Deep learning (CSE 40301)
Principles of Deep learning (IE40801)

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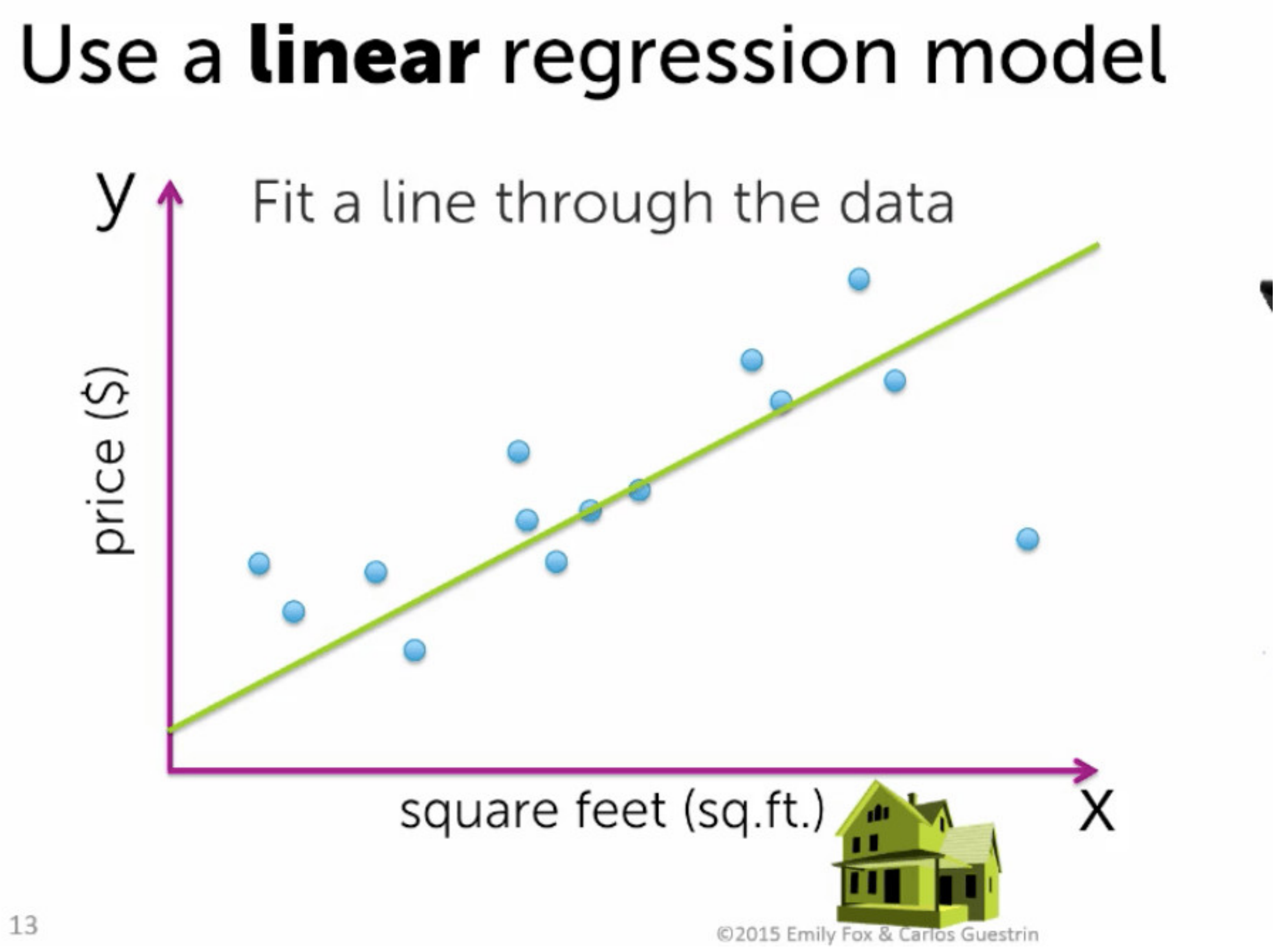
Brief history of deep learning

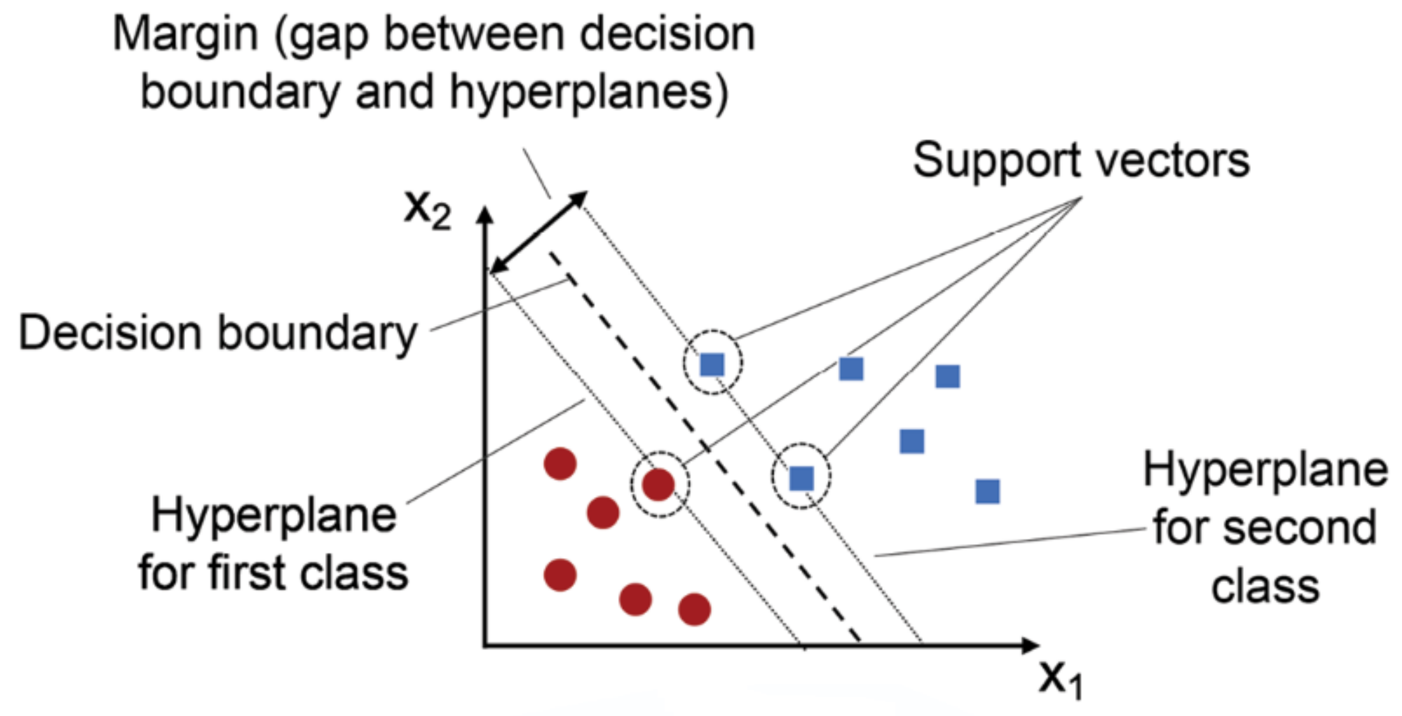
- **Linear regression** (Legendre & Gauss, 1805) and **logistic regression**
  - Simplest forms of regression or classification to model the relationship between input variables and a continuous output or a label
  - Example: Predicting house prices based on features like area, number of rooms.
  - Example: Predicting if an email is spam or not.
  - More statistics than AI (ML)



* https://www.linkedin.com/pulse/predicting-house-prices-using-linear-regression-along-muhammet-ergenc

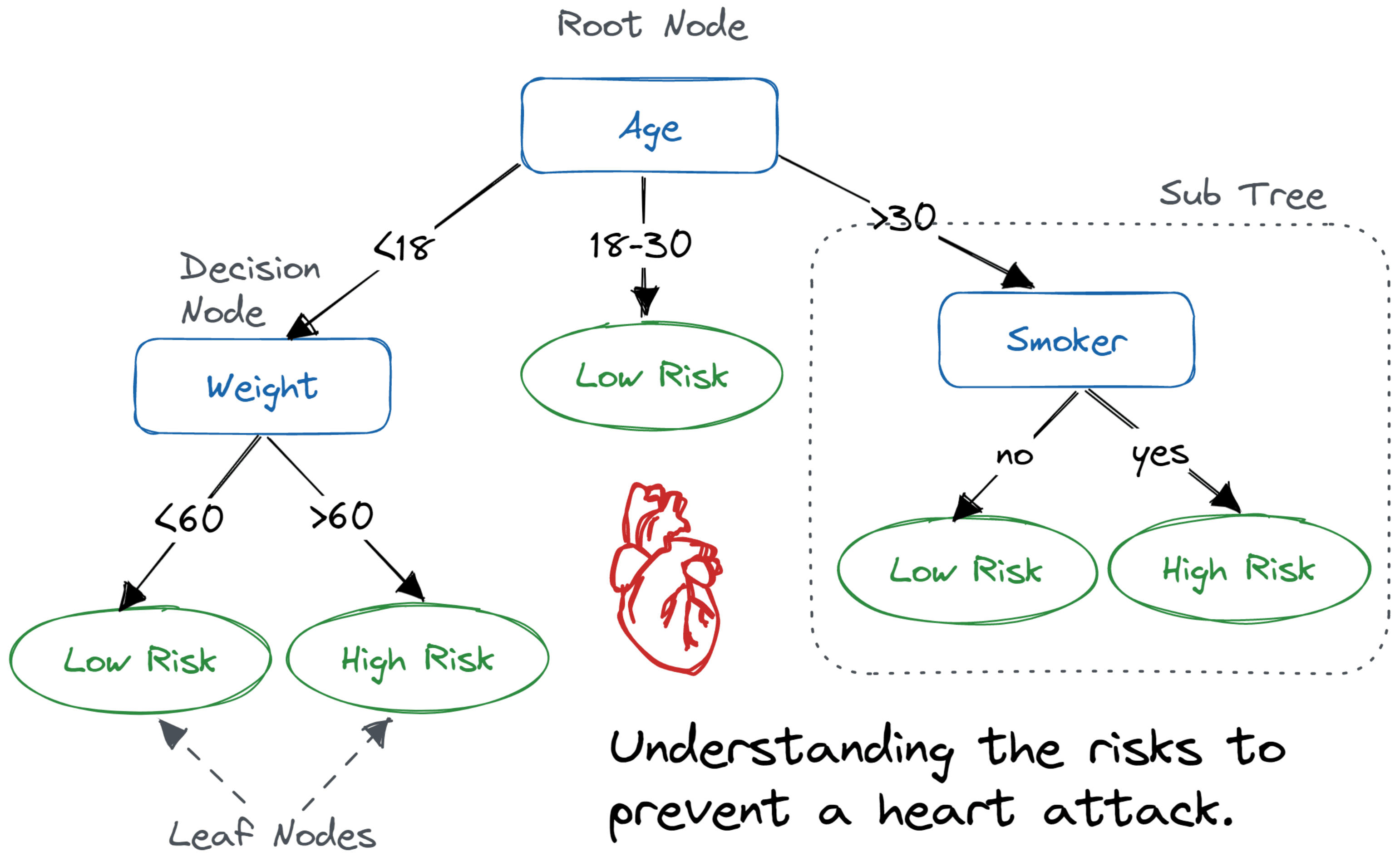
Brief history of deep learning

- **Support Vector Machines** (SVM) (Vladimir Vapnik, 1990s)
  - A powerful classifier that finds the hyperplane separating different classes with the maximum margin
  - Used in image classification, handwriting recognition, etc.



https://vitalflux.com/classification-model-svm-classifier-python-example/

- **Decision Trees and Ensembles**
  - Tree-based methods and advancements in ensemble methods like Random Forests and Gradient Boosting
  - Limitations 1: Heavy reliance on handcrafted features.
  - Limitations 2: Poor scalability and limited ability to handle large, complex data like images or raw text.



https://www.datacamp.com/tutorial/decision-tree-classification-python

Before the deep learning take-off

- Before 2012
  - SVM-variants were the model-of-choice !
  - With handcrafted features such as SIFT (Scale-Invariant Feature Transform) or HOG (Histogram of Oriented Gradients)
- Since 2012
  - AlexNet breakthrough in 2012
  - Dramatically changed the landscape
    - reducing error rates by a significant margin
    - surpassed traditional methods in virtually every domain,
    - from image classification to natural language processing.

