AI70601/CSE61101

3D Vision and Machine Perception

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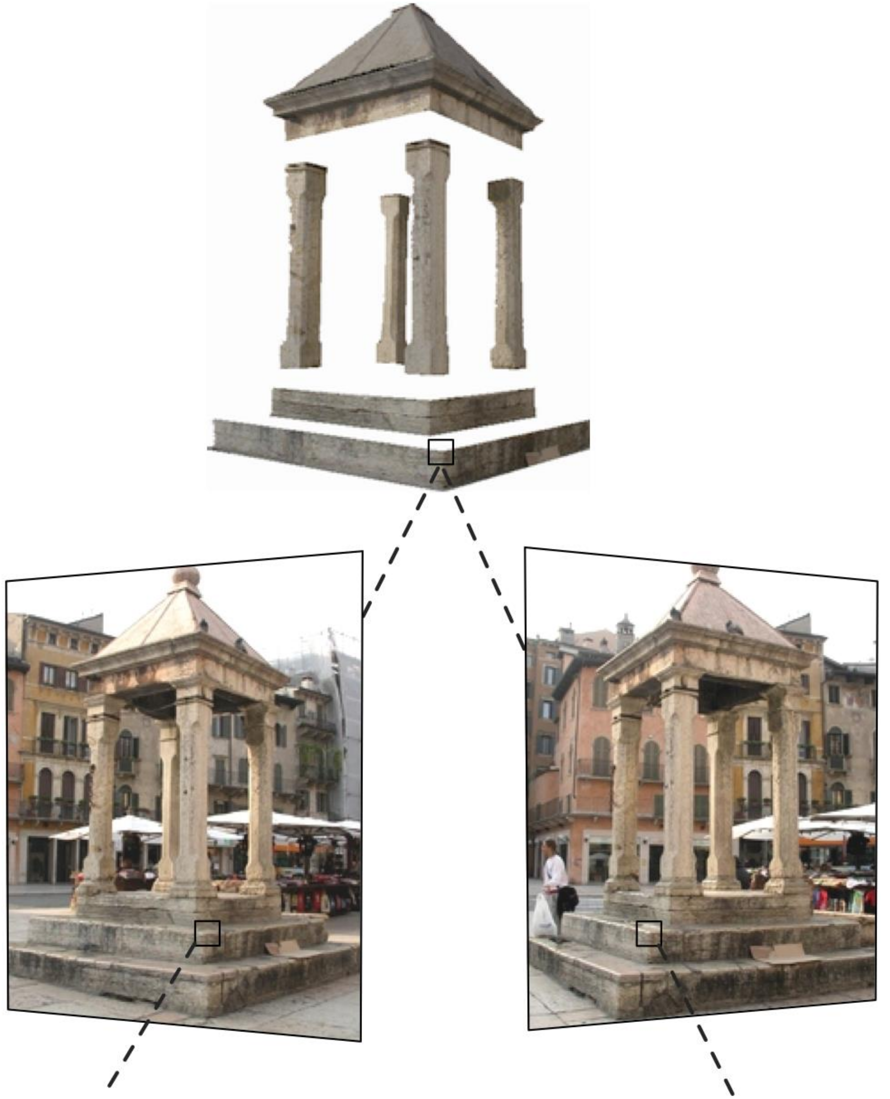
Some materials, figures, and slides (used for this course) are from textbooks, published papers, and other open lectures

Two-view geometry

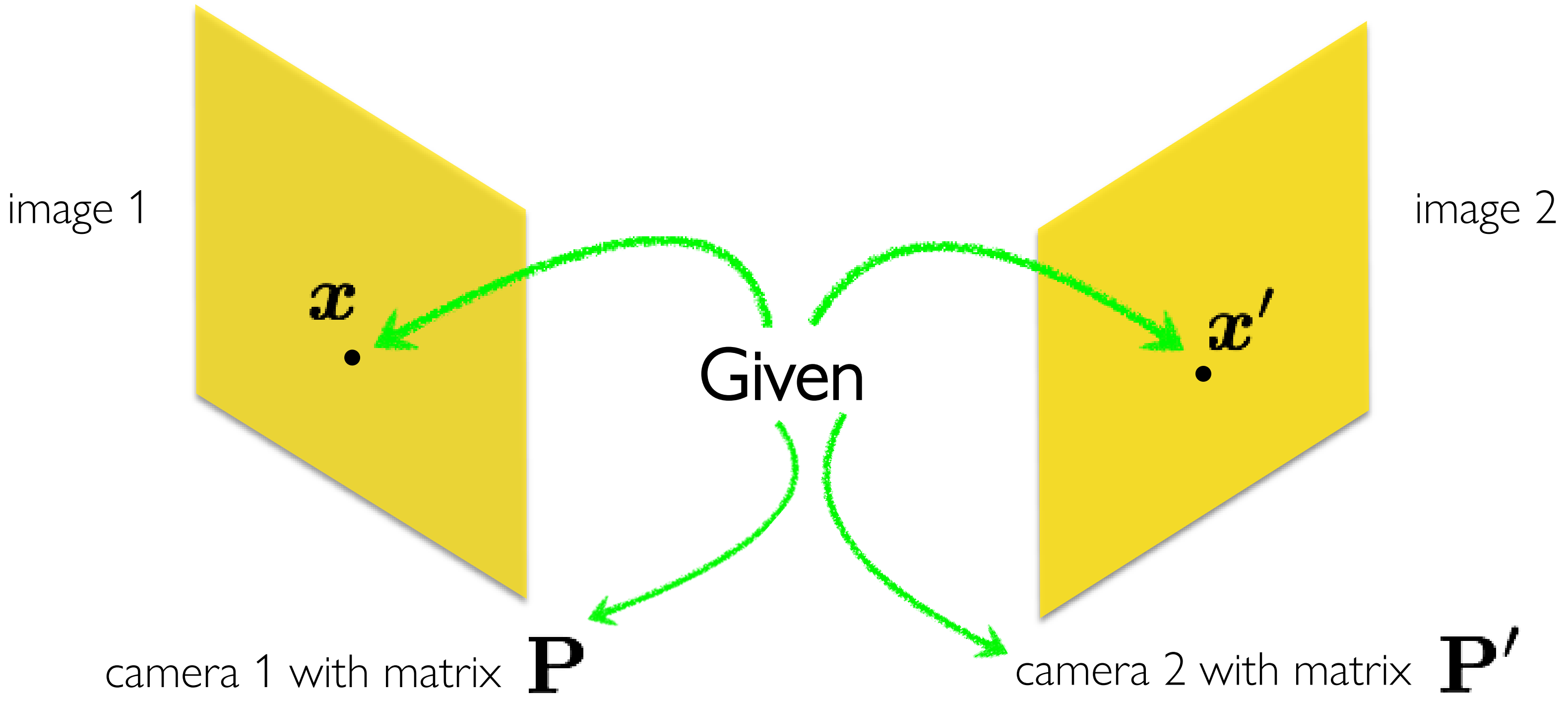


Contents

- Triangulation.
- Epipolar geometry.
- Essential matrix.
- Fundamental matrix.
- 8-point algorithm.

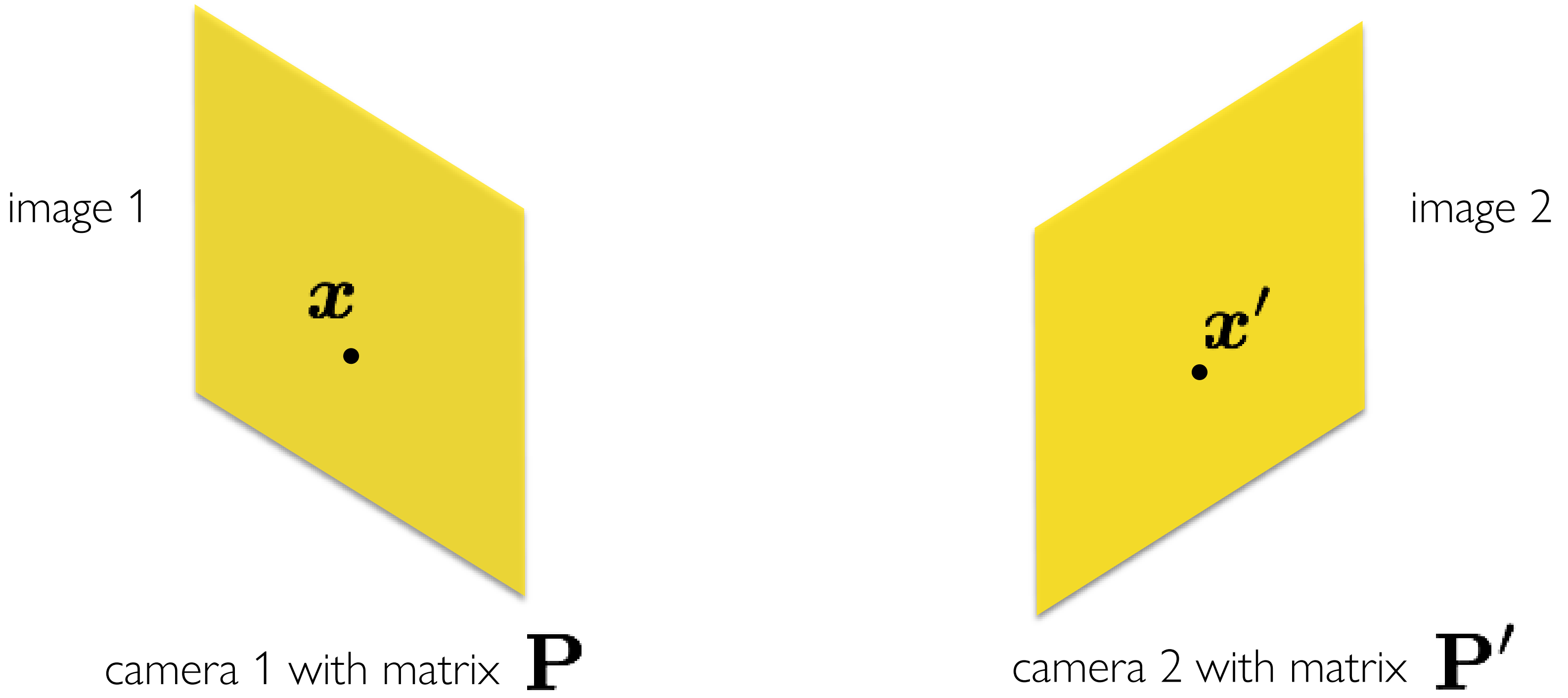
Triangulation

Triangulation



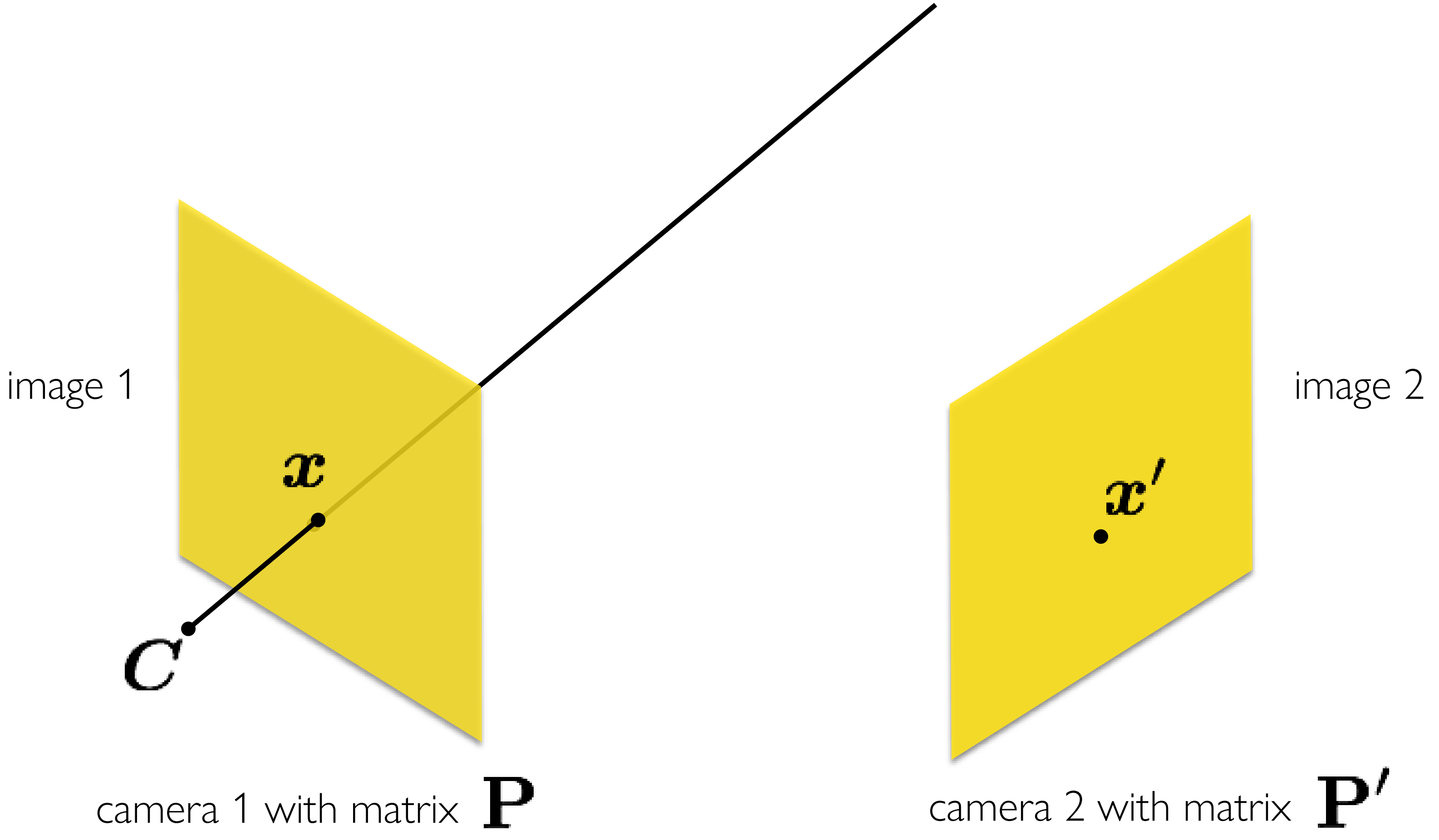
Triangulation

- Which 3D points map to **x**?



