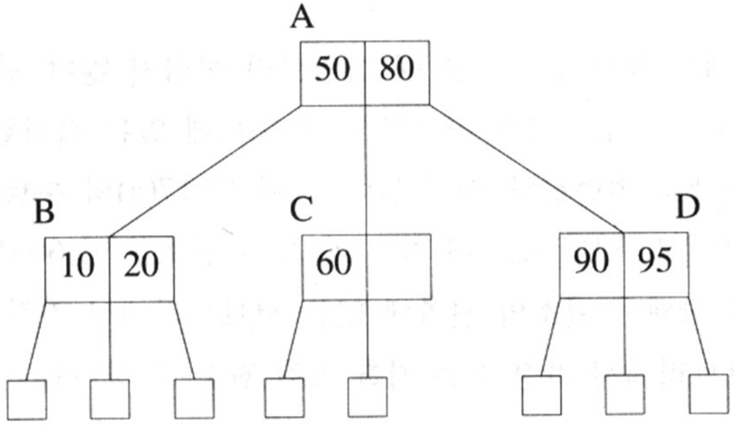
Example



(a) Initial 2-3 tree



(b) 70 deleted

delete 70: node C has 1 element left, ok

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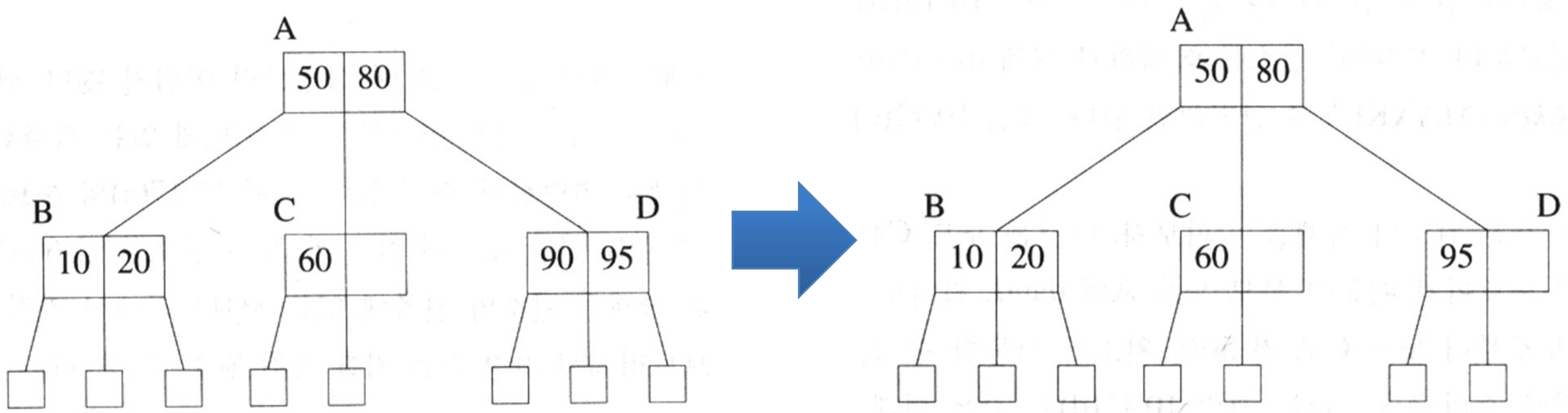
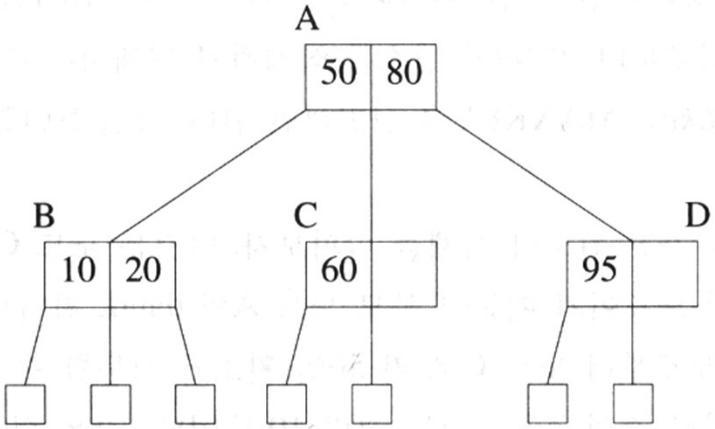
Example



