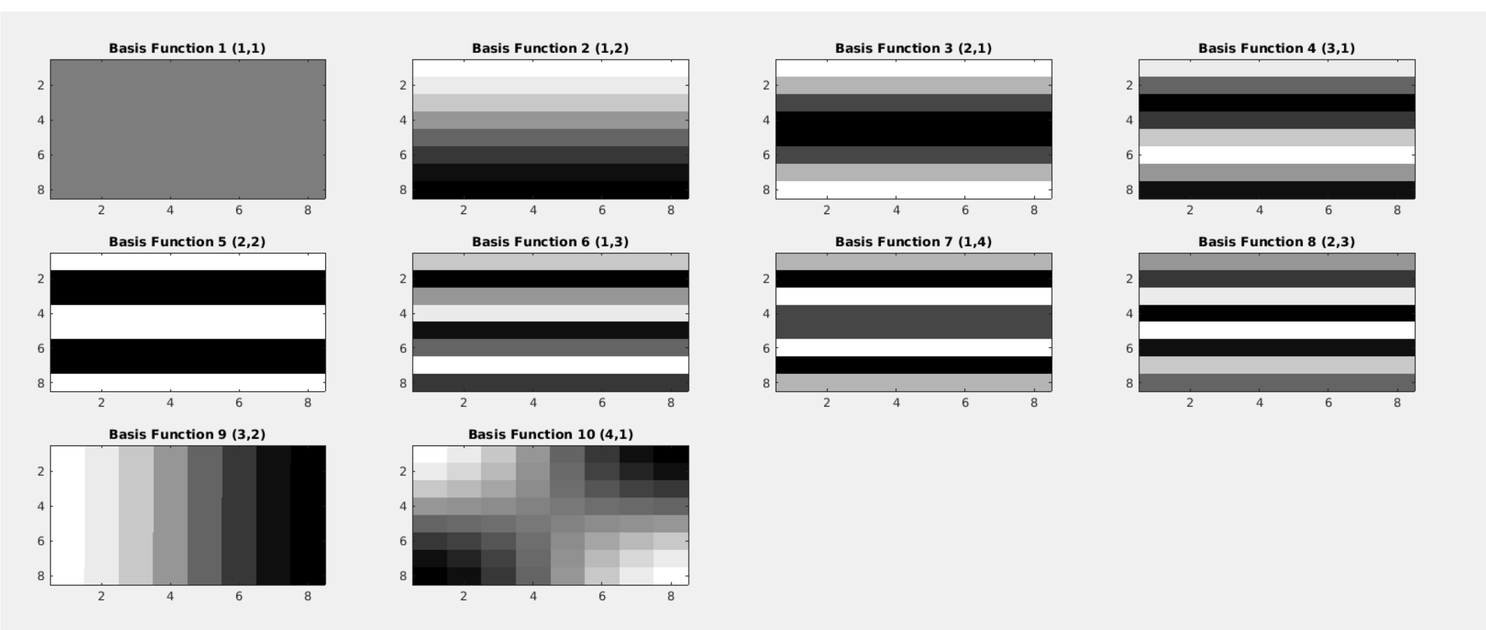
Final Project

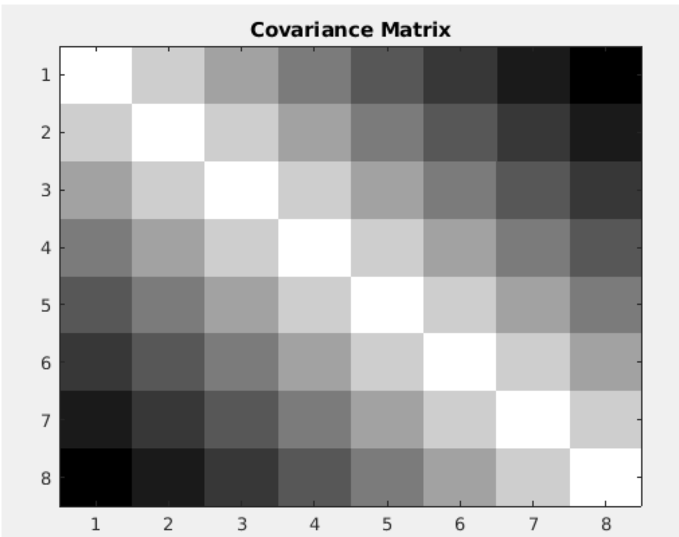
1. Principal Component Analysis

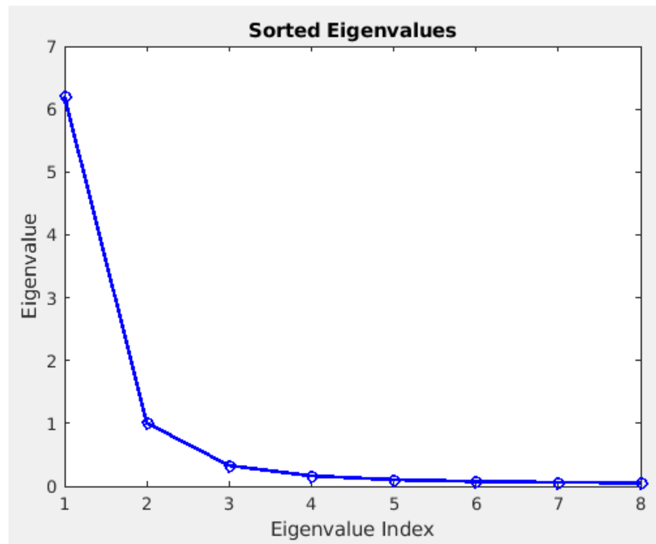
1.1 Below we display the Y-channel image.

1.2 a) In the above figure, we show the Absolute Error plot and the reconstructed image. As we can see, the reconstruction quality is very good. Below, we show the top K (in our case 10, in zigzag order) 2D DCT basis.

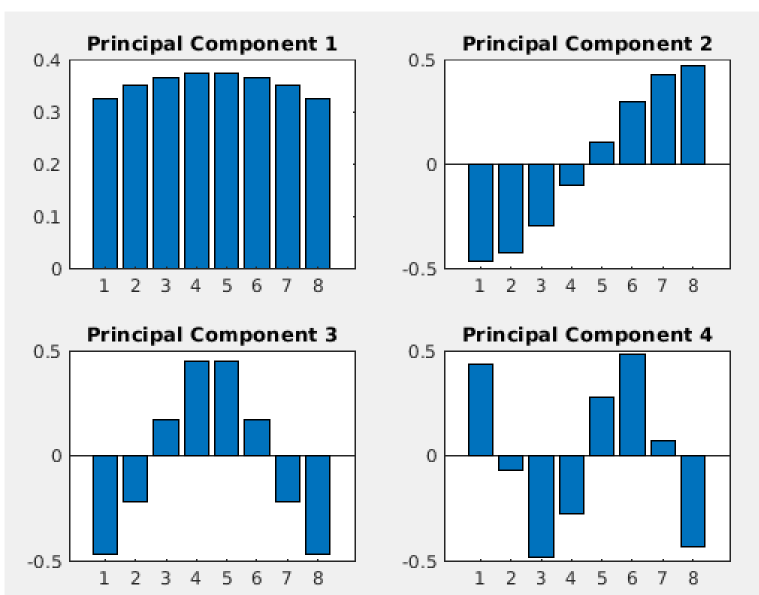


1.2 b) Here we plot the covariance matrix and the eigenvalues. As expected, there are only a few eigenvalues with value higher than or equal to 1, which indicates we can significantly compress our data without losing too much information. One thing to note is that we calculated the eigenvectors and eigenvalues of the C covariance matrix, which is 8 by 8. Thus, at most we will have 8 eigenvalues and eigenvectors (we applied PCA compression on each 8 by 8 image block in a row-wise manner).