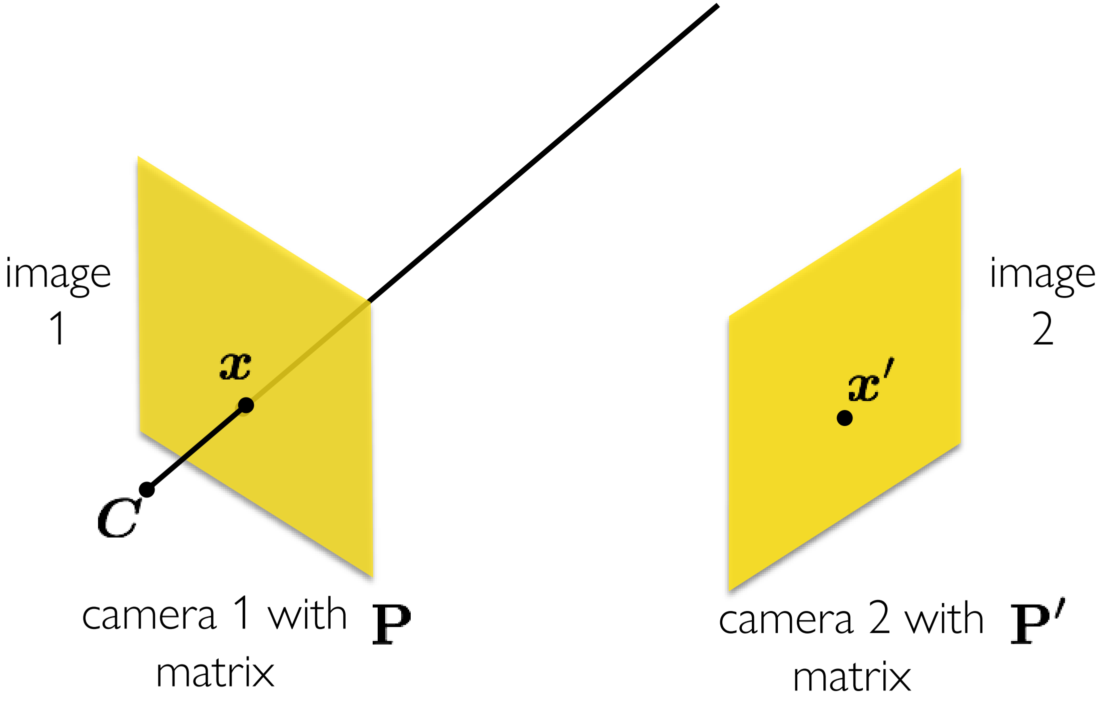
Triangulation

- How can you compute this ray?



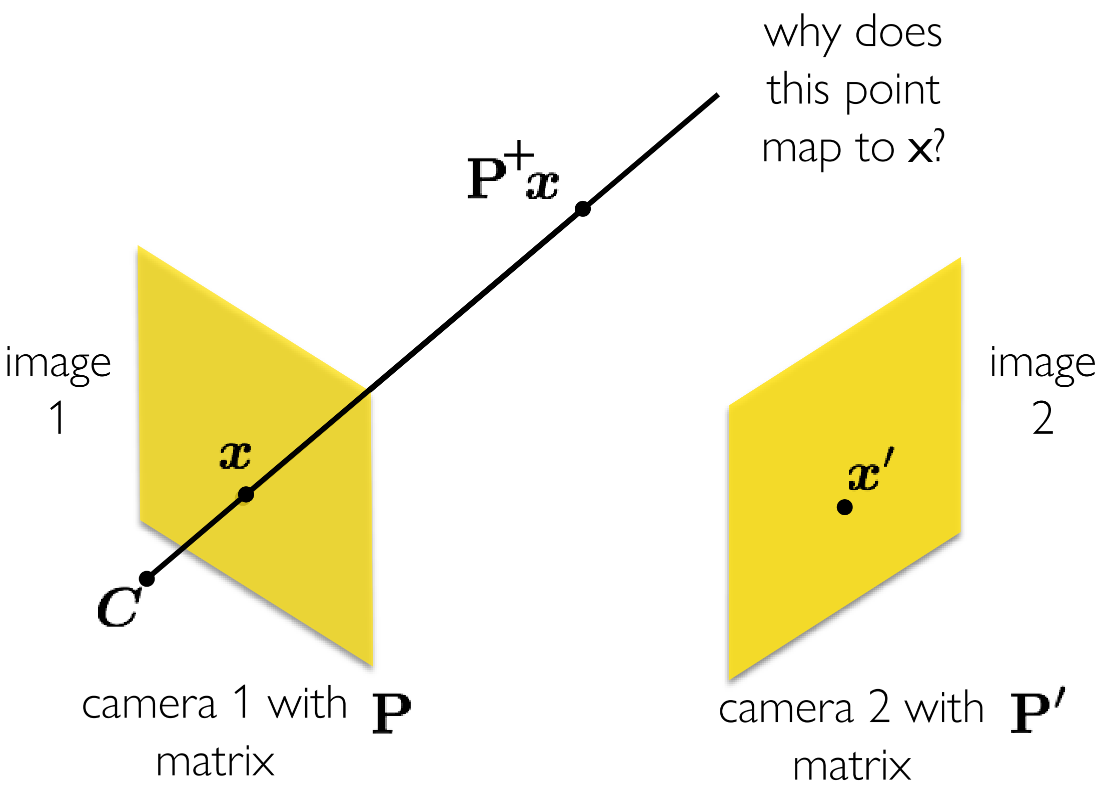
Triangulation

- Create two points on the ray:
  - Find the camera center; and



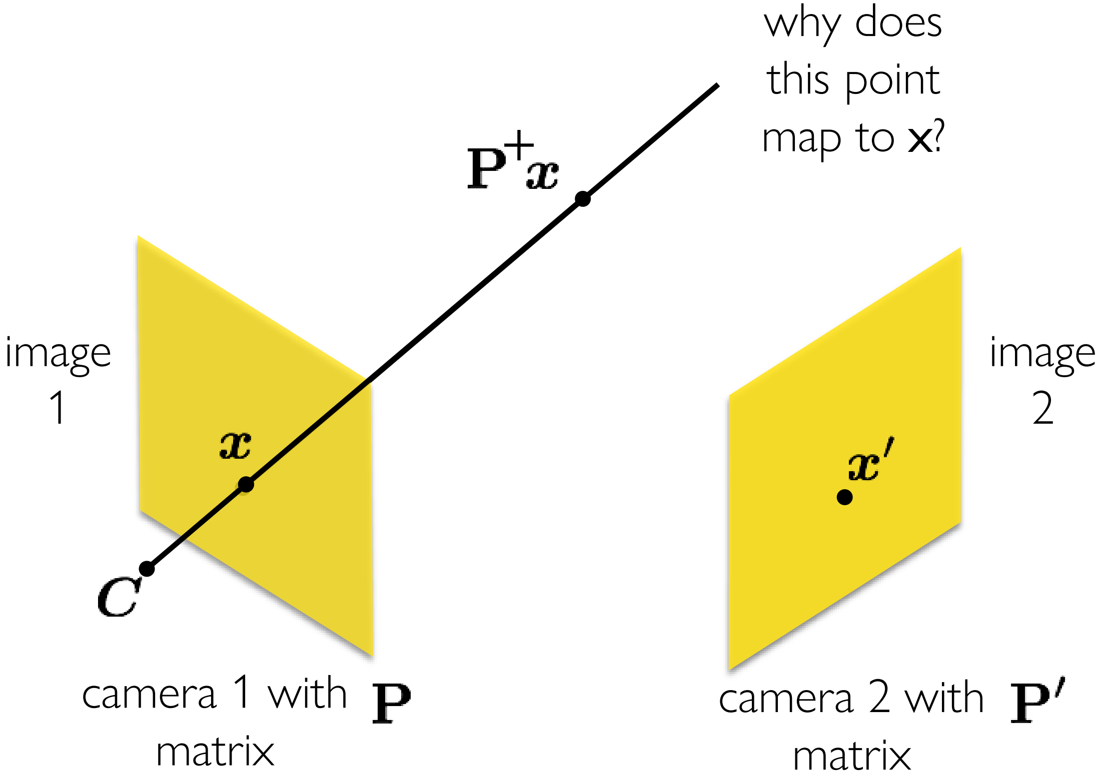
Triangulation

- Create two points on the ray:
  - Find the camera center; and
  - Apply the pseudo-inverse of P on x.



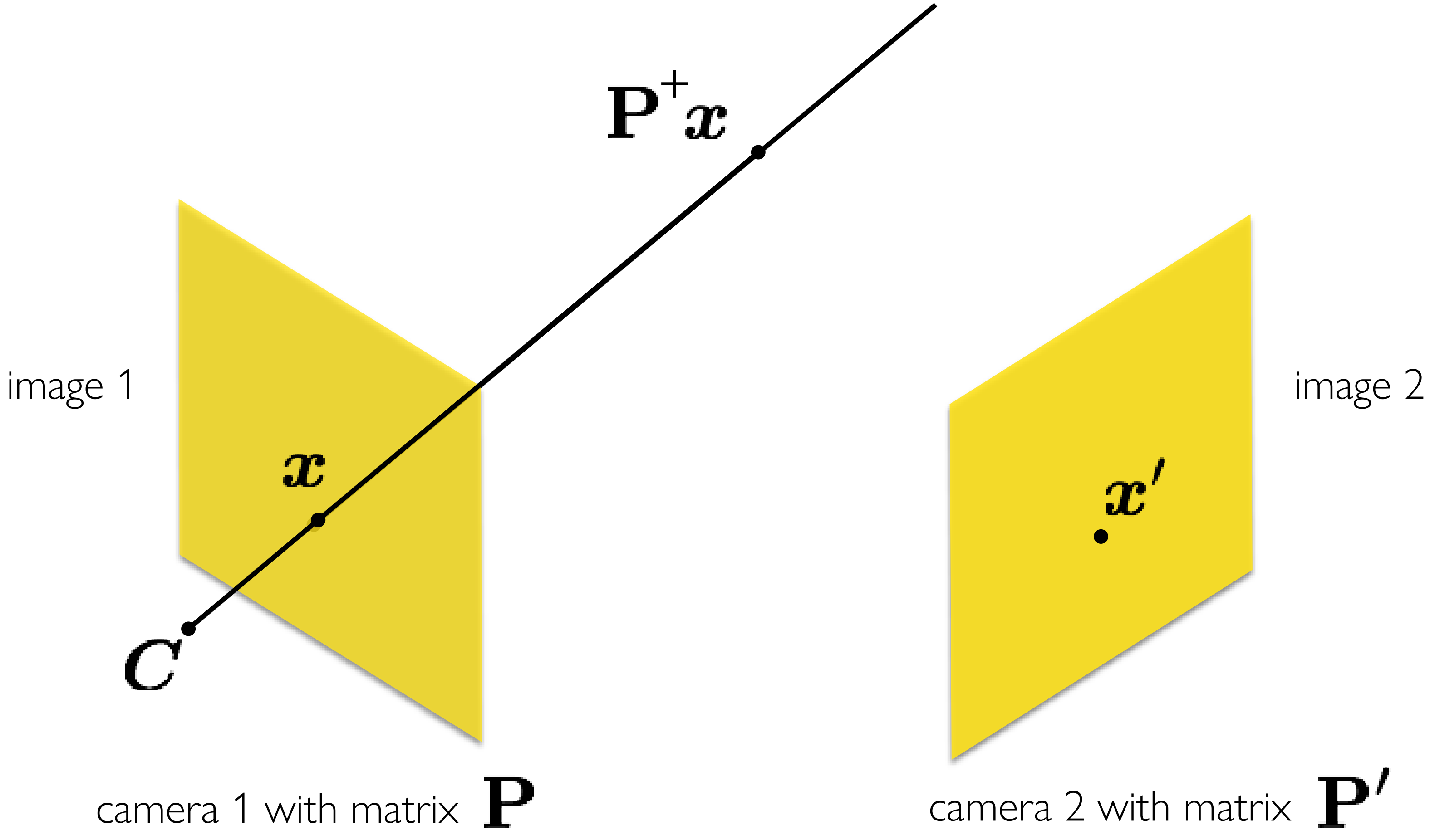
Triangulation

- Create two points on the ray:
  - Find the camera center; and
  - Apply the pseudo-inverse of P on x.
  - Then connect the two points.
- **This procedure is called back-projection**



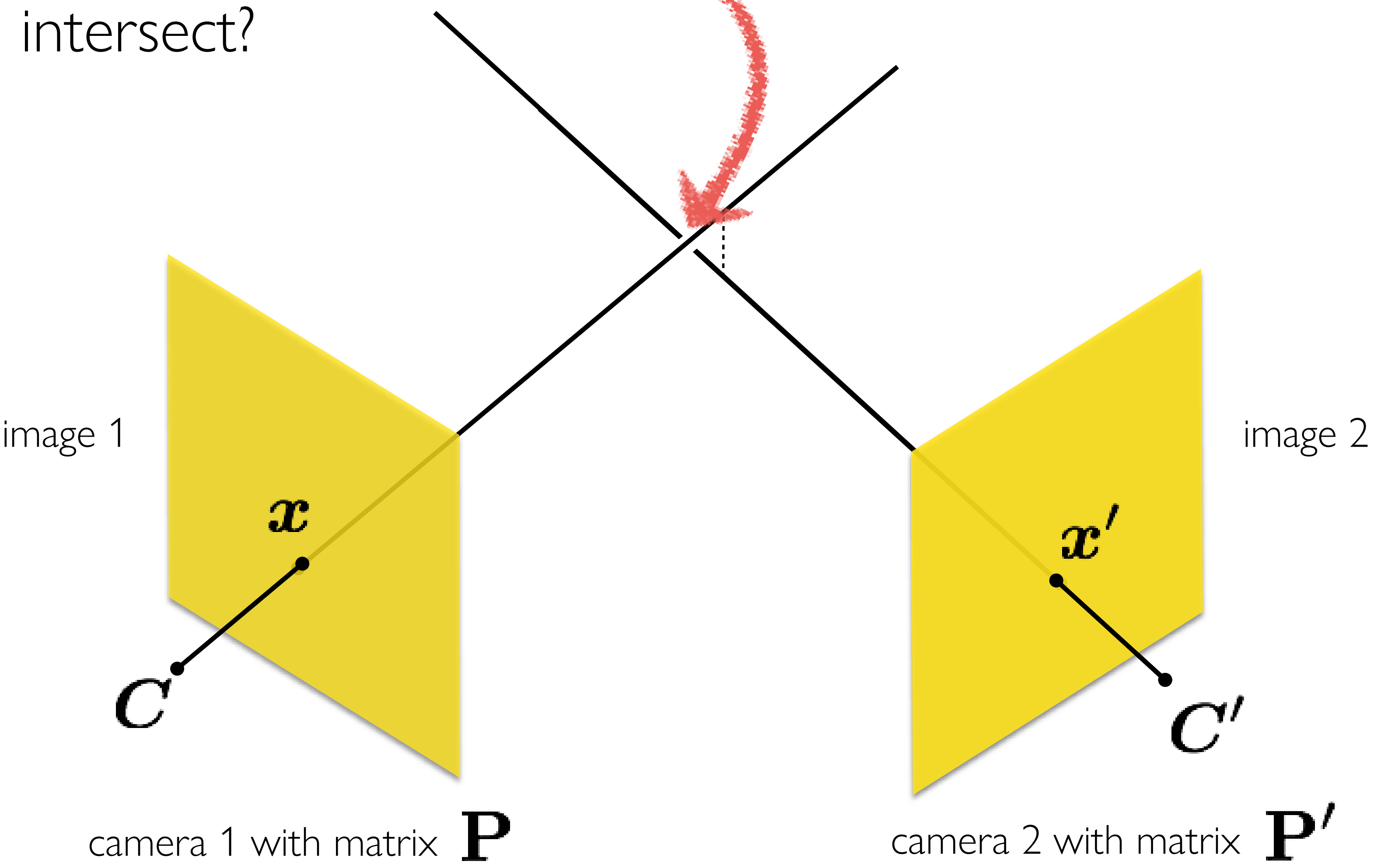
Triangulation

- How do we find the exact point on the ray?