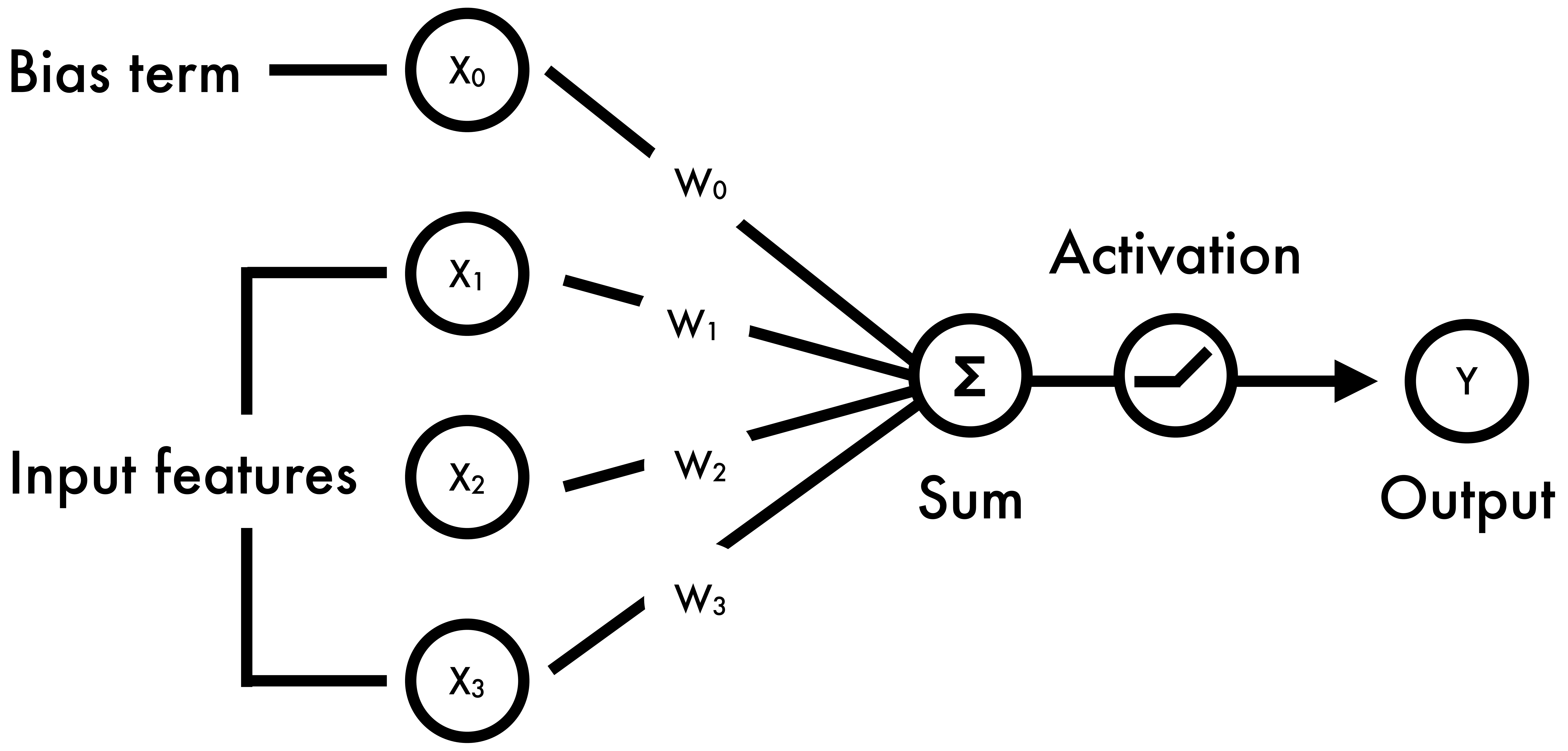
ImageNet Leaderboard: Pre-Deep Learning & Early Deep Learning Models

Year	Model	Top-5 Error Rate	Notes
2010	SIFT + SVM	28.2%	SVM + handcrafted features (e.g., SIFT, HOG). First ImageNet competition.
2011	Improved SIFT + Fisher Vectors	25.7%	Handcrafted features with Fisher vectors. Top-performing non-deep model.
2011	Deep Learning (LeCun's Lab)	~26%	Early CNN approach from LeCun's lab. Limited computational power.
2011	Handcrafted + Shallow CNNs	~25%	Small CNNs combined with handcrafted features, still underperforming.
2012	AlexNet (Deep CNN)	16.4%	First deep learning breakthrough. Massive performance leap (8.5% gain).
2013	ZFNet (Deep CNN)	14.8%	Improved on AlexNet with better architecture (deconvolution visualization).
2014	GoogLeNet (Inception Network)	6.7%	Introduced the Inception module, made networks deeper and more efficient.
2015	ResNet (Deep Residual Network)	3.6%	Introduced residual connections to solve vanishing gradients.
2016	ResNet-152	3.0%	Extended ResNet to 152 layers, further reducing error.

Why? Scalability!

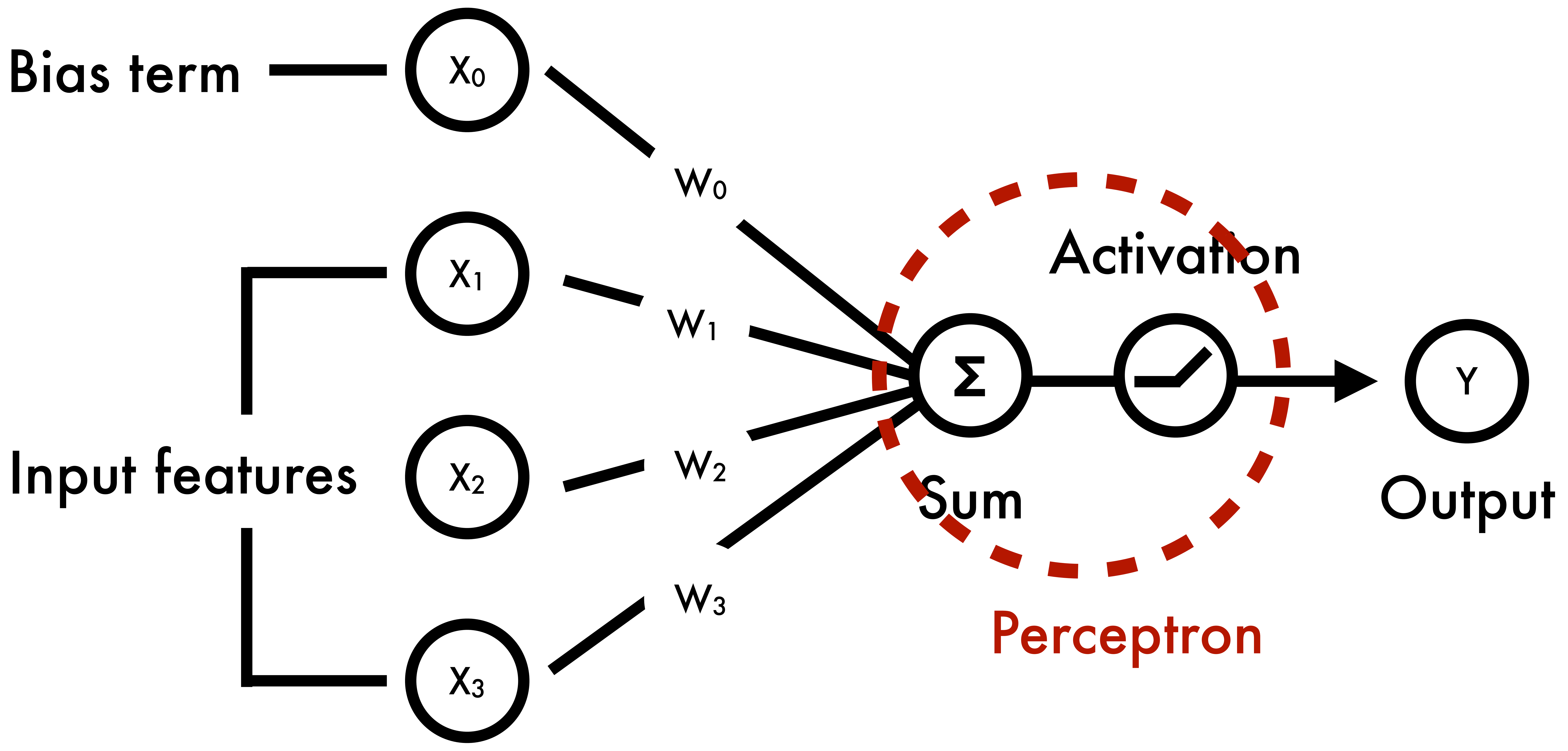
- More data
  - The availability of massive datasets (e.g., ImageNet) allowed deep learning models to generalize better by learning from a wide variety of examples
- Larger model
  - Deep learning models, especially deep neural networks, have a large number of parameters, enabling them to capture more complex patterns and relationships in the data
- More compute
  - The rise of powerful GPUs and specialized hardware (e.g., TPUs) made it feasible to train large models efficiently, overcoming previous computational barriers

Perceptron



Bias term
X_0
W_0
X_1
W_1
Activation
Σ
Y
Input features
X_2
W_2
Sum
Output
W_3
X_3

Perceptron


Bias term
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Perceptron

Perceptron ≈ Linear regression

Linear Regression in Matrix Form:

$$\mathbf{Y} = \mathbf{X}\mathbf{w} + \epsilon$$

Where:

- $\mathbf{Y} \in \mathbb{R}^{n \times 1}$: The vector of target values (outputs) with n samples.
- $\mathbf{X} \in \mathbb{R}^{n \times d}$: The matrix of input features (with n samples and d features). Each row is a feature vector corresponding to a data sample.
- $\mathbf{w} \in \mathbb{R}^{d \times 1}$: The vector of weights (coefficients) that we aim to learn.
- $\epsilon \in \mathbb{R}^{n \times 1}$: The vector of error terms (residuals).