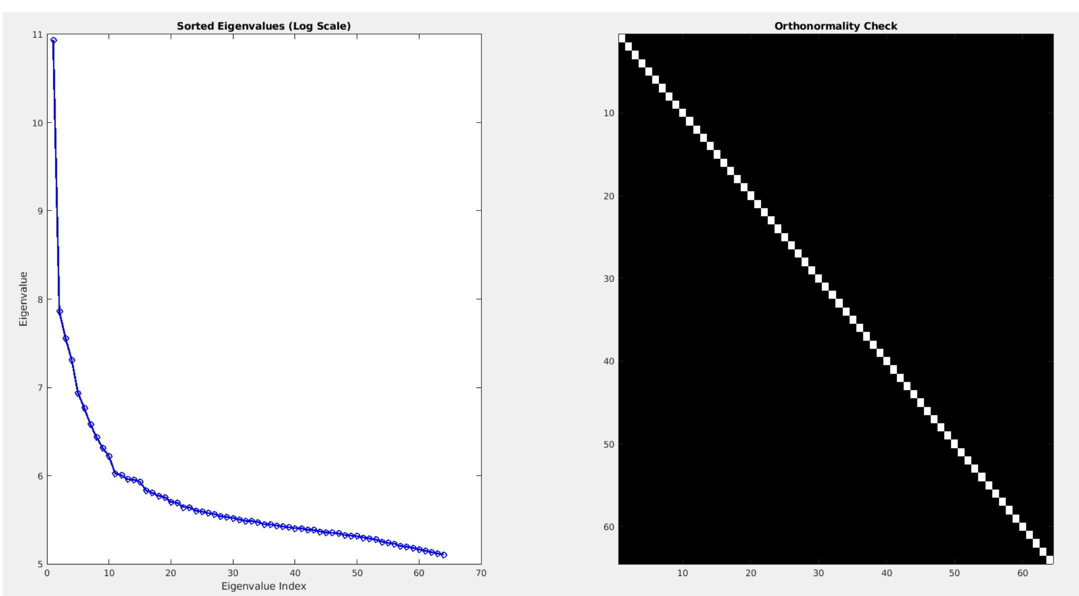


Below, we show the values of top 4 Principal Components; the x axis refers to the specific index inside the vector and the y axis refers to the value.

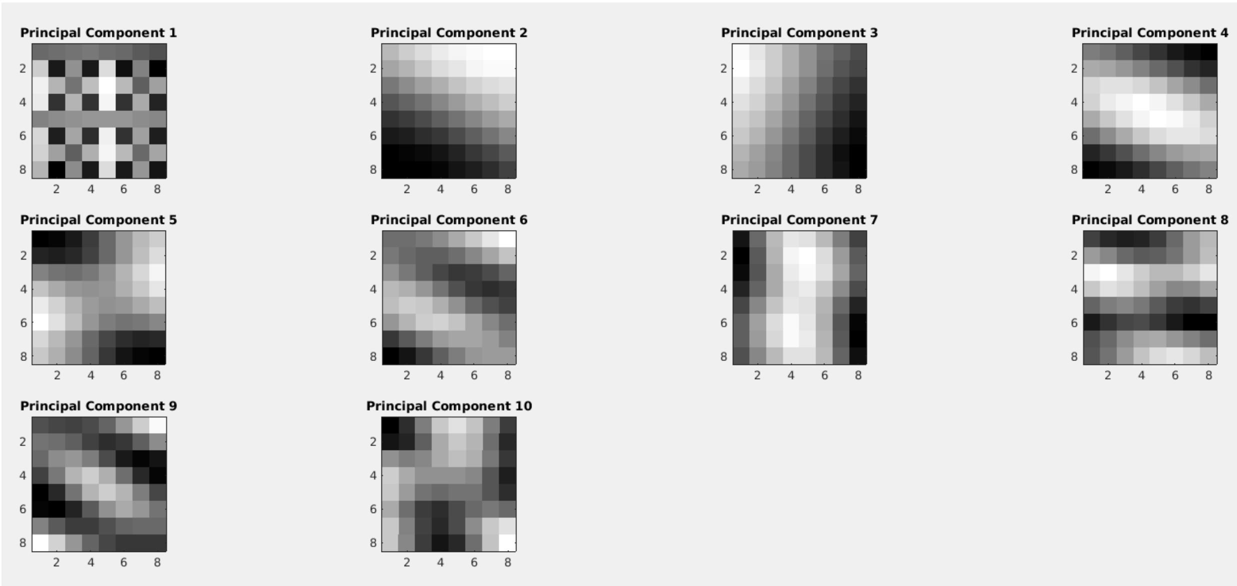


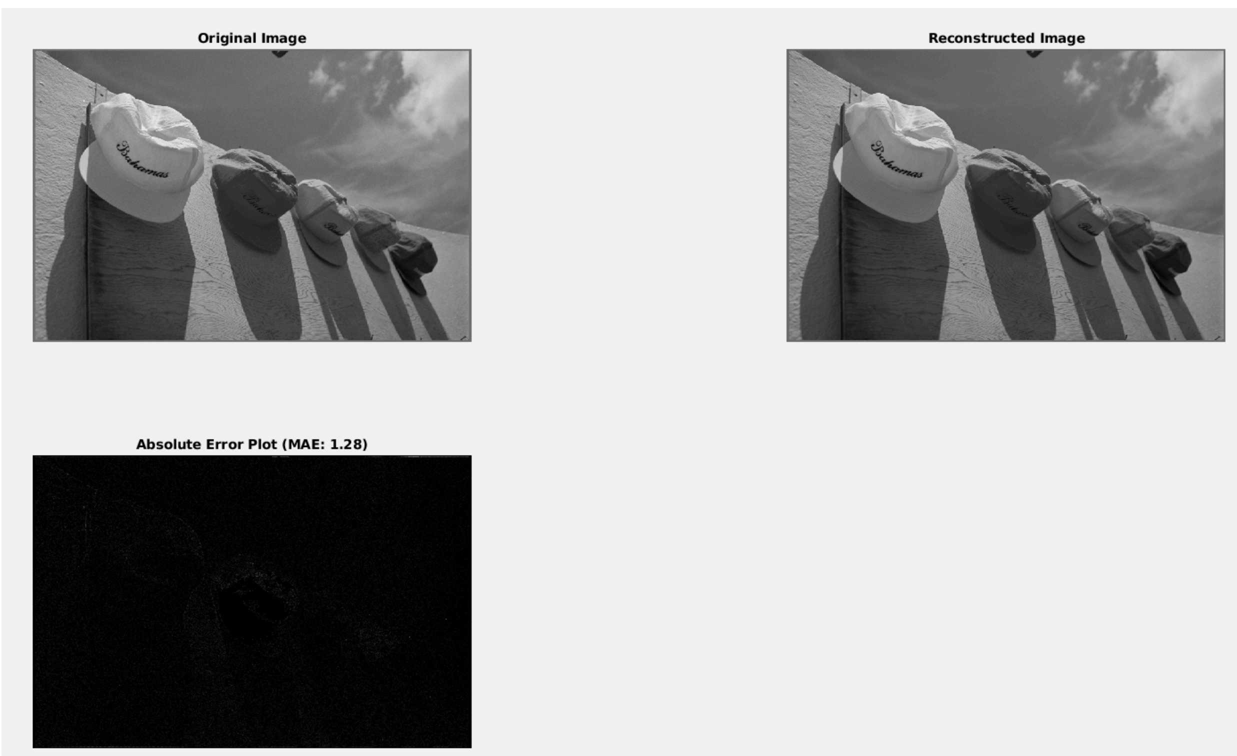
Below, we show the reconstructed image and the absolute error plot using 4 Principal Components. As we can see, the reconstruction quality is still pretty good.

1.2 **c)** The size of the X matrix was 64 by 98304, and the size of covariance matrix C was 64 by 64. Below, we show the eigenvalues of the C matrix and the orthogonality check. As we can see, the eigenvectors are all orthogonal to each other.