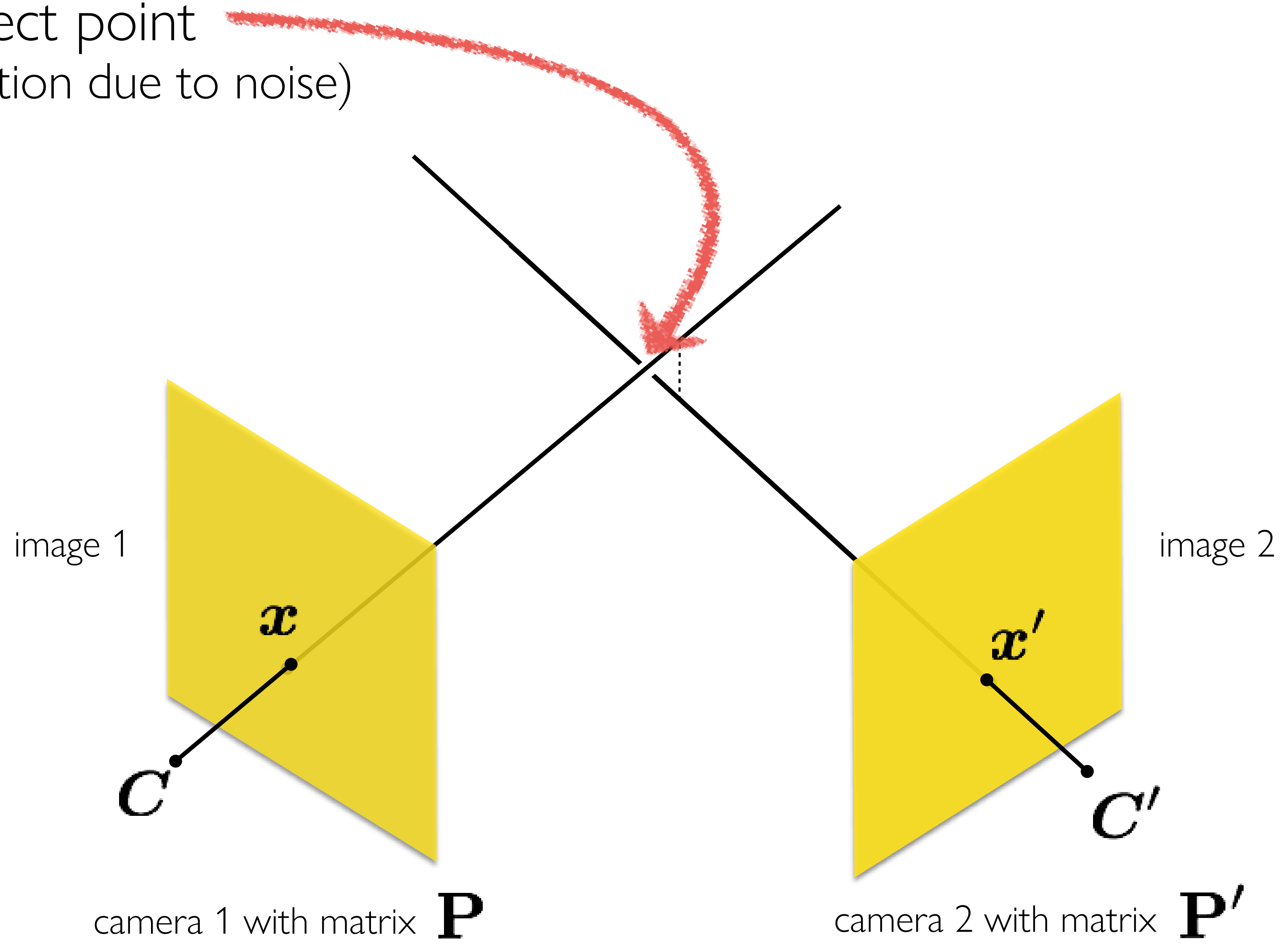
Triangulation

- Find 3D object point
- Will the lines intersect?



Triangulation

- Find 3D object point
  (no single solution due to noise)



Triangulation

- Given a set of (noisy) matched points : $\{\boldsymbol{x}_i, \boldsymbol{x}'_i\}$

- And camera matrices : $\mathbf{P}, \mathbf{P}'$

- Estimate the 3D point : $\mathbf{X}$

- Can we compute X from a single correspondence x?

$$\underset{\text{known}}{\mathbf{x}} = \underset{\text{known}}{\mathbf{P}}\boldsymbol{X}$$

$$\mathbf{x} = \mathbf{P}\boldsymbol{X}$$

homogeneous
coordinate

- This is a similarity relation because it involves homogeneous coordinates

$$\mathbf{x} = \alpha\mathbf{P}\boldsymbol{X}$$

homogeneous
coordinate

- Same ray direction but differs by a scale factor

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \alpha \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

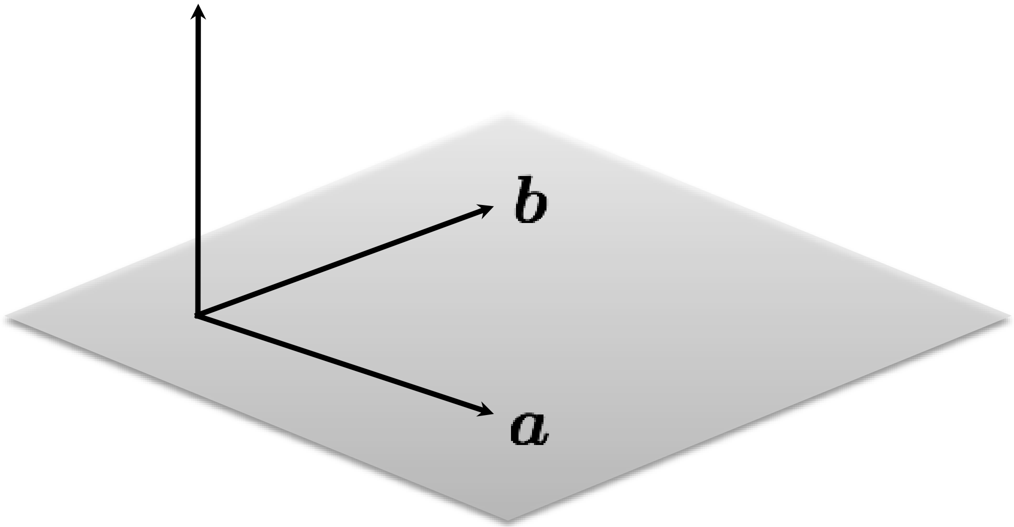
- How do we solve for unknowns in a similarity relation?

Linear algebra reminder: cross product

- Vector (cross) product
    - Takes two vectors and returns a vector <u>perpendicular</u> to both

$$\boldsymbol{a} \times \boldsymbol{b} = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$$

$$\boldsymbol{c} = \boldsymbol{a} \times \boldsymbol{b}$$



$$\boldsymbol{c} \cdot \boldsymbol{a} = 0 \qquad \boldsymbol{c} \cdot \boldsymbol{b} = 0$$

- Cross product of two vectors in the same direction is zero vector

$$\boldsymbol{a} \times \boldsymbol{a} = 0$$

- Remember this!!!

Linear algebra reminder: cross product

- Cross product

$$\boldsymbol{a} \times \boldsymbol{b} = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$$

- Can be also written as a matrix multiplication

$$\boldsymbol{a} \times \boldsymbol{b} = [\boldsymbol{a}]_\times \boldsymbol{b} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

skew symmetric

- Back to triangulation

$$\mathbf{x} = \alpha \mathbf{P} \boldsymbol{X}$$

- Same direction but differs by a scale factor
- How can we rewrite this using vector products?

- Back to triangulation

$$\mathbf{x} = \alpha \mathbf{P} \boldsymbol{X}$$

- Same direction but differs by a scale factor

$$\mathbf{x} \times \mathbf{P} \boldsymbol{X} = \mathbf{0}$$

- Cross product of two vectors of same direction is zero
  (this equality removes the scale factor)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \alpha \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \alpha \begin{bmatrix} \text{---} & \boldsymbol{p}_1^\top & \text{---} \\ \text{---} & \boldsymbol{p}_2^\top & \text{---} \\ \text{---} & \boldsymbol{p}_3^\top & \text{---} \end{bmatrix} \begin{bmatrix} | \\ \boldsymbol{X} \\ | \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \alpha \begin{bmatrix} \boldsymbol{p}_1^\top \boldsymbol{X} \\ \boldsymbol{p}_2^\top \boldsymbol{X} \\ \boldsymbol{p}_3^\top \boldsymbol{X} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \times \begin{bmatrix} \boldsymbol{p}_1^\top \boldsymbol{X} \\ \boldsymbol{p}_2^\top \boldsymbol{X} \\ \boldsymbol{p}_3^\top \boldsymbol{X} \end{bmatrix} = \begin{bmatrix} y\boldsymbol{p}_3^\top \boldsymbol{X} - \boldsymbol{p}_2^\top \boldsymbol{X} \\ \boldsymbol{p}_1^\top \boldsymbol{X} - x\boldsymbol{p}_3^\top \boldsymbol{X} \\ x\boldsymbol{p}_2^\top \boldsymbol{X} - y\boldsymbol{p}_1^\top \boldsymbol{X} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

- Using the fact that the cross product should be zero

$$\mathbf{x} \times \mathbf{P}\boldsymbol{X} = \mathbf{0}$$

$$\begin{bmatrix} y\boldsymbol{p}_3^\top \boldsymbol{X} - \boldsymbol{p}_2^\top \boldsymbol{X} \\ \boldsymbol{p}_1^\top \boldsymbol{X} - x\boldsymbol{p}_3^\top \boldsymbol{X} \\ x\boldsymbol{p}_2^\top \boldsymbol{X} - y\boldsymbol{p}_1^\top \boldsymbol{X} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

- Third line is a linear combination of the first and second lines.
  (x times the first line plus y times the second line)

- One 2D to 3D point correspondence give you $\boxed{}$ equations

- Using the fact that the cross product should be zero

$$\mathbf{x} \times \mathbf{P}\boldsymbol{X} = \mathbf{0}$$

$$\begin{bmatrix} y\boldsymbol{p}_3^\top \boldsymbol{X} - \boldsymbol{p}_2^\top \boldsymbol{X} \\ \boldsymbol{p}_1^\top \boldsymbol{X} - x\boldsymbol{p}_3^\top \boldsymbol{X} \\ x\boldsymbol{p}_2^\top \boldsymbol{X} - y\boldsymbol{p}_1^\top \boldsymbol{X} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

- Third line is a linear combination of the first and second lines.
  (x times the first line plus y times the second line)

- One 2D to 3D point correspondence give you 2 equations

$$\begin{bmatrix} y\boldsymbol{p}_3^\top \boldsymbol{X} - \boldsymbol{p}_2^\top \boldsymbol{X} \\ \boldsymbol{p}_1^\top \boldsymbol{X} - x\boldsymbol{p}_3^\top \boldsymbol{X} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

remove third row, and rearrange as system on unknowns

$$\begin{bmatrix} y\boldsymbol{p}_3^\top - \boldsymbol{p}_2^\top \\ \boldsymbol{p}_1^\top - x\boldsymbol{p}_3^\top \end{bmatrix} \boldsymbol{X} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\mathbf{A}_i \boldsymbol{X} = \mathbf{0}$$

- Now we can make a system of linear equations
  (two lines for each 2D point correspondence)

- Concatenate the 2D points from both images

two rows from camera one

two rows from camera two

$$\begin{bmatrix} y\boldsymbol{p}_3^\top - \boldsymbol{p}_2^\top \\ \boldsymbol{p}_1^\top - x\boldsymbol{p}_3^\top \\ y'\boldsymbol{p}_3'^\top - \boldsymbol{p}_2'^\top \\ \boldsymbol{p}_1'^\top - x'\boldsymbol{p}_3'^\top \end{bmatrix} \boldsymbol{X} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

sanity check! dimensions?

$$\mathbf{A}_i \boldsymbol{X} = \mathbf{0}$$

- How do we solve homogeneous linear system?