Perceptron ≈ Linear regression

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- **Input matrix $\mathbf{X}$:**

$$\mathbf{X} = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1d} \\ x_{21} & x_{22} & \dots & x_{2d} \\ \vdots & \vdots & \dots & \vdots \\ x_{n1} & x_{n2} & \dots & x_{nd} \end{bmatrix}$$

  Where each row represents a sample, and each column represents a feature.

- **Weight vector $\mathbf{w}$:**

$$\mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_d \end{bmatrix}$$

  These are the coefficients for each feature.

- **Target vector $\mathbf{Y}$:**

$$\mathbf{Y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

Closed-form solution using SSE

$$SSE = ||\mathbf{y} - \mathbf{X}\mathbf{w}||^2$$

$$SSE = (\mathbf{y} - \mathbf{X}\mathbf{w})^\top(\mathbf{y} - \mathbf{X}\mathbf{w})$$

$$SSE = \mathbf{y}^\top\mathbf{y} - 2\mathbf{y}^\top\mathbf{X}\mathbf{w} + \mathbf{w}^\top\mathbf{X}^\top\mathbf{X}\mathbf{w}$$

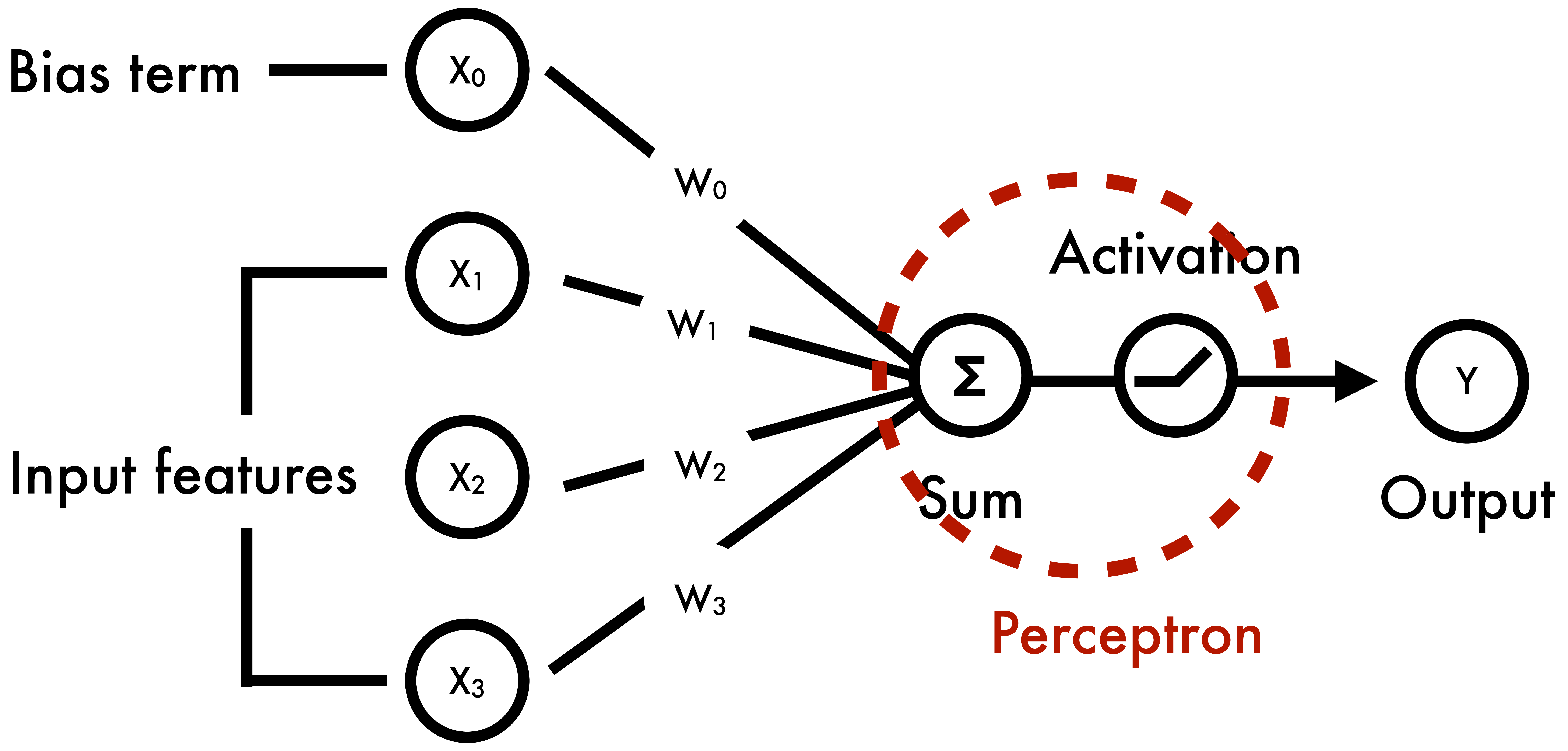
$$\frac{\partial SSE}{\partial \mathbf{w}} = -2\mathbf{X}^\top\mathbf{y} + 2\mathbf{X}^\top\mathbf{X}\mathbf{w} = 0$$

$$\mathbf{X}^\top\mathbf{X}\mathbf{w} = \mathbf{X}^\top\mathbf{y}$$

$$\mathbf{w} = (\mathbf{X}^\top\mathbf{X})^{-1}\mathbf{X}^\top\mathbf{y}$$

Pseudo-inverse

Perceptron


Bias term
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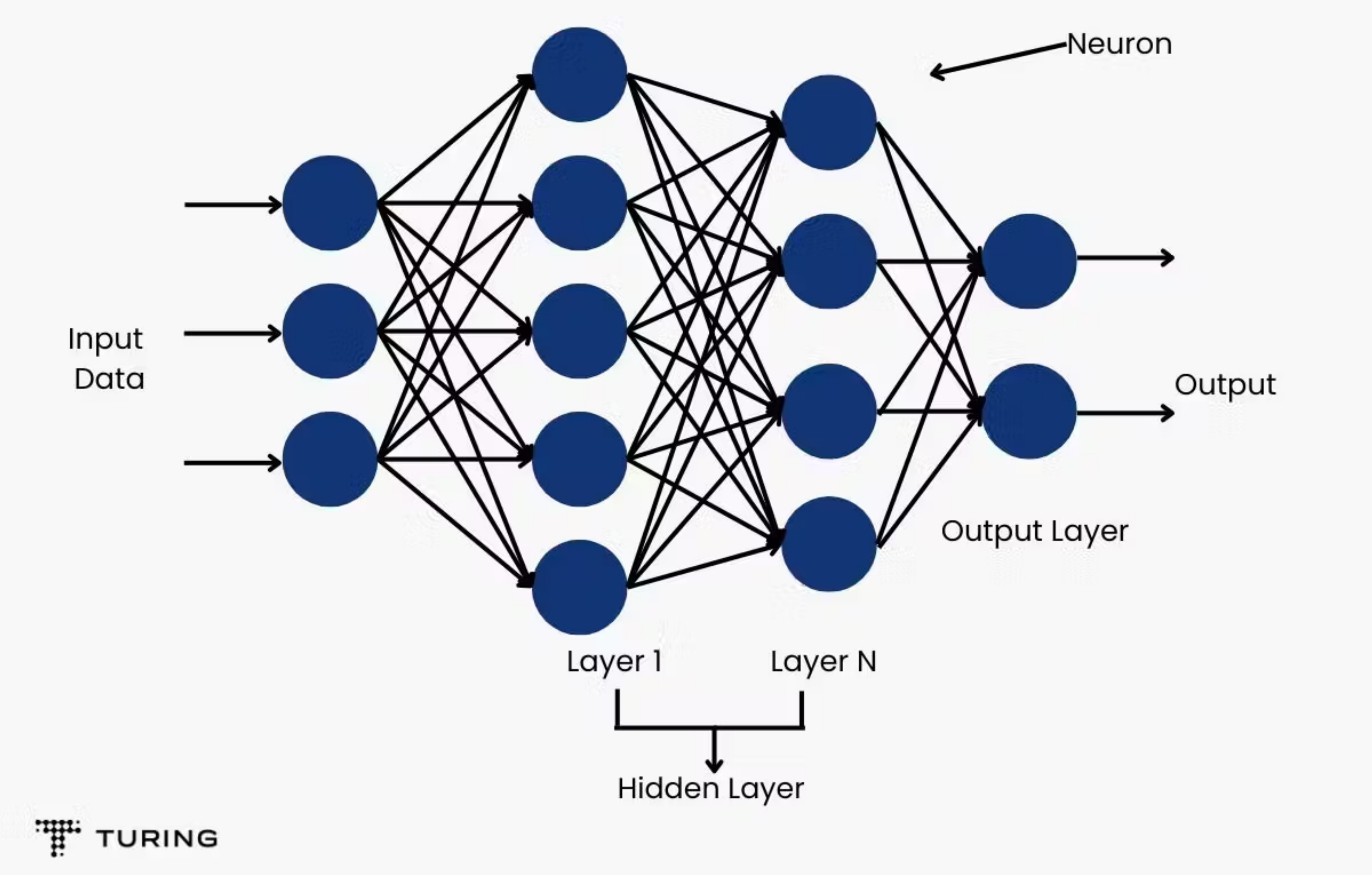
Output
Perceptron

Multi Layer Perceptron

- Also known as feed-forward network (FNN)
- **Spread** multiple perceptrons horizontally
  - -> (Hidden) layers
- and
- **Stack** multiple layers vertically
- That’s it!!!
- Works for both regression and classification
  - And much more actually…



https://www.turing.com/kb/explanation-of-deep-neural-network-multilayer-perceptron-deep-q-network

MLP ≈ Deep learning

- Understanding how MLP works means that you know (almost) everything about deep learning!!!
- By spreading and stacking hundreds of thousands of (even millions of) perceptrons,
- By feeding an enormous amount of data,
- By adopting a simple learning algorithm, e.g., back propagation,
- Deep learning has been, is, and will be achieving some amazing things
  - Regression, classification
  - Generative models
  - Deep reinforcement learning algorithms

Wait...

