CSE221

Trees

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Acknowledgment: The content of this file is based on the slides of the textbook as well as the slides provided in former lectures at UNIST.

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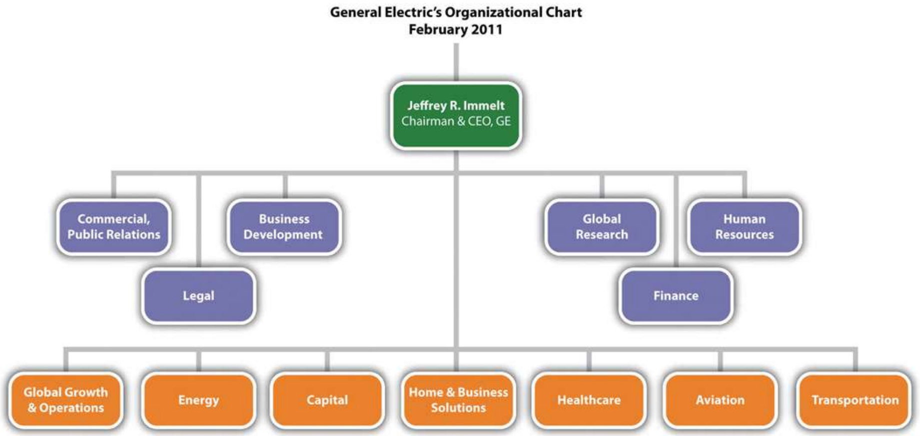
Outline

- Tree representation
- Binary trees

Linear Lists and Trees

- Linear lists are useful for serially ordered data
  - (a,b,c,d,e)
  - Days of week
  - Students in a class
- Trees are useful for nonlinear (hierarchically) ordered data
  - Structure of a company
  - Lineage



Trees

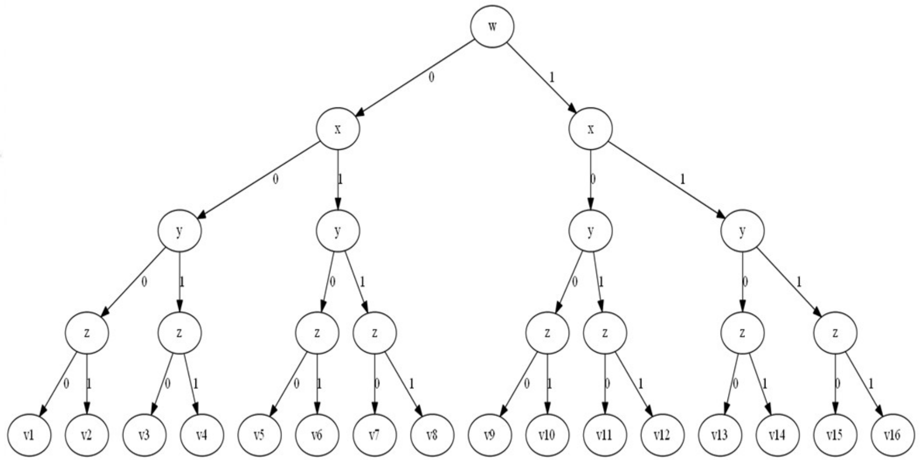
- Definition
  - Finite set T of nodes storing elements in a parent-child relationship
    - If T is nonempty, there is a specially designated node, the root, that has no parent
    - Each node v in T has a <u>unique</u> parent w
    - Every node with parent w is a child of w
- By definition, a tree can be empty

Trees

root

leaf

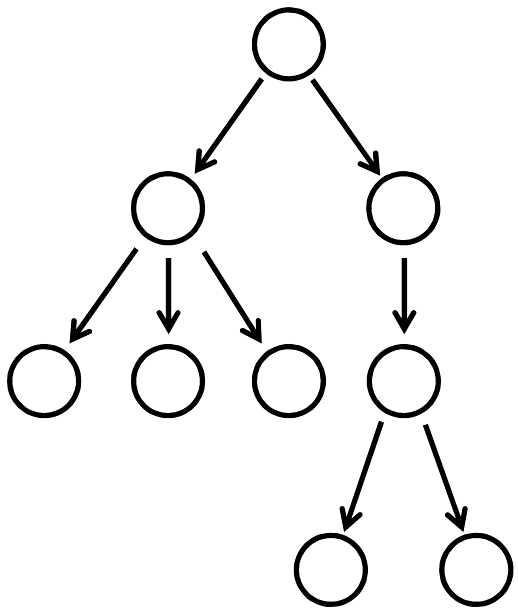
root



leaf

Terminology

- Node
- Degree of a node X
  - # of children of a node X
- Internal nodes
  - Node with at least one child
  - Degree > 0
- Leaf (external) nodes
  - Node with no child
  - Degree 0 nodes



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Terminology

- Level (depth) of a node
  - Root node : 0
  - Child of a node whose level is n : n+1
- Degree of a tree
  - Max degree of nodes in the tree
- Height of a tree
  - Maximum level of nodes in the tree