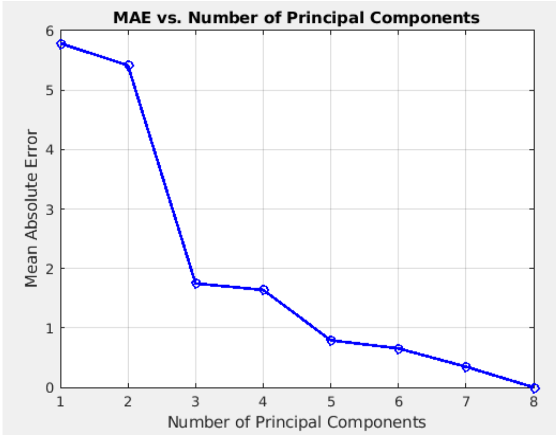


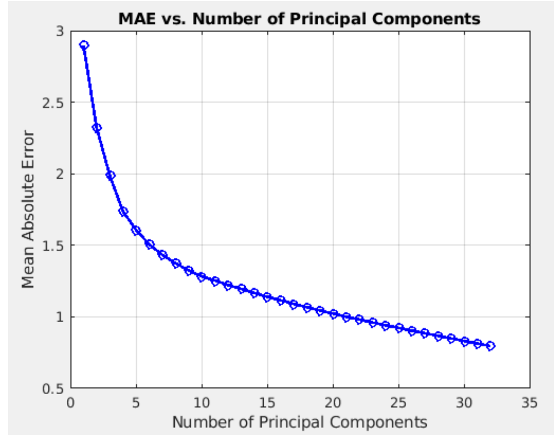
Below, we show the K (in this case, 10) most important 2D principal components and then display the original image, reconstructed image based on the PCs and the absolute error.





1.3 Below, we show the representation accuracy for the methods in b) and c). The below figures show expected results: as we increase the number of PCs, then the reconstruction error decreases.



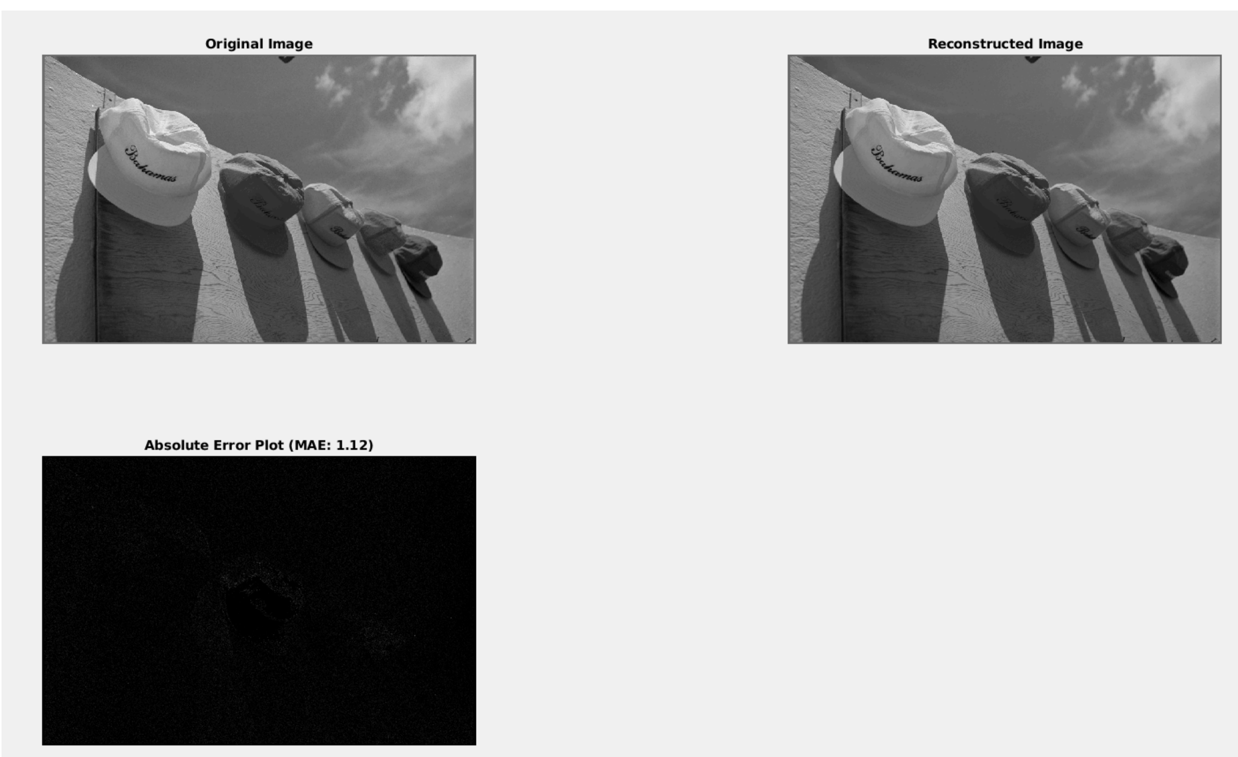


Below, we show reconstruction results for the three methods when changing the number of components from 16 to 4 (8 is skipped because of taking too much extra space). As in method b), the maximum number of eigenvectors was 8 due to our covariance matrix and implementation, so we decided to choose 6 and 4 PCs (instead of 16 and 4) for that method. From the results, we can observe that as DCT gives the best results among the other methods and as we decrease the number of PCs, then the reconstruction quality decreases in all methods.

K = 16 (DCT)

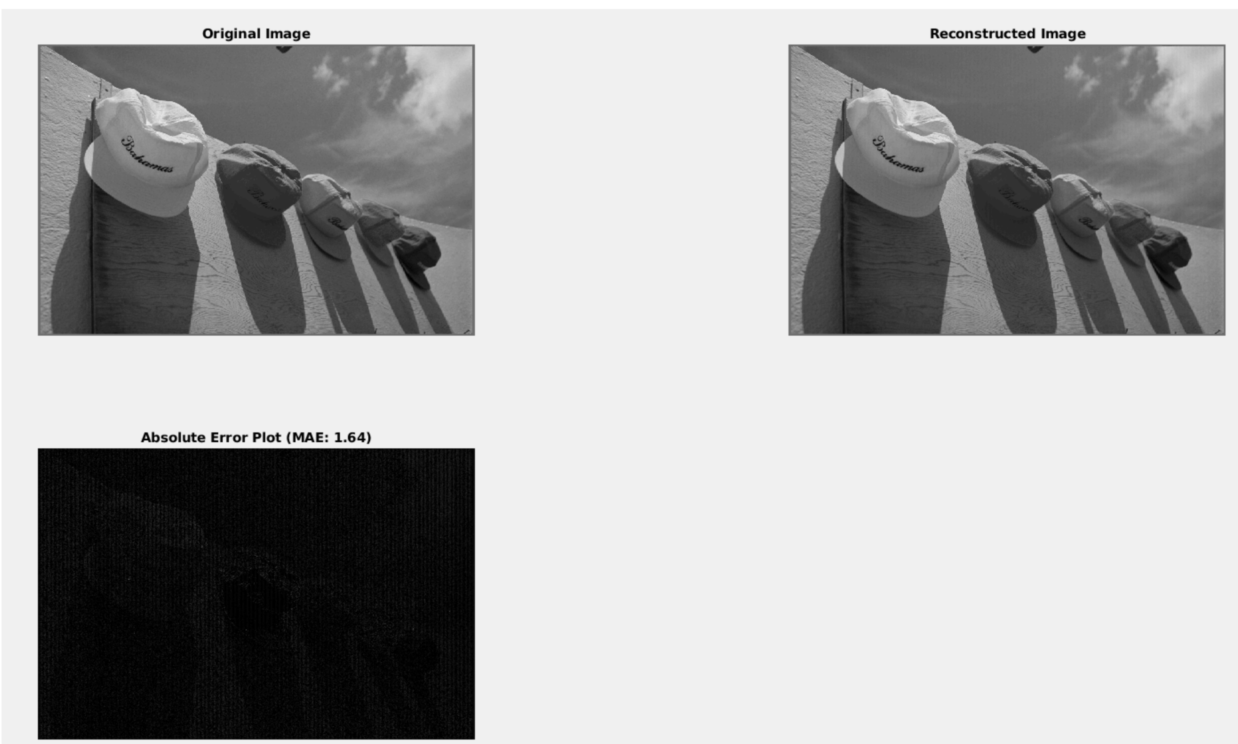
K = 6 (PCA with 1D AR model)

