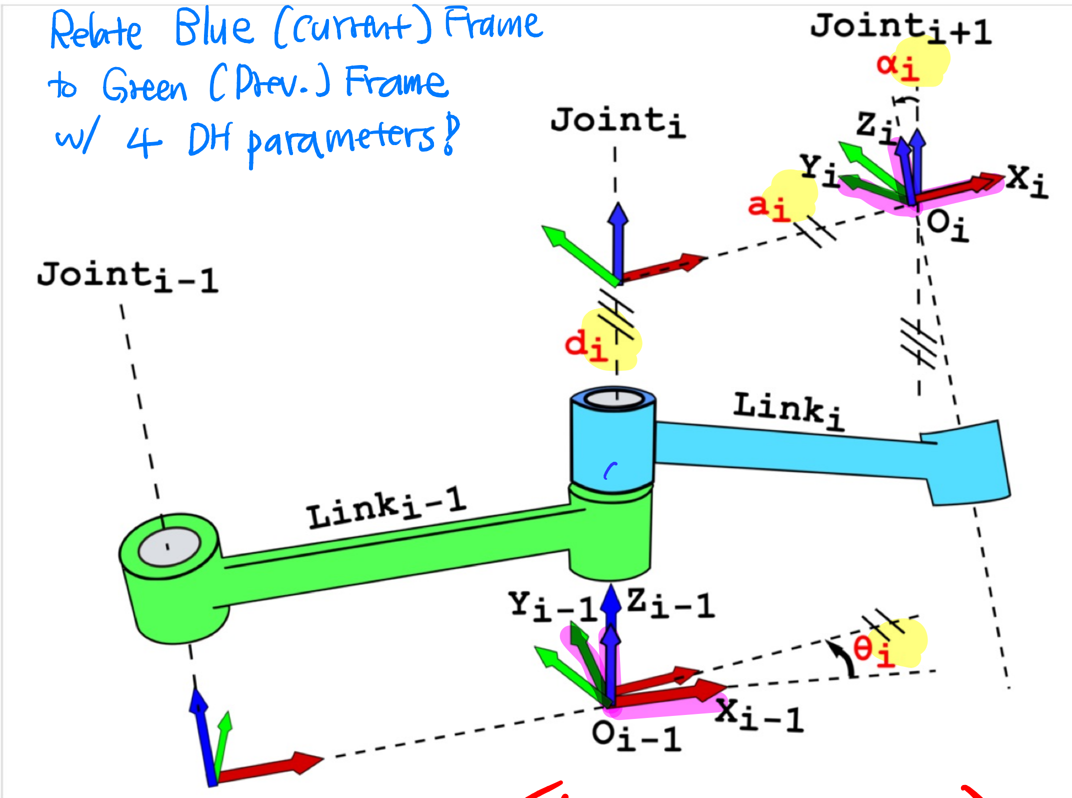
[Denavit–Hartenberg (DH) Parameters / Convention]

- An approach to solve Forward Kinematics.

Relate Blue (current) Frame
to Green (Prev.) Frame
w/ 4 DH parameters?



$\langle \theta_i \quad d_i \quad a_i \quad \alpha_i \rangle$

Assumption) $x_n \perp z_{n-1}$, x_n intersects z_{n-1}

- $[Z_i] = Trans_{z_i}(d_i)\, Rot_{z_i}(\theta_i) = Rot_{z_i}(\theta_i)\, Trans_{z_i}(d_i)$
- $[X_i] = Trans_{x_i}(a_i)\, Rot_{x_i}(\alpha_i) = Rot_{x_i}(\alpha_i)\, Trans_{x_i}(a_i)$

$$\Rightarrow T_{n-1}^{n} = [Z_{n-1}][X_n]$$

- Note that

$\rightarrow \text{Trans}_{z_{n-1}}(d_n) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $\text{Rot}_{z_{n-1}}(\theta_n) = \begin{bmatrix} c\theta_n & -s\theta_n & 0 & 0 \\ s\theta_n & c\theta_n & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$\text{Trans}_{x_n}(a_n) = \begin{bmatrix} 1 & 0 & 0 & a_n \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $\text{Rot}_{x_{n-1}}(\alpha_n) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha_n & -s\alpha_n & 0 \\ 0 & s\alpha_n & c\alpha_n & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$\therefore T_{n-1}^{n} = \begin{bmatrix} c\theta_n & -s\theta_n c\alpha_n & s\theta_n s\alpha_n & a_n c\theta_n \\ s\theta_n & c\theta_n c\alpha_n & -c\theta_n s\alpha_n & a_n s\theta_n \\ 0 & s\alpha_n & c\alpha_n & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then $T_1^n = \prod_{i=1}^{n-1} T_i^{i+1}$: FK Solved!

[Inverse Kinematics]

$\Rightarrow$ Given $T_1^n = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}$, find $q = \begin{bmatrix} q_1 \\ \vdots \\ q_n \end{bmatrix}$ for n joints of robot.

* There can be (1) no
(2) multiple solutions *

Numerical Solution!

$\Rightarrow$ Jacobian Matrix.

for $f : (x_1, \dots x_n) \mapsto (f_1, \dots f_m)$

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix} \in \mathbb{R}^{m \times n}$$

$x \rightarrow$ Joints

$f \rightarrow$ result from FK.

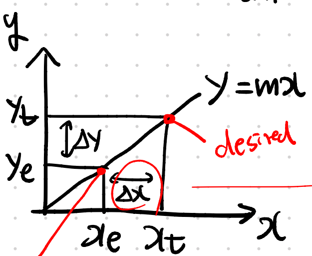
$y = Ax$

$A^T y = A^T A x$

$(A^T A)^{-1} A^T y = x$

Use psuedo inverse, if non-invertible

$J^{+} = (J^T J)^{-1} J^T$

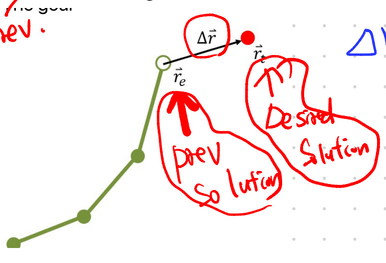


prev.

$\Delta y = m \Delta x$

$\Rightarrow \Delta x = \frac{1}{m} \Delta y$

likewise $\Rightarrow \Delta r = J \Delta \theta$



$$\Delta r = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} = J \Delta \theta = \begin{bmatrix} \frac{dx}{d\theta_1} & \cdots & \frac{dx}{d\theta_n} \\ \frac{dy}{d\theta_1} & \cdots & \frac{dy}{d\theta_n} \\ \frac{dz}{d\theta_1} & \cdots & \frac{dz}{d\theta_n} \end{bmatrix} \begin{bmatrix} \Delta\theta_1 \\ \vdots \\ \Delta\theta_n \end{bmatrix}$$

$\therefore \Delta\theta = J^{-1} \Delta r$

Then, we do

$\theta_{n+1} \leftarrow \theta_n + J^{-1}(r_t - FK(\theta_n))$

Given Target Position.