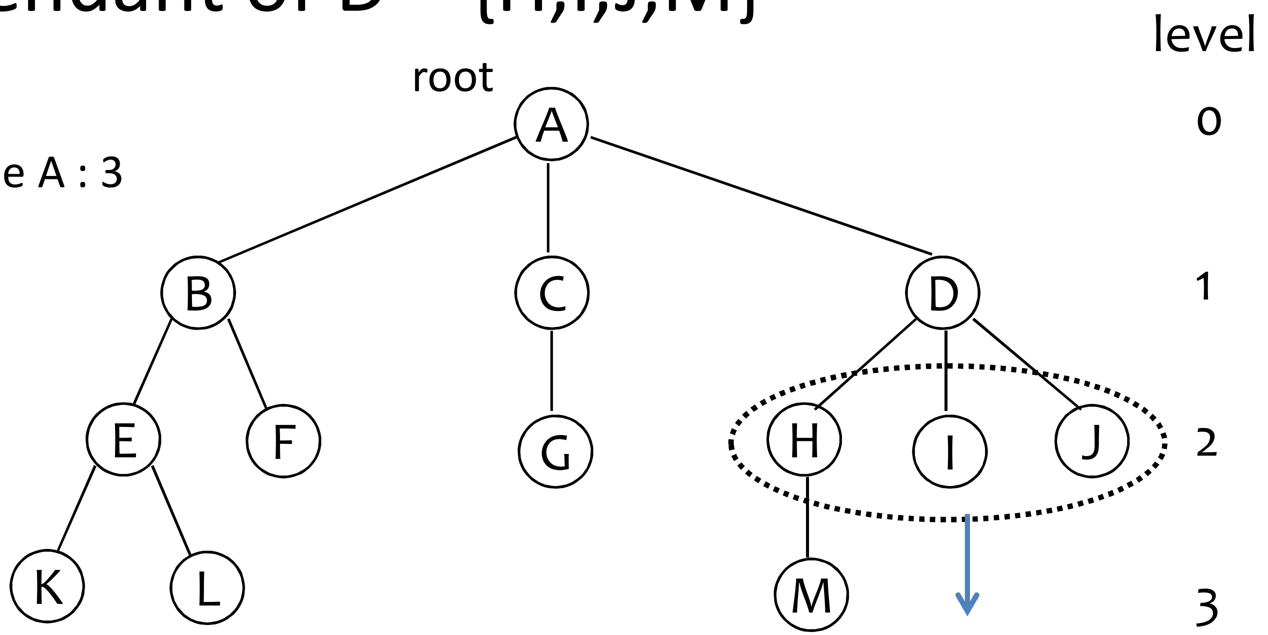
Terminology

- Subtree of X
  - Tree whose root is one of the children of X
- Ancestors of node X
  - All nodes in the paths from X to the root
- Descendants of node X
  - All nodes in the subtrees of node X