- Concatenate the 2D points from both images

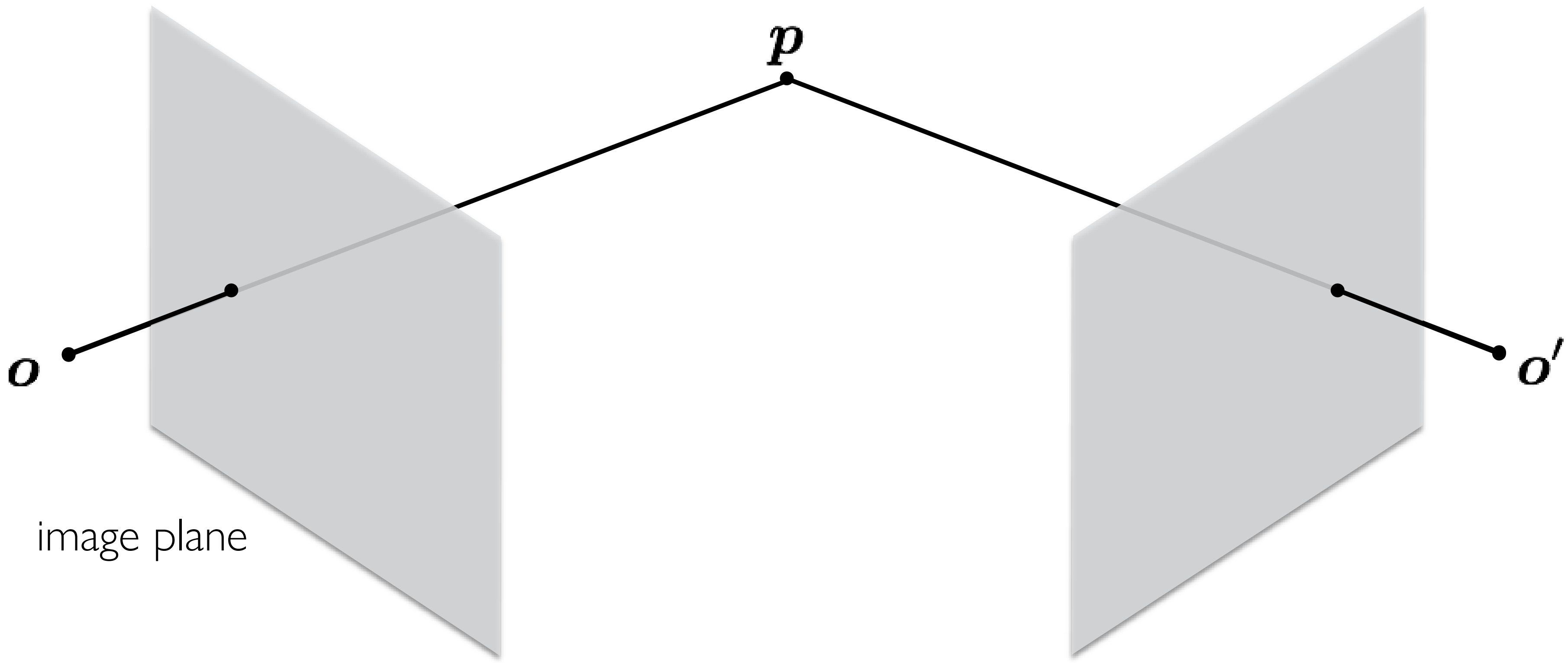
$$\begin{bmatrix} y\boldsymbol{p}_3^\top - \boldsymbol{p}_2^\top \\ \boldsymbol{p}_1^\top - x\boldsymbol{p}_3^\top \\ y'\boldsymbol{p}_3'^\top - \boldsymbol{p}_2'^\top \\ \boldsymbol{p}_1'^\top - x'\boldsymbol{p}_3'^\top \end{bmatrix} \boldsymbol{X} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{A}_i \boldsymbol{X} = \mathbf{0}$$

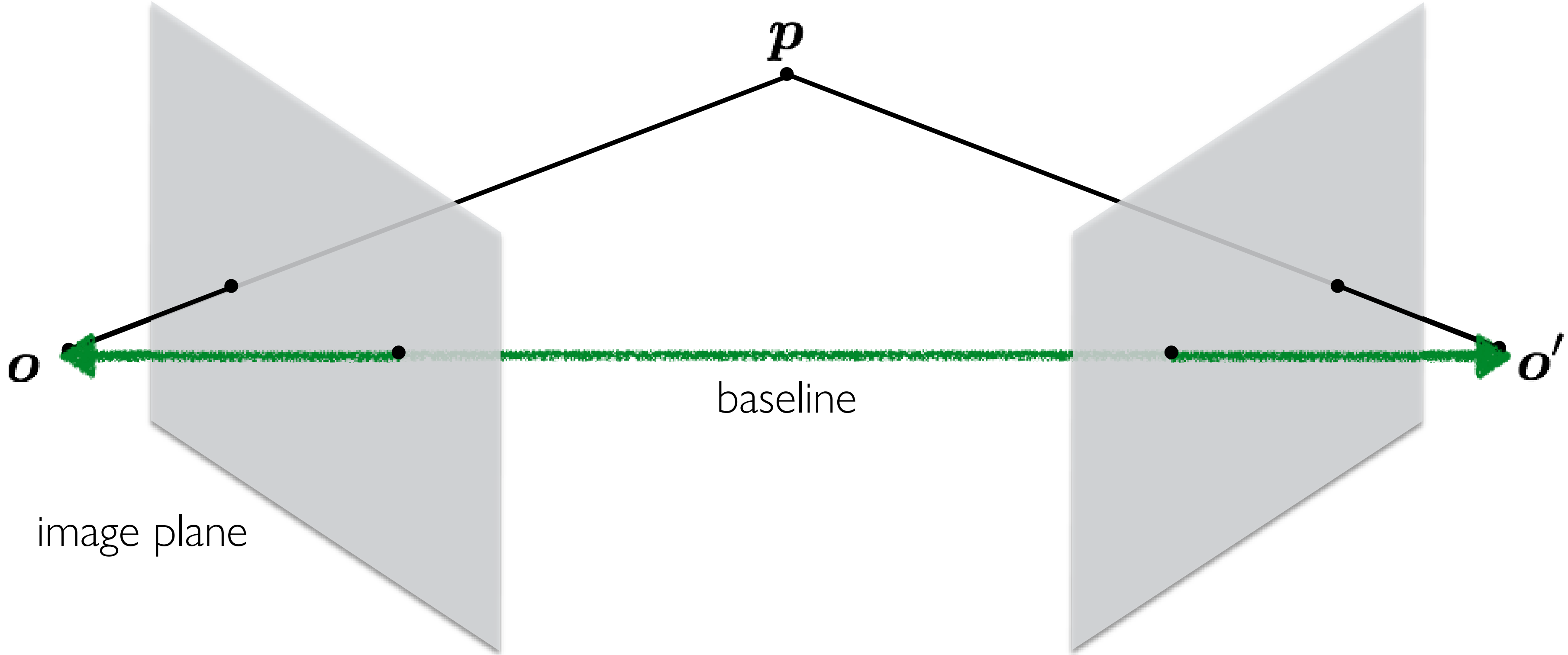
- How do we solve homogeneous linear system? S V D !