(b) 70 deleted (c) 90 deleted

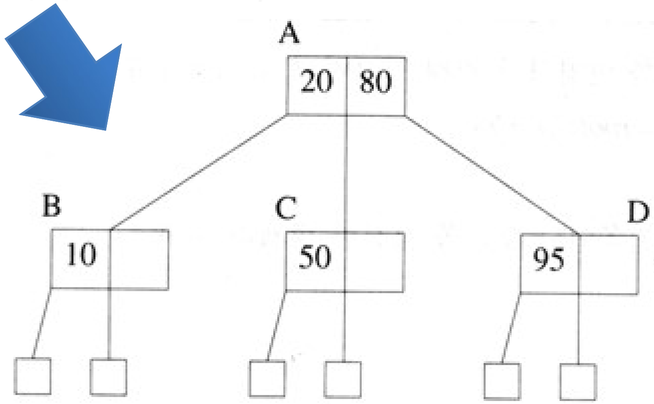
delete 90: node D has 1 element left, ok

Example

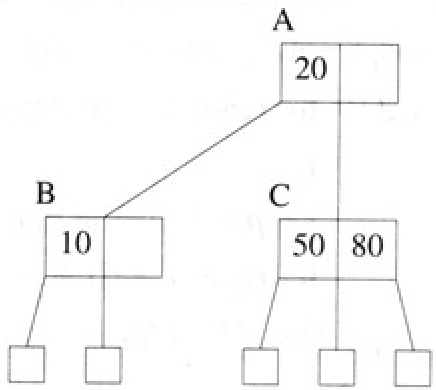


(c) 90 deleted

delete 95: combine
→ by pushing 80 down, we keep all external nodes in the same level



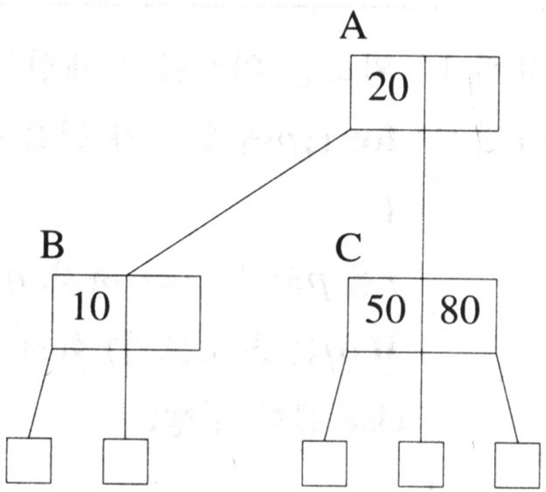
(d) 60 deleted



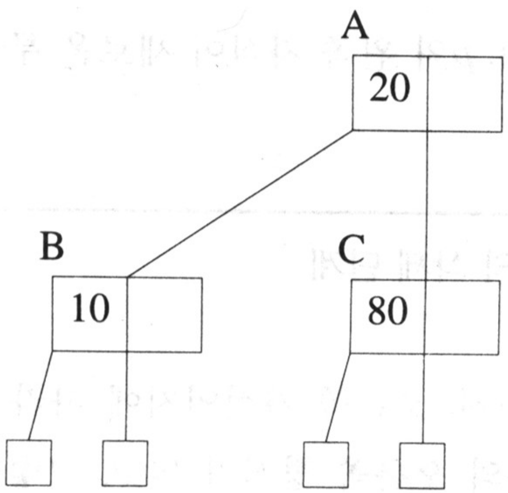
(e) 95 deleted

delete 60: rotation right

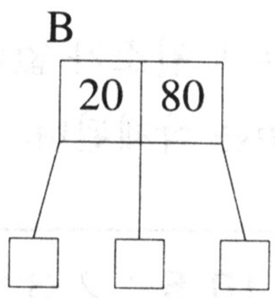
Example



(e) 95 deleted



(f) 50 deleted



(g) 10 deleted

delete 50: node C has 1 element left, ok

delete 10: combine

Questions?