K = 16 (PCA)

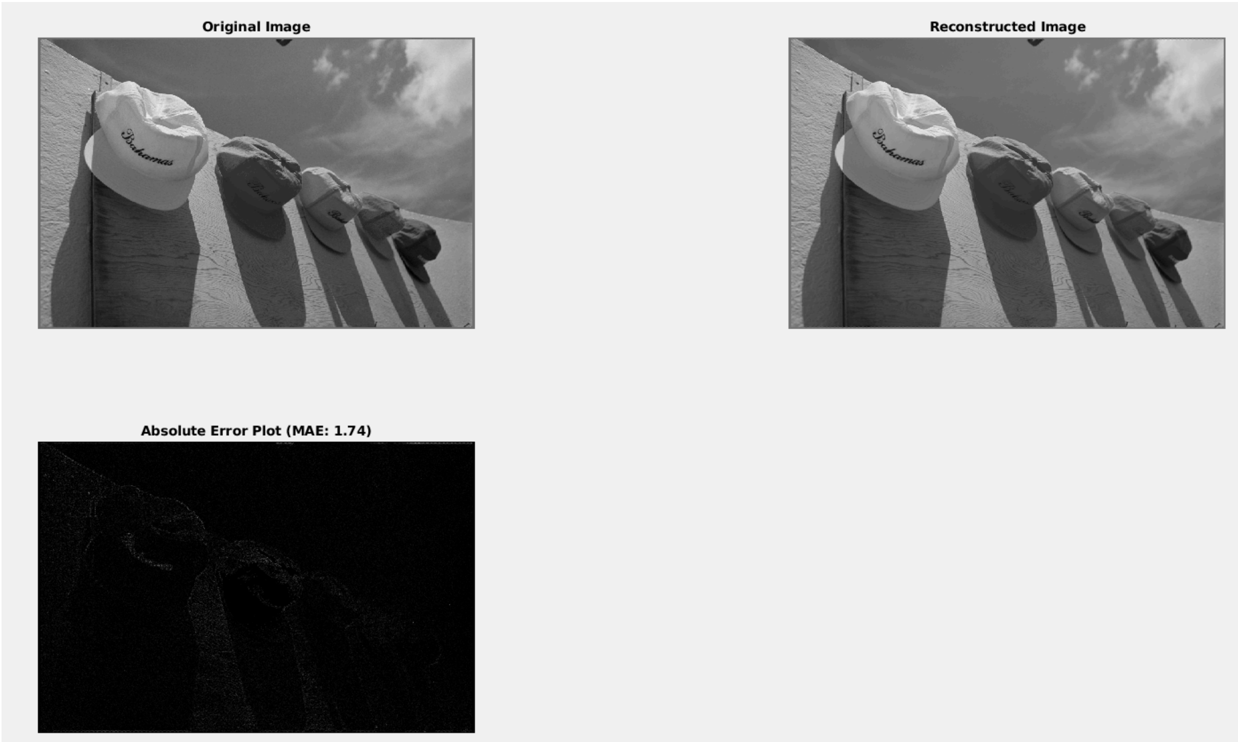


K = 4 (DCT)

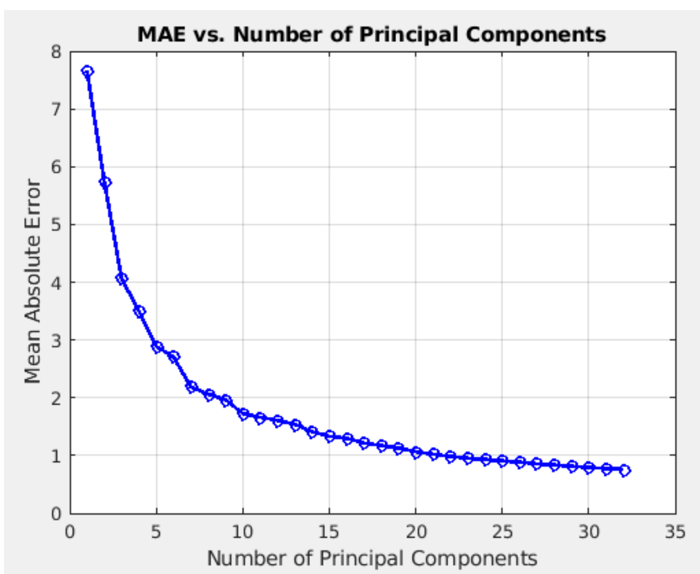
K = 4 (PCA with 1D AR Model)



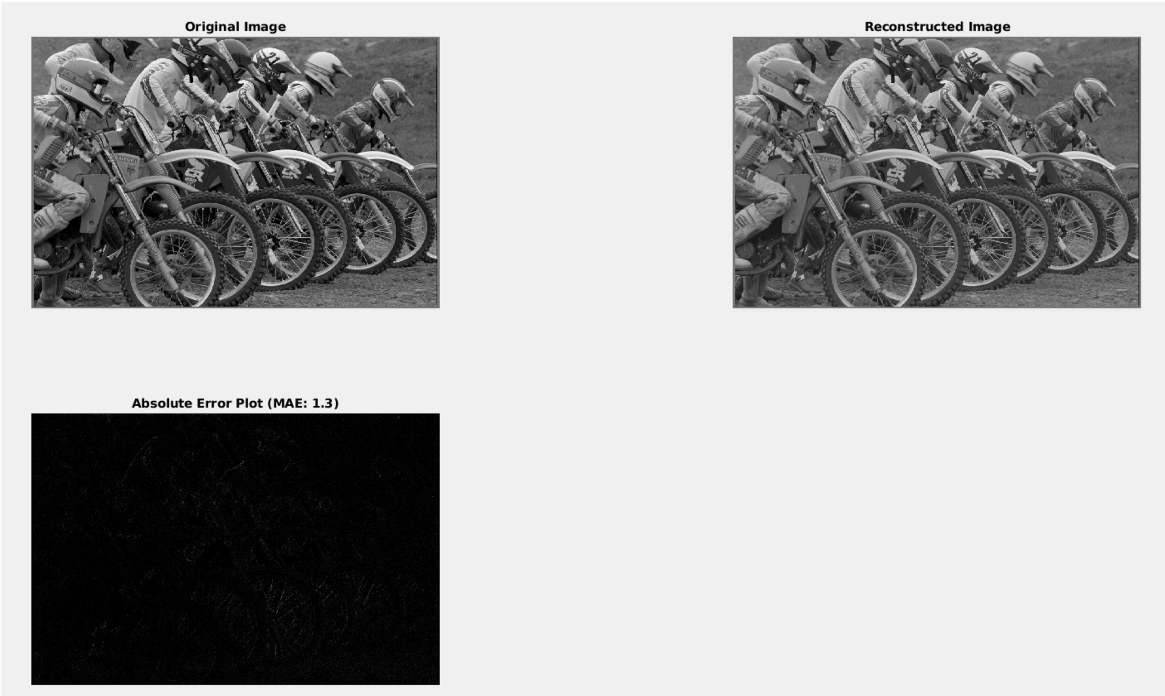
K = 4 (PCA)