Example: Tree Terminology

- Ancestor of M = {H,D,A}
- Descendant of D = {H,I,J,M}

Tree height: 3
Degree of node A : 3

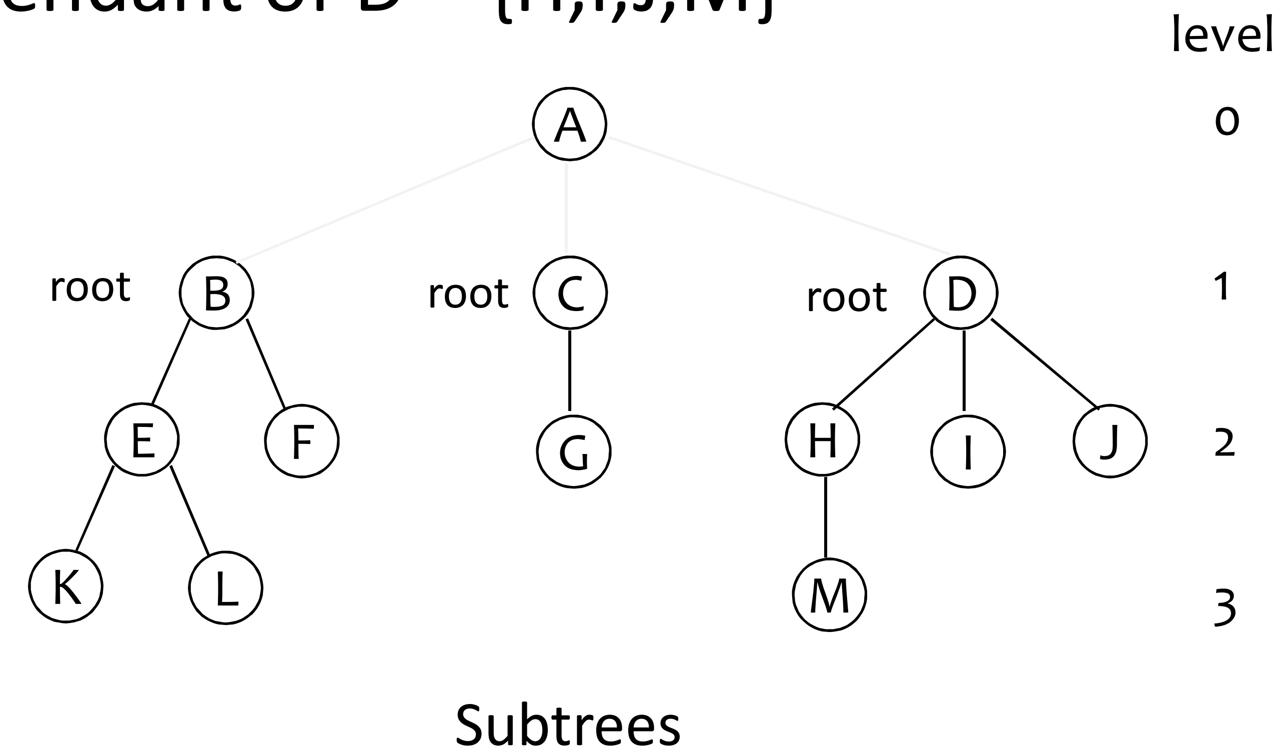


Leaf nodes : K, L, F, G, M, I, J

sibling
children of D

Example: Tree Terminology

- Ancestor of M = {H,D,A}
- Descendant of D = {H,I,J,M}



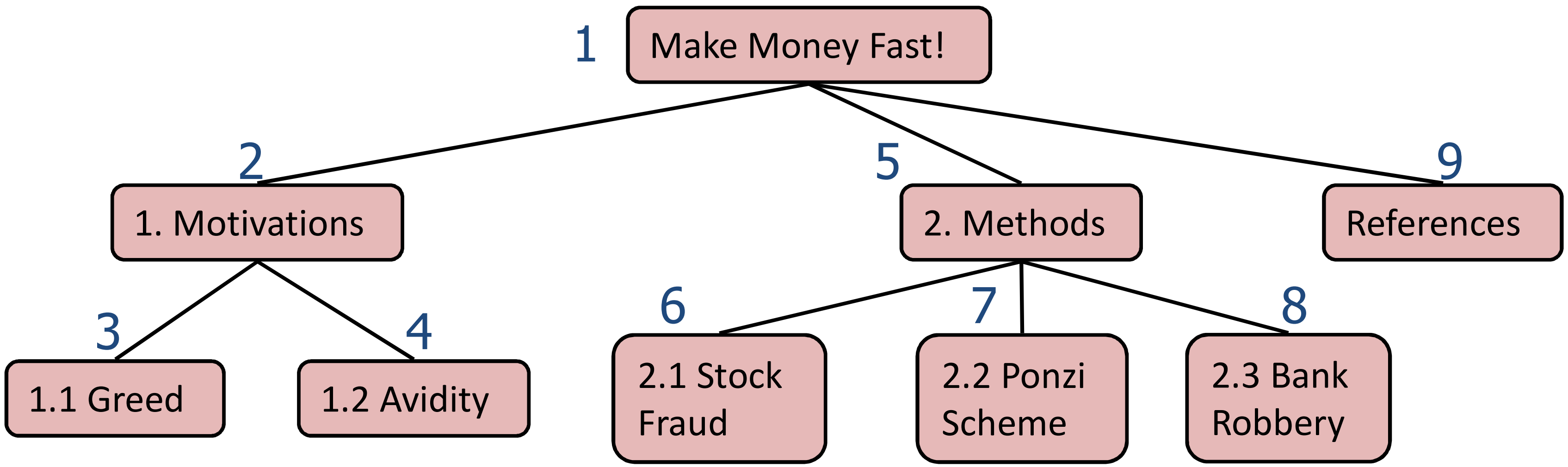
Tree ADT

- We use positions to abstract nodes
- Generic methods:
  - integer size()
  - boolean empty()
- Accessor methods:
  - position root()
  - list<position> positions()
- Position-based methods:
  - position p.parent()
  - list<position> p.children()
- Query methods:
  - boolean isRoot()
  - boolean isExternal()
- Additional update methods may be defined by data structures implementing the Tree ADT

Preorder Traversal

- A traversal visits the nodes of a tree in a systematic manner
- In a preorder traversal, a node is visited before its descendants

Algorithm *preorder*(*v*)
    visit(*v*)
    for each child *w* of *v*
        preorder (*w*)



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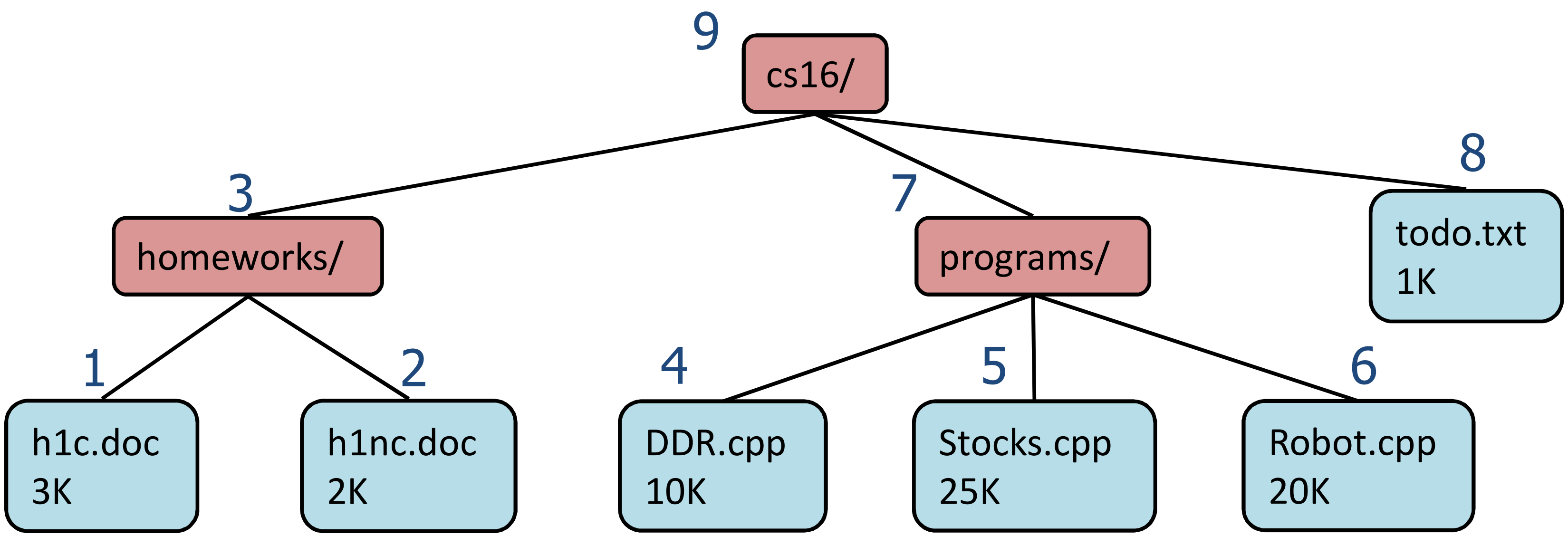
Postorder Traversal

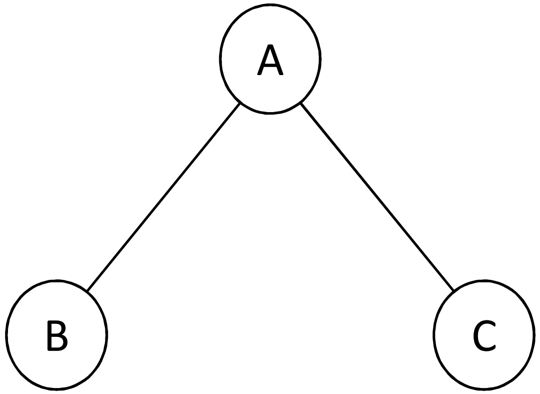
- In a postorder traversal, a node is visited after its descendants

Algorithm *postorder*(v)
  for each child w of v
    postorder (w)
  visit(v)



