3D Vision and Machine Perception

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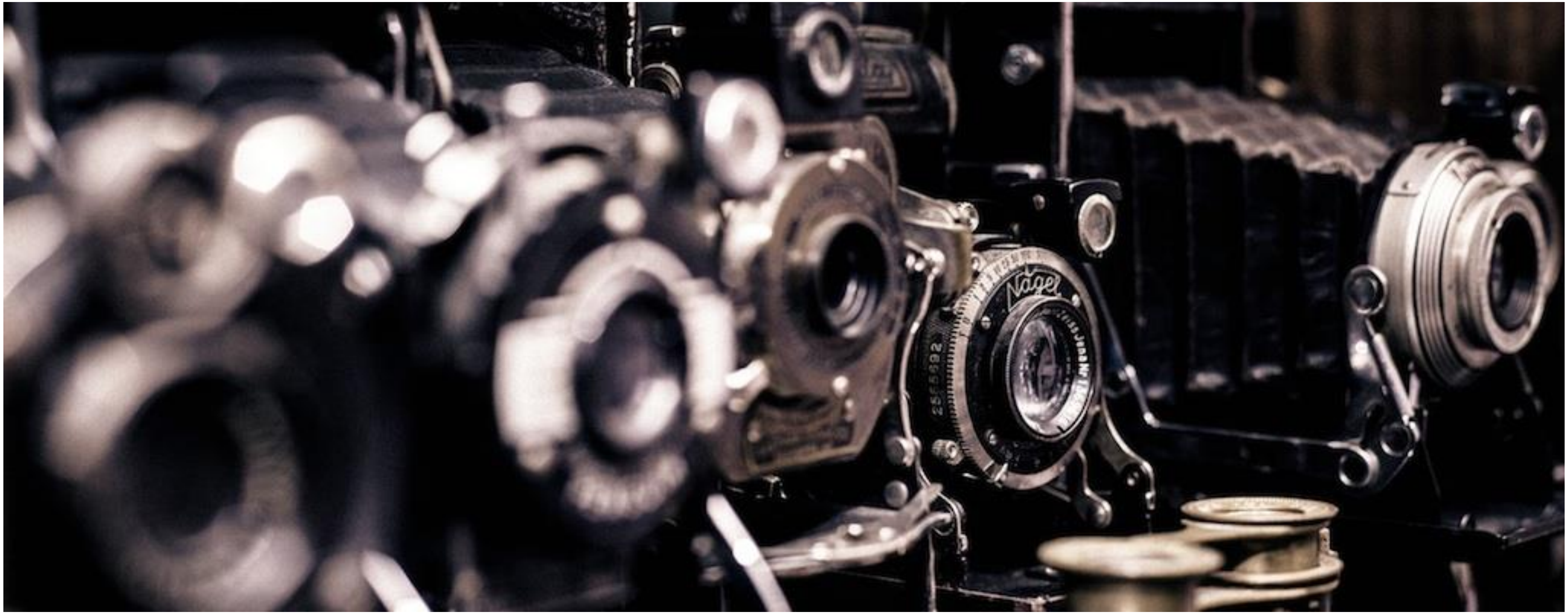
Geometric camera models

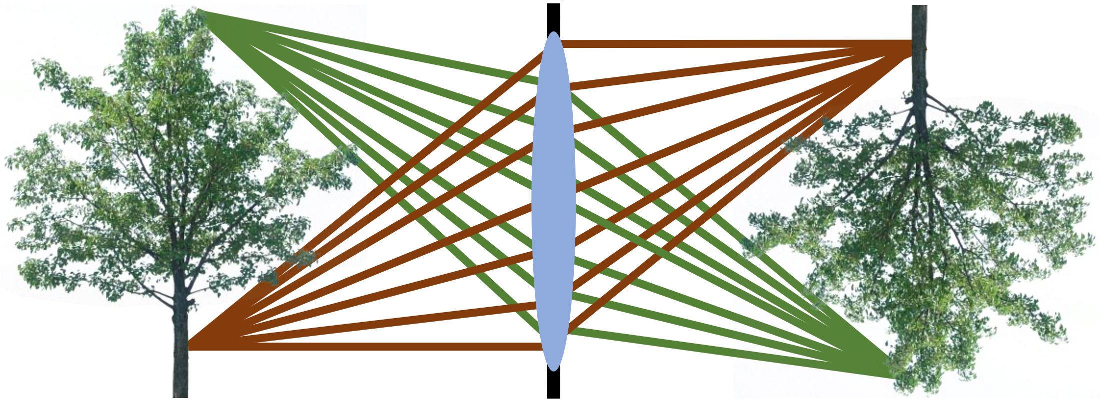


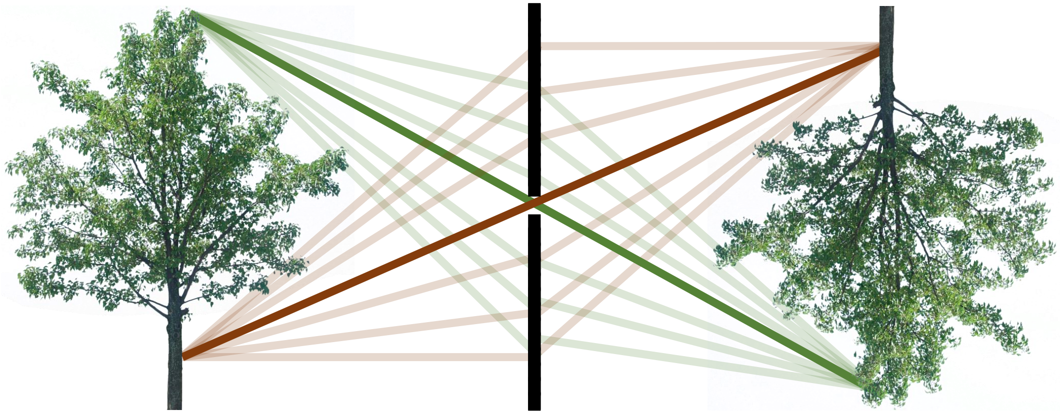
Contents

- Some motivational imaging experiments
- Pinhole camera
- Camera matrix

Recap: Describing both lens and pinhole cameras

- We can derive properties and descriptions that hold for both camera models if:
  - We use only central rays.
  - We assume the lens camera is in focus.
  - We assume that the focus distance of the lens camera is equal to the focal length of the pinhole camera.





Recap: Describing both lens and pinhole cameras

- We can derive properties and descriptions that hold for both camera models if:
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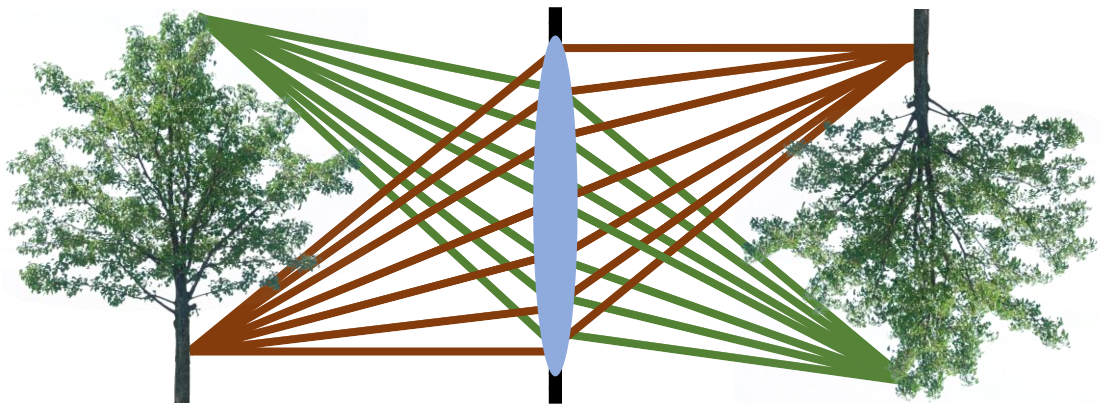
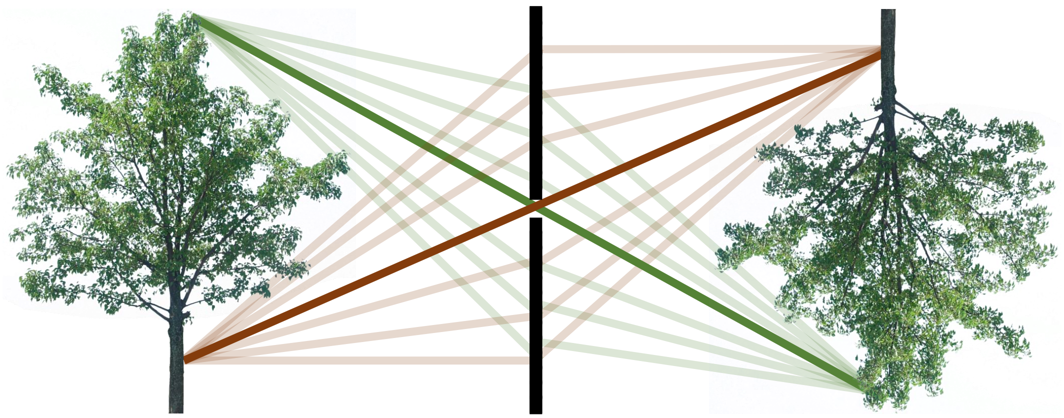
  - We assume the lens camera is in focus.
  - We assume that the focus distance of the lens camera is equal to the focal length of the pinhole camera.



- Remember: focal length f refers to different things for lens and pinhole cameras.
  - In this lectures, we use it to refer to the aperture-sensor distance, as in the pinhole camera case.