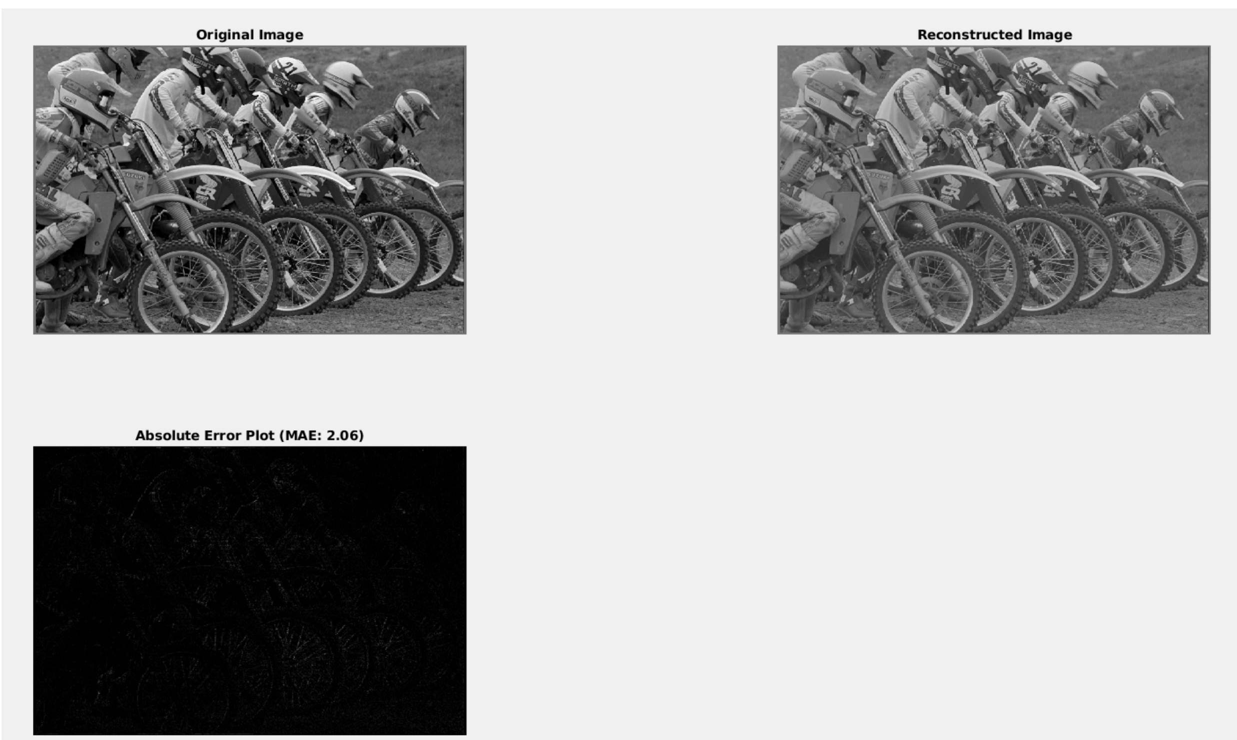
1.4 We repeated the method c) for the image k05.pgm, and got the following results (we did not run other methods to save space). Surprisingly, even if PCs come from the image k03.pgm, we can still successfully reconstruct the image k05.pgm by only using those PCs.



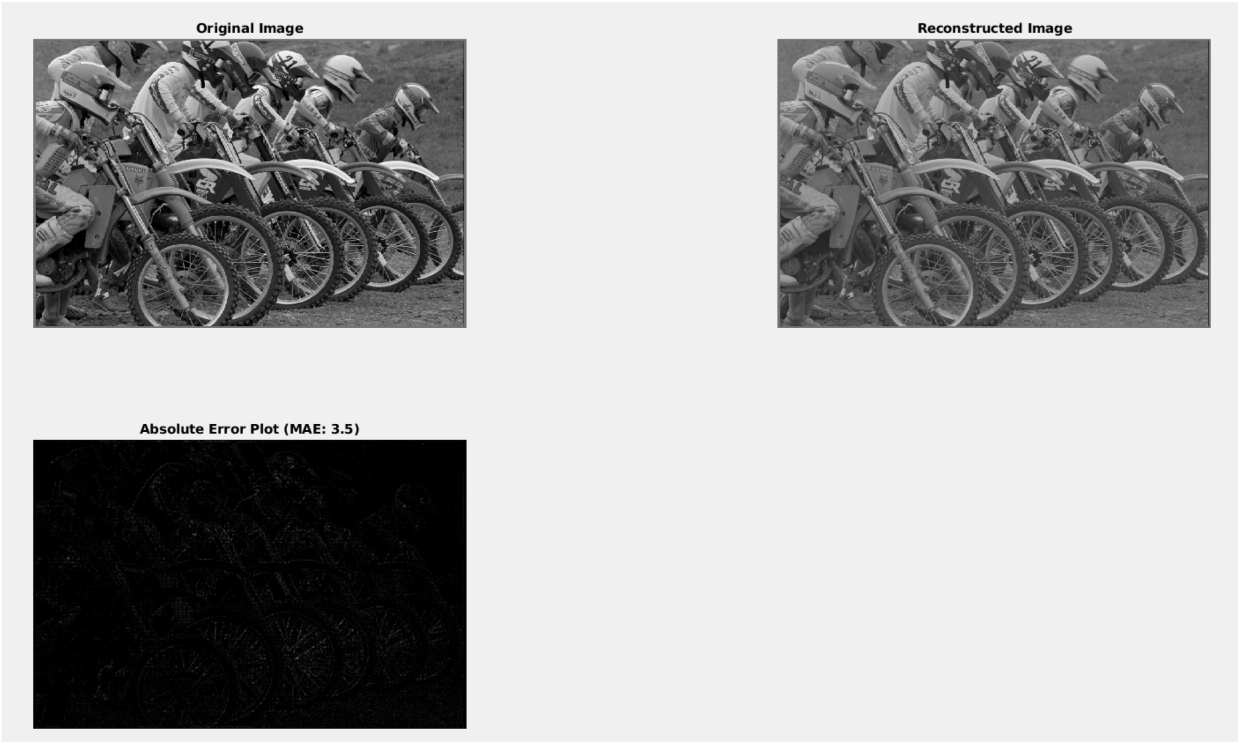
K = 16 (PCA)



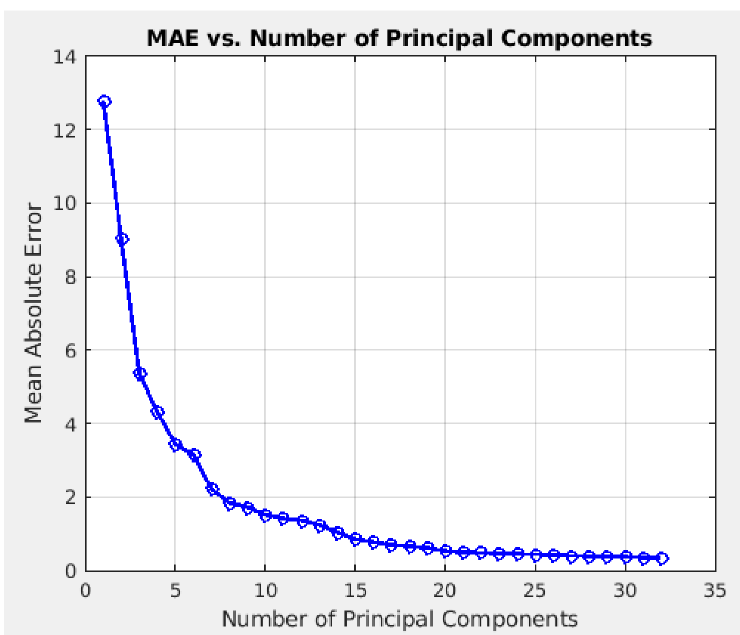
K = 8 (PCA)