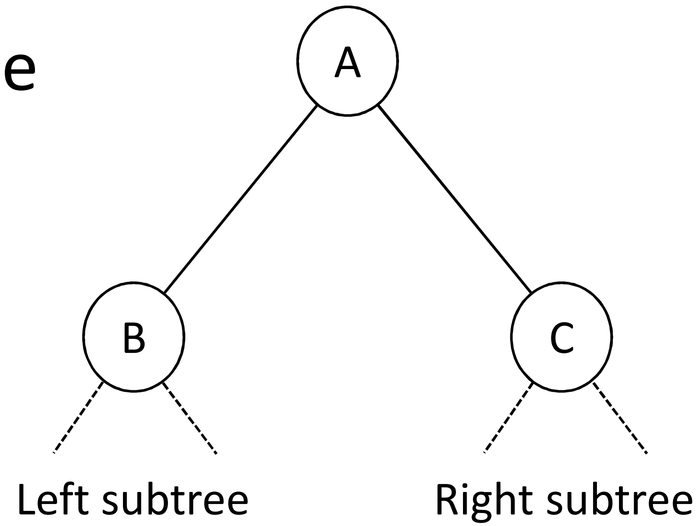
Tree Representation

- Binary tree
  - k=2
  - Left child – right child



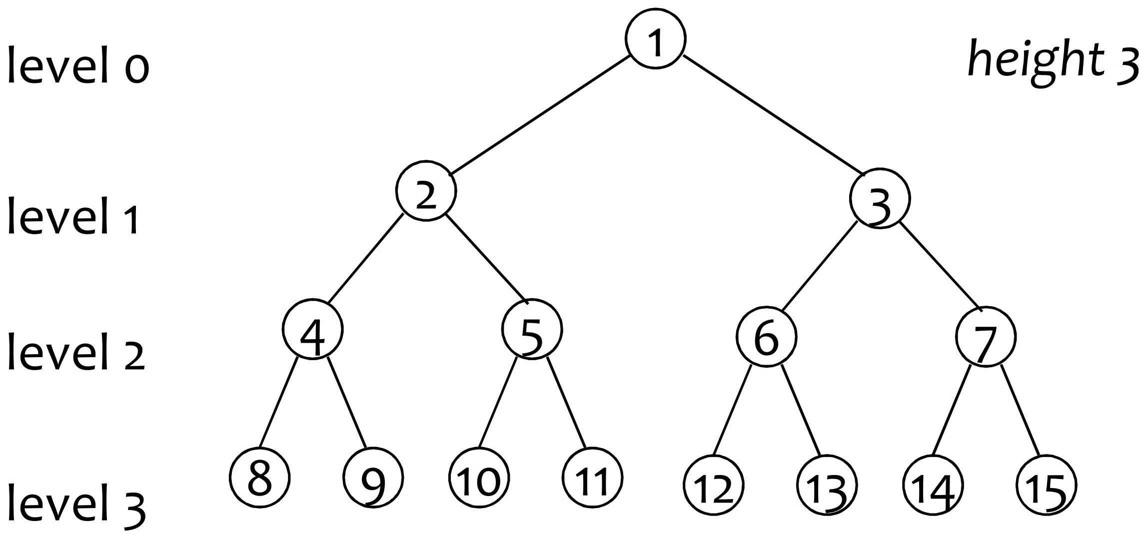
Binary Trees

- Definition
  - Finite set of nodes that consist of a root and two disjoint binary trees called the left subtree and the right subtree
  - Or empty tree



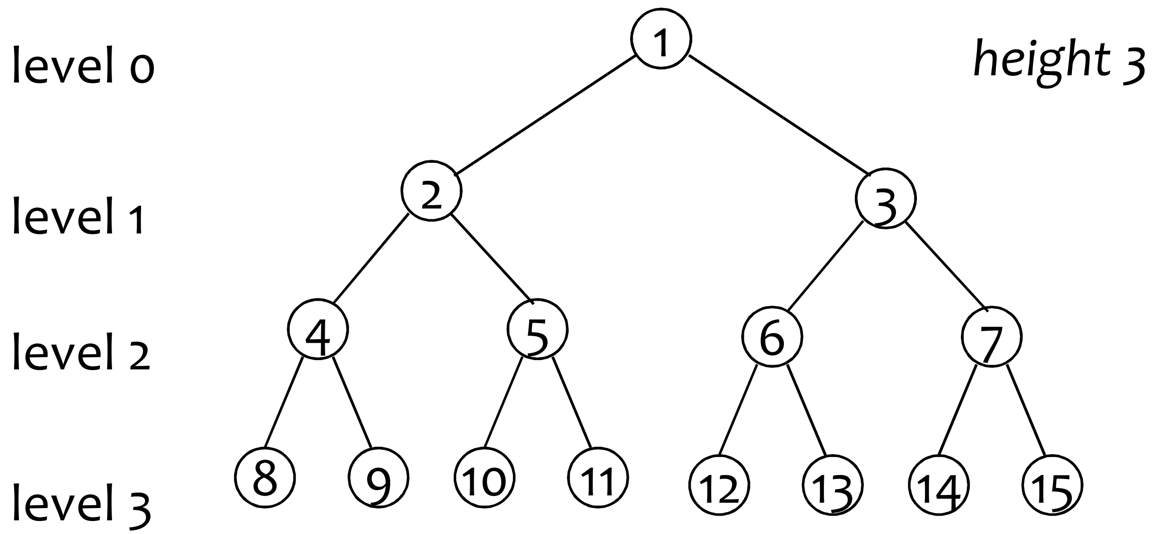
Properties of Binary Trees

- Maximum # of nodes on level i
  - 2^i, i>=0
- Maximum # of nodes in a binary tree of height k
  - $1 + 2 + 2^2 + 2^3 \ldots + 2^k = 2^{k+1}-1$



Full Binary Tree

- For height k, the tree has 2^{k+1}-1 nodes
  - Example: a full binary tree for height 3



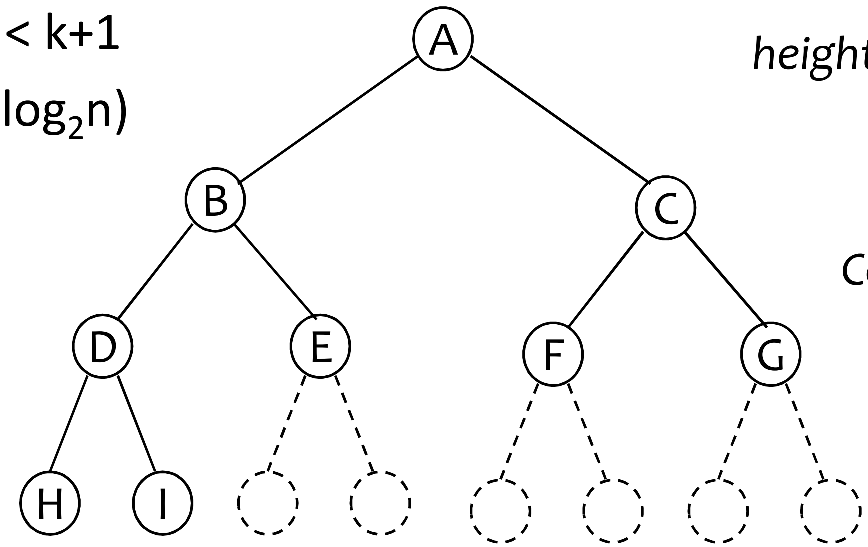
Complete Binary Tree

- All levels except the last level are fully filled
- All leaf nodes are filled from left to right
- Height of a complete binary tree (n nodes)