Epipolar geometry

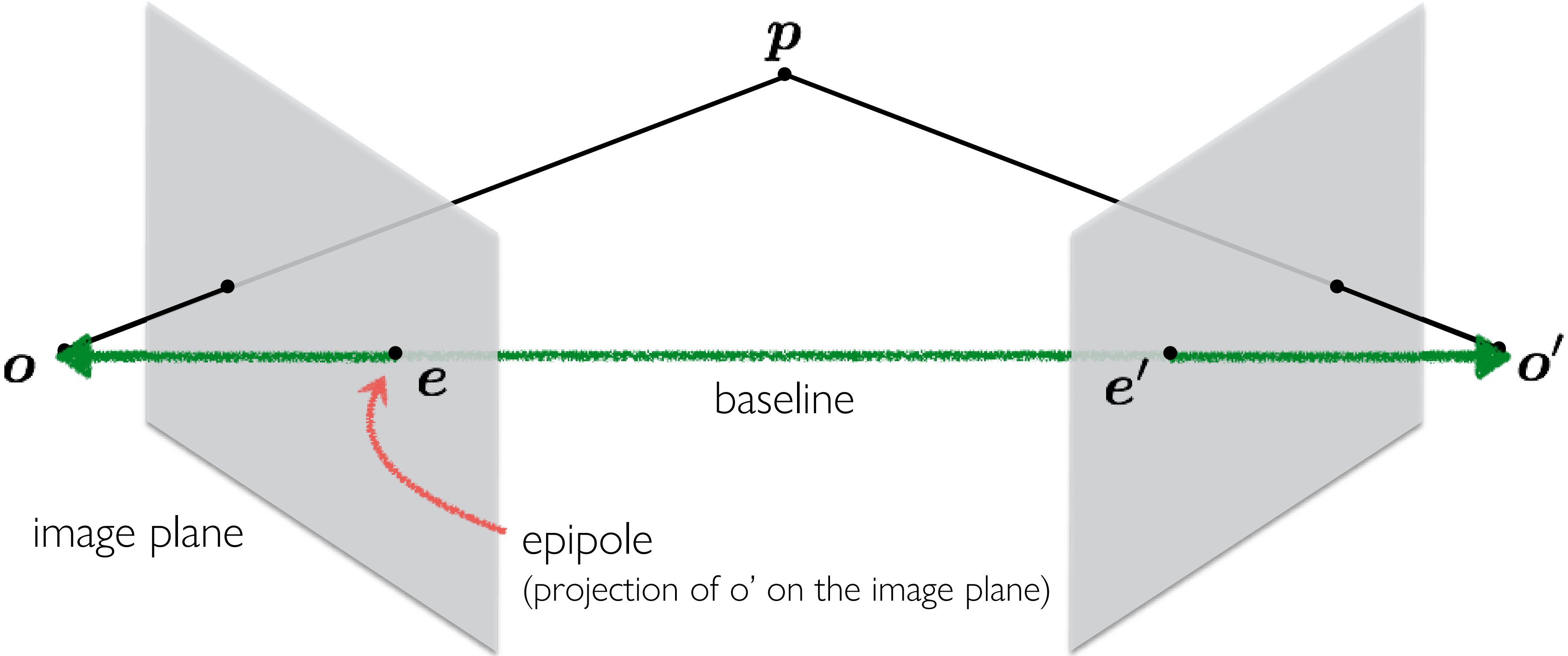
Epipolar geometry



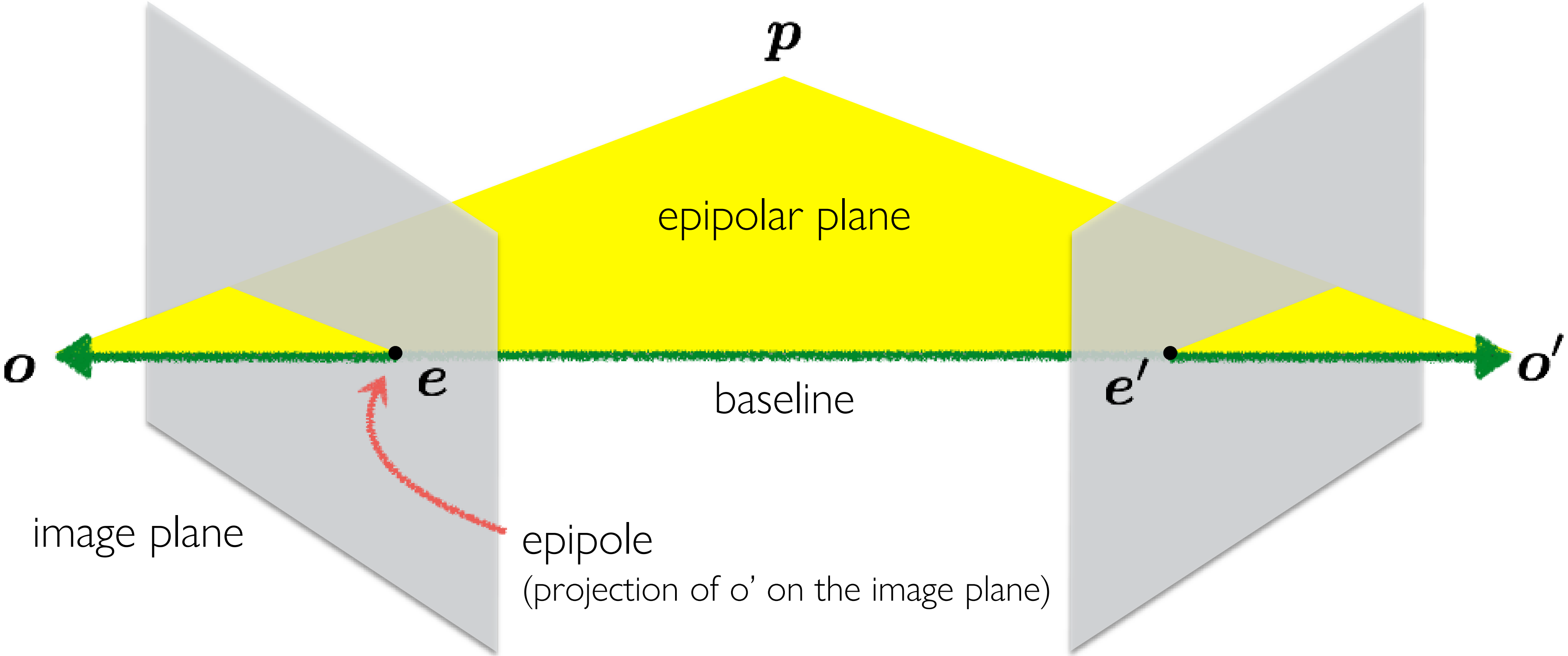
Epipolar geometry



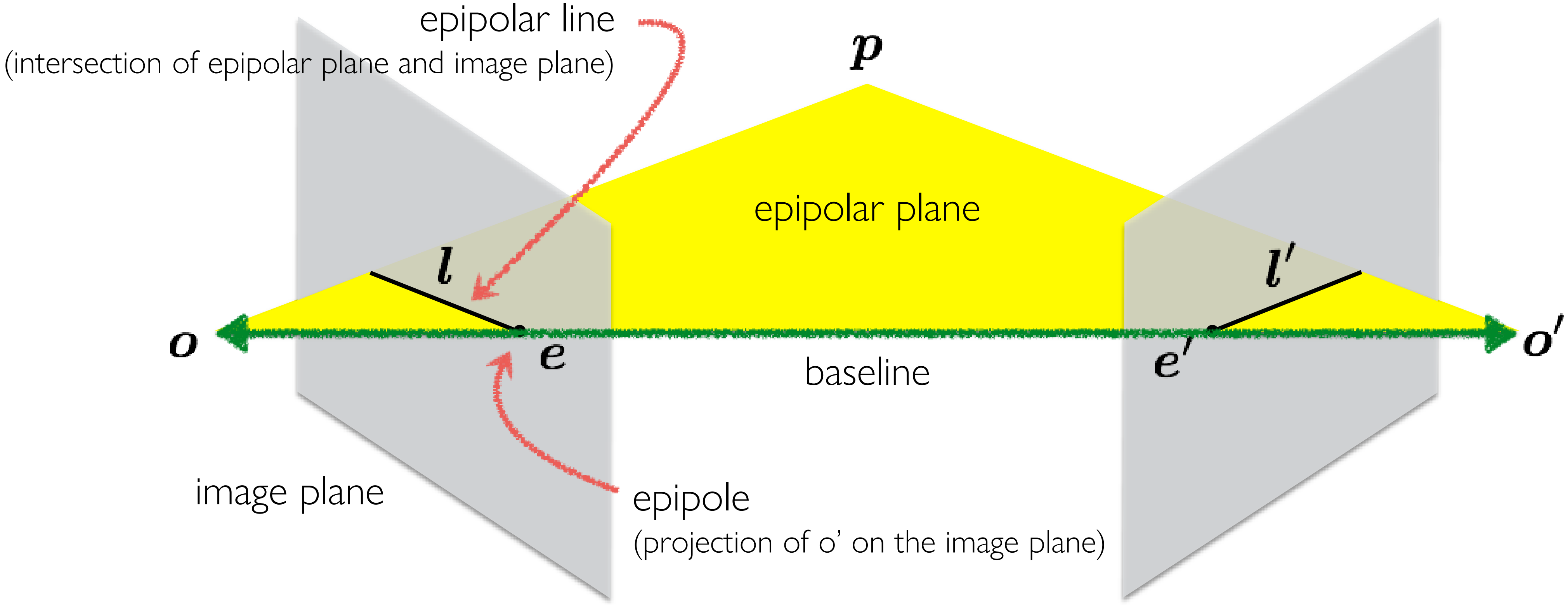
Epipolar geometry



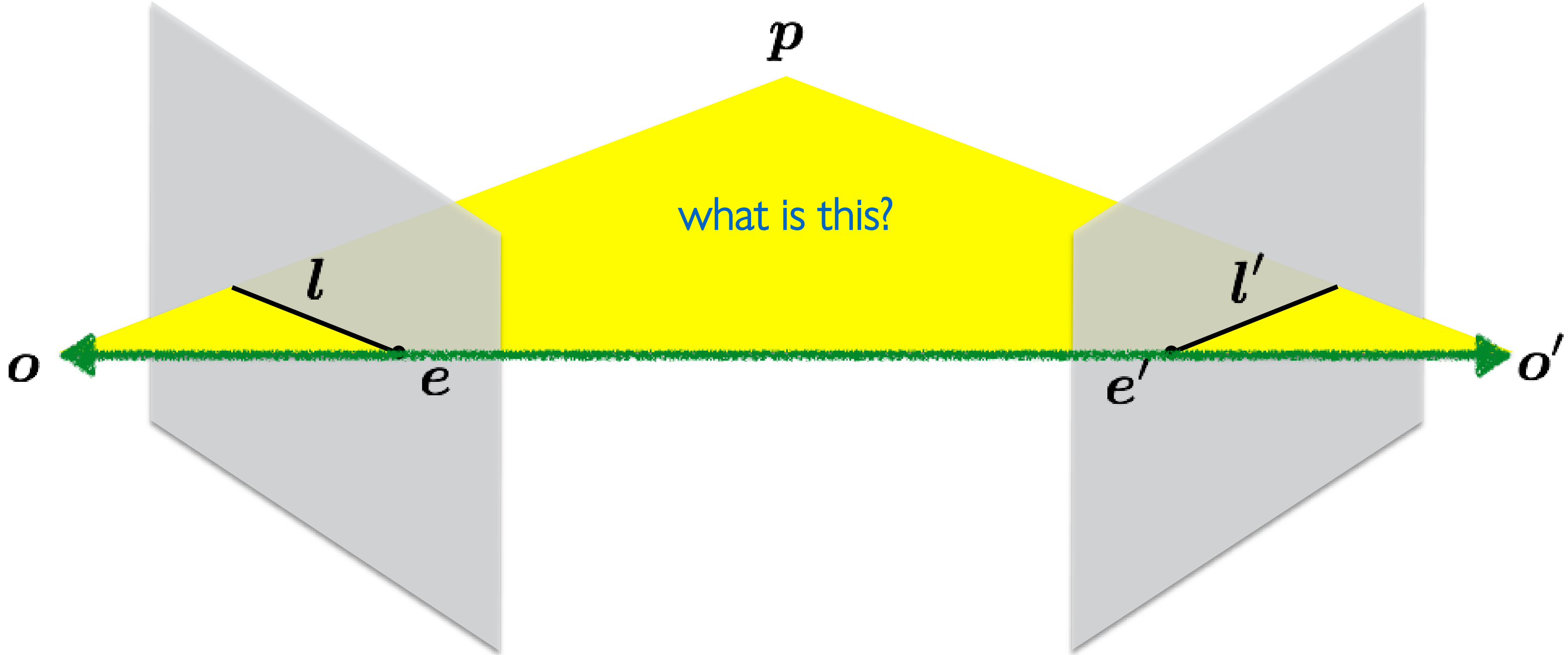
Epipolar geometry



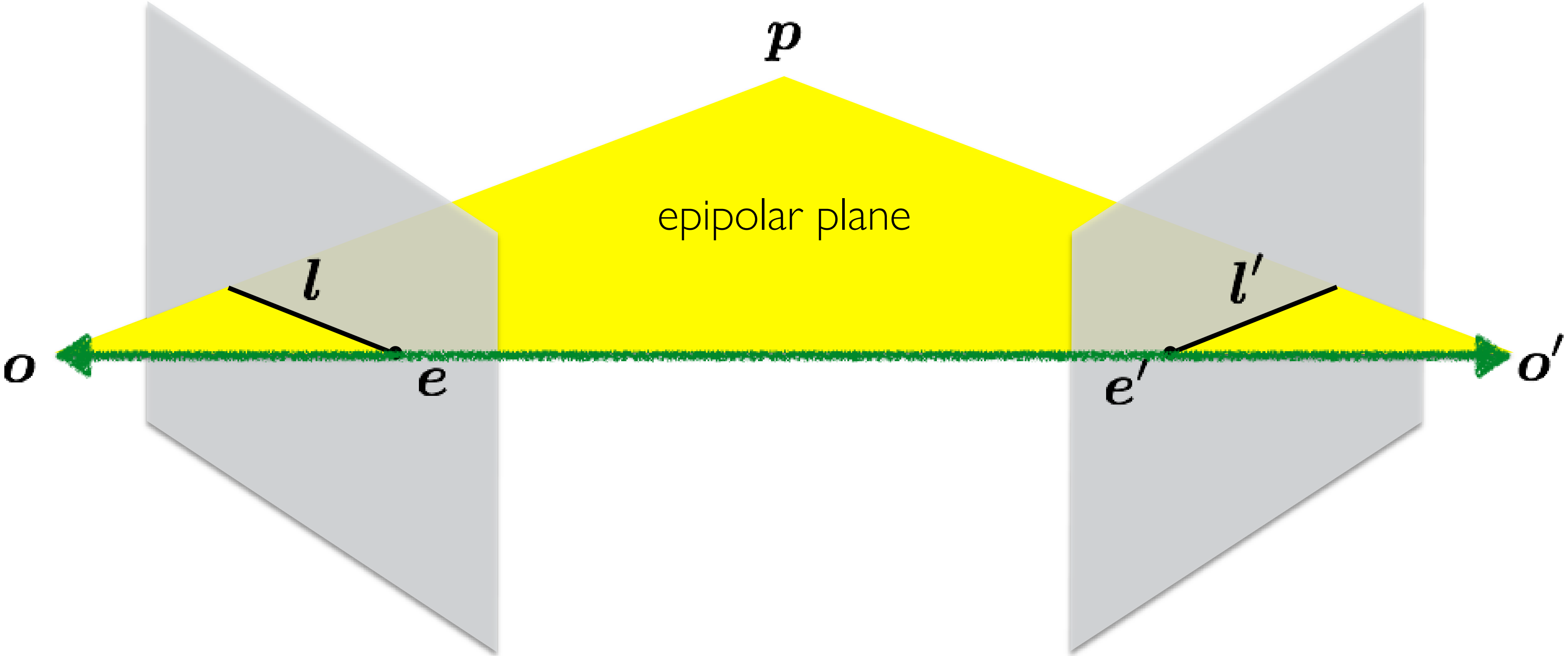
Epipolar geometry



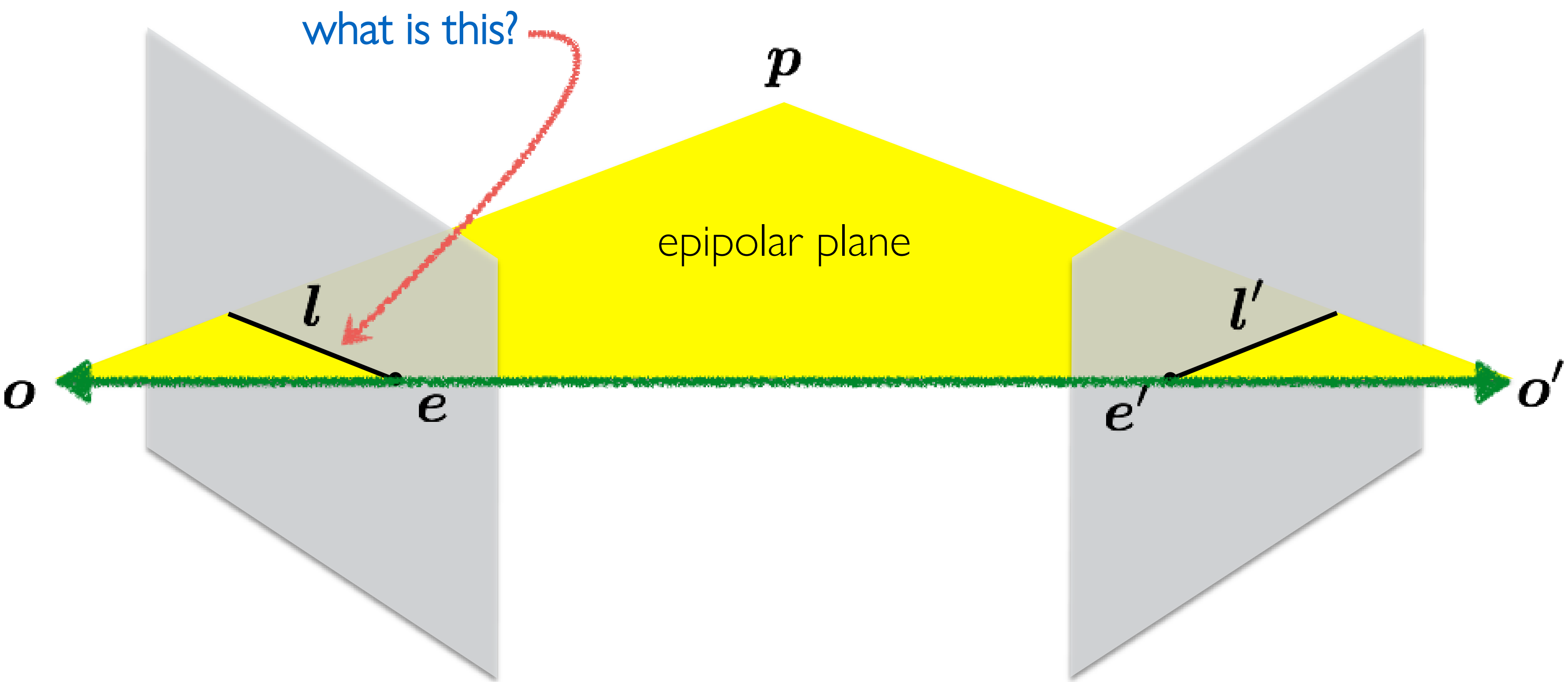
Quiz



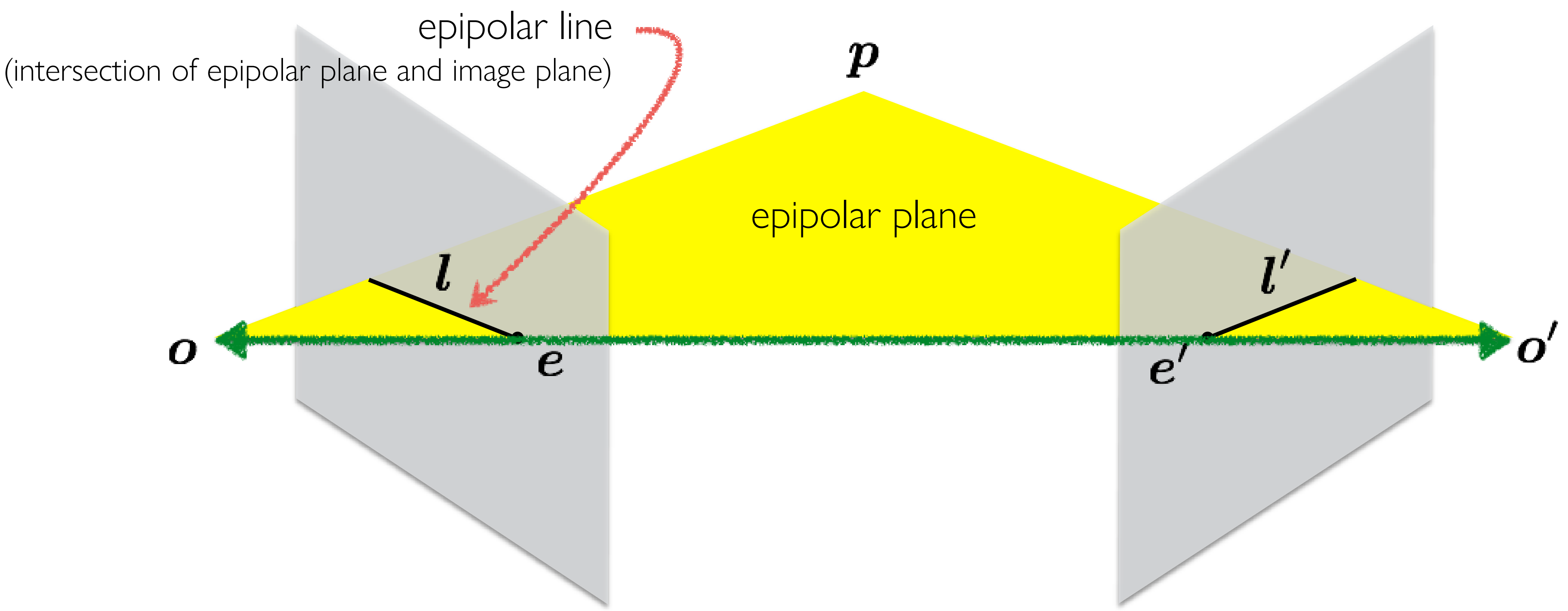
Quiz



Quiz

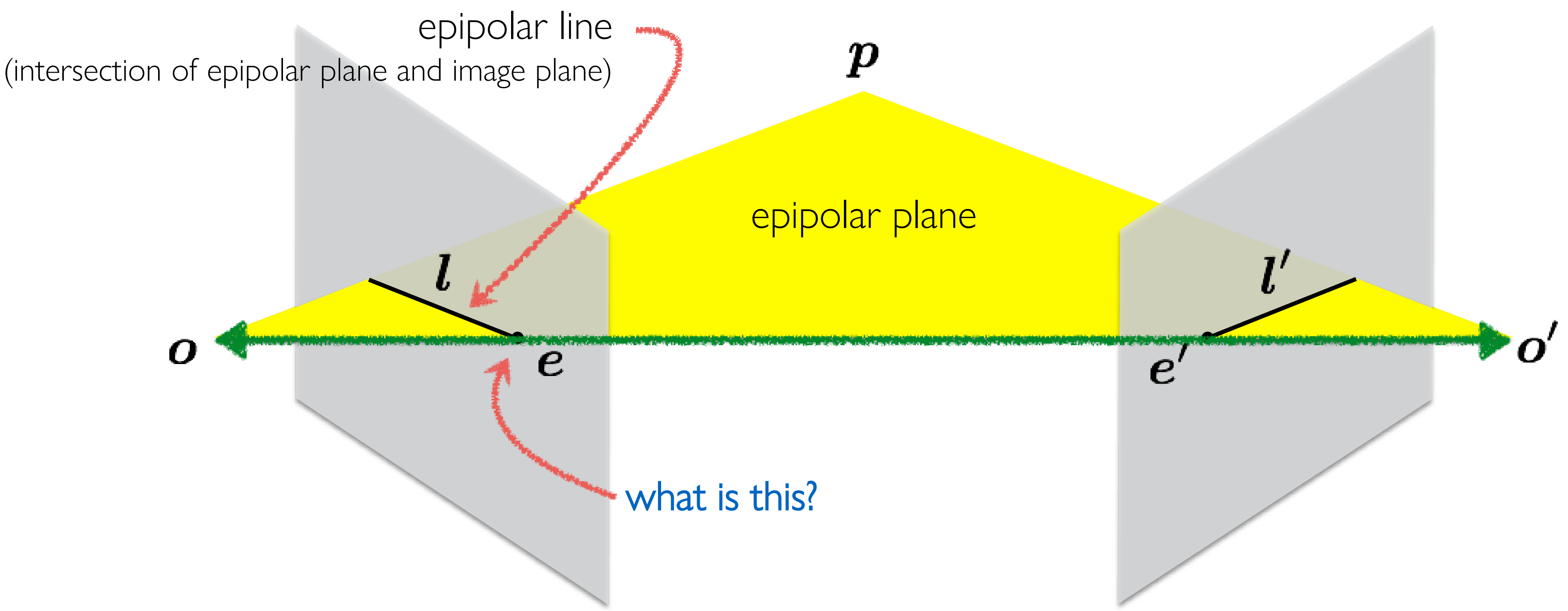

what is this?
$\boldsymbol{p}$
epipolar plane
$\boldsymbol{l}$
$\boldsymbol{l}'$
$\boldsymbol{o}$
$\boldsymbol{e}$
$\boldsymbol{e}'$
$\boldsymbol{o}'$

Quiz



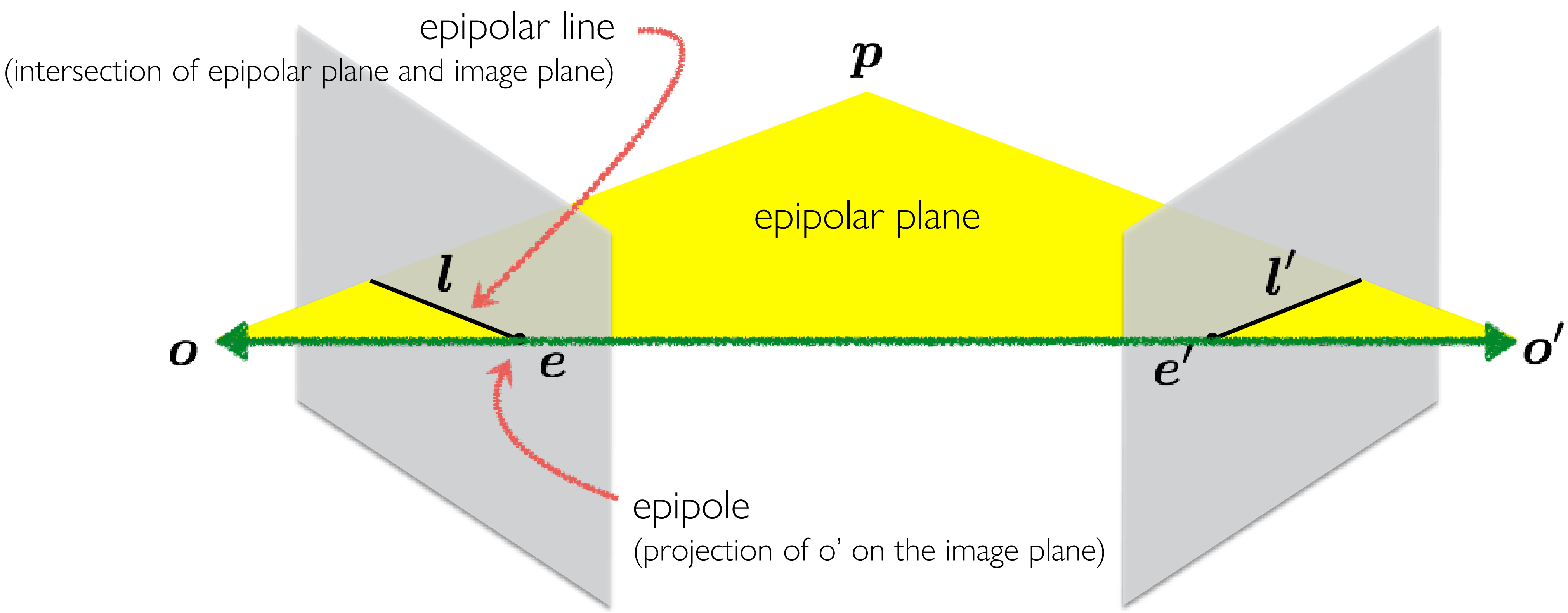
epipolar line
(intersection of epipolar plane and image plane)
$\boldsymbol{p}$
epipolar plane