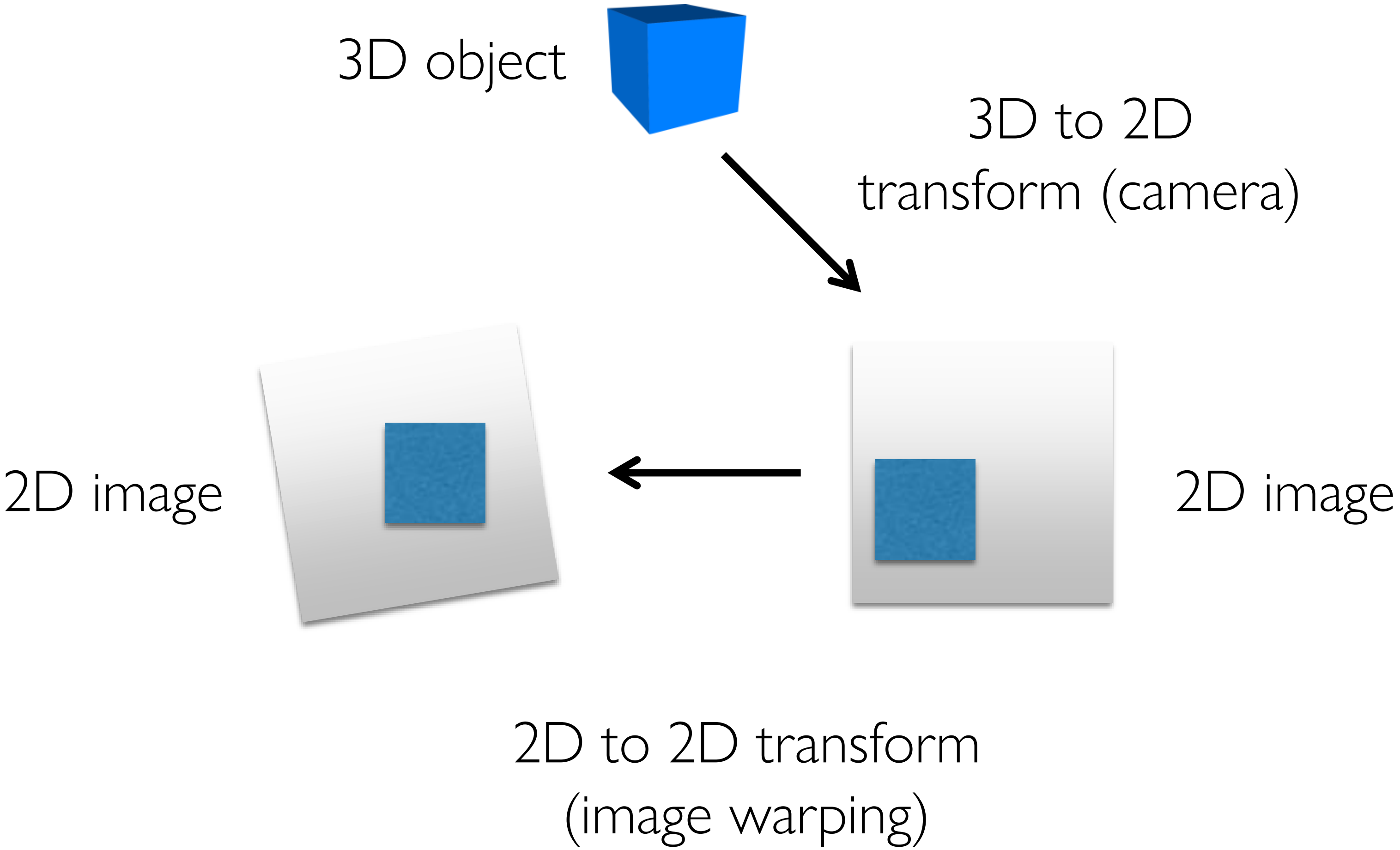


Camera matrix

The camera as a coordinate transformation

- A camera is a mapping from:
  The 3D world

- To: A 2D image



The camera as a coordinate transformation

- A camera is a mapping from:
  The 3D world

homogeneous coordinates

$$\underset{\substack{\text{2D image}\\\text{point}}}{\mathbf{x}} = \underset{\substack{\text{camera}\\\text{matrix}}}{\mathbf{P}}\underset{\substack{\text{3D world}\\\text{point}}}{\mathbf{X}}$$

- To: A 2D image

- What are the dimensions of each variable?

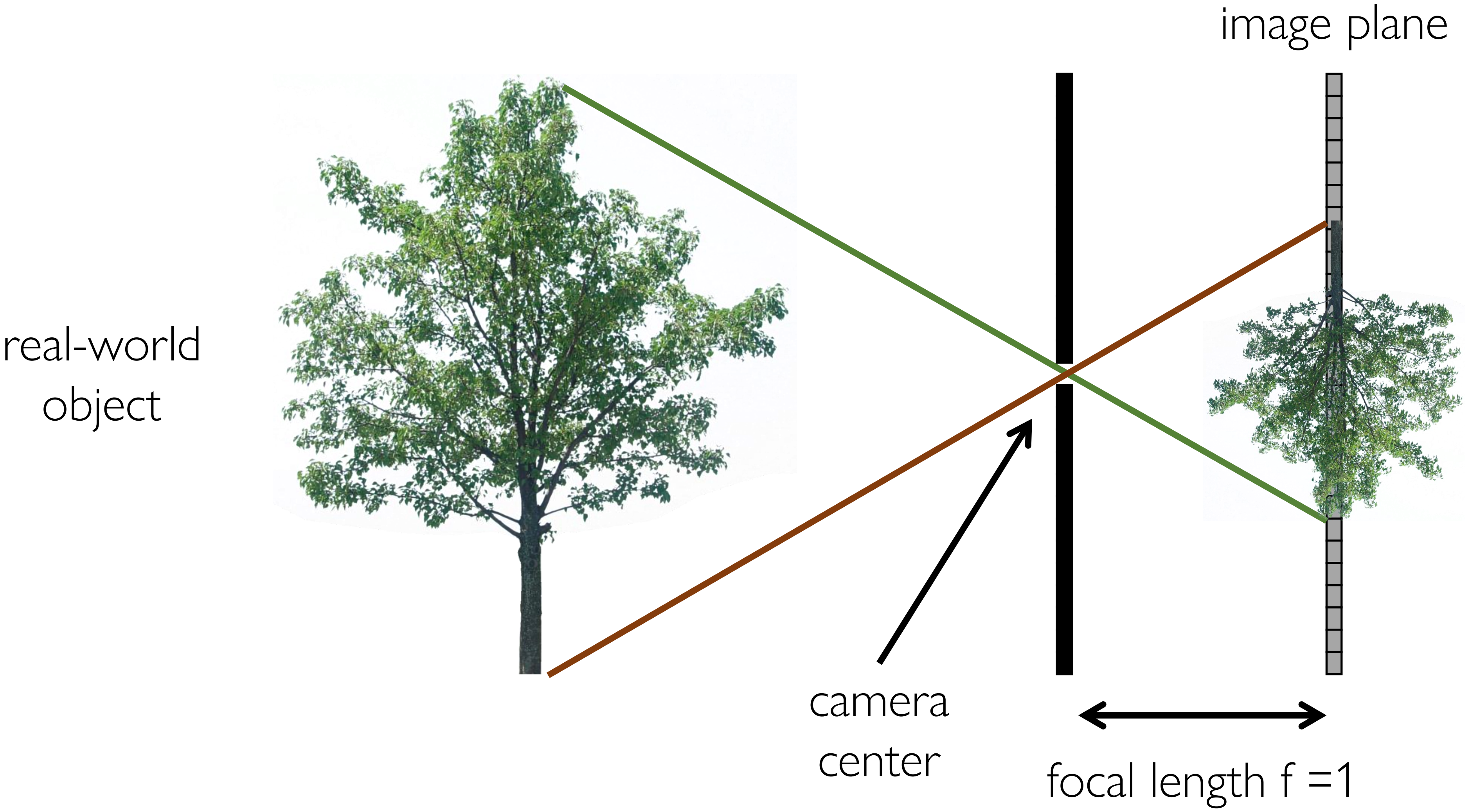
The camera as a coordinate transformation

$$\mathbf{x} = \mathbf{P}\mathbf{X}$$

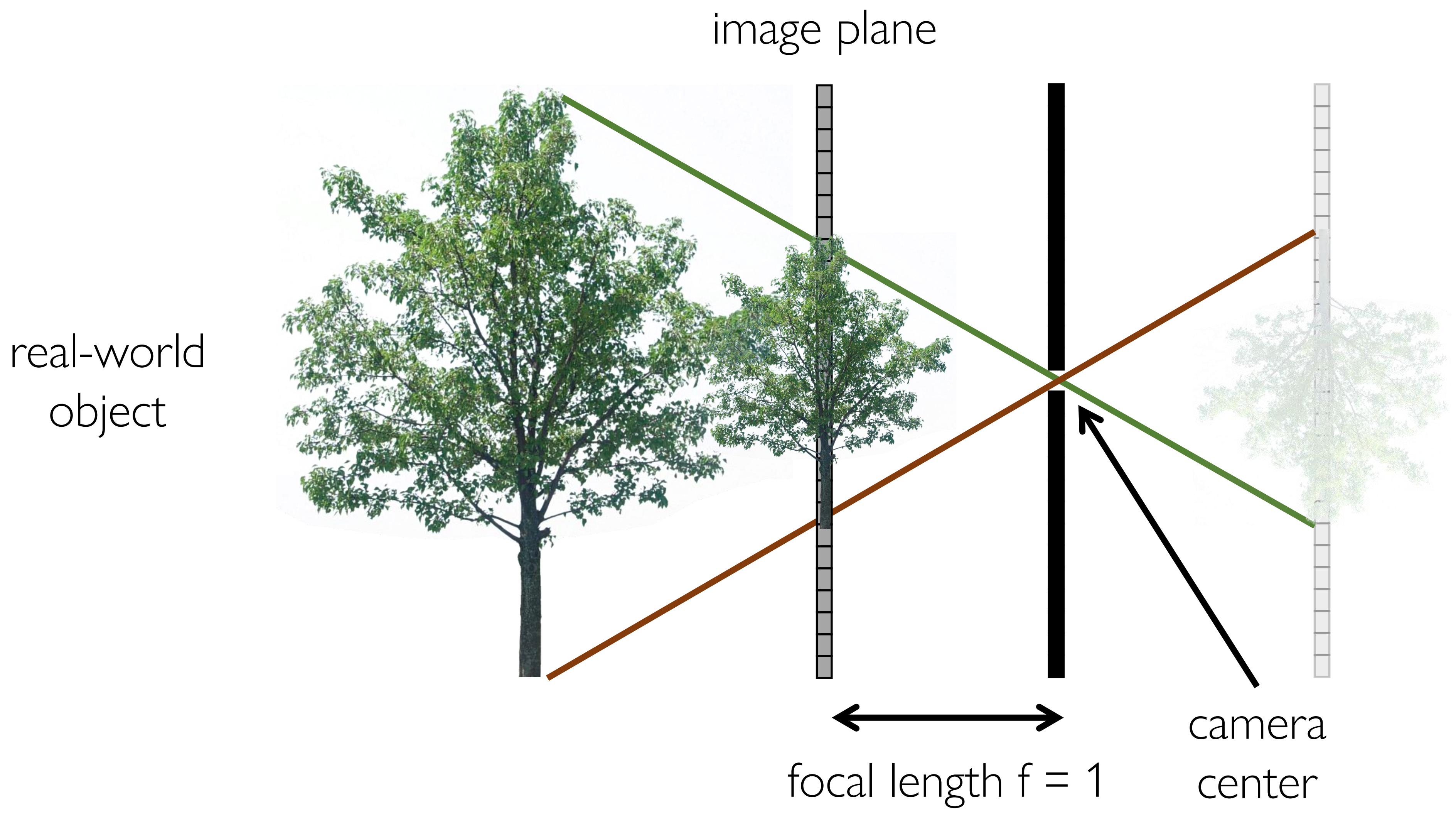
$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