$2^k \leq n < 2^{k+1}$

$\rightarrow k \leq \log_2 n < k+1$

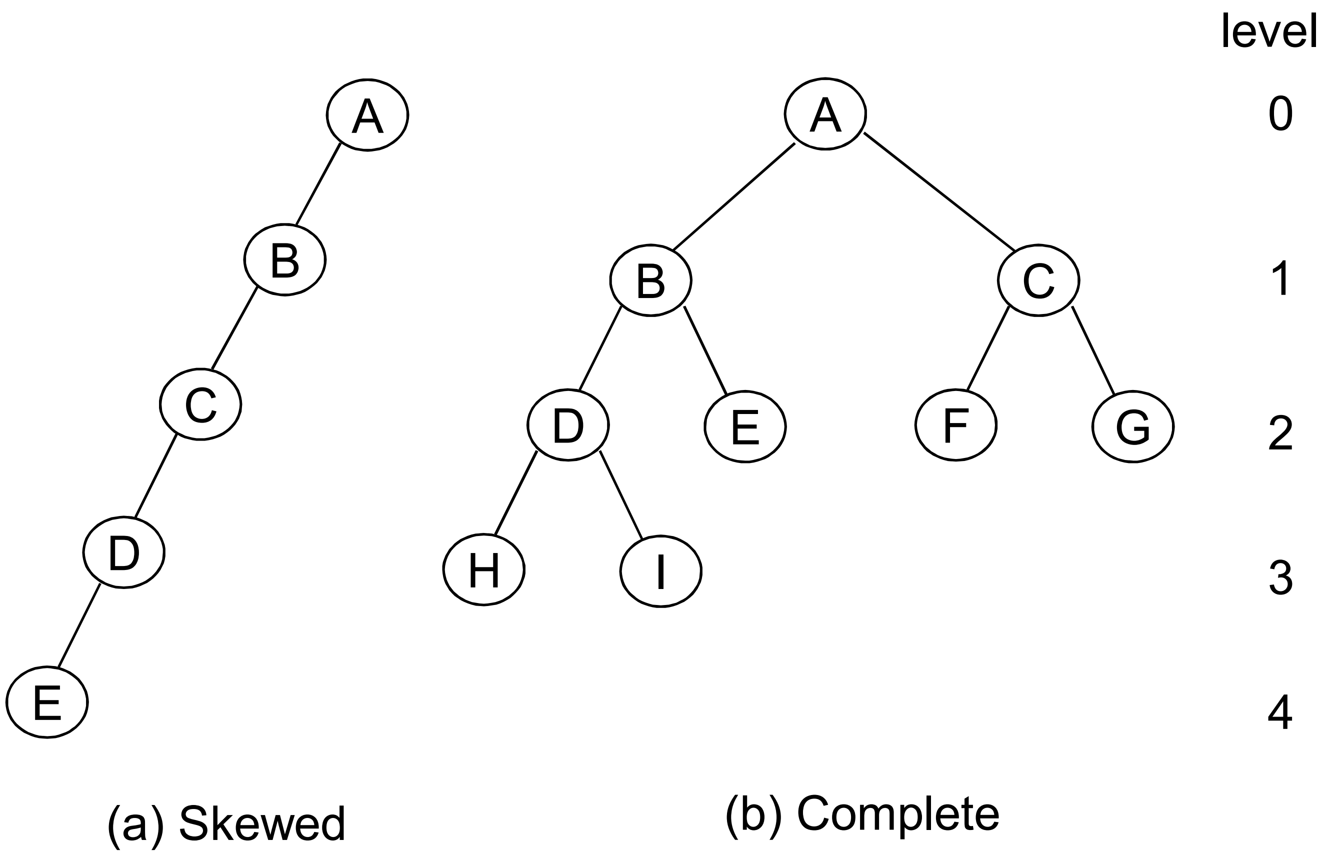
$\rightarrow k = \mathrm{floor}(\log_2 n)$



height 3

Complete Binary tree

Binary Trees



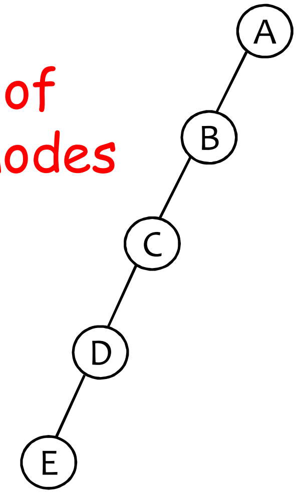
Tree Representation

- Linked-list based node structure for binary tree

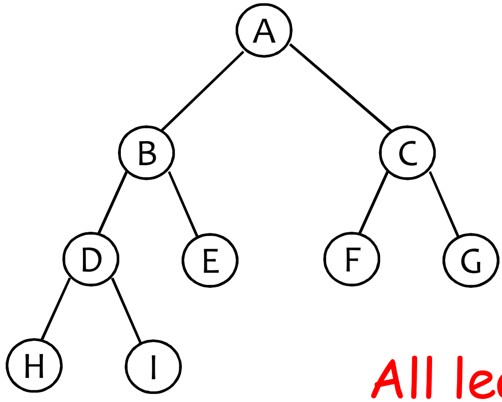
data	$CHILD_1$	$CHILD_2$

- All fields are efficiently used?

Right childs of
all internal nodes
are empty



(a)



All leaf nodes
have empty childs

(b)

Tree Representation

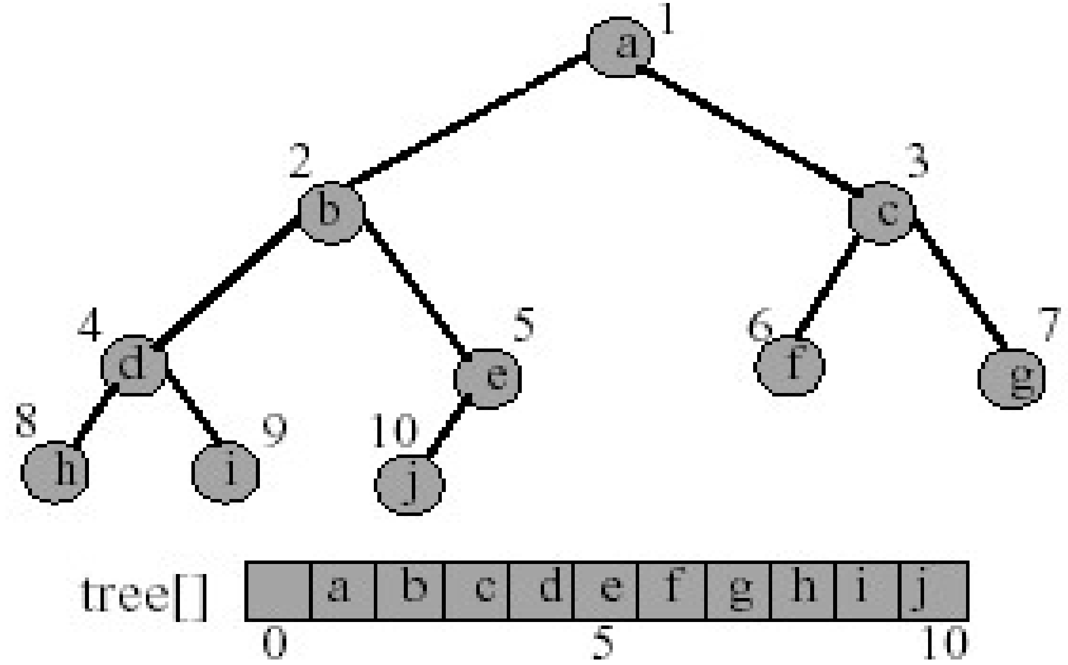
- Linked-list based node structure for a tree of degree k