- Linear regression ≈ Perceptron
- Perceptron -> MLP
- MLP ≈ Deep learning

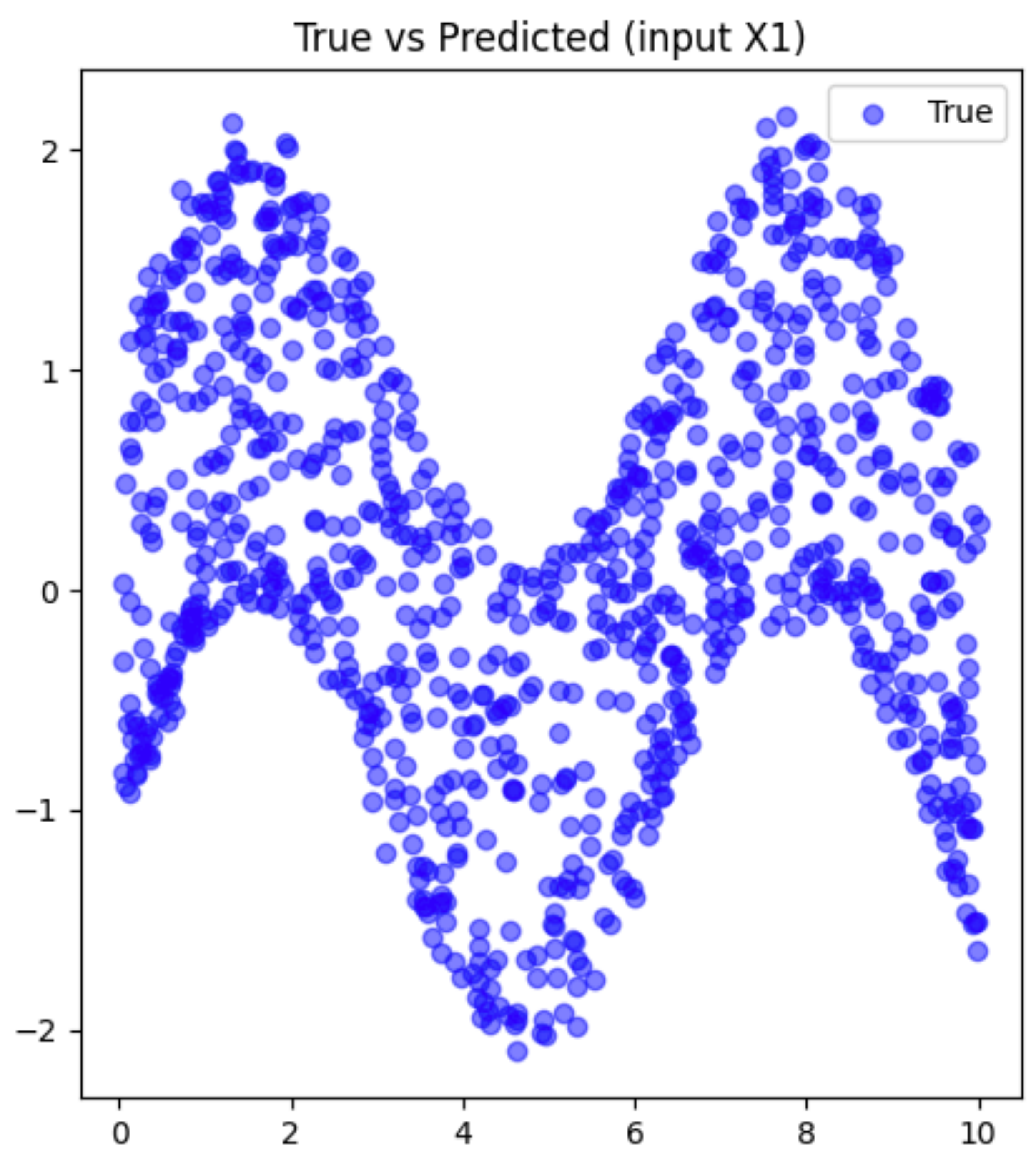
- Deep learning eventually reduces to a perceptron?!

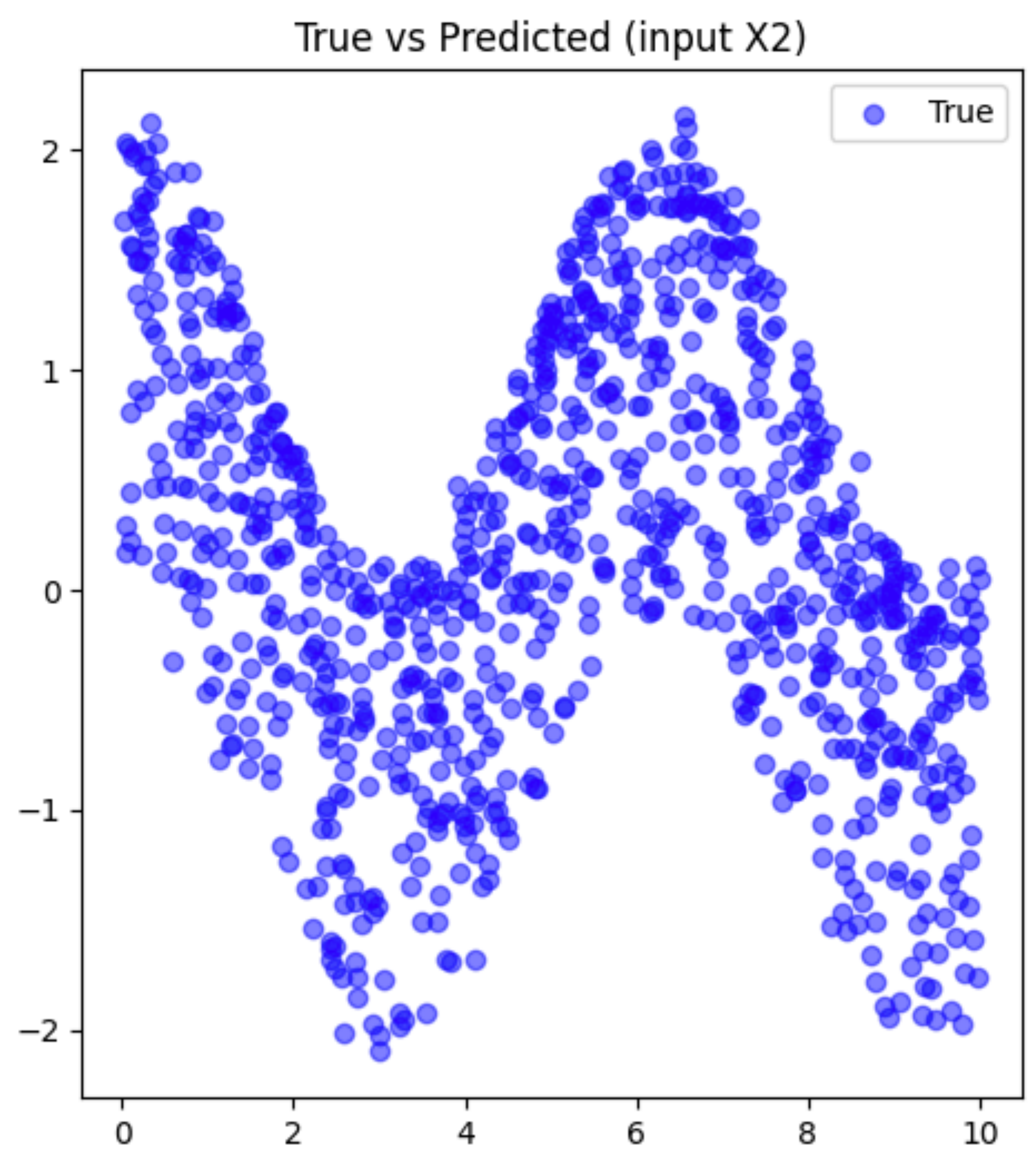
Wait...

- $Y = WX + B$
- $= W(WX + B) + B$
- $= W(W(WX + B) + B)$
- $= WX + B$
- Thus, eventually, spreading and stacking multiple perceptrons means absolutely, nothing
- It’s still just a linear projection!

Experiment

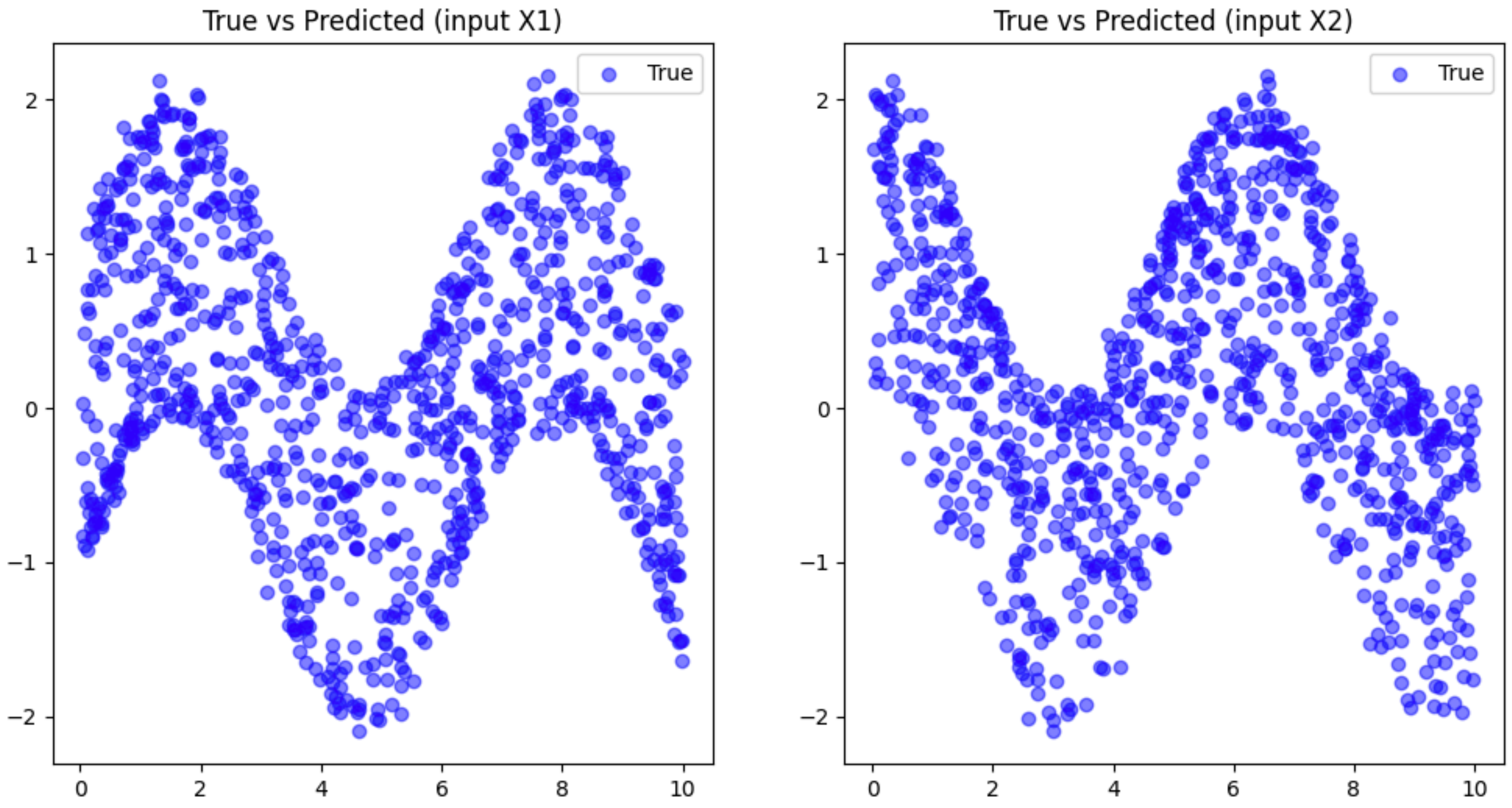
```python
# Generate random inputs (X) and target outputs (Y)
X = np.random.rand(n_samples, 2) * 10  # inputs in range [0, 10]
Y = np.sin(X[:, 0]) + np.cos(X[:, 1]) + np.random.randn(n_samples) * 0.1  # target with some noise
```