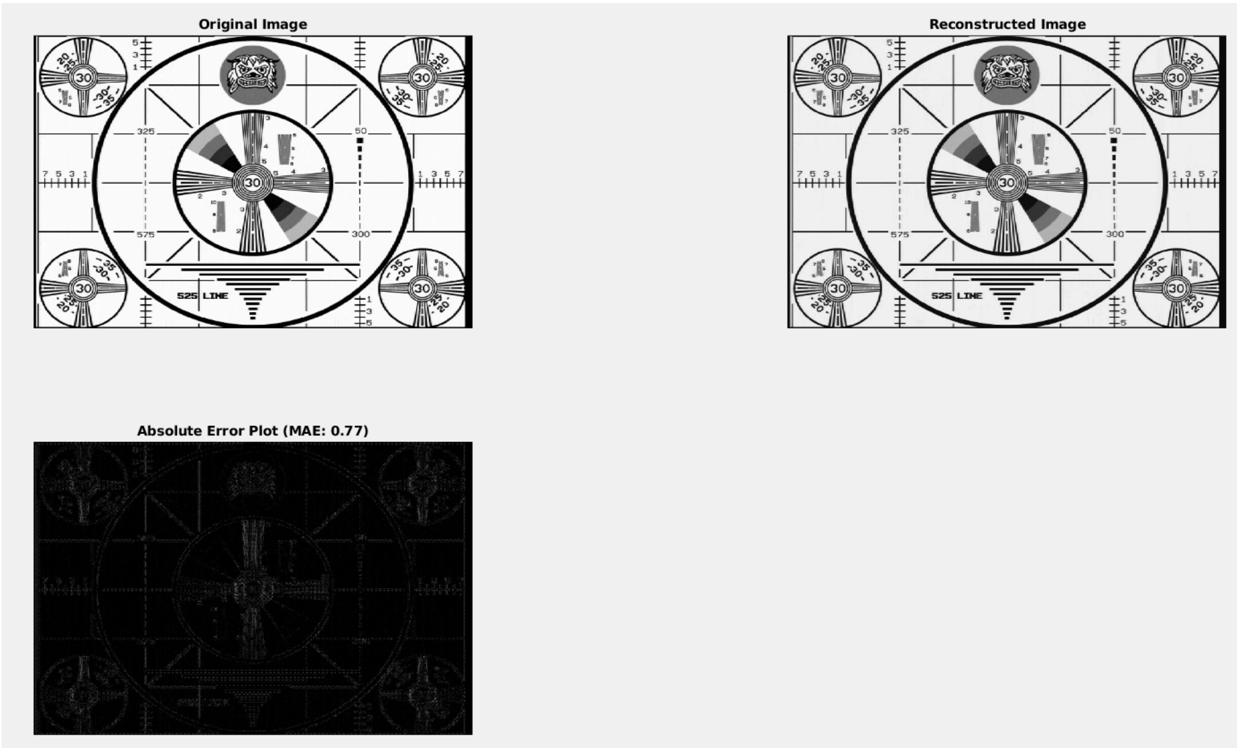
K=4 (PCA)



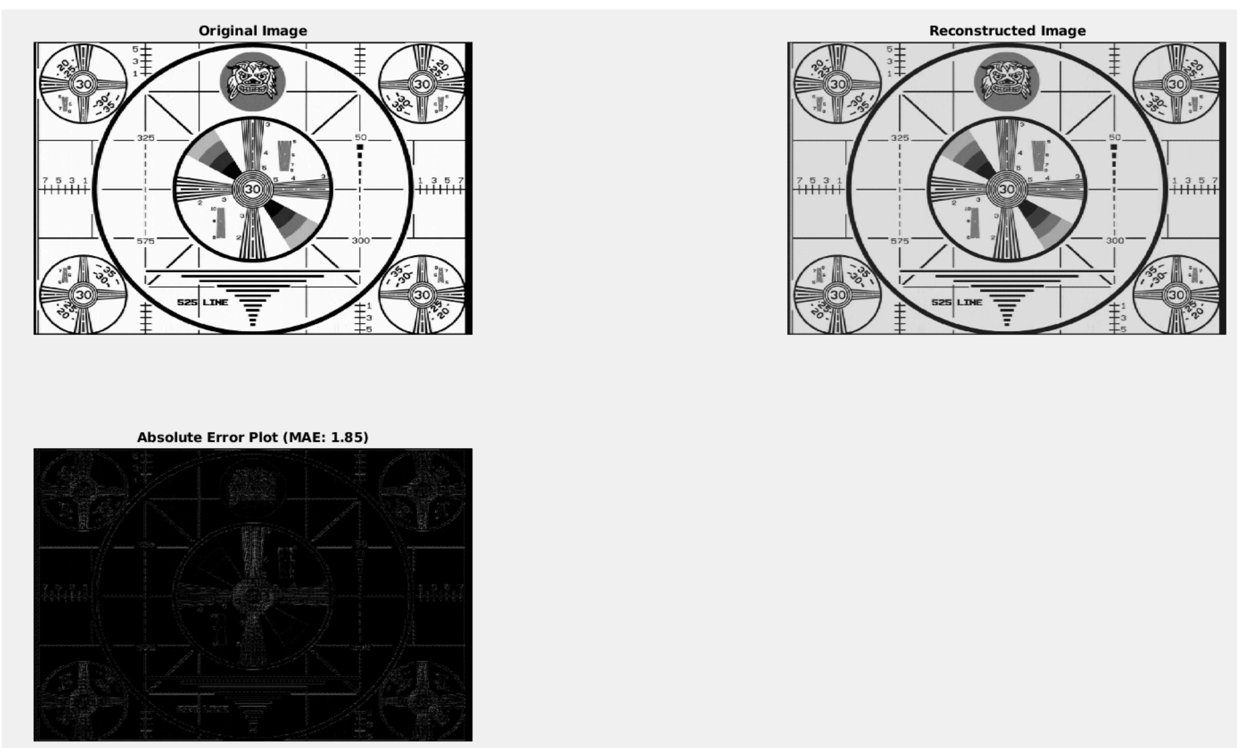
1.5 We performed the same process that we had in 1.3 to the image lionhead.bmp also. In this case, even though the image is very different compared to k03.pgm, we can successfully reconstruct this new image by reusing the PCs of k03.pgm.



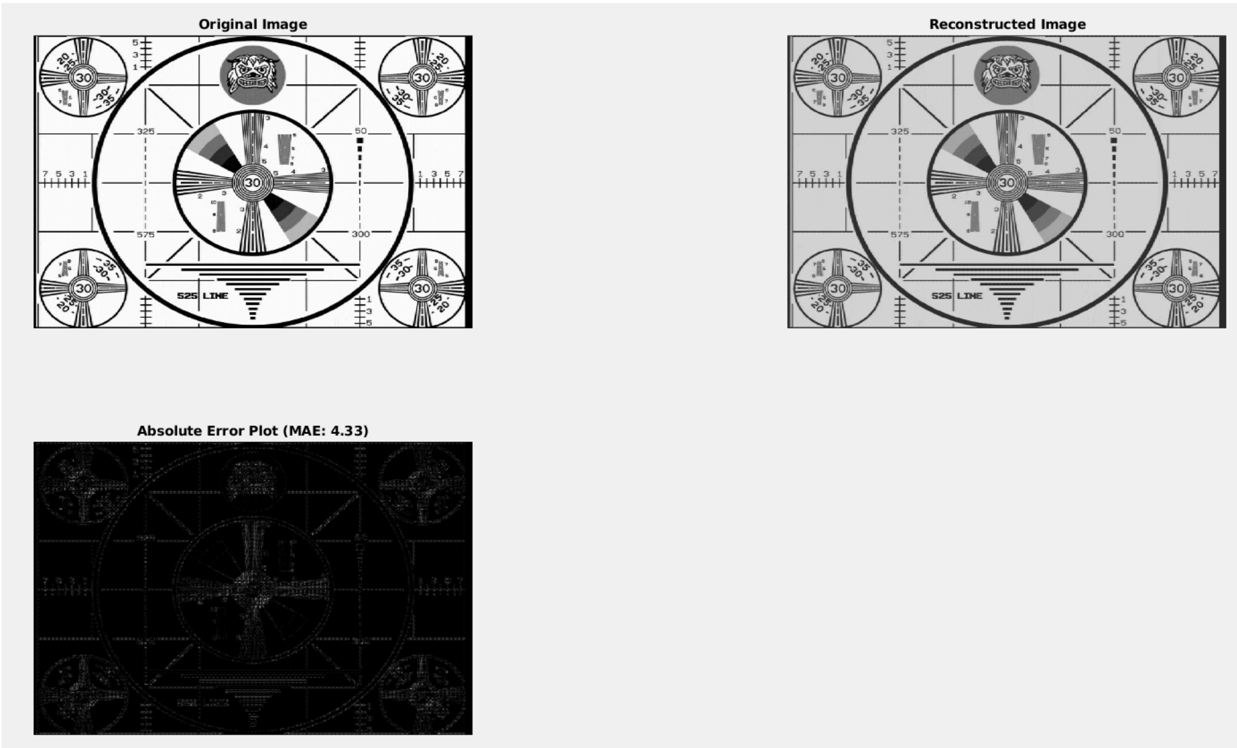
K = 16 (PCA)