homogeneous image coordinates 3 x 1	camera matrix 3 x 4	homogeneous world coordinates 4 x 1

The pinhole camera



The (rearranged) pinhole camera



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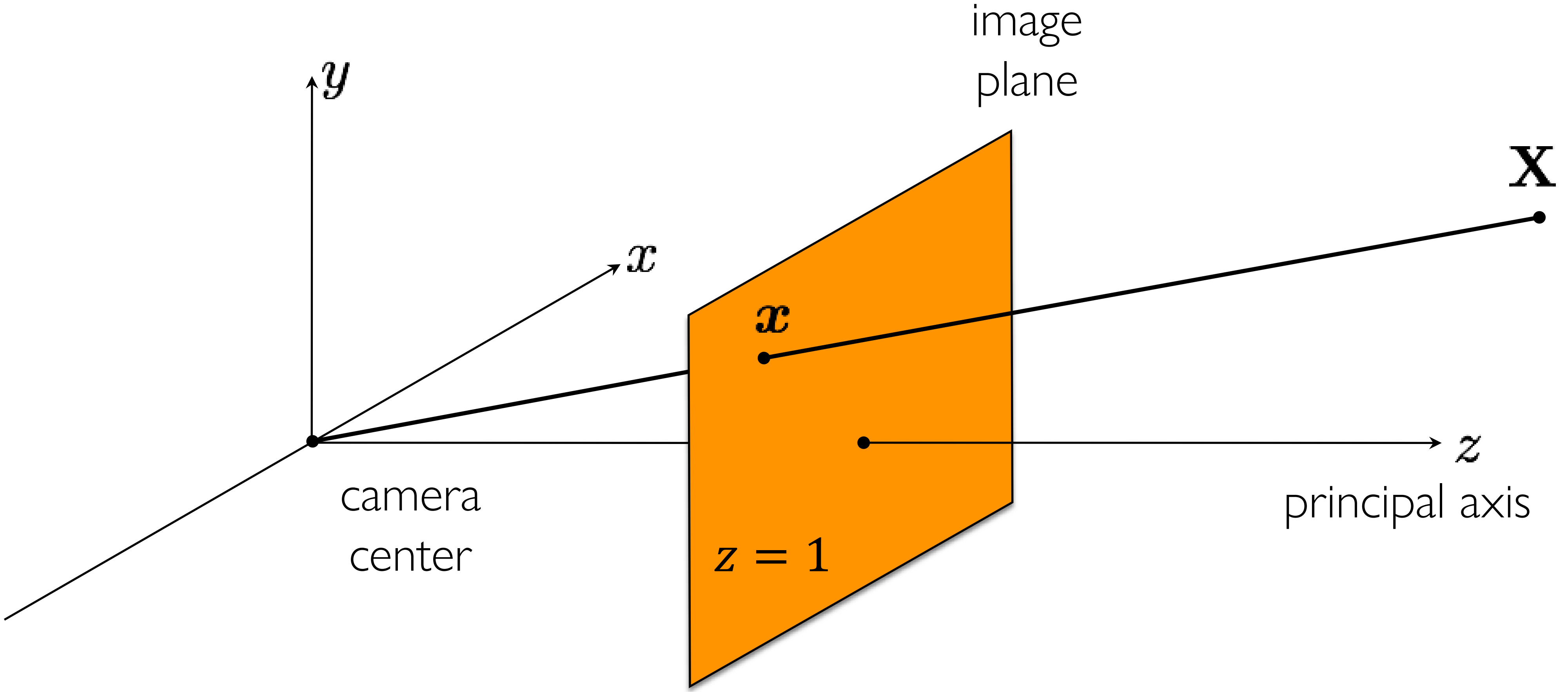
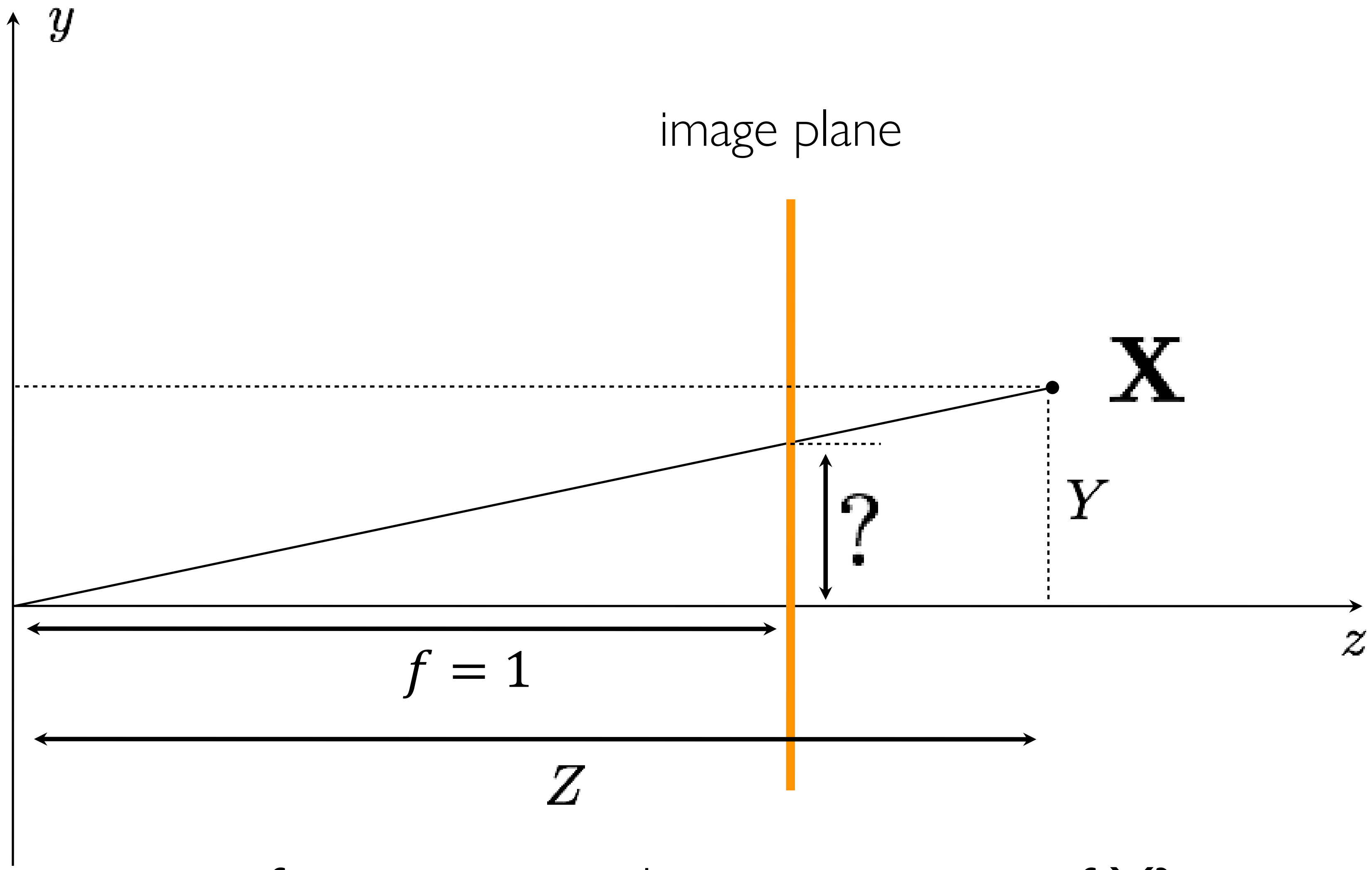


- What is the equation for image coordinate **x** in terms of X?