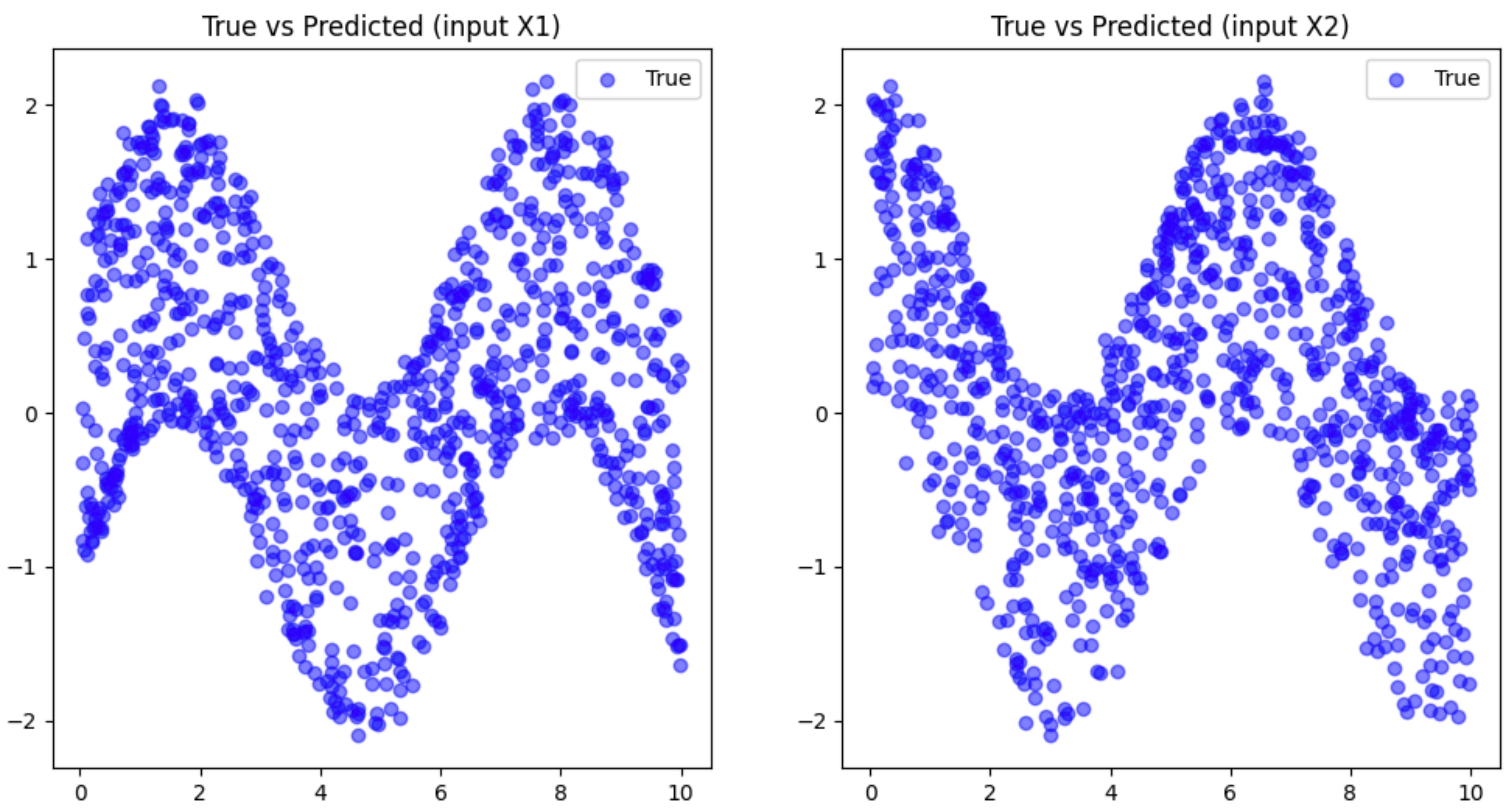
Experiment

- 1000 training datapoints
- Input dimension: 2 (2 features)
- 2 hidden layers
- A little bit of noise
- 64 perceptrons for each hidden layer