K = 8 (PCA)



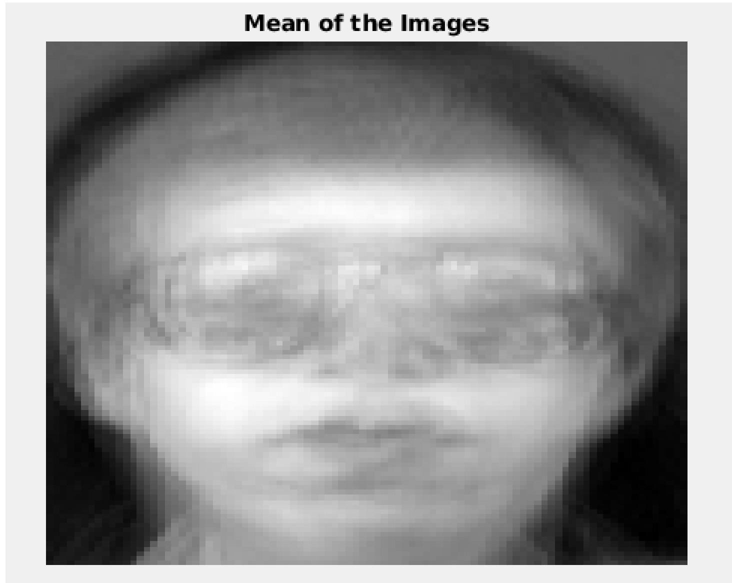
K = 4 (PCA)



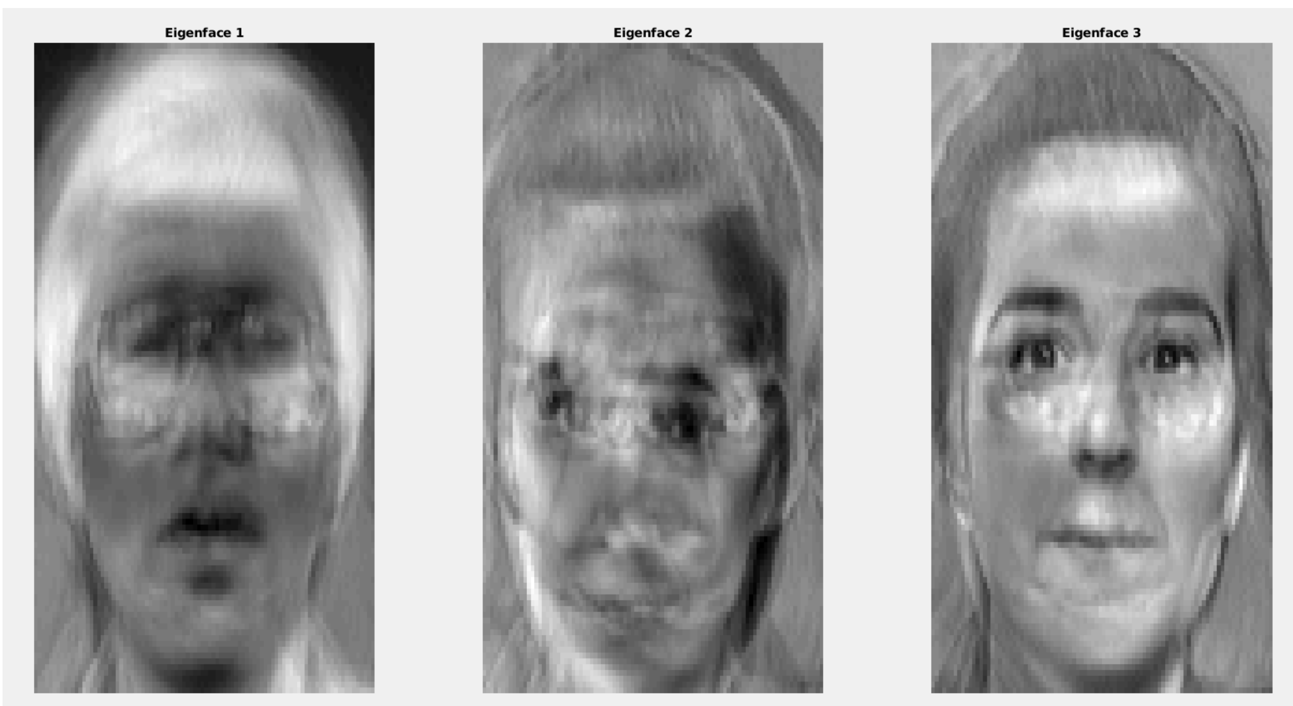
2. Detection

2.1. Size of the data matrix is 10304 by 20. Here are all the samples.

Here we show the mean image constructed by taking the mean of the given images.

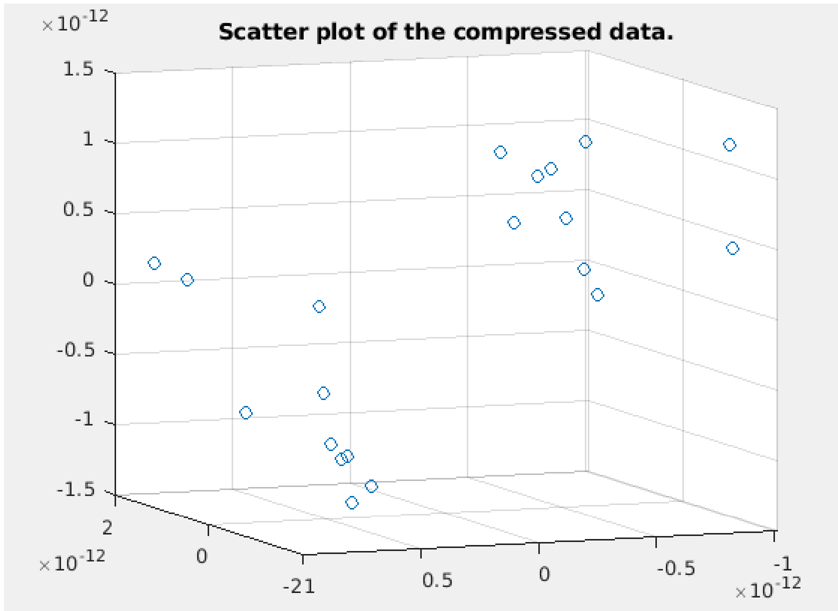


Here, we show the three eigenfaces constructed from reshaping the eigenvectors:



As we can see, those eigenvectors have inherent meaning and represent the most important features in the face dataset.

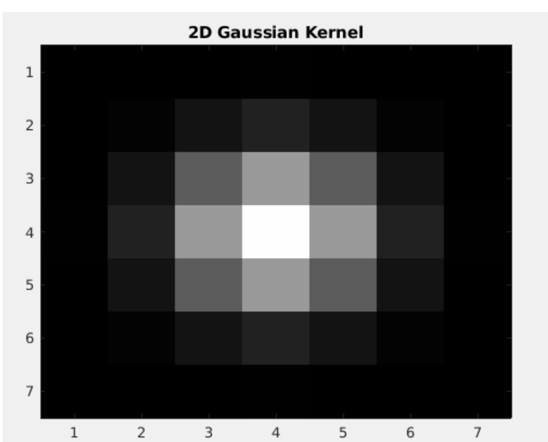
2.2 Here we show the scatter plot of our data after projecting it into lower dimensional space using the principal components. As we can see, there is a clear division in the dataset, and two clusters emerge in this 3 dimensional space; one cluster formed by the male face samples and the other by female face samples.