data	$CHILD_1$	$CHILD_2$	…	$CHILD_k$

- Inherent space waste by zero child pointers
  - Total # of nodes : n
  - Total # of child pointers : nk
  - Total # of child pointers used : n-1
    - Root cannot be pointed by a child pointer
  - Total # of zero child pointers : nk–(n-1) = n(k-1)+1

Binary Tree Representation using Array

- Array representation
  - Each node is stored at the array position corresponding to the number assigned to it



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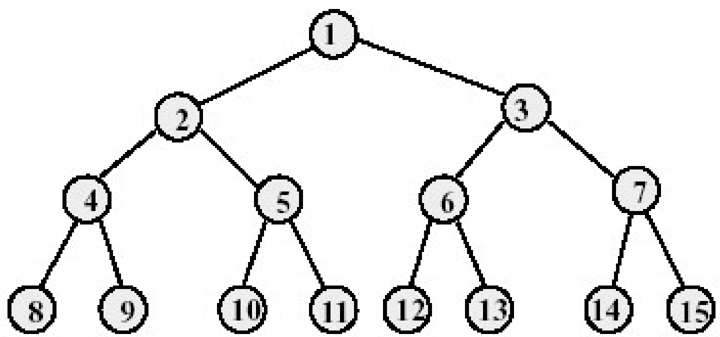
Binary Tree Representation using Array

- If a complete binary tree with n nodes is represented sequentially
  - parent(i) is at $\lfloor i/2 \rfloor$ if i != 1
    - i=1 is root and no parent exists
  - leftChild(i) is 2i if 2i <= n
  - rightChild(i) is 2i+1 if 2i+1 <= n
- Example
  - leftChild(5) = 10
  - parent(7) = $\lfloor 3.5 \rfloor = 3$

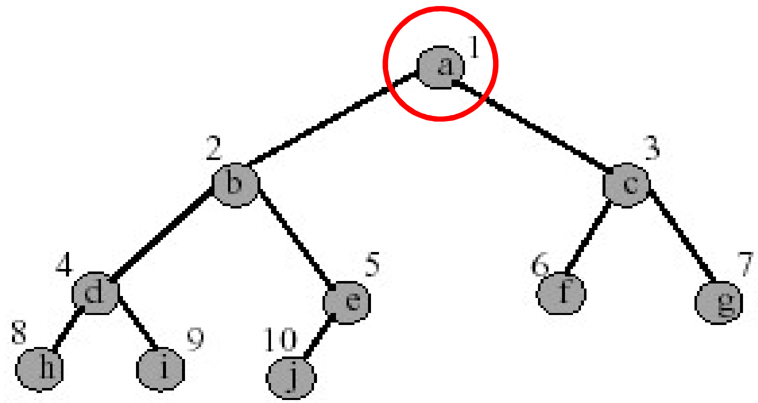


Binary Tree Representation using Array

- Traverse array

a : T(1)