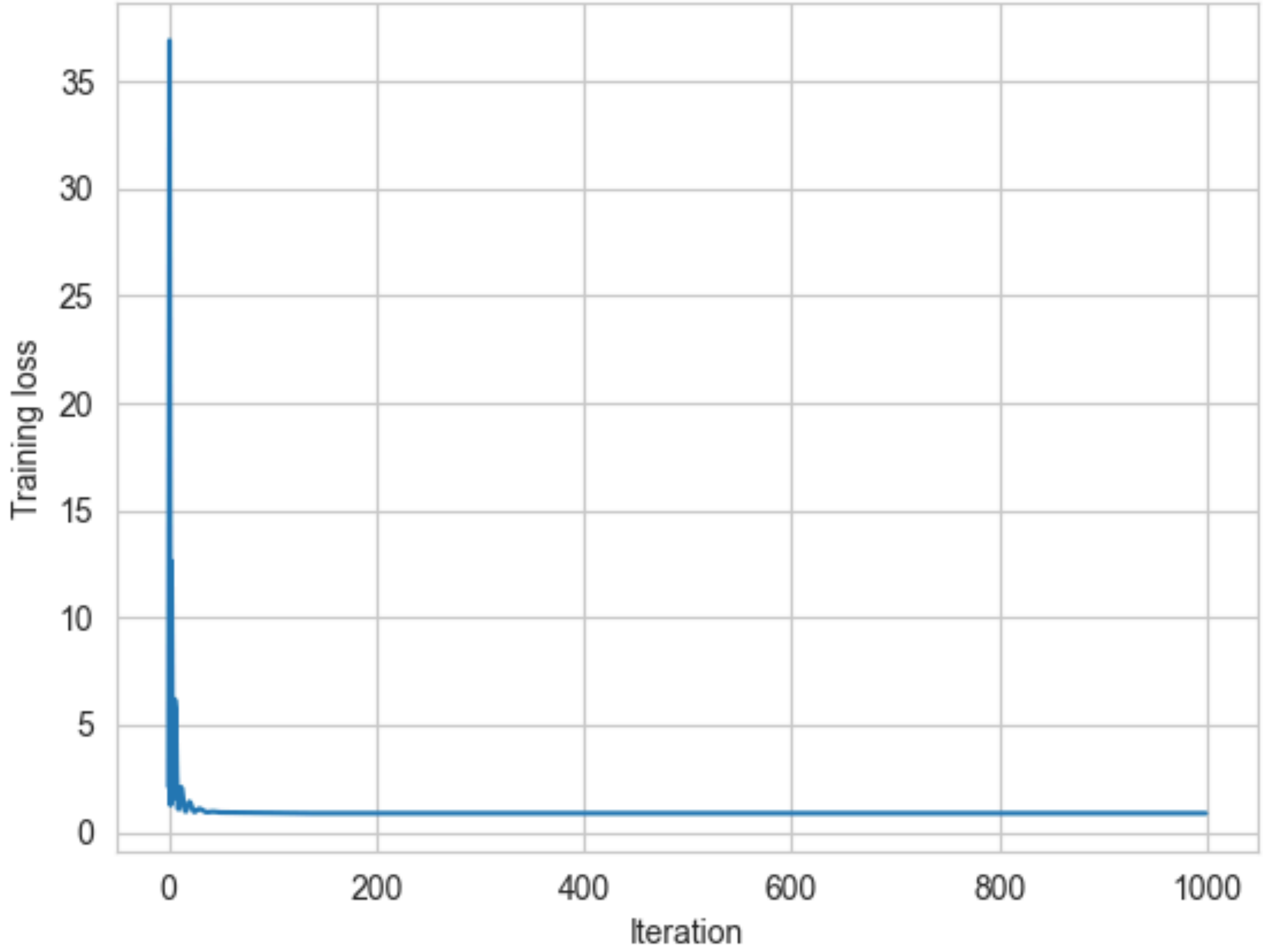
Experiment

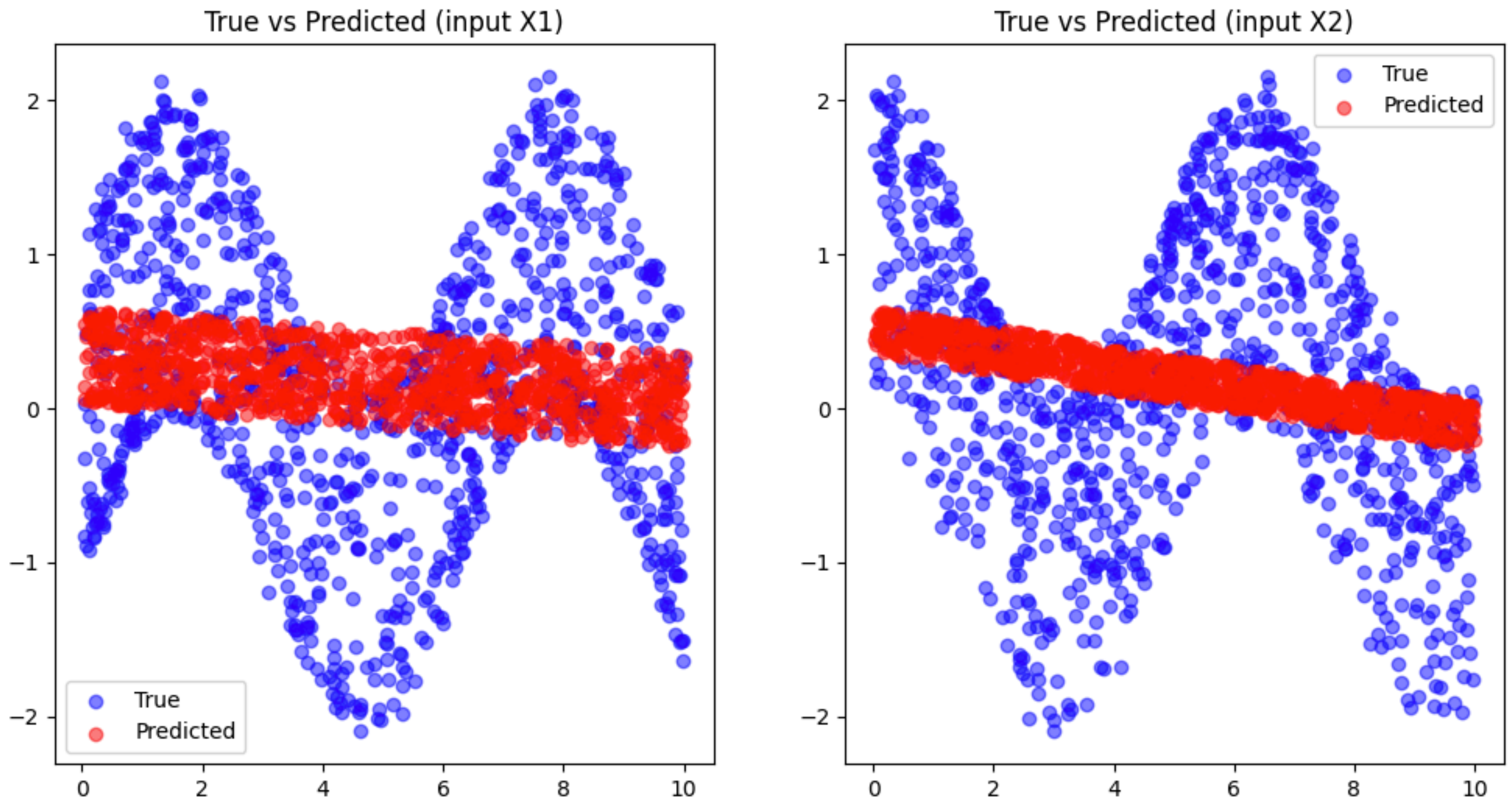
- 1000 training datapoints
- Input dimension: 2 (2 features)
- 2 hidden layers
- A little bit of noise
- 64 perceptrons for each hidden layer
- For a deep learning model of this size, fitting 1,000 2-dimensional datapoints is a piece of cake!



Experimental results

- 2-hidden layer MLP with 128 hidden perceptrons is indeed just a linear regression!

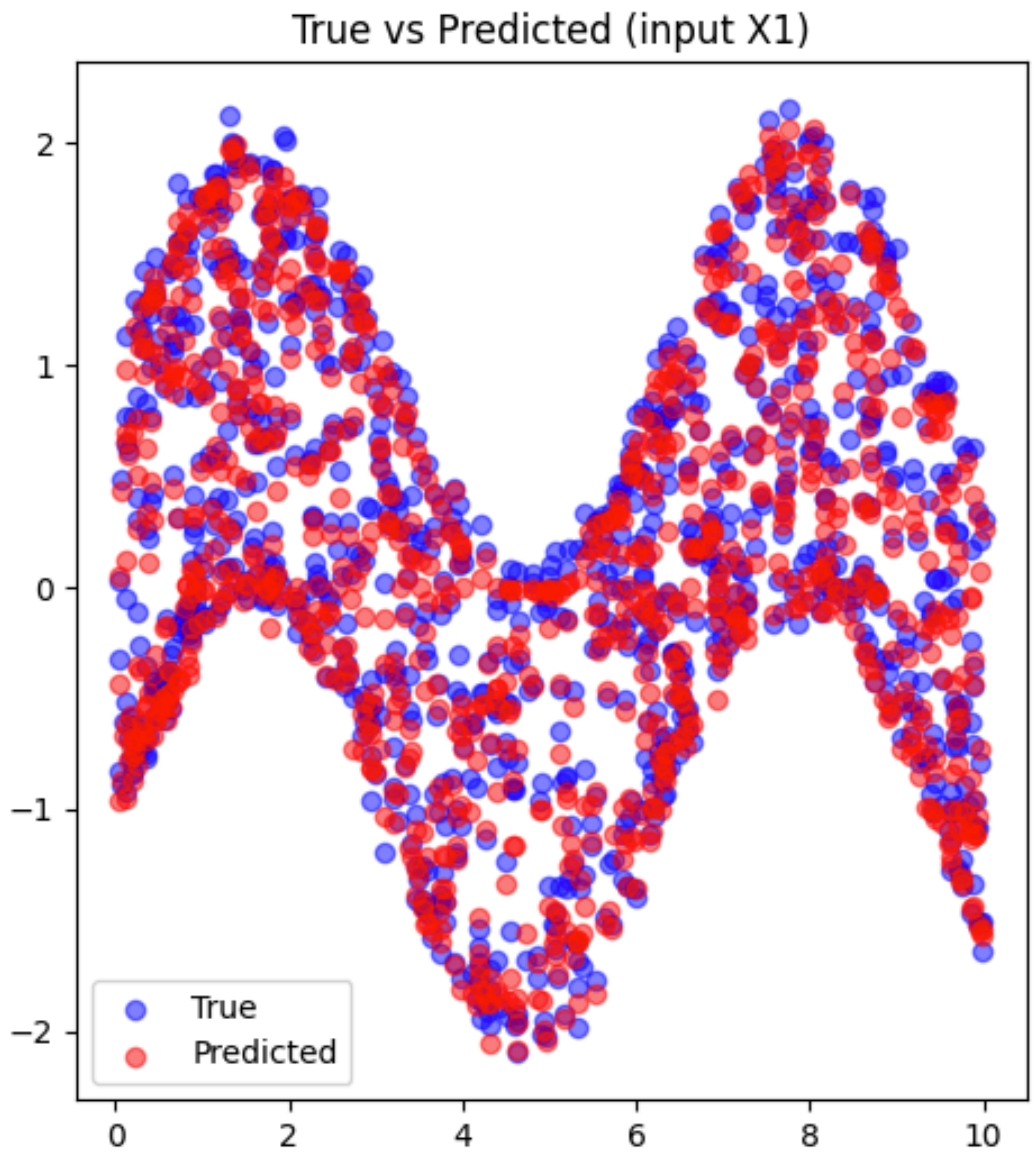




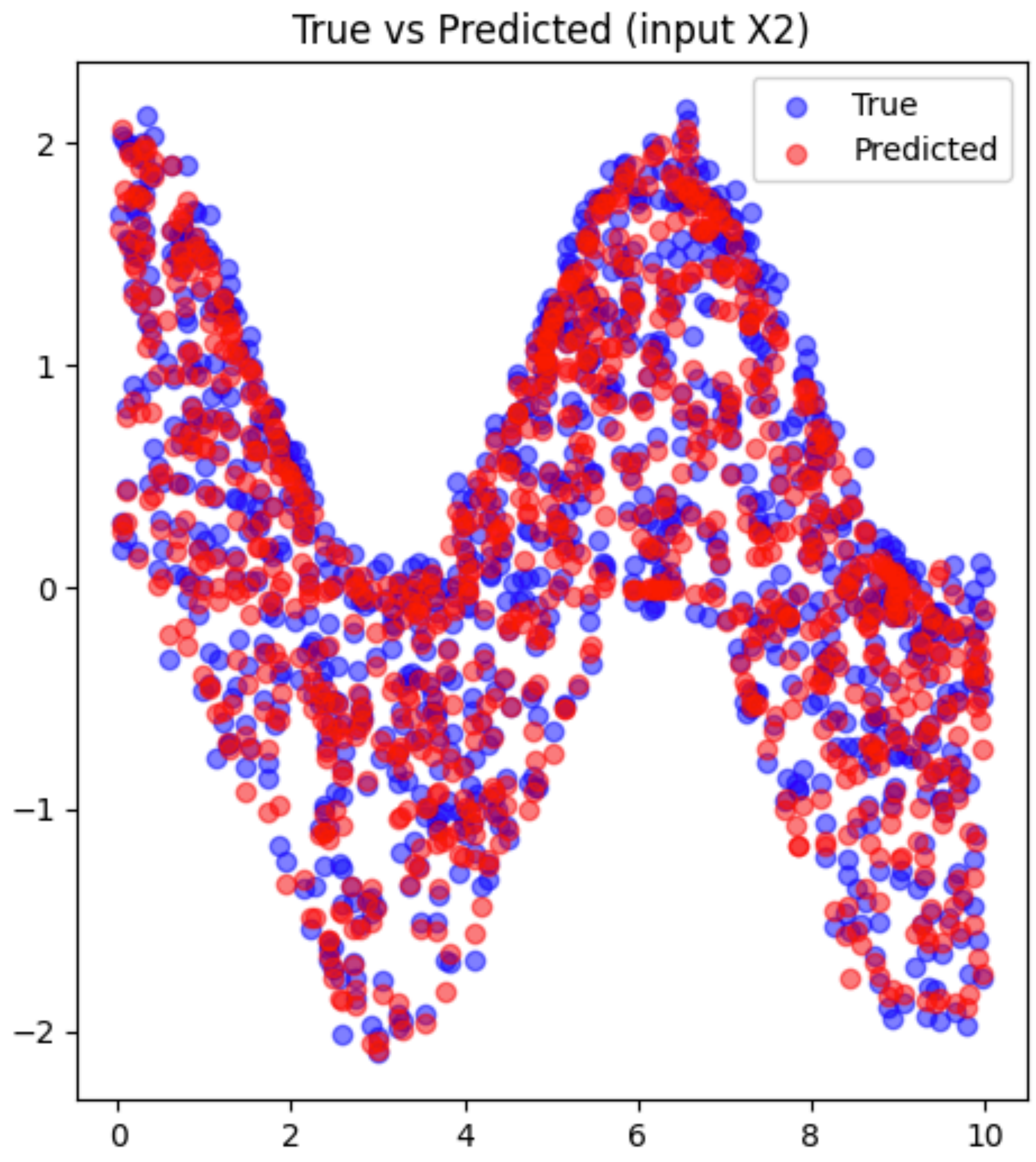
Of course, there are more concerns

- More data
  - Fit different modalities
    - Sequential data
    - Image data
- Larger model
  - Bigger model is not always the answer
  - Overfitting? Overparameterization?
- More compute
  - Closed-form solution
  - Learning algorithm