The 2D view of the (rearranged) pinhole camera



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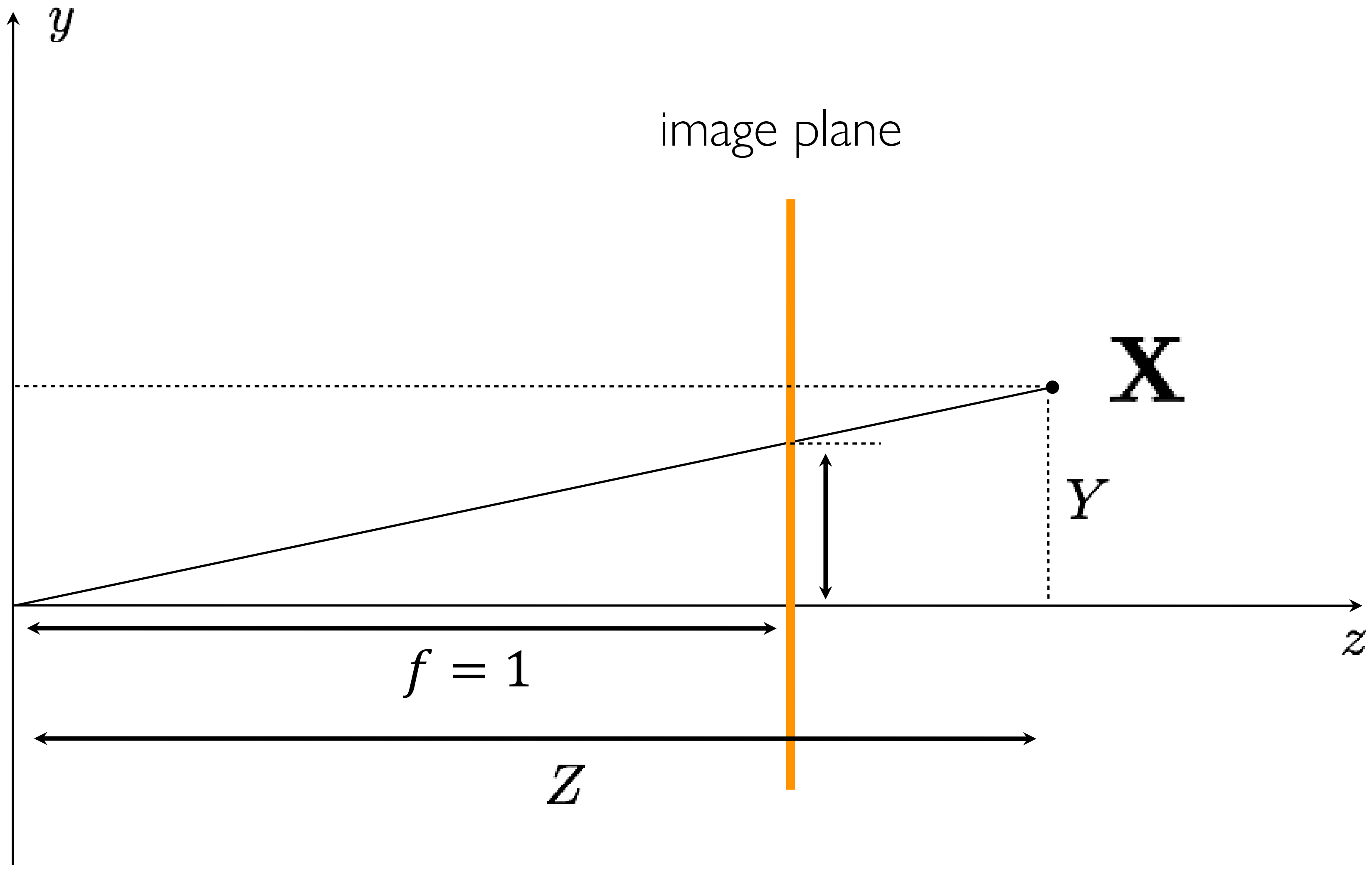
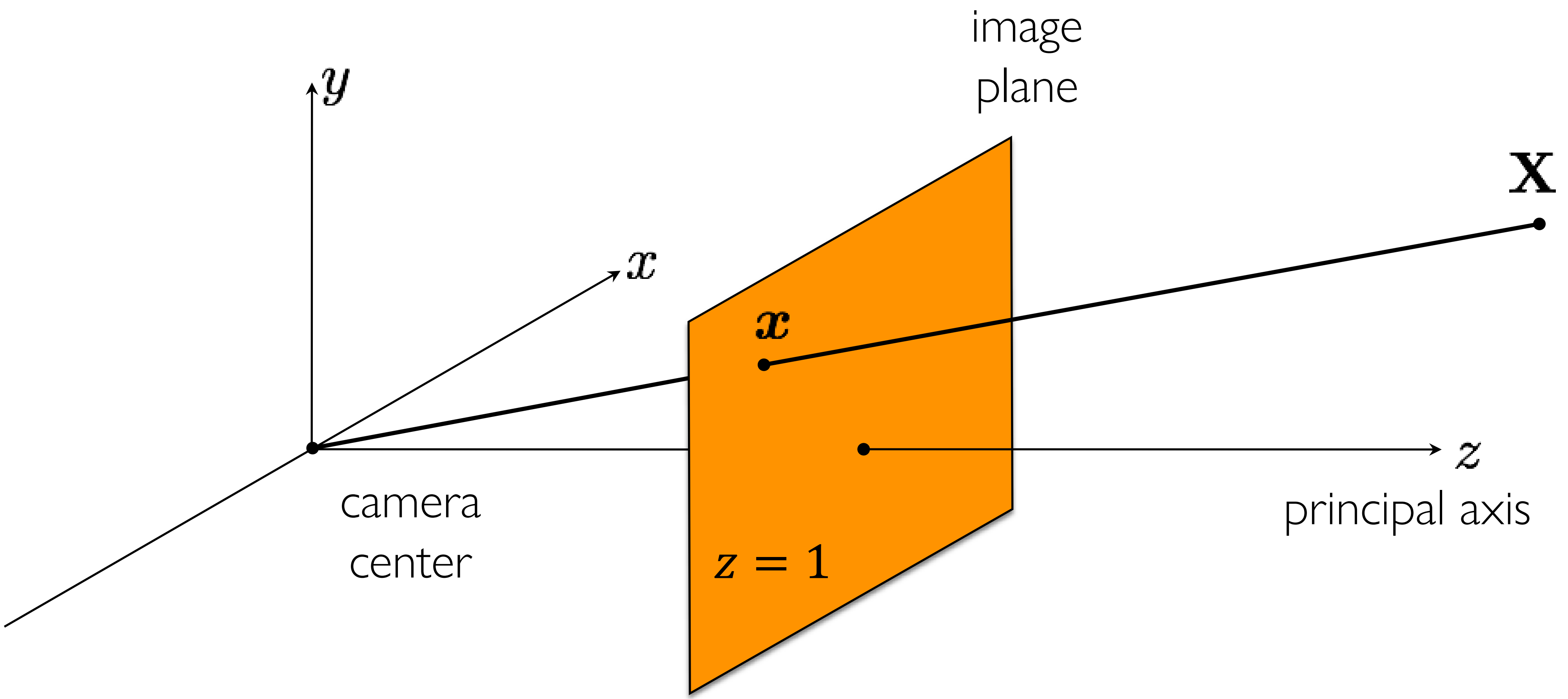
The 2D view of the (rearranged) pinhole camera



$$[X \quad Y \quad Z]^T \rightarrow [X/Z \quad Y/Z]$$

The (rearranged) pinhole camera



$$\boldsymbol{x} = \mathbf{PX}$$

- What is the camera matrix P for a pinhole camera?

The pinhole camera matrix

- Relationship from similar triangles:

$$[X \quad Y \quad Z]^T \rightarrow [X/Z \quad Y/Z]$$

- General camera model in homogeneous coordinates:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

- What does the pinhole camera projection look like?

$$\mathbf{P} = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix}$$

The pinhole camera matrix

- Relationship from similar triangles:

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- What does the pinhole camera projection look like?

the perspective projection matrix

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The pinhole camera matrix

- Relationship from similar triangles:

$$[X \quad Y \quad Z]^T \rightarrow [X/Z \quad Y/Z]$$

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$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

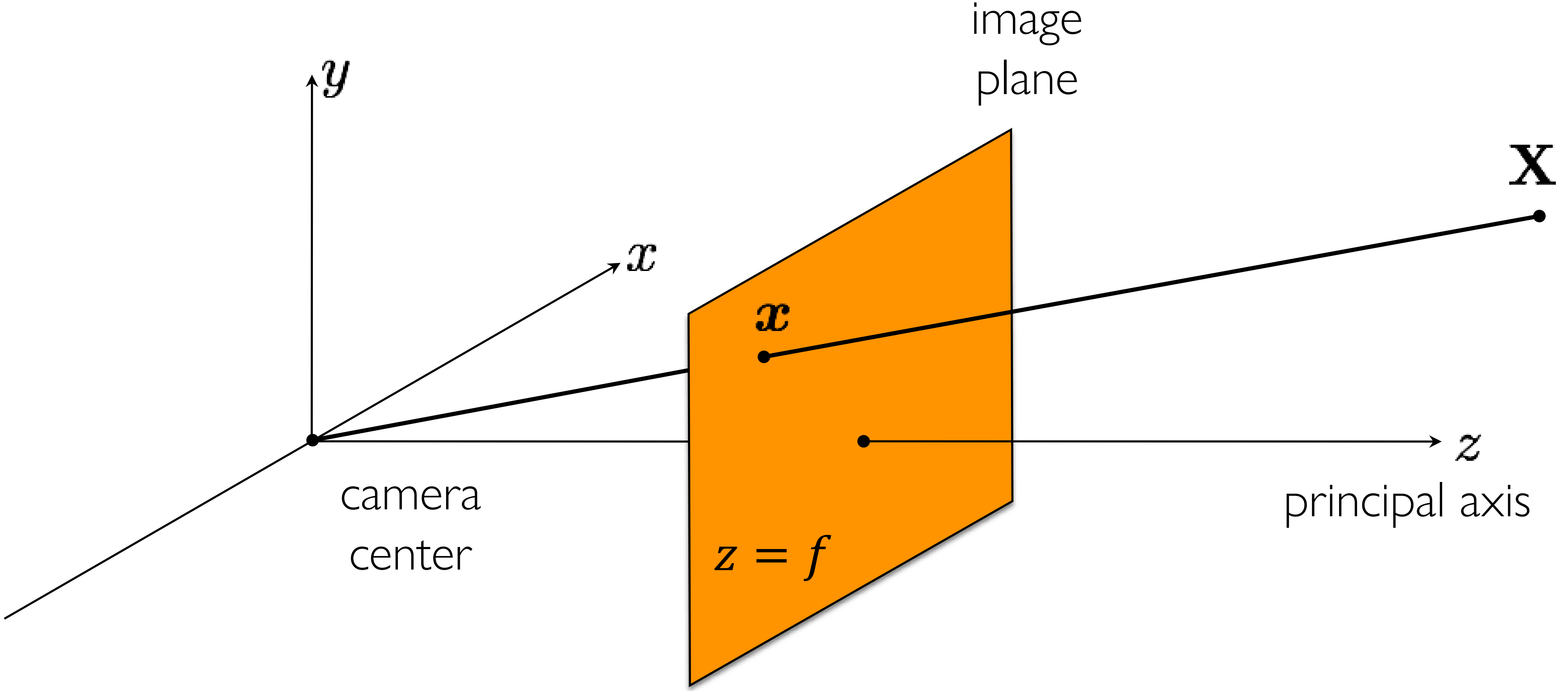
- What does the pinhole camera projection look like?