$\boldsymbol{l}$
$\boldsymbol{l'}$
$\boldsymbol{o}$
$\boldsymbol{e}$
$\boldsymbol{e'}$
$\boldsymbol{o'}$

Quiz

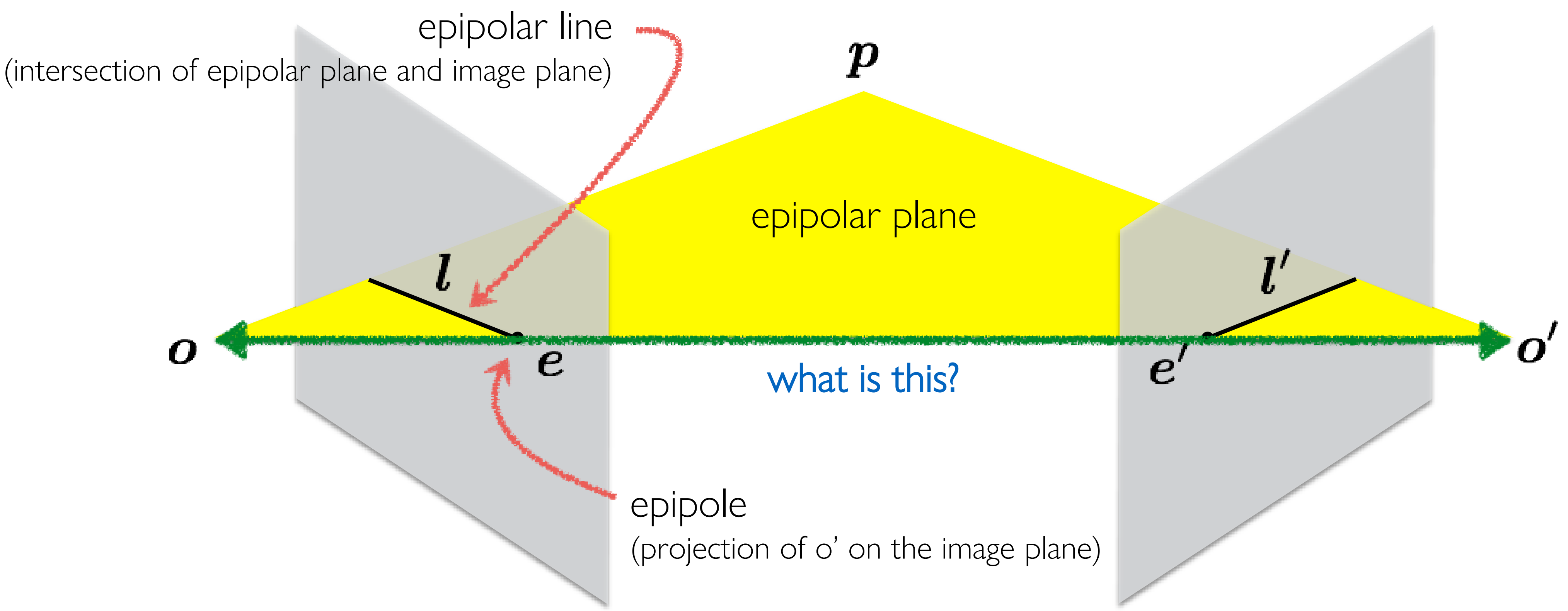


epipolar line
(intersection of epipolar plane and image plane)
$\boldsymbol{p}$
epipolar plane
$\boldsymbol{l}$
$\boldsymbol{l}'$
$\boldsymbol{o}$
$\boldsymbol{e}$
$\boldsymbol{e}'$
$\boldsymbol{o}'$
what is this?

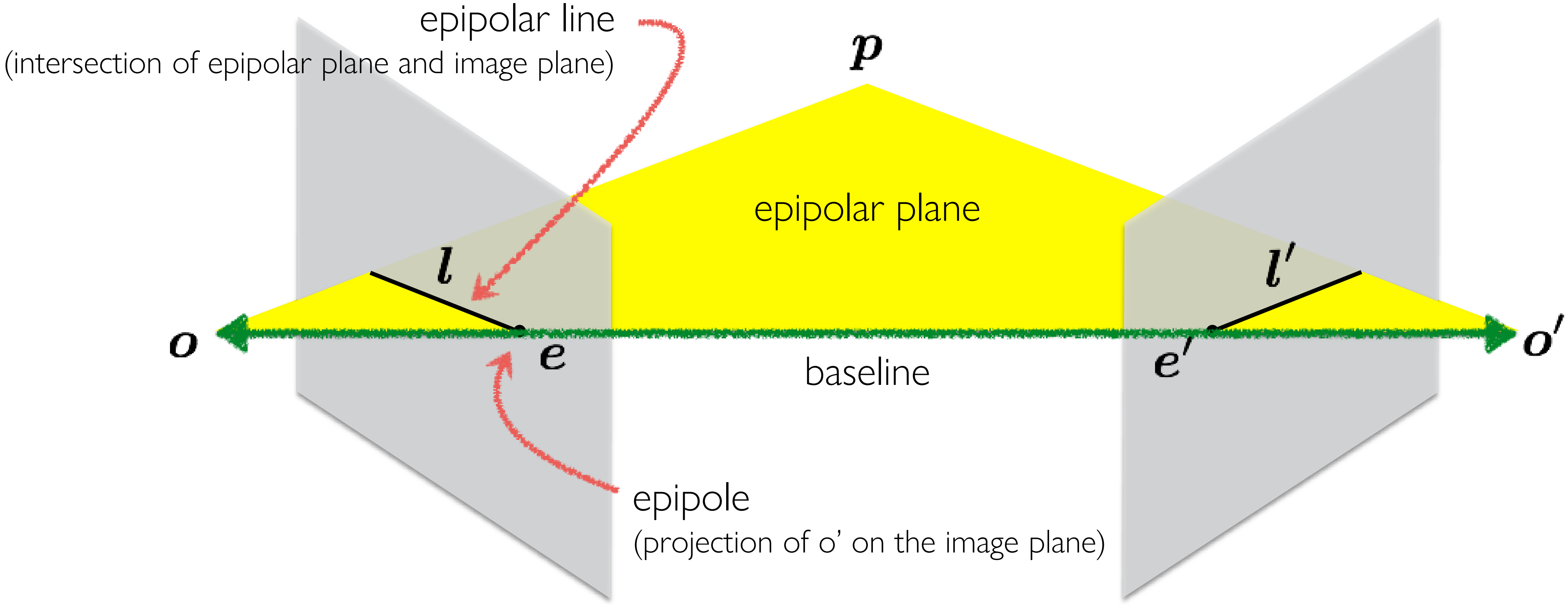
Quiz


epipolar line
(intersection of epipolar plane and image plane)
p
epipolar plane
l
l'
o
e
e'
o'
epipole
(projection of o’ on the image plane)

Quiz

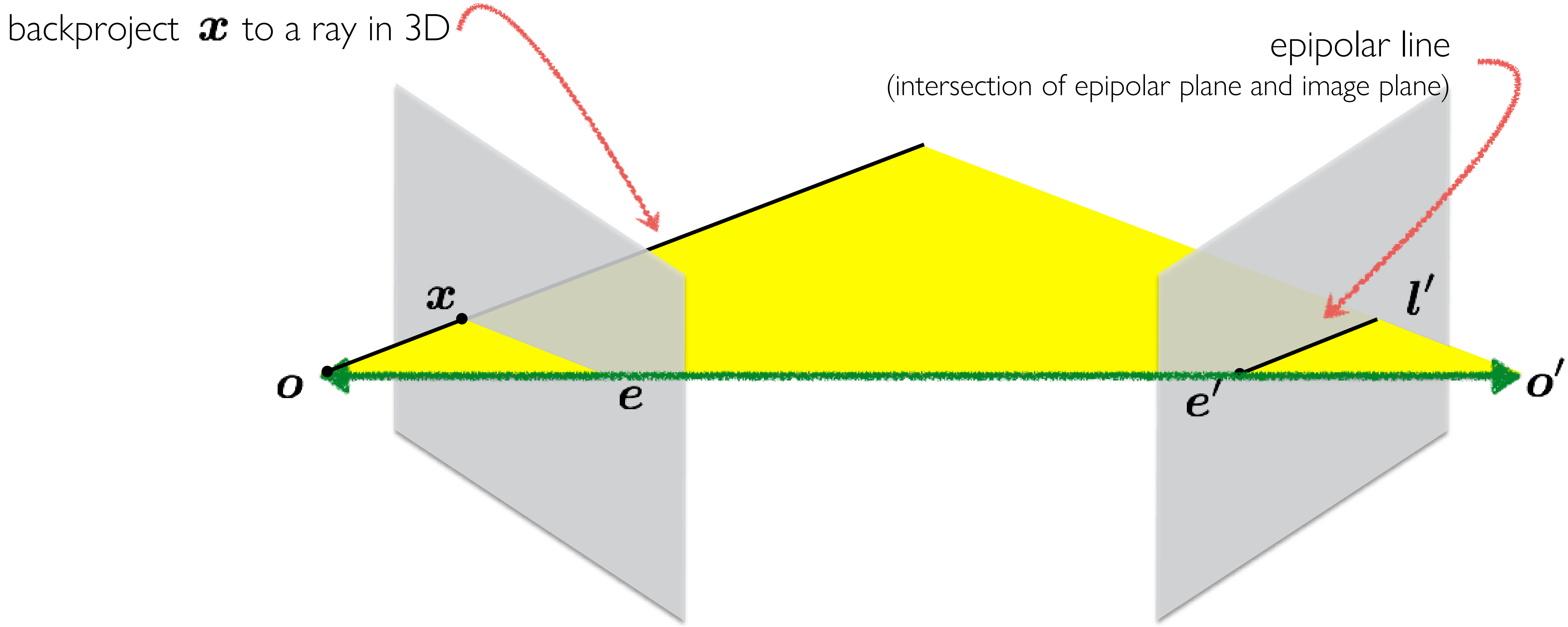

epipolar line
(intersection of epipolar plane and image plane)
p
epipolar plane
l
l'
o
e
what is this?
e'
o'
epipole
(projection of o’ on the image plane)