W/ ReLU

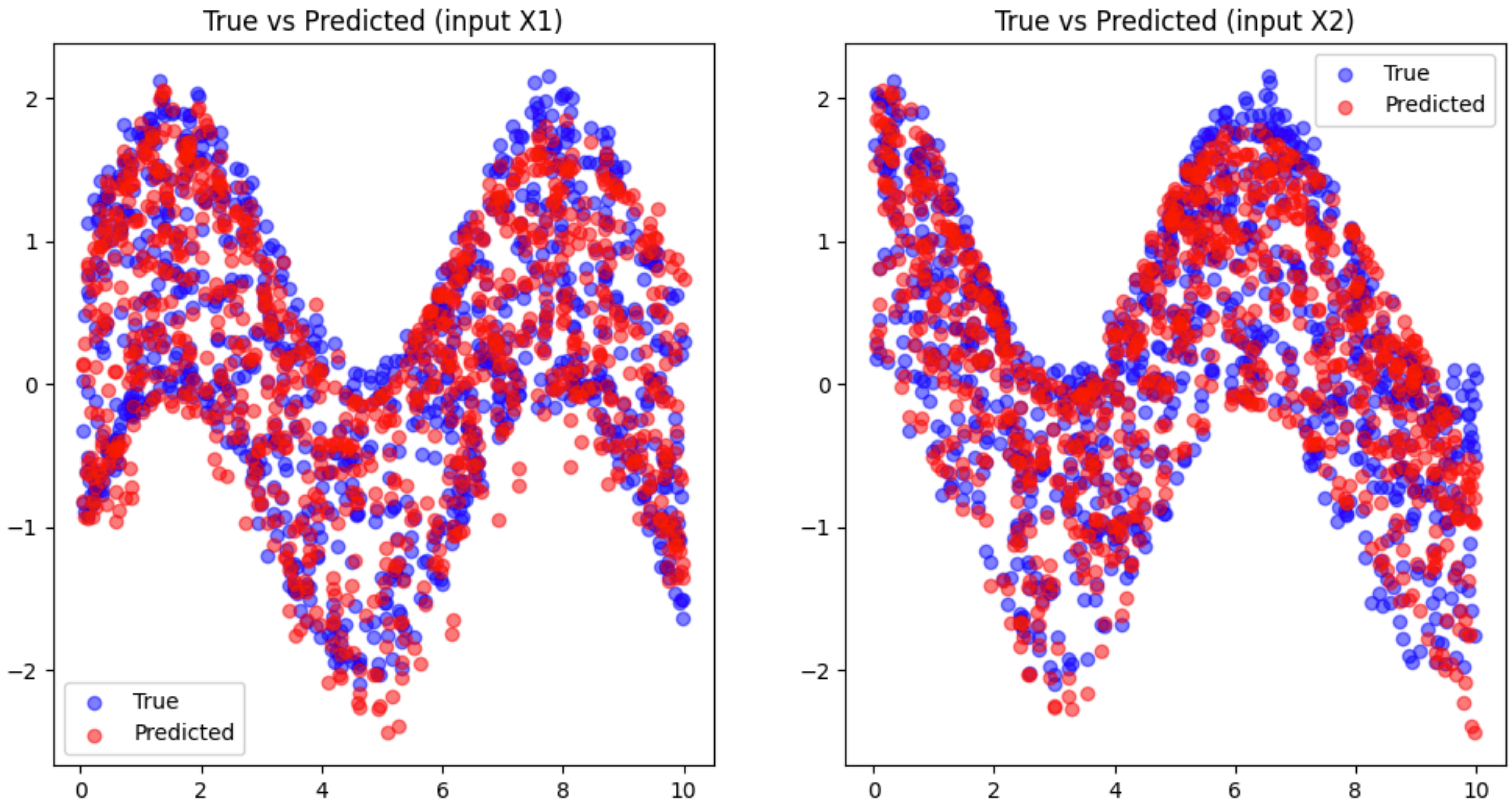


True vs Predicted (input X1)
True
Predicted
0
2
4
6
8
10
−2
−1
0
1
2

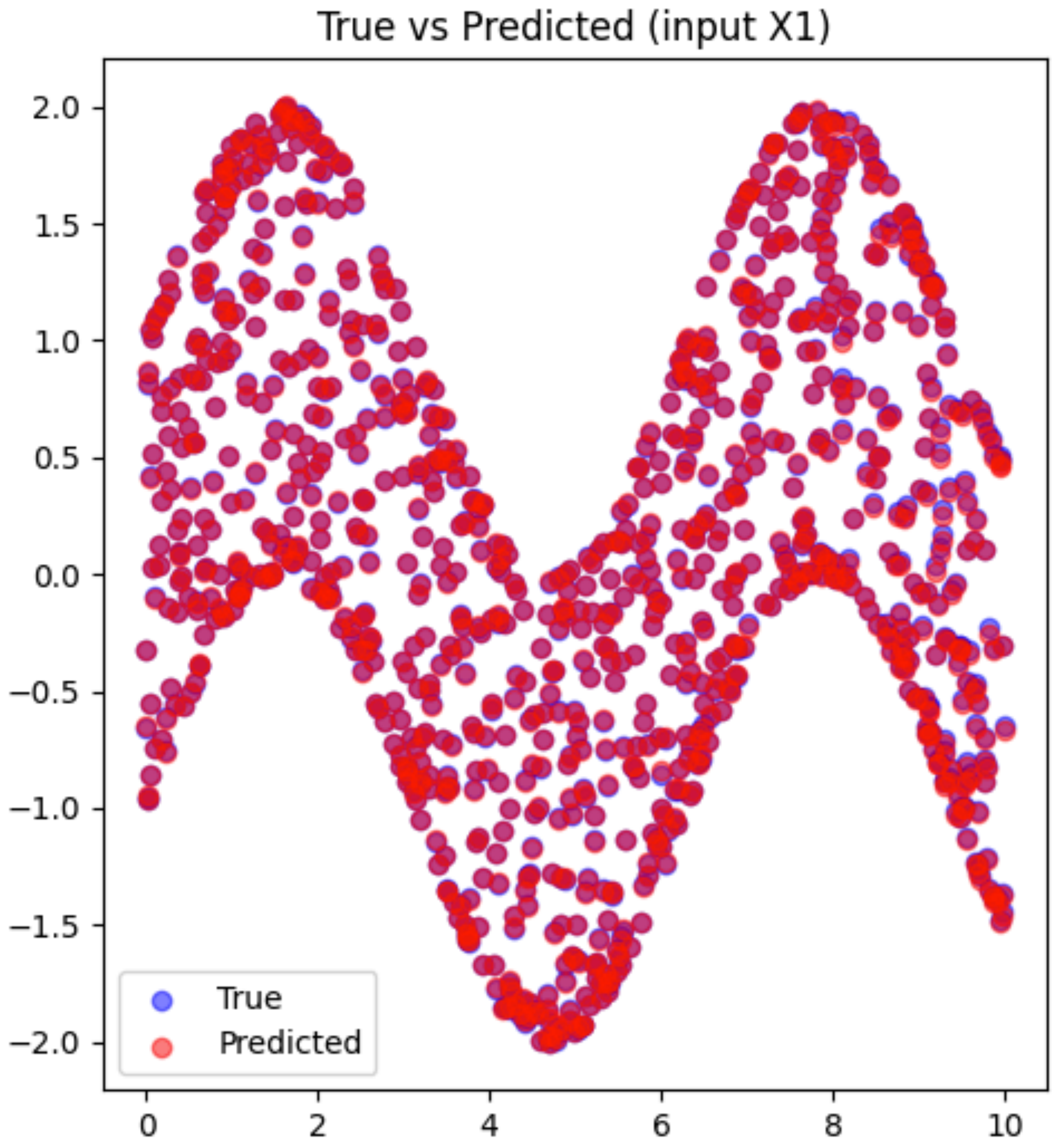


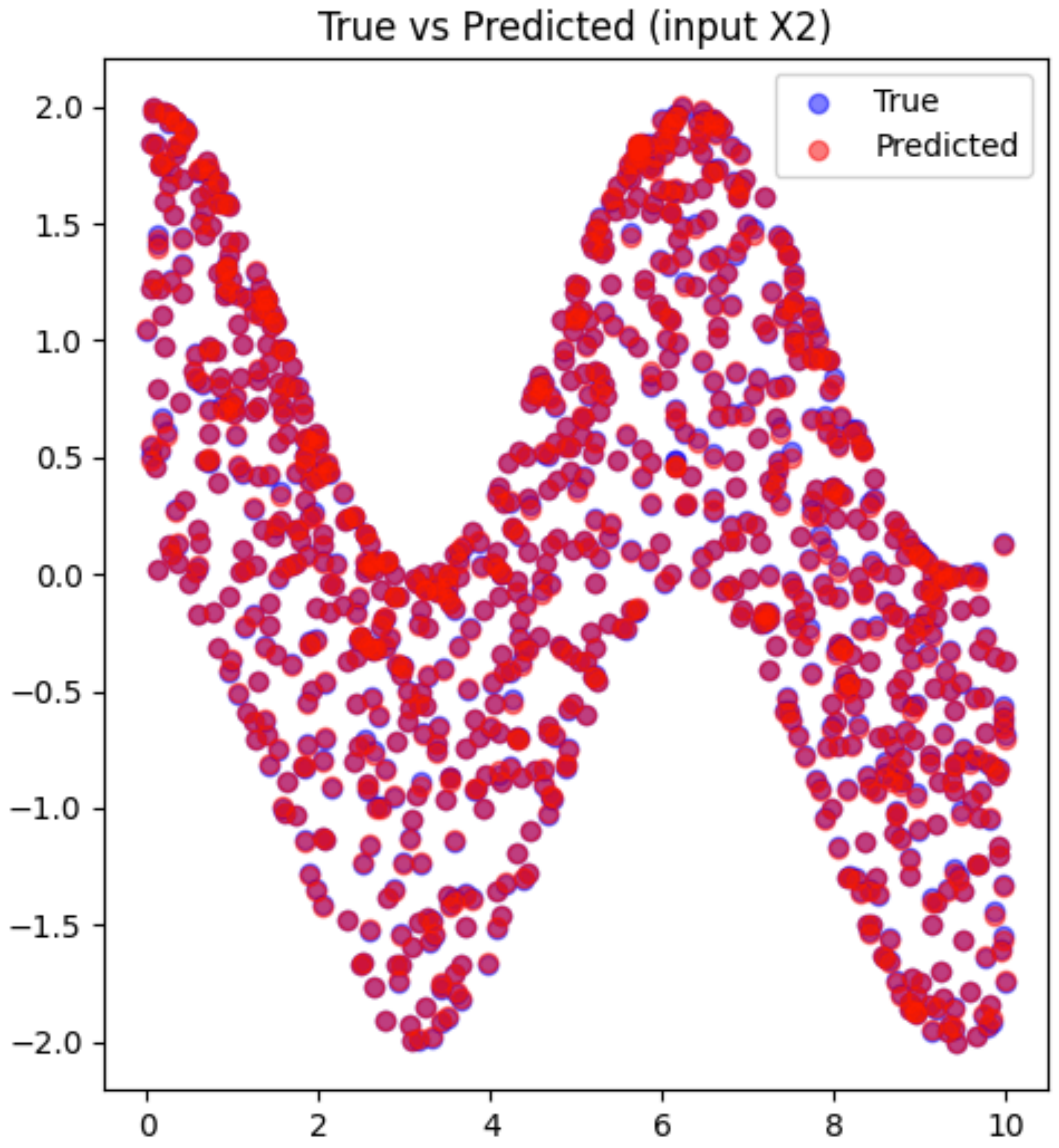
True vs Predicted (input X2)
True
Predicted
0
2
4
6
8
10
−2
−1
0
1
2