the perspective projection matrix

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = [\mathbf{I} \quad | \quad \mathbf{0}]$$

alternative way to write the same thing

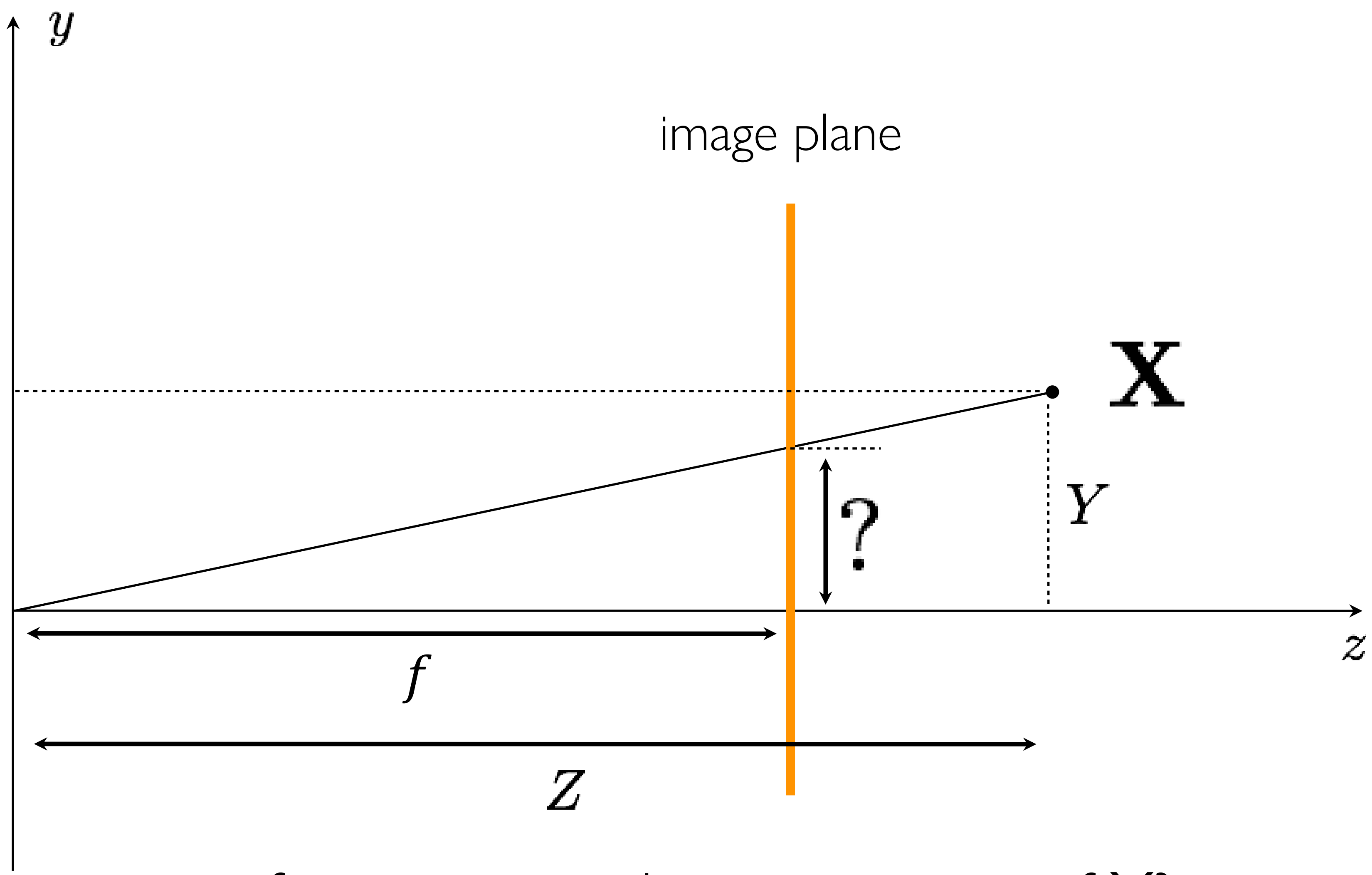
More general case: arbitrary focal length



$$\boldsymbol{x} = \mathbf{PX}$$

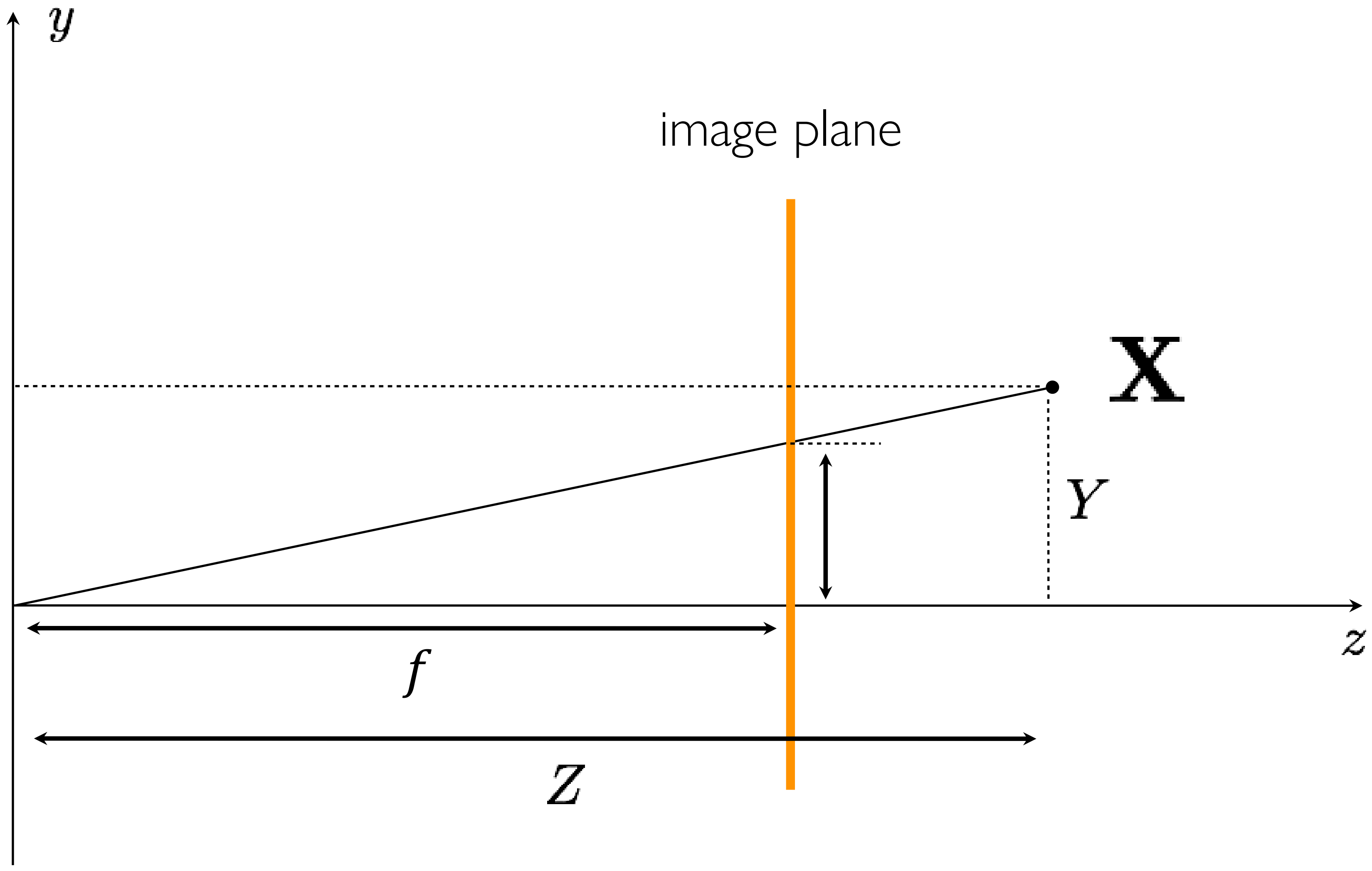
- What is the camera matrix P for a pinhole camera?

More general (2D) case: arbitrary focal length



- What is the equation for image coordinate **x** in terms of X?