Quiz



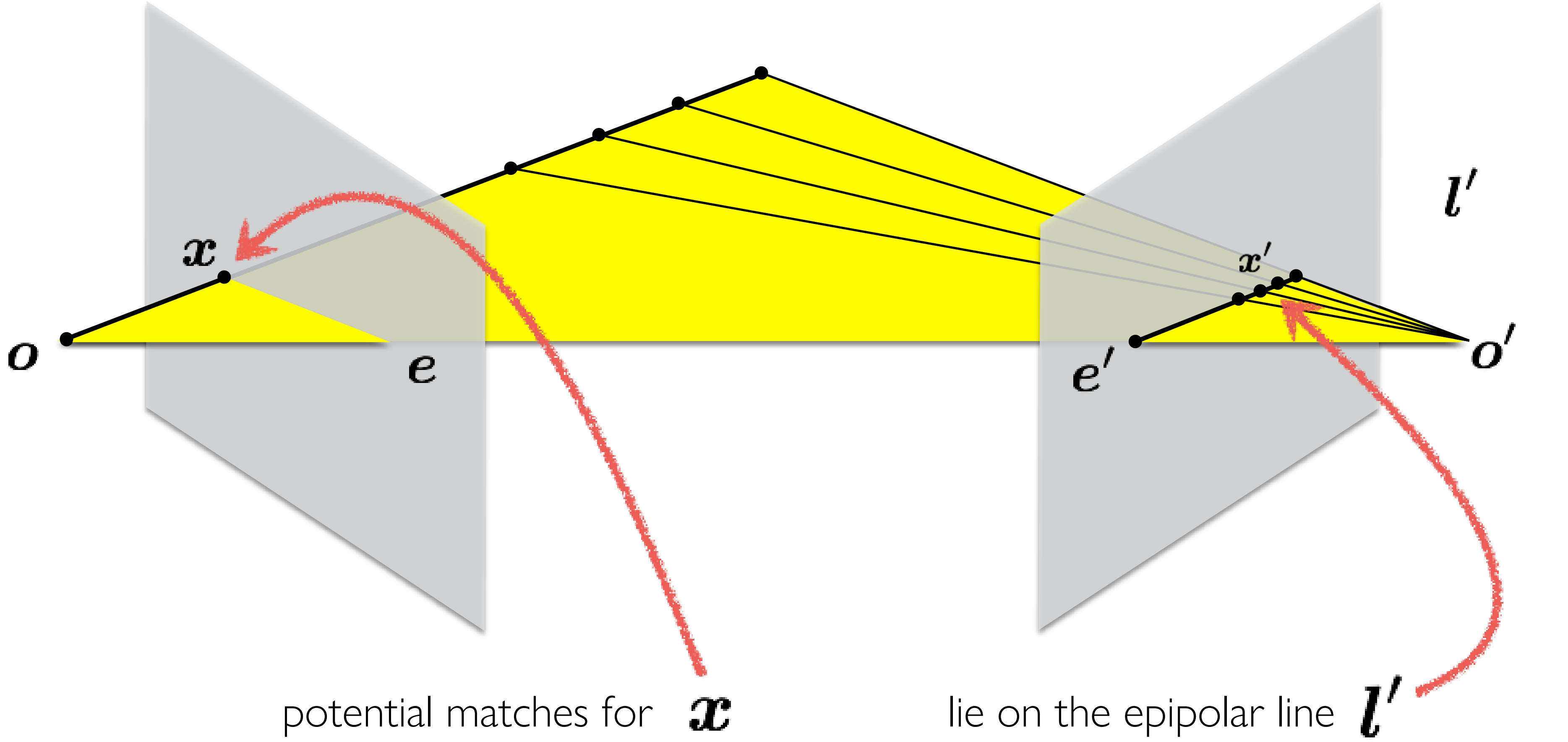
epipolar line
(intersection of epipolar plane and image plane)
$\boldsymbol{p}$
epipolar plane
$\boldsymbol{l}$
$\boldsymbol{l}'$
$\boldsymbol{o}$
$\boldsymbol{e}$
baseline
$\boldsymbol{e}'$
$\boldsymbol{o}'$
epipole
(projection of o’ on the image plane)

Epipolar constraint

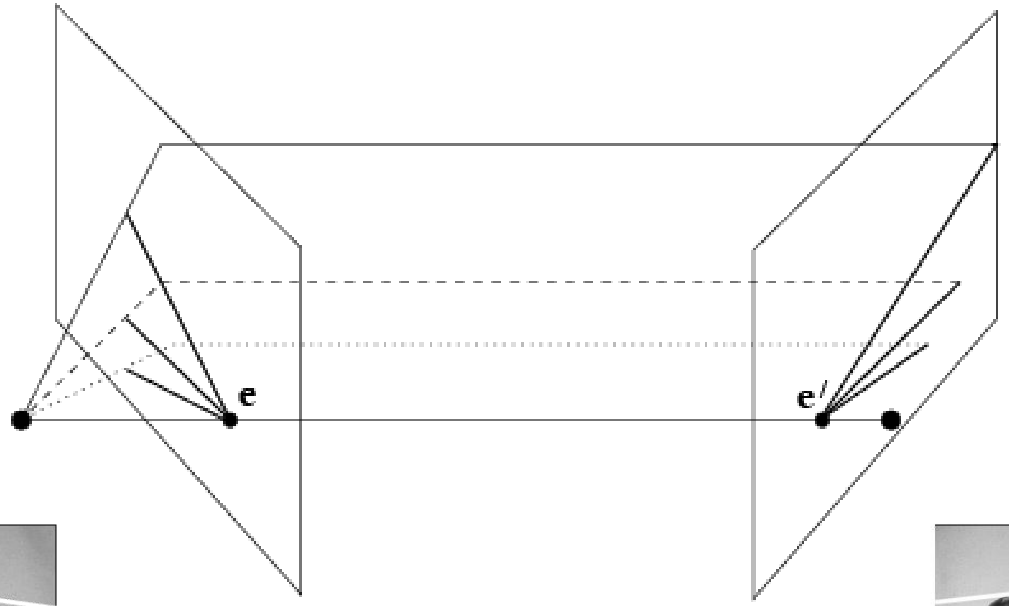


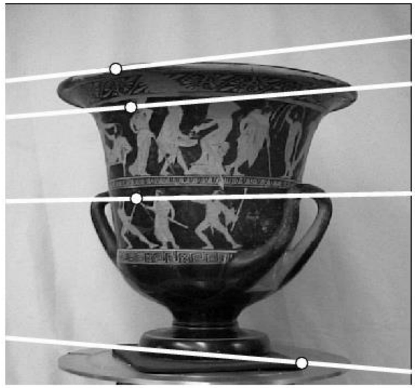
- Another way to construct the epipolar plane, this time given $\boldsymbol{x}$

Epipolar constraint



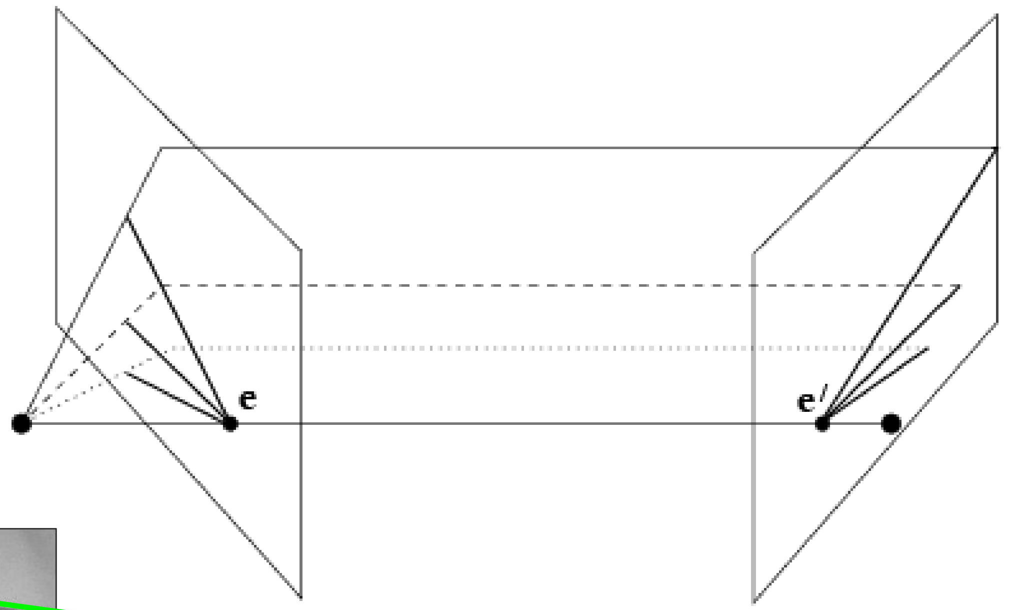
Converging cameras

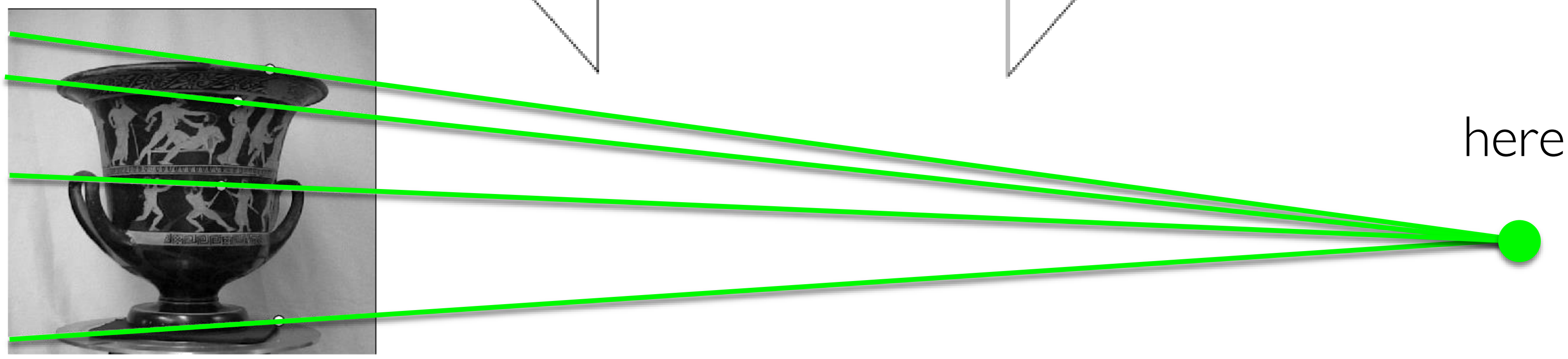




- Where is the epipole in this image?

Converging cameras

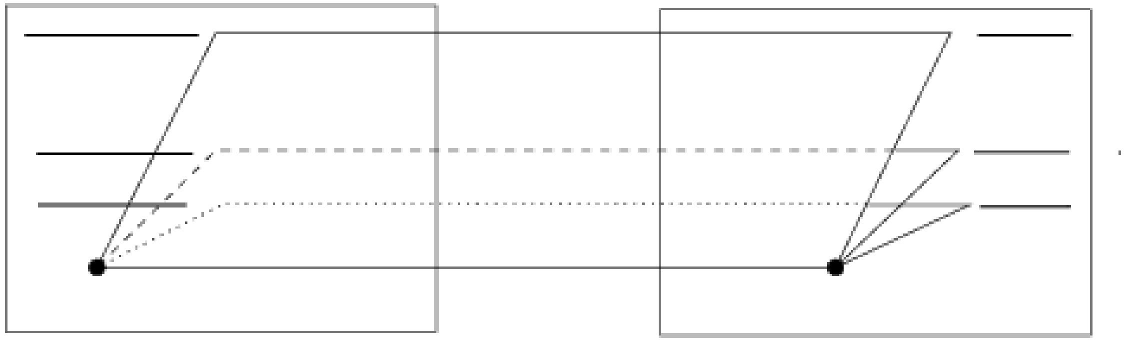




here!