W/ Sigmoid


True vs Predicted (input X1)
True
Predicted
True vs Predicted (input X2)
True
Predicted

W/ ReLU, 100K datapoints, 100K iter





MNIST dataset

- Handwritten numbers from 0 to 9
  - Different writing styles
  - Some numbers are hard to be differentiated even for humans
  - Can be downloaded using PyTorch, which is a python’s machine learning library

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```python
import torch
from torchvision import datasets, transforms
from torchvision.utils import save_image

train_kwargs = {'batch_size': 64}

transform=transforms.Compose([
    transforms.ToTensor(),
    # transforms.Normalize((0.1307,), (0.3081,))
    ])

train_dataset = datasets.MNIST('mnist_data', train=True, download=True, transform=transform)
test_dataset = datasets.MNIST('mnist_data', train=False, transform=transform)

print(train_dataset)
print(test_dataset)

train_loader = torch.utils.data.DataLoader(train_dataset, **train_kwargs)
test_loader = torch.utils.data.DataLoader(test_dataset, **train_kwargs)

data = next(iter(train_loader))

print(data[0].shape)
print(data[1], data[1].shape)

save_image(data[0], 'mnist_samples.png')
```

MNIST dataset

- Handwritten numbers from 0 to 9
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  - Some numbers are hard to be differentiated even for humans
  - Can be downloaded using PyTorch, which is a python's machine learning library
- 60,000 train set; 10,000 test set
  - 6,000, 1,000 train, test images
  - For each class

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```
Dataset MNIST
    Number of datapoints: 60000
    Root location: mnist_data
    Split: Train
    StandardTransform
Transform: Compose(
               ToTensor()
           )
Dataset MNIST
    Number of datapoints: 10000
    Root location: mnist_data
    Split: Test
    StandardTransform
Transform: Compose(
               ToTensor()
           )
```

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  - For each class
- Each is 28 * 28 black and white image

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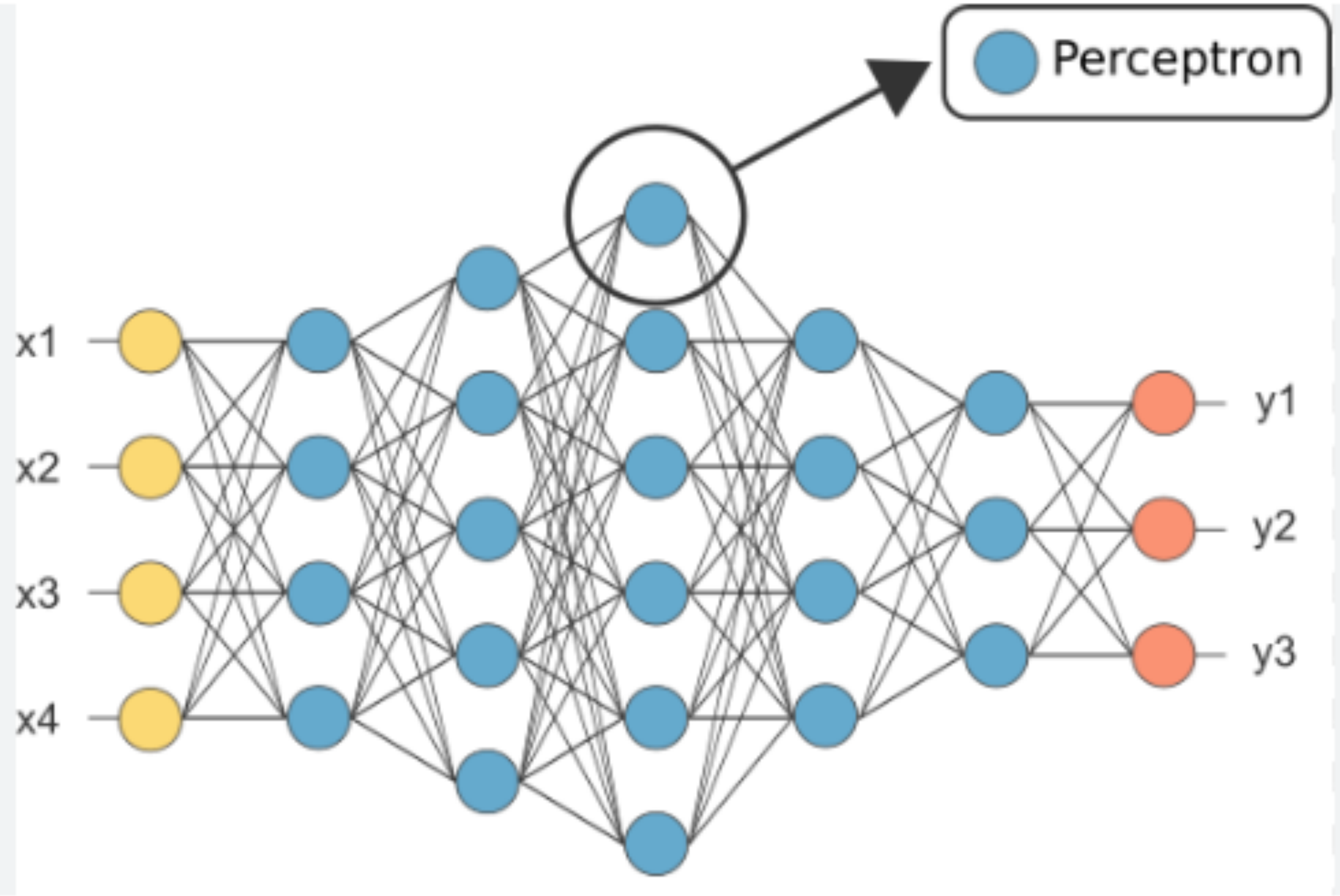
```

```
torch.Size([64, 1, 28, 28])
tensor([5, 0, 4, 1, 9, 2, 1, 3, 1, 4, 3, 5, 3, 6, 1, 7, 2, 8, 6, 9, 4, 0, 9, 1,
        1, 2, 4, 3, 2, 7, 3, 8, 6, 9, 0, 5, 6, 0, 7, 6, 1, 8, 7, 9, 3, 9, 8, 5,
        9, 3, 3, 0, 7, 4, 9, 8, 0, 9, 4, 1, 4, 4, 6, 0]) torch.Size([64])
```

The simplest deep learning structure

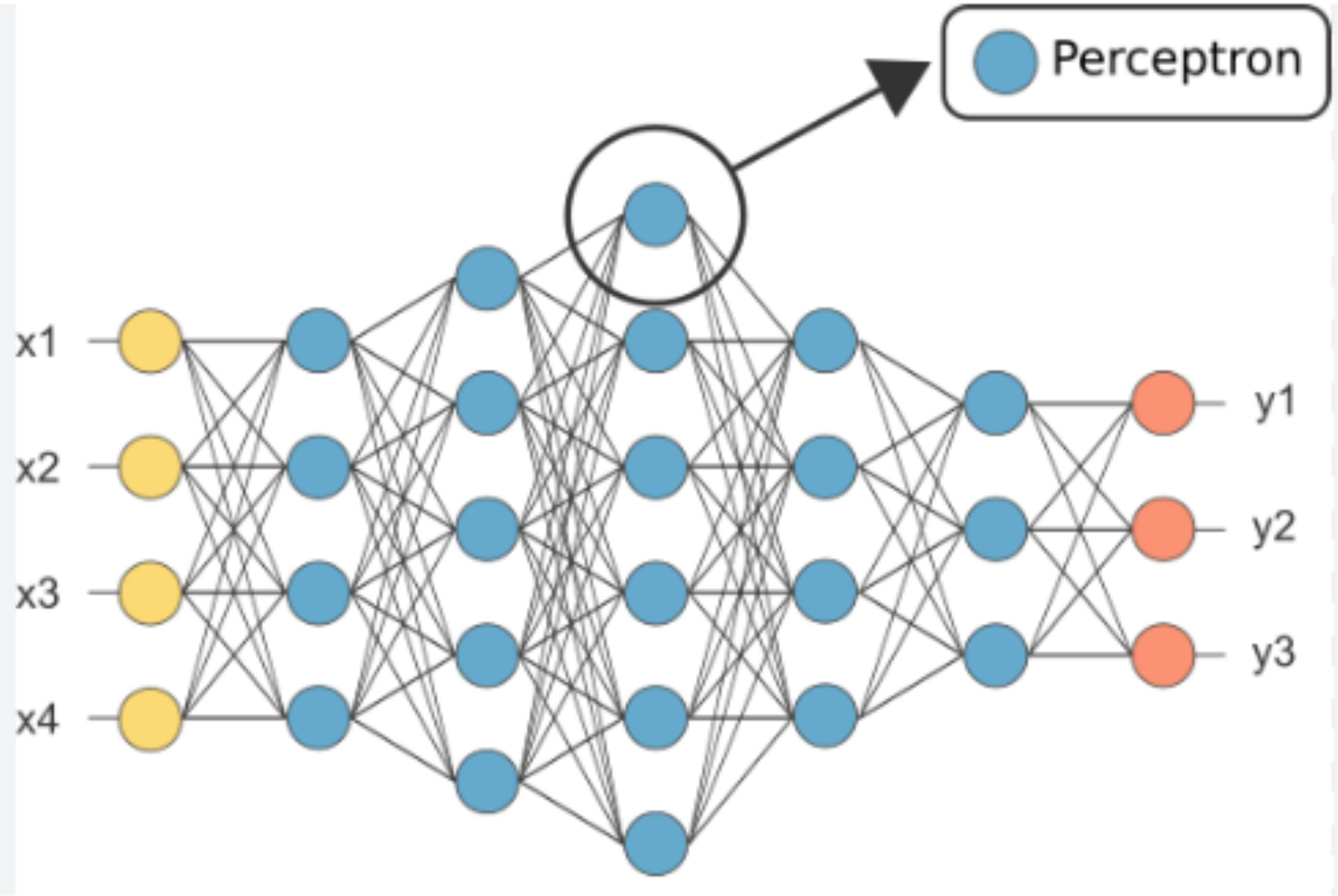
- Feedforward neural network (FNN)
- 3 types of layers:
  - Input (Yellow), hidden (Blue), output (Red)
  - Each layer consists of multiple perceptrons (neurons)
  - Layers-layers are connected with weights
  - Fully-connected:
    - All perceptrons from n th to n+1 th layer are all (fully) connected
- "Learning" is a process of which the weights of the neural networks are being optimized



https://medium.com/@b.terryjack/introduction-to-deep-learning-feed-forward-neural-networks-ffnns-a-k-a-c688d83a309d

The simplest deep learning structure

- Feedforward neural network (FNN)
- 3 types of layers
- "Learning" is a process of which the weights of the neural networks are being optimized
  - All weights and the output values of the perceptrons are scalar values
  - First, only the input perceptrons have legit values
    - Weights are randomly initialized
  - Using the input values as well as all the other (randomized) weights, you compute the output value of the upper-layer perceptrons



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