More general (2D) case: arbitrary focal length



$$[X \quad Y \quad Z]^\top \mapsto [fX/Z \quad fY/Z]^\top$$

The pinhole camera matrix for arbitrary focal length

- Relationship from similar triangles:

$$[X \quad Y \quad Z]^\top \mapsto [fX/Z \quad fY/Z]^\top$$

- General camera model in homogeneous coordinates:

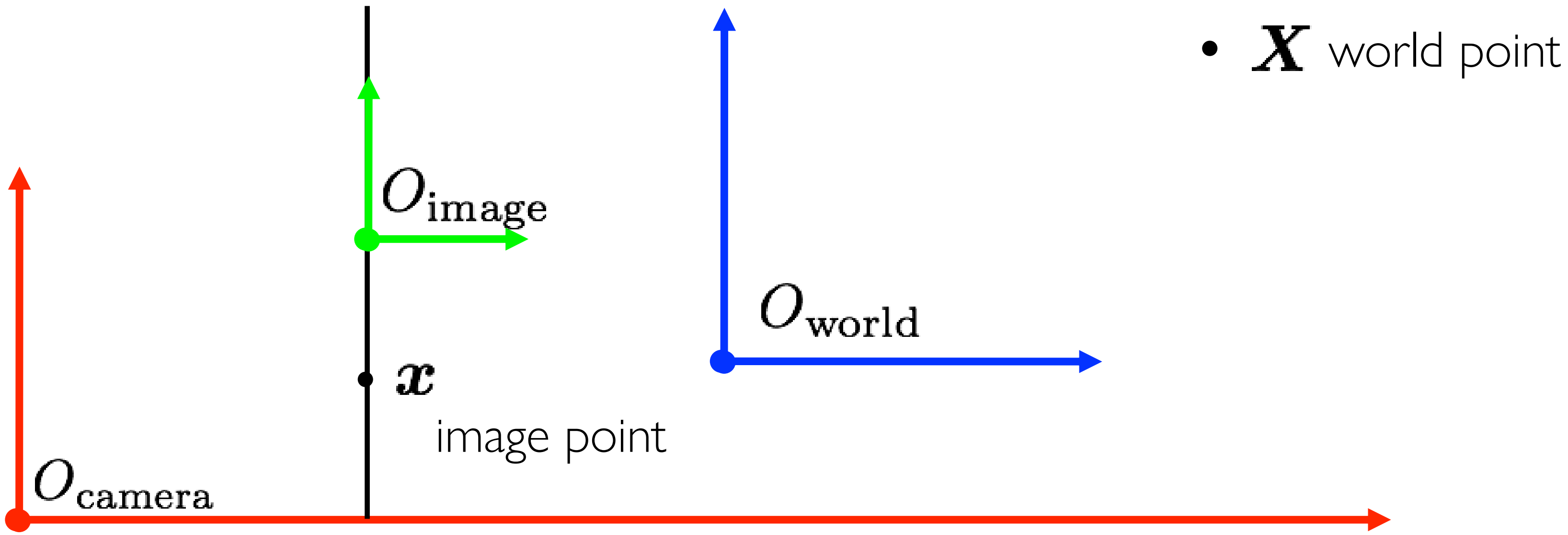
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

- What does the pinhole camera projection look like?

$$\mathbf{P} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

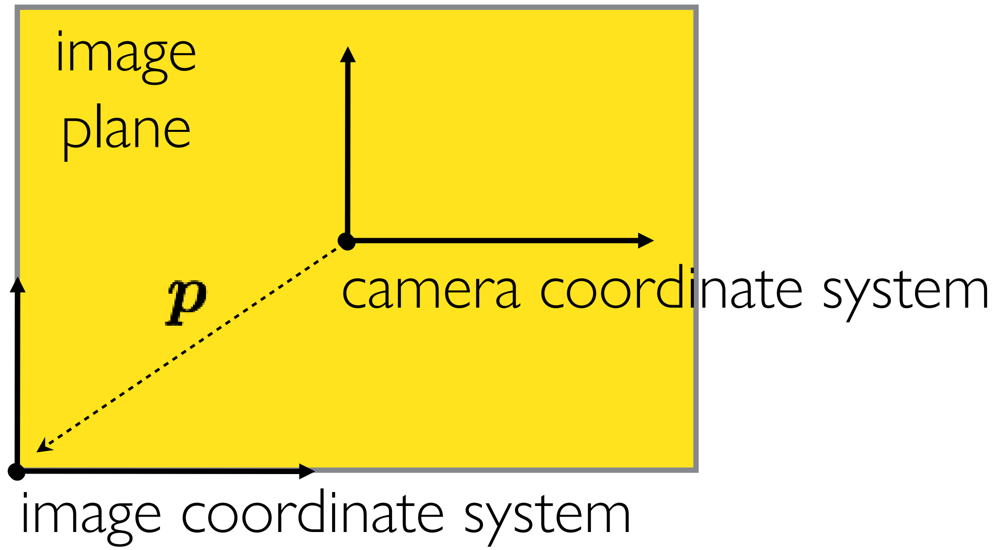
Generalizing the camera matrix

- In general, the camera and image have **different** coordinate systems.



Generalizing the camera matrix

- In particular, the camera origin and image origin may be different:

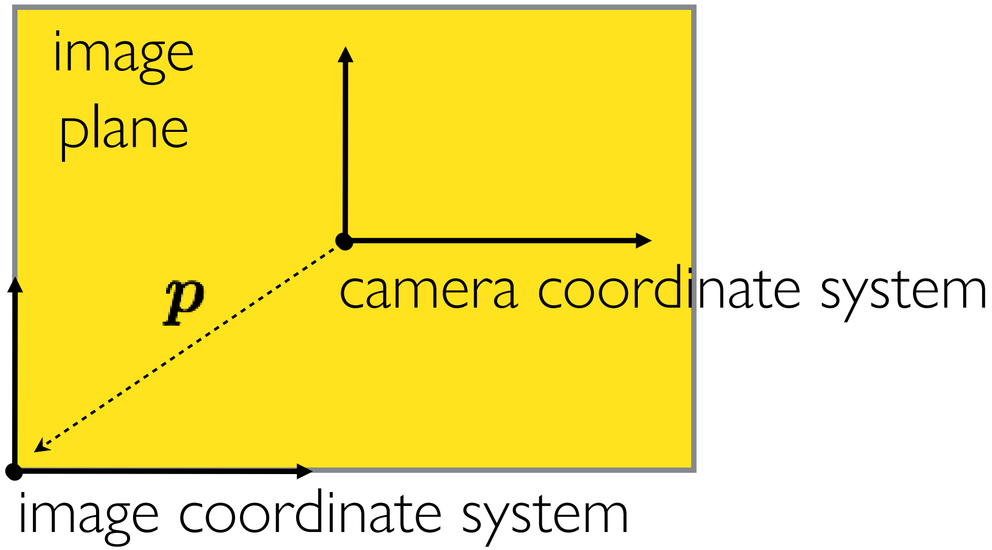


- How does the camera matrix change?

$$\mathbf{P} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Generalizing the camera matrix

- In particular, the camera origin and image origin may be different:



- How does the camera matrix change?

shift vector transforming camera origin to image origin

$$\mathbf{P} = \left[\begin{array}{cccc} f & 0 & p_x & 0 \\ 0 & f & p_y & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

Camera matrix decomposition

- We can decompose the camera matrix like this:

$$\mathbf{P} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \left[\begin{array}{ccc:c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

- What does each part of the matrix represent?

Camera matrix decomposition

- We can decompose the camera matrix like this:

$$\mathbf{P} = \left[\begin{array}{ccc} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{array}\right] \left[\begin{array}{ccc:c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

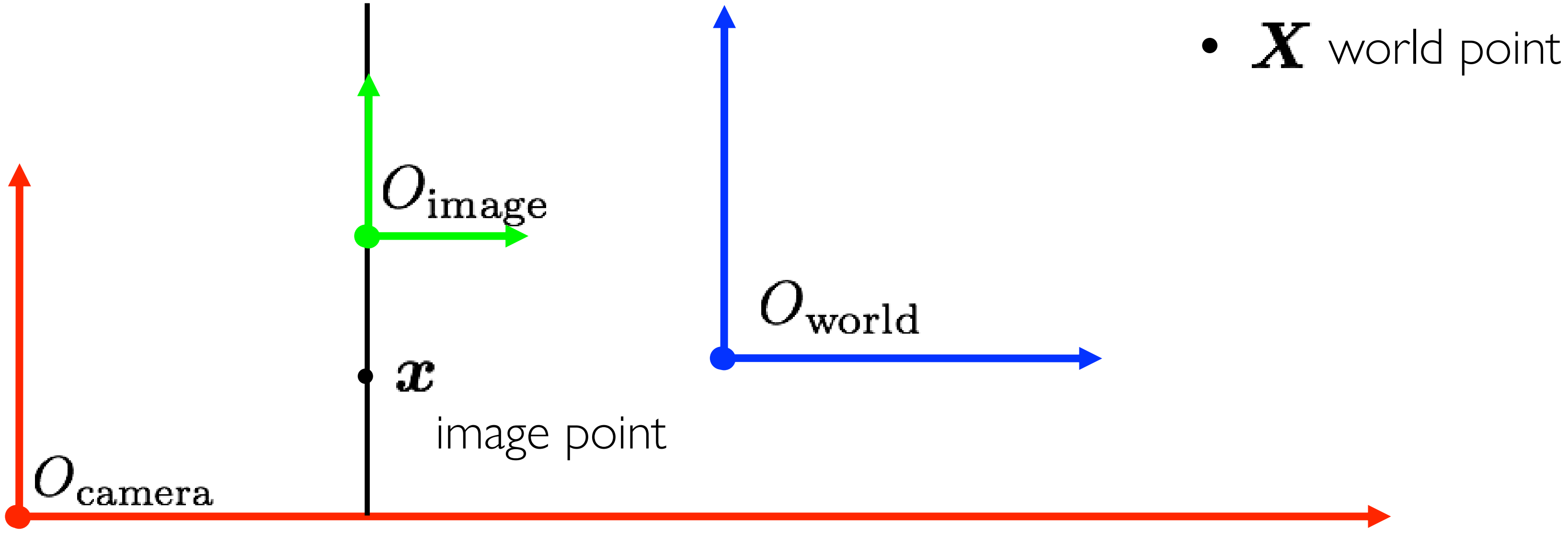
(homogeneous) transformation from 2D to 2D, accounting for not unit focal length and origin shift

(homogeneous) perspective projection from 3D to 2D, assuming image plane at z = 1 and shared camera/image origin

also written as: $\mathbf{P} = \mathbf{K}[\mathbf{I}|\mathbf{0}]$ where $\mathbf{K} = \left[\begin{array}{ccc} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{array}\right]$