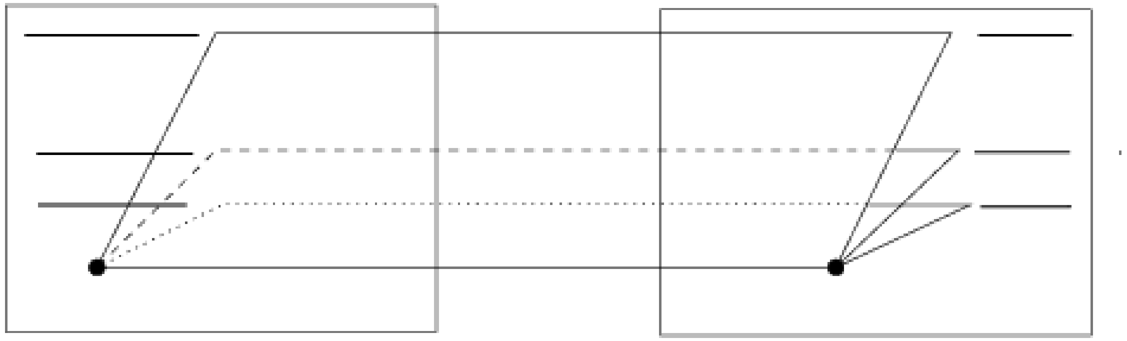
- Where is the epipole in this image? It's not always in the image

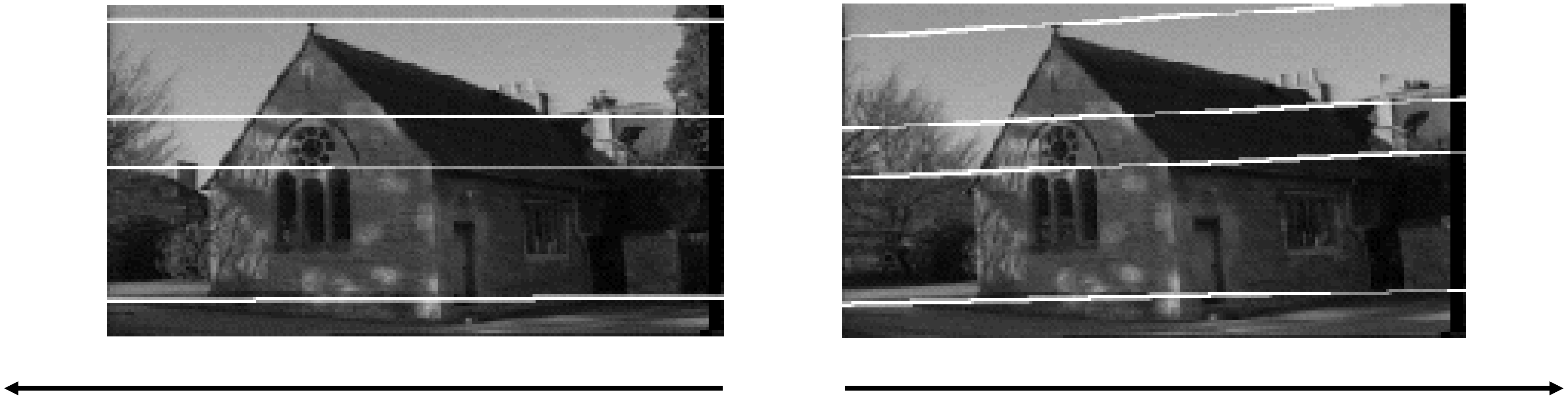
Parallel cameras



- Where is the epipole?

Parallel cameras

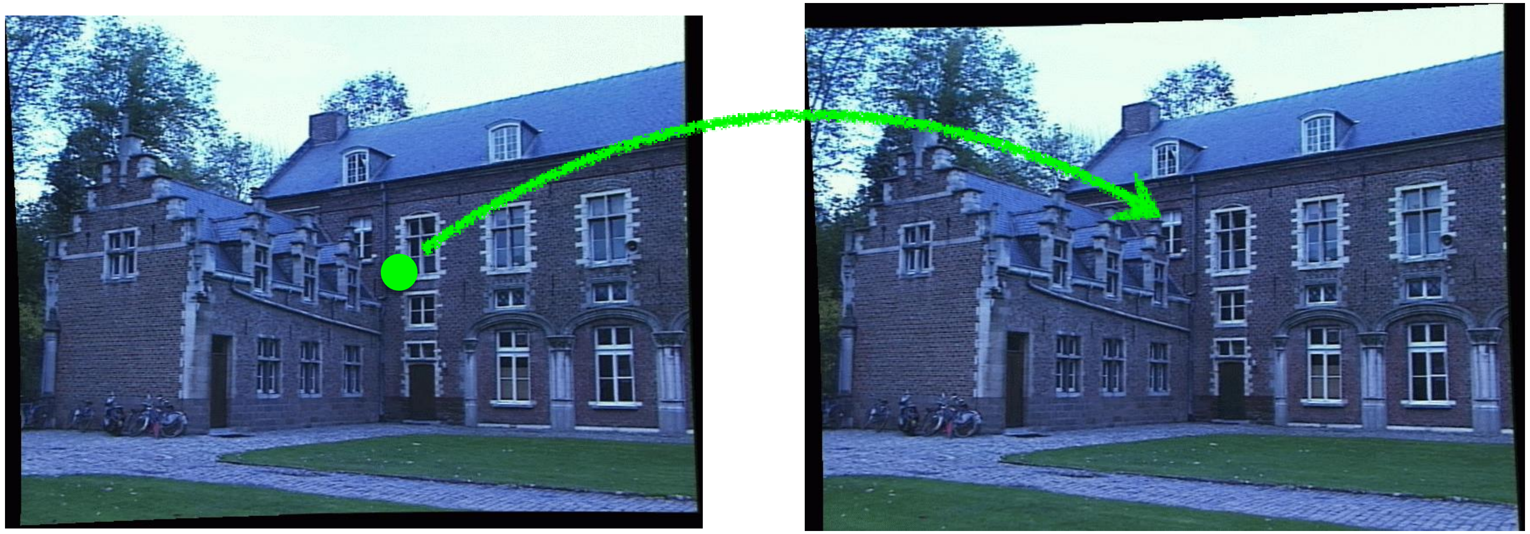




- Epipole at infinity

- The epipolar constraint is an important concept for stereo vision

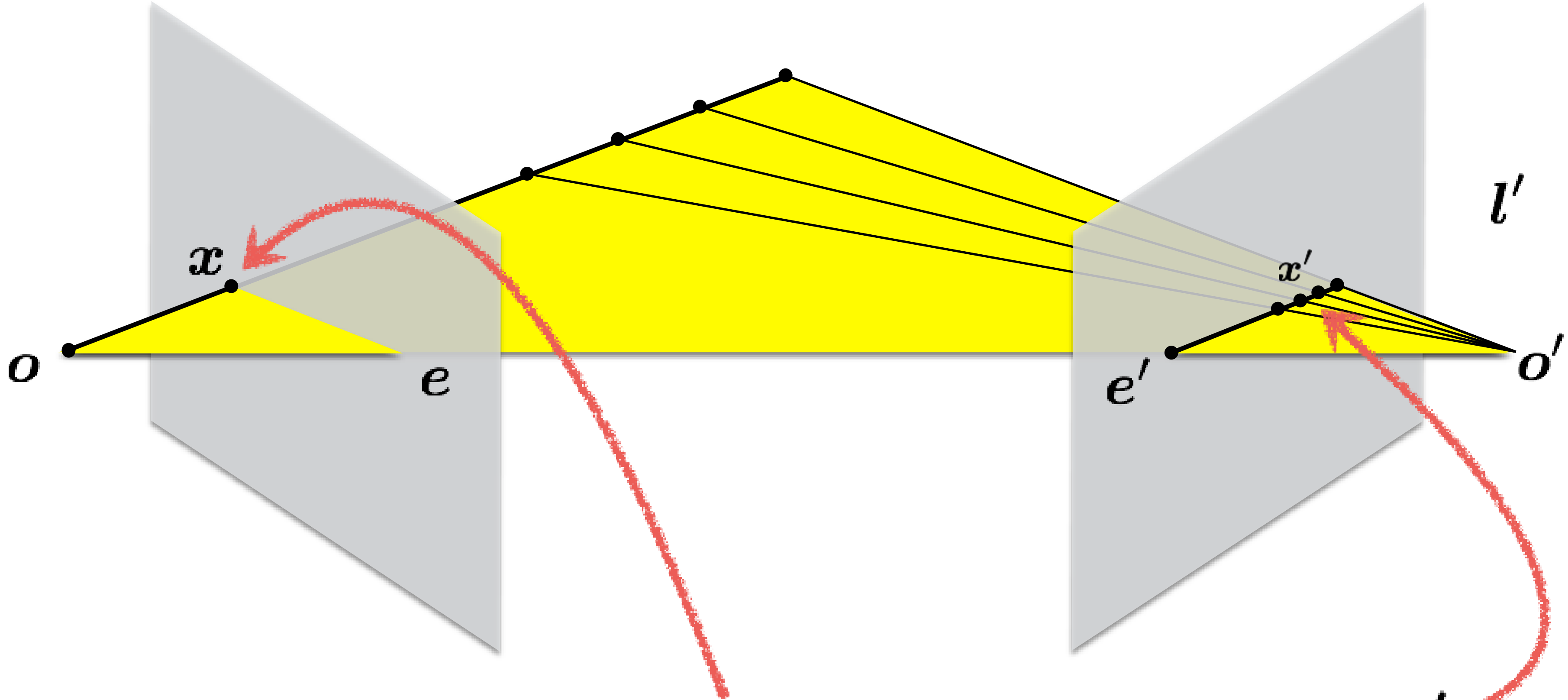
task: match point in left image to point in right image



left image　　　　right image

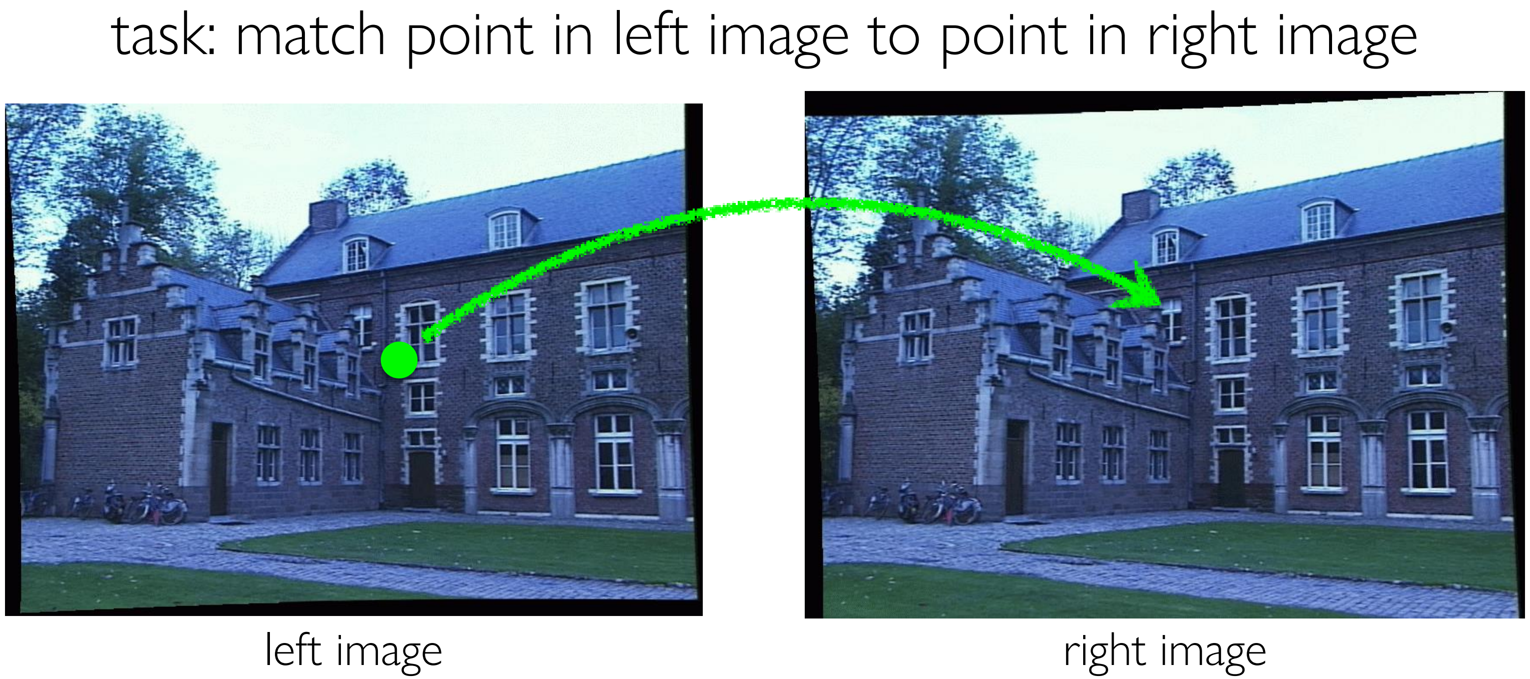
- How would you do it?

Epipolar constraint



potential matches for $\boldsymbol{x}$ lie on the epipolar line $\boldsymbol{l}'$

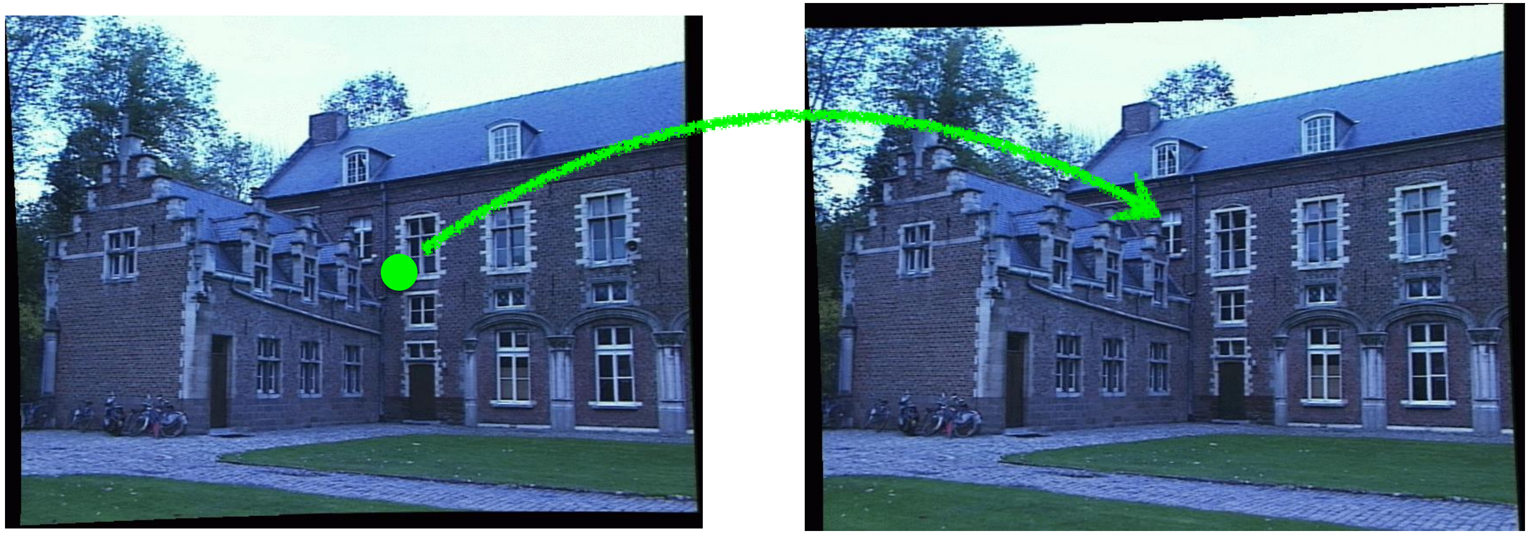
- The epipolar constraint is an important concept for stereo vision



- Want to avoid search over entire image
- Epipolar constraint reduces search to a single line

- The epipolar constraint is an important concept for stereo vision

task: match point in left image to point in right image



left image　　　　right image

- Want to avoid search over entire image
- Epipolar constraint reduces search to a single line
- How do you compute the epipolar line?