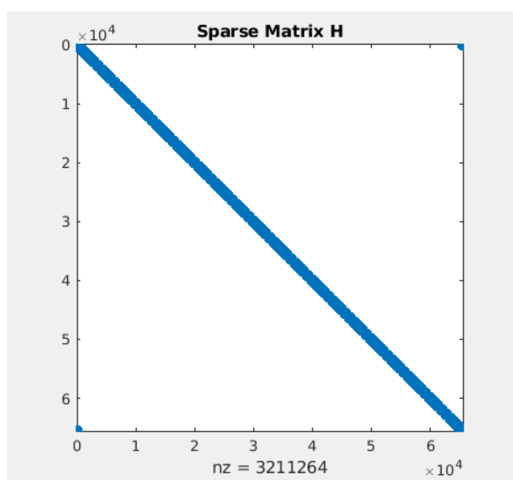


2.3-2.4. We trained a small SVM model on the 16 face samples and then evaluated it on 4 remaining samples to detect male faces. The model could achieve a perfect score, giving Precision (power) and Recall equal 1.0.

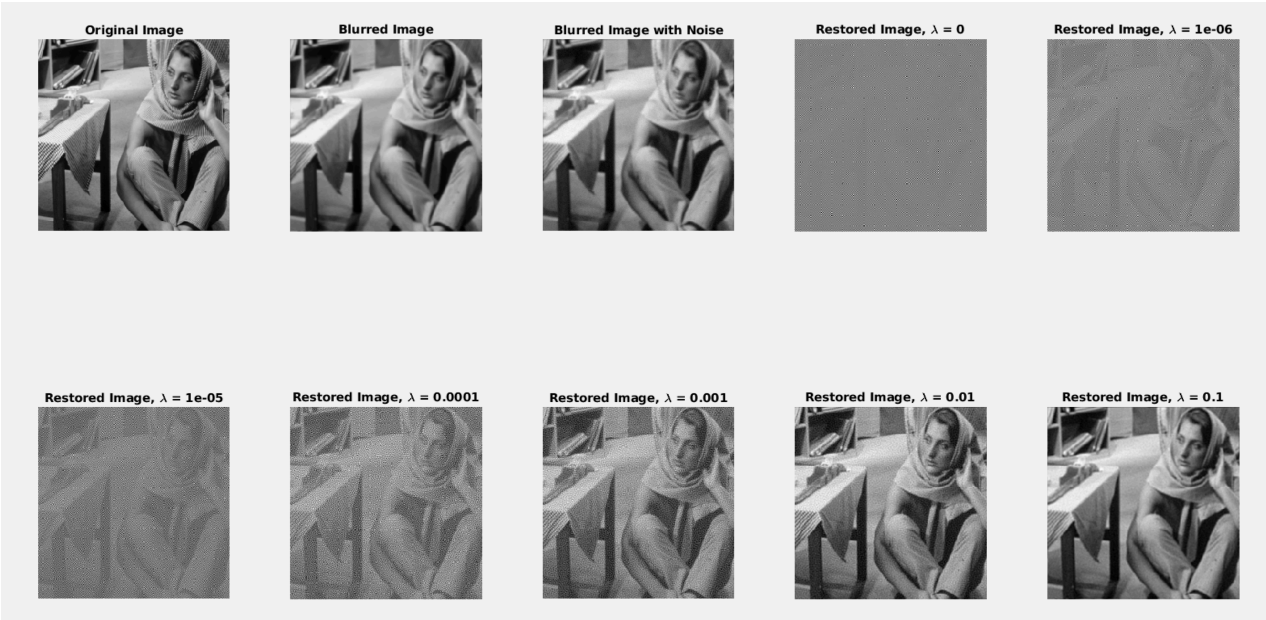
3 Restoration

3.1 Here is the plot of the 3D Gaussian Kernel and the sparse matrix H.

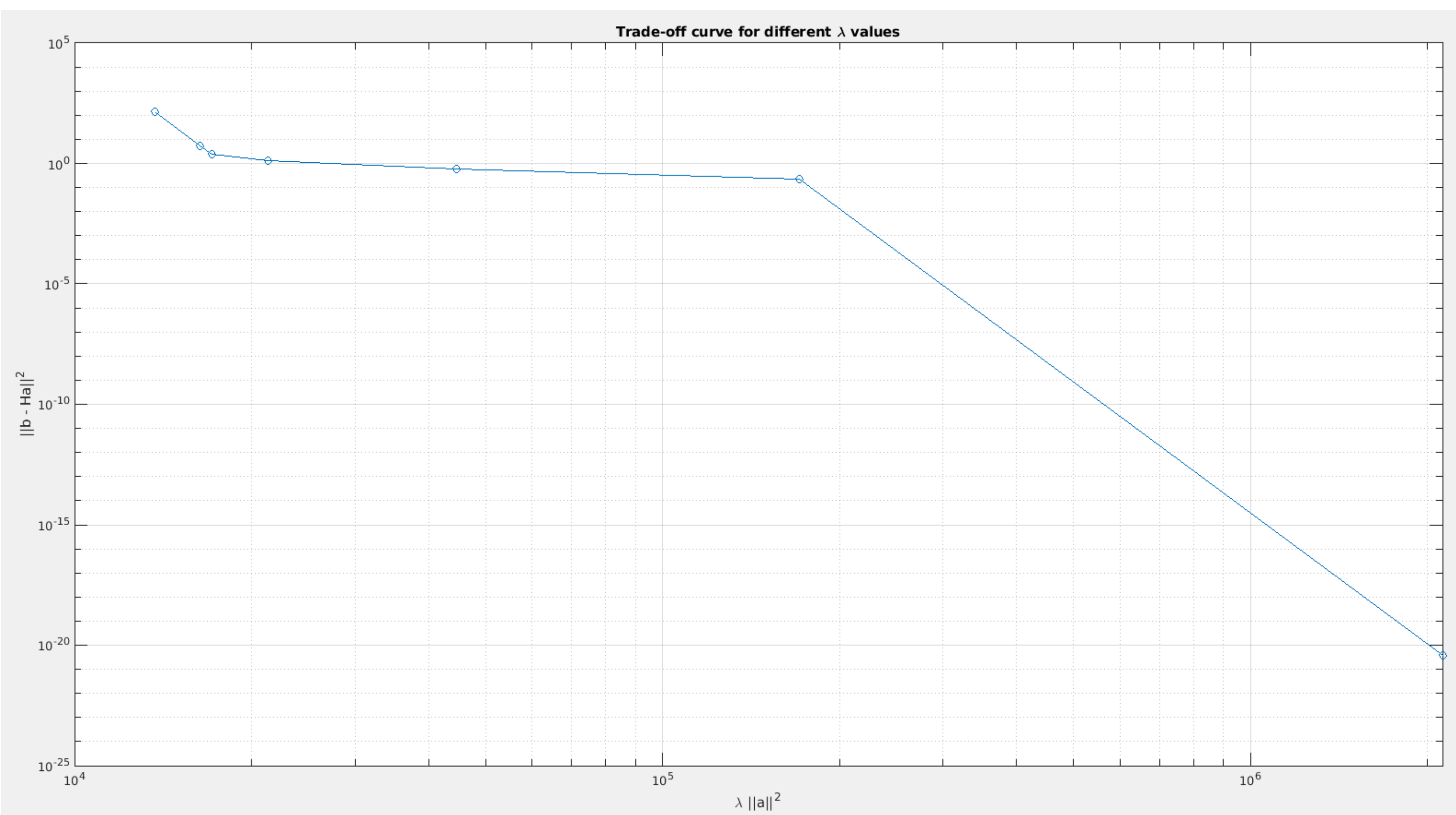




3.2-3.3 Here we plot the original and all the other images that we got after transformation. We can see that for restoring the image, the value of lambda is important; as it increases in our case, the quality of the restoration also increases.



Below, we plot the trade-off curve for different lambda values. To choose the lambda value, we decided to rely on the reconstruction loss such MAE between the restored image and the original image. Using such a criterion, we chose the lambda equal to 0.01 as it had the lowest MAE value.



Here we plot the cost that we are trying to minimize and the iteration number of each method. We can see that Steepest Descent and Conjugate Gradient methods are much better then the Gradient Descent approach as they reach minimum extremely fast.

We also show the restored image quality depending on the algorithm. The Gradient Descent did not show very good reconstruction results compared to the other approaches.