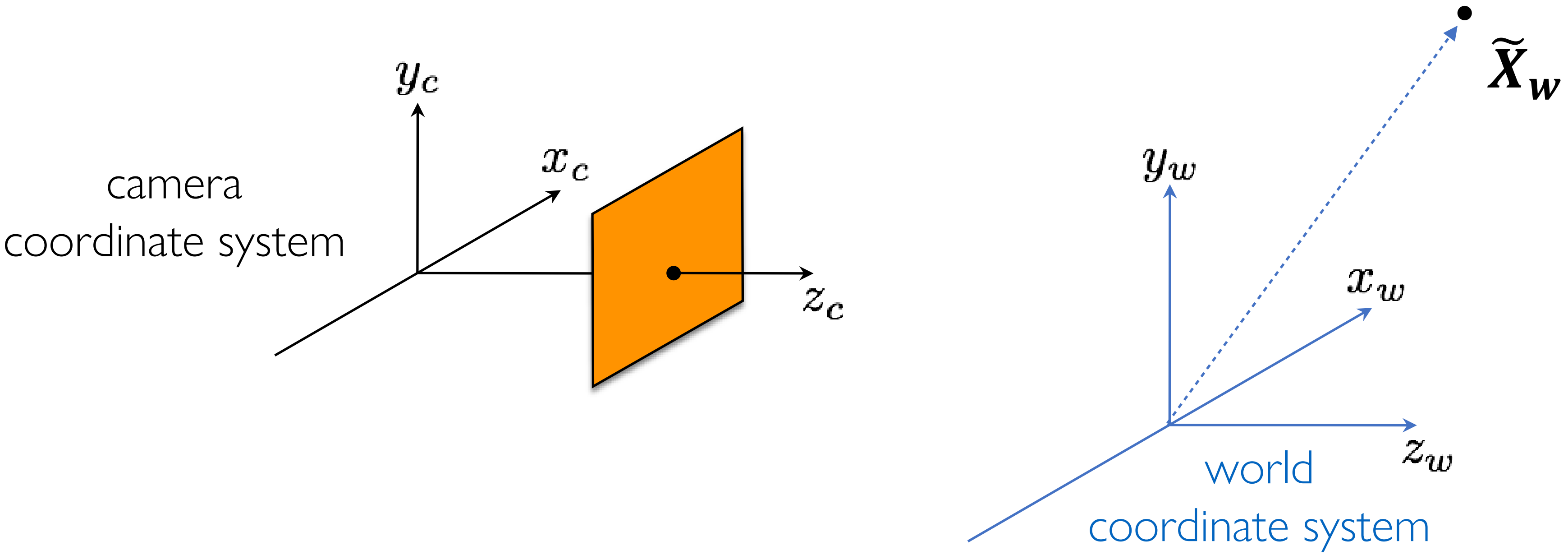
Generalizing the camera matrix

- In general, there are three, generally different, coordinate systems



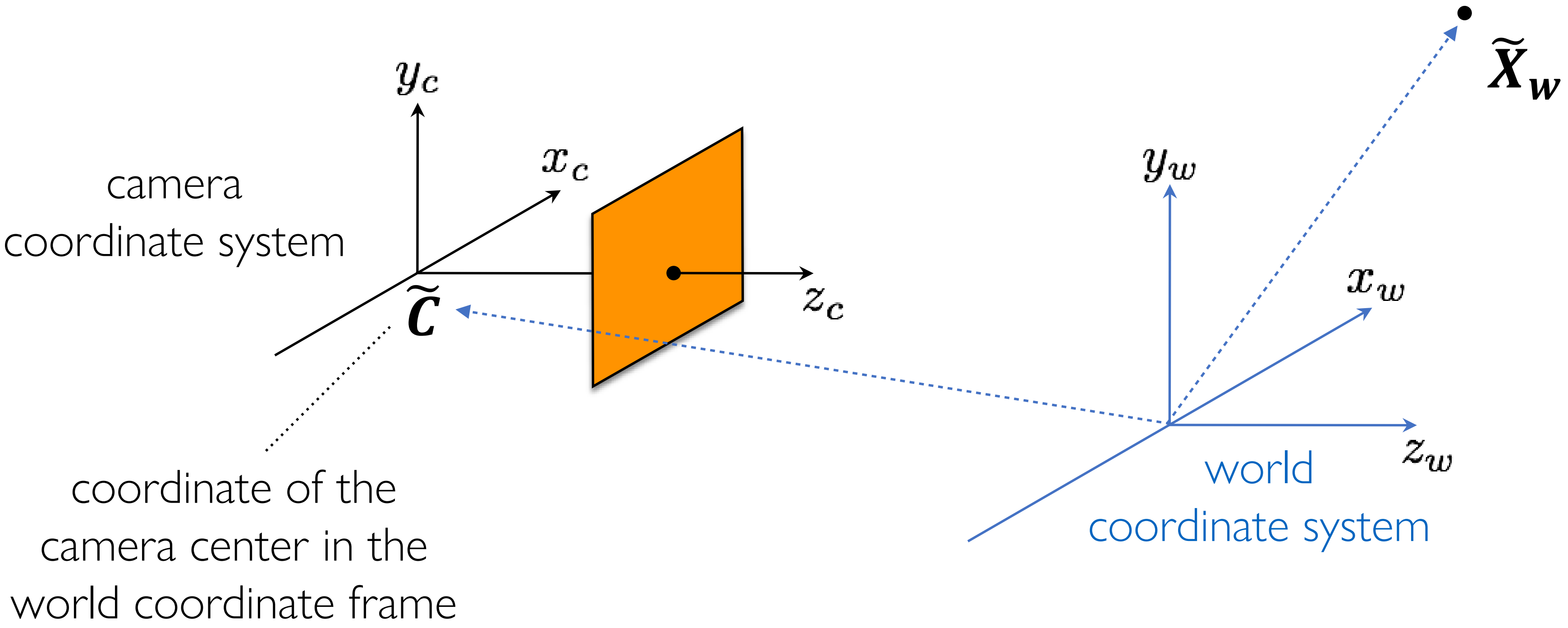
- We need to know the transformations between them.