	a	b	c	d	e	f	g	h	i	j				
0	1	2	3	4	5	6	7	8	9	10				

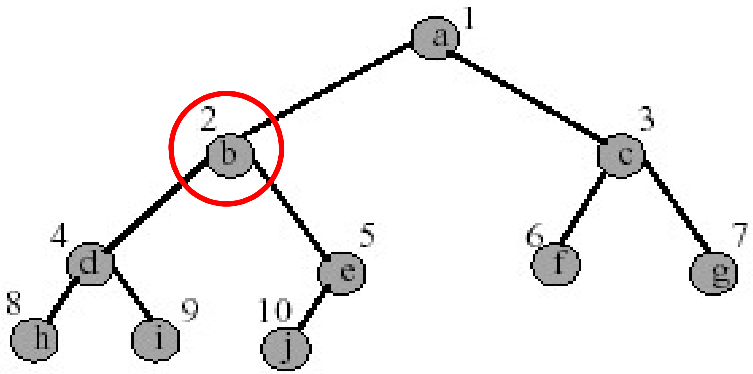


Binary Tree Representation using Array

- Traverse array

  a -> leftChild = T(2*1) = T(2) = b

	a	b	c	d	e	f	g	h	i	j				
0	1	2	3	4	5	6	7	8	9	10				

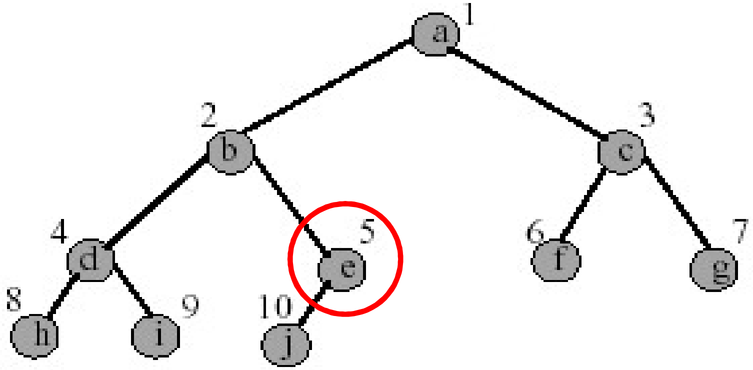


Binary Tree Representation using Array

- Traverse array

  b -> rightChild = T(2*2+1) = T(5) = e

	a	b	c	d	e	f	g	h	i	j				
0	1	2	3	4	5	6	7	8	9	10				

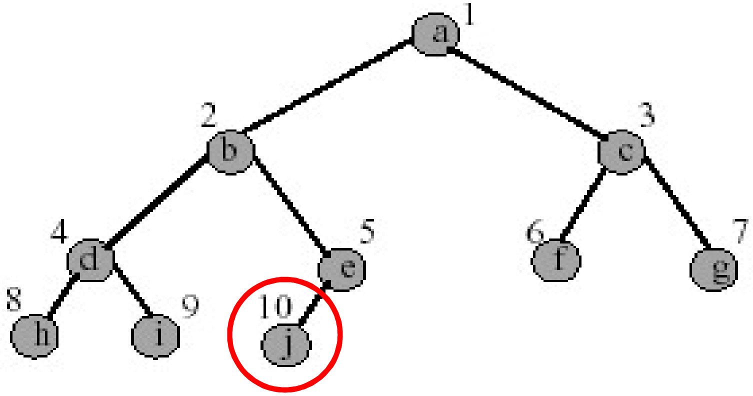


Tree Representation using Array

- Traverse array

e -> leftChild = T(2*5) = T(10) = j

	a	b	c	d	e	f	g	h	i	j				
0	1	2	3	4	5	6	7	8	9	10				



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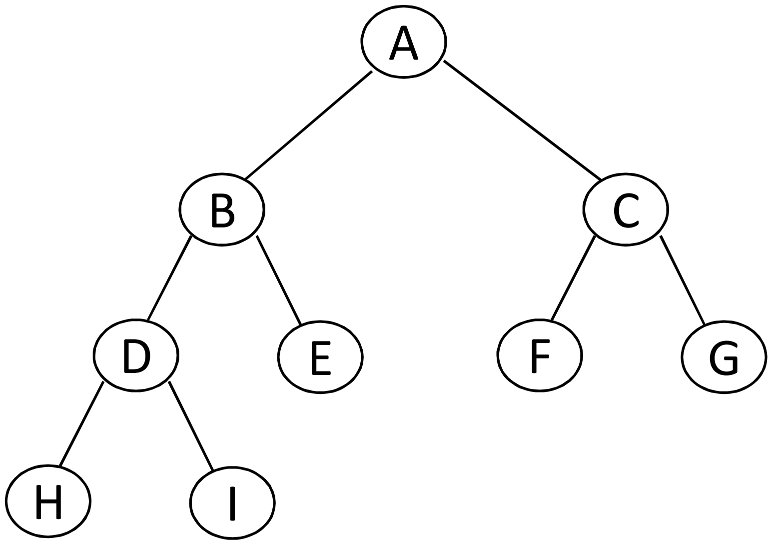
Binary Tree Representation using Array

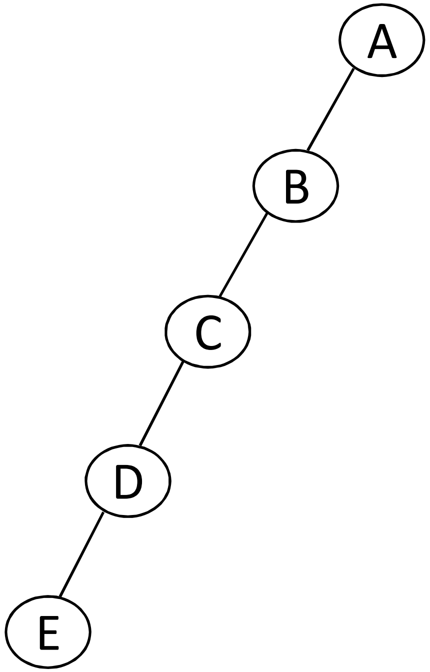
- Space usage
  - Best case : binary tree is complete



[0]	-
[1]	A
[2]	B
[3]	C
[4]	D
[5]	E
[6]	F
[7]	G
[8]	H
[9]	I

Binary Tree Representation using Array

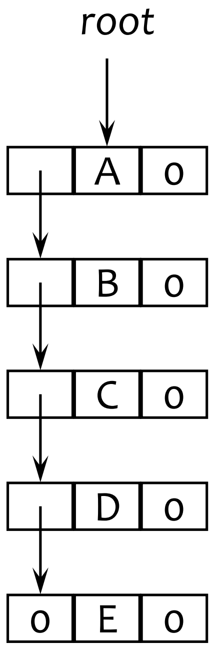
- Space usage
  - Worst case : binary tree is skewed
    - k+1 is used out of 2^{k+1}-1 (height k)



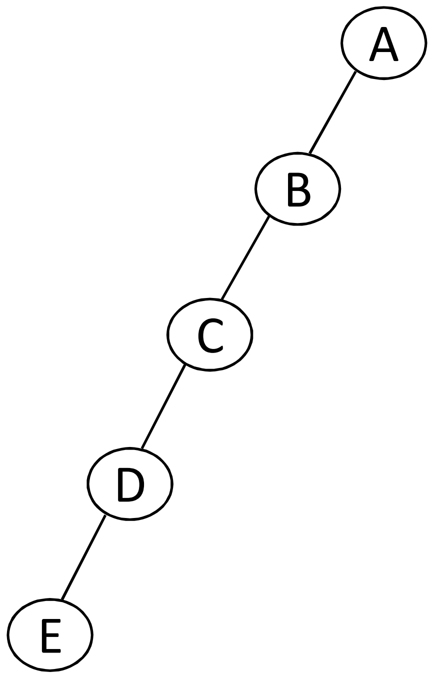
[0]	-
[1]	A
[2]	B
[3]	-
[4]	C
[5]	-
[6]	-
[7]	-
[8]	D
[9]	-
. . .	. . .
[16]	E

Array v.s. Linked List for Binary Trees

- Worst case in array-based binary tree



Linked list based
(space = 15)

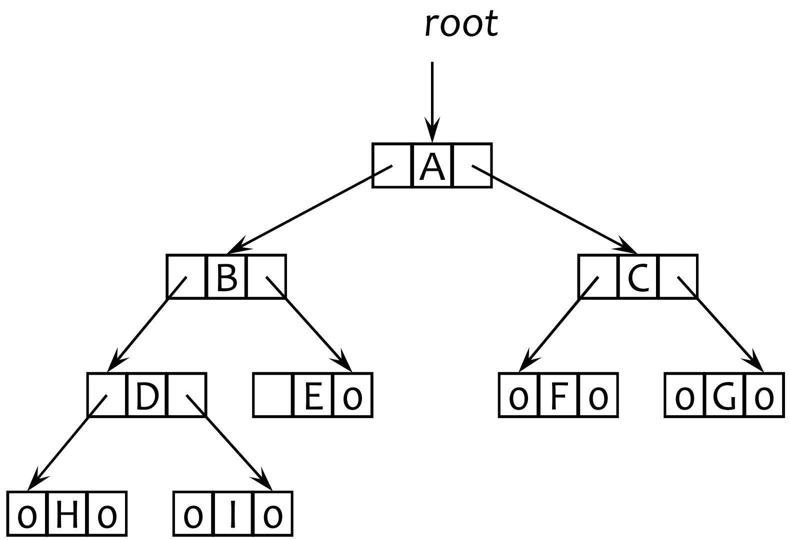


Array based
(space = 32: right skew,
17: left skew)