World-to-camera coordinate system transformation

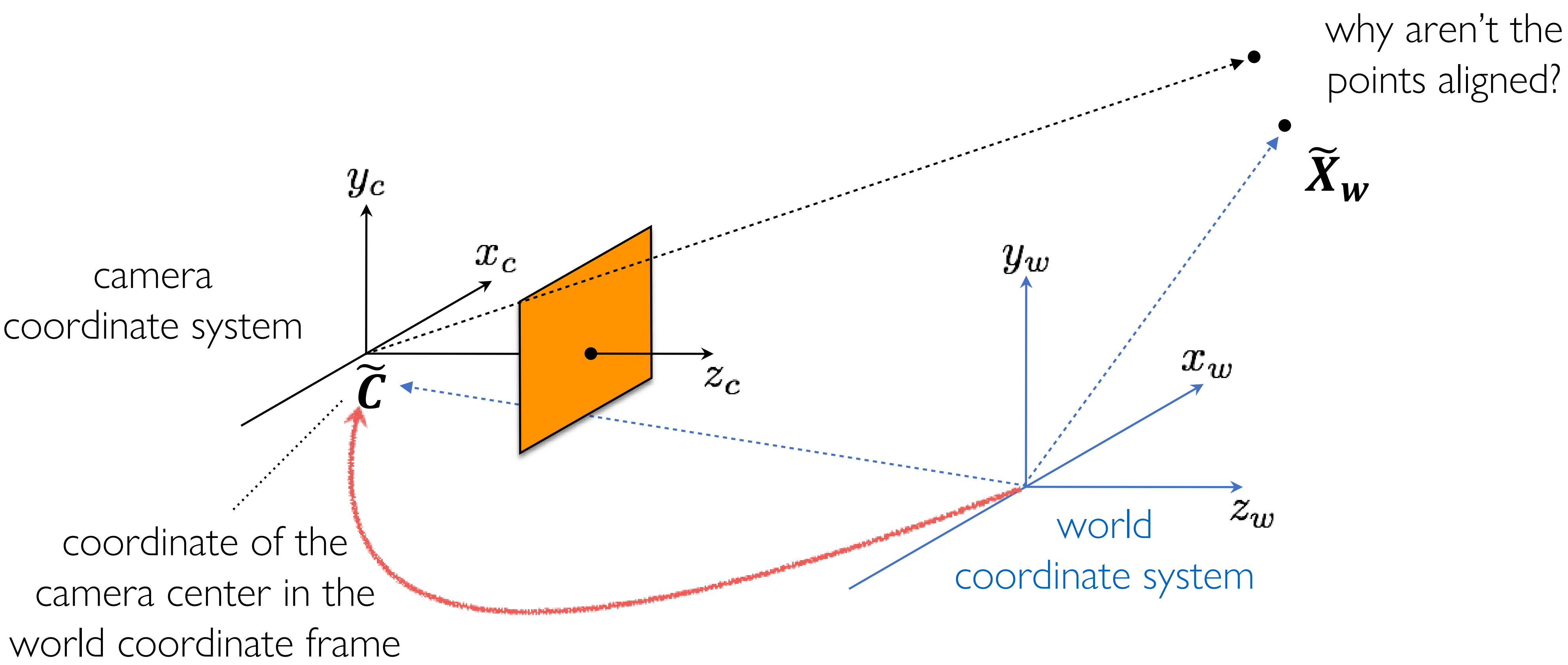


tilde means
heterogeneous
coordinates

World-to-camera coordinate system transformation



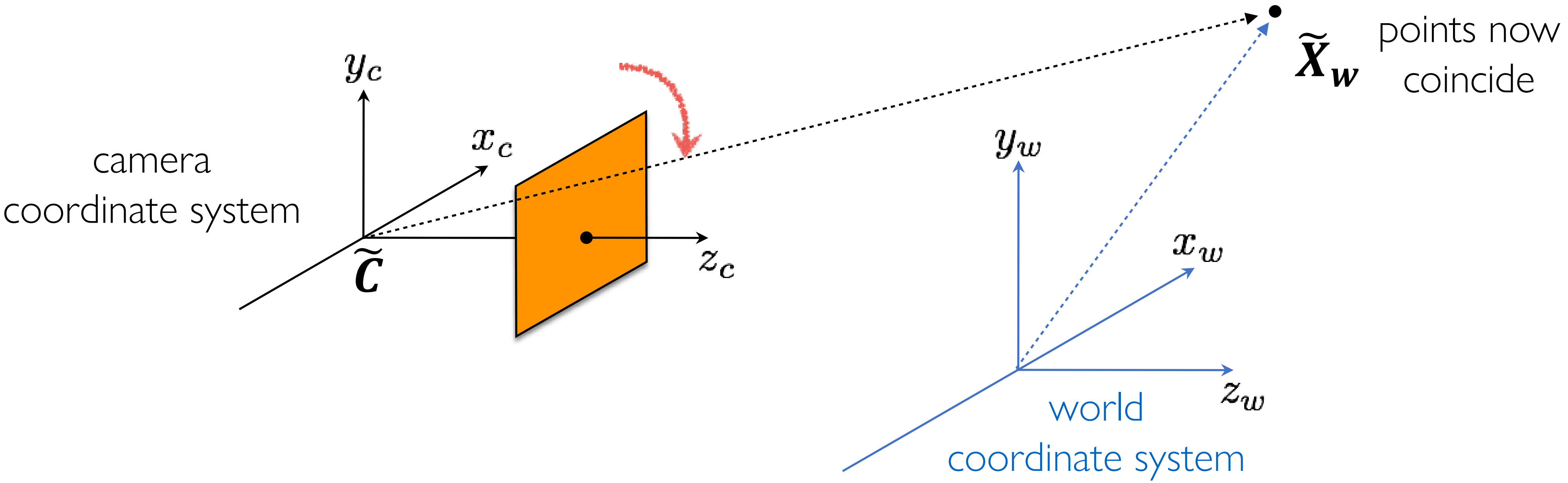
World-to-camera coordinate system transformation



$$(\tilde{X}_w - \tilde{C})$$

translate

World-to-camera coordinate system transformation



$$\boldsymbol{R} \cdot \left(\widetilde{\boldsymbol{X}}_{\boldsymbol{w}} - \widetilde{\boldsymbol{C}}\right)$$

rotate translate

Modeling the coordinate system transformation

- In heterogeneous coordinates, we have:

$$\widetilde{\mathbf{X}}_{\mathbf{c}} = \mathbf{R} \cdot \left(\widetilde{\mathbf{X}}_{\mathbf{w}} - \tilde{\mathbf{C}}\right)$$

- How do we write this transformation in homogeneous coordinates?

Modeling the coordinate system transformation

- In heterogeneous coordinates, we have:

$$\widetilde{\mathbf{X}}_{\mathbf{c}} = \mathbf{R} \cdot \left(\widetilde{\mathbf{X}}_{\mathbf{w}} - \tilde{\mathbf{C}}\right)$$

- In homogeneous coordinates, we have:

$$\begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{R} & -\mathbf{RC} \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} \quad \text{or} \quad \mathbf{X_c} = \begin{bmatrix} \mathbf{R} & -\mathbf{R}\tilde{\mathbf{C}} \\ \mathbf{0} & 1 \end{bmatrix} \mathbf{X_w}$$

Incorporating the transform in the camera matrix

- The previous camera matrix is for homogeneous 3D coordinates in camera coordinate systems:

$$\mathbf{x} = \mathbf{P}\mathbf{X_c} = \mathbf{K}[\mathbf{I}|\mathbf{0}]\mathbf{X_c}$$

- We also just derived:

$$\mathbf{X_c} = \begin{bmatrix} \mathbf{R} & -\mathbf{R}\tilde{\mathbf{C}} \\ \mathbf{0} & 1 \end{bmatrix} \mathbf{X_w}$$

Putting it all together

- We can write everything into a single projection:

$$\mathbf{x} = \mathbf{P}\mathbf{X}_{\mathbf{w}}$$

- The camera matrix now looks like:

$$\mathbf{P} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} [\mathbf{I} \quad | \quad \mathbf{0}] \begin{bmatrix} \mathbf{R} & -\mathbf{R}\tilde{\mathbf{C}} \\ \mathbf{0} & 1 \end{bmatrix}$$

intrinsic parameters (3 x 3):
correspond to camera internals
(image-to-image transformation)

perspective projection (3 x 4):
maps 3D to 2D points
(camera-to-image transformation)

extrinsic parameters (4 x 4):
correspond to camera externals
(world-to-camera transformation)

Putting it all together

- We can write everything into a single projection:

$$\mathbf{x} = \mathbf{P}\mathbf{X_w}$$

- The camera matrix now looks like:

$$\mathbf{P} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \left[\mathbf{R} \;\vdots\; -\mathbf{RC}\right]$$

intrinsic parameters (3 x 3):
correspond to camera internals
(sensor not at f = 1 and origin shift)

extrinsic parameters (3 x 4):
correspond to camera externals
(world-to-camera transformation)

General pinhole camera matrix

- We can decompose the camera matrix like this:

$$\mathbf{P} = \mathbf{KR}[\mathbf{I}| - \mathbf{C}]$$

(translate first then rotate)

- Another way to write the mapping:

$$\mathbf{P} = \mathbf{K}[\mathbf{R}|\mathbf{t}]$$

where $\mathbf{t} = -\mathbf{RC}$

(rotate first then translate)

General pinhole camera matrix

$$\mathbf{P} = \mathbf{K}[\mathbf{R}|\mathbf{t}]$$

$$\mathbf{P} = \left[\begin{array}{ccc} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{array}\right] \left[\begin{array}{ccc:c} r_1 & r_2 & r_3 & t_1 \\ r_4 & r_5 & r_6 & t_2 \\ r_7 & r_8 & r_9 & t_3 \end{array}\right]$$

intrinsic parameters

extrinsic parameters

$$\mathbf{R} = \left[\begin{array}{ccc} r_1 & r_2 & r_3 \\ r_4 & r_5 & r_6 \\ r_7 & r_8 & r_9 \end{array}\right] \qquad \mathbf{t} = \left[\begin{array}{c} t_1 \\ t_2 \\ t_3 \end{array}\right]$$

3D rotation

3D translation

Recap

- What is the size and meaning of each term in the camera matrix?

$$\mathbf{P} = \mathbf{K}\mathbf{R}[\mathbf{I}| - \mathbf{C}]$$

? ? ? ?

Recap

- What is the size and meaning of each term in the camera matrix?

$$\mathbf{P} = \mathbf{K}\mathbf{R}[\mathbf{I}| - \mathbf{C}]$$

3x3
intrinsics

?

?

?

Recap

- What is the size and meaning of each term in the camera matrix?

$$\mathbf{P} = \mathbf{K}\mathbf{R}[\mathbf{I}| - \mathbf{C}]$$

3x3
intrinsics

3x3
3D rotation

?

?

Recap

- What is the size and meaning of each term in the camera matrix?

$$\mathbf{P} = \mathbf{K}\mathbf{R}[\mathbf{I}| - \mathbf{C}]$$

3x3
intrinsics

3x3
3D rotation

3x3
identity

?

Recap

- What is the size and meaning of each term in the camera matrix?

$$\mathbf{P} = \mathbf{K}\mathbf{R}[\mathbf{I}| - \mathbf{C}]$$

3x3	3x3	3x3	3x1
intrinsics	3D rotation	identity	3D camera center

Quiz

- The camera matrix relates what two quantities?

Quiz

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$$\boldsymbol{x} = \mathbf{P}\mathbf{X}$$

homogeneous 3D points to 2D image points

Quiz

- The camera matrix relates what two quantities?

$$\boldsymbol{x} = \mathbf{PX}$$

homogeneous 3D points to 2D image points

- The camera matrix can be decomposed into?

Quiz

- The camera matrix relates what two quantities?

$$\boldsymbol{x} = \mathbf{P}\mathbf{X}$$

homogeneous 3D points to 2D image points

- The camera matrix can be decomposed into?

$$\mathbf{P} = \mathbf{K}[\mathbf{R}|\mathbf{t}]$$

intrinsic and extrinsic parameters

More general camera matrices

- The following is the standard camera matrix we saw.

$$\mathbf{P} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \quad \left[\mathbf{R} \;\middle|\; -\mathbf{RC}\right]$$

More general camera matrices

- CCD camera: pixels may not be square?

$$\mathbf{P} = \begin{bmatrix} \alpha_x & 0 & p_x \\ 0 & \alpha_y & p_y \\ 0 & 0 & 1 \end{bmatrix} \left[\mathbf{R} \;\vdots\; -\mathbf{RC}\right]$$

- How many degrees of freedom?

More general camera matrices

- CCD camera: pixels may not be square?

$$\mathbf{P} = \begin{bmatrix} \alpha_x & 0 & p_x \\ 0 & \alpha_y & p_y \\ 0 & 0 & 1 \end{bmatrix} \left[\mathbf{R} \;\middle|\; -\mathbf{RC} \right]$$

- How many degrees of freedom?
  - 10 DOF

More general camera matrices

- Finite projective camera: sensor be skewed.

$$\mathbf{P} = \begin{bmatrix} \alpha_x & s & p_x \\ 0 & \alpha_y & p_y \\ 0 & 0 & 1 \end{bmatrix} \left[\mathbf{R} \;\middle|\; -\mathbf{RC} \right]$$

- How many degrees of freedom?

More general camera matrices

- Finite projective camera: sensor be skewed.

$$\mathbf{P} = \begin{bmatrix} \alpha_x & s & p_x \\ 0 & \alpha_y & p_y \\ 0 & 0 & 1 \end{bmatrix} \left[\mathbf{R} \;\middle|\; -\mathbf{RC} \right]$$

- How many degrees of freedom?
  - 11 DOF