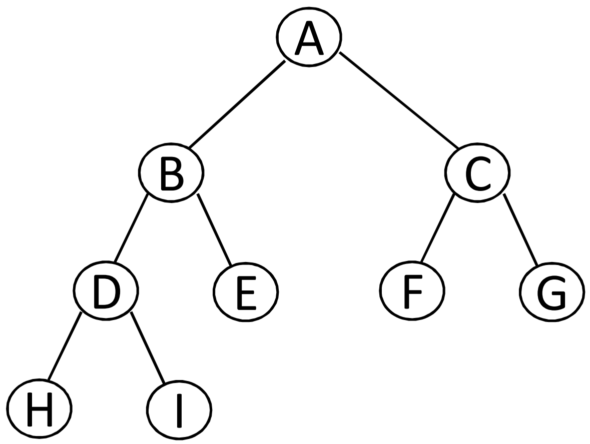
[0]	-
[1]	A
[2]	B
[3]	-
[4]	C
[5]	-
[6]	-
[7]	-
[8]	D
[9]	-
. . .	. . .
[16]	E

Array v.s. Linked List for Binary Trees

- Best case in array-based binary tree



Linked list based
(space = 27)



Array based
(space = 10)

[0]	-
[1]	A
[2]	B
[3]	C
[4]	D
[5]	E
[6]	F
[7]	G
[8]	H
[9]	I

Tree Using Linked List

- Pros
  - Efficient memory management in many incomplete binary trees
  - Node insert / delete can be fast
    - Array-based tree requires shifting elements
- Cons
  - Extra memory to store pointers for complete trees
  - Traversing parent needs extra pointer per node

data	PARENT	$CHILD_1$	…	$CHILD_k$

Binary Tree Class

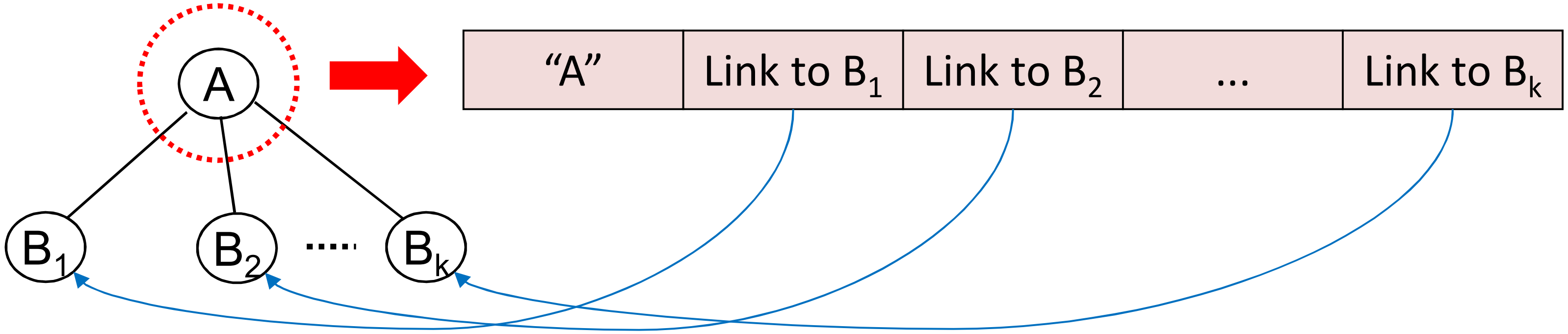
```
class TreeNode{
private:
        TreeNode *LeftChild;
        char data;
        TreeNode *RightChild;
};

class Tree {
public:
        // Tree operations
        ...
private:
        TreeNode *root;
};
```

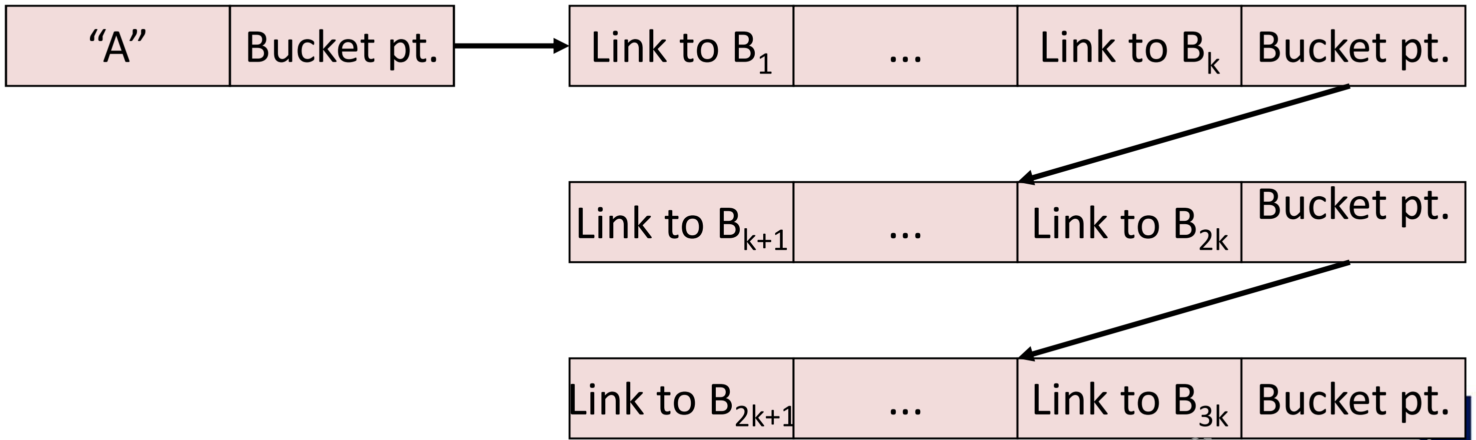
Can We Make Node Size Flexible?

- A lot of engineering trick is possible
  - Bucket size k can be arbitrary, but not 1 (↑overhead)
  - Dynamically add a bucket to have more childs
  - Can merge buckets if there are too many buckets

Increasing Degree Dynamically



If node A has to include any number of children



Questions?