Multivariate Data Analysis

(MGT513, BAT531, TIM711)

Lecture 3

Chapter 5

Exploratory Factor Analysis (EFA)

What is Exploratory Factor Analysis?

- Exploratory Factor Analysis (EFA)
  - Examines the interrelationships among a large number of variables and then attempts to explain them in terms of their common underlying dimensions.
  - Identifies the common factors and explain their relationship
  - The observed variance in each variable = a few of (unobservable) common factors + a specific factor (unrelated to any factor)

What is Exploratory Factor Analysis?

- Latent Traits or Unobservable Characteristics
  - For length or weight, such a distinction may be unnecessary because the property is almost perfectly Observable
  - However, for attitudes, belief, perceptions, and other psychological notions, our measurement instruments are imperfect

Types of Factor Analysis

Exploratory Factor Analysis (EFA)

- used to discover the factor structure of a construct and examine its reliability. It is data driven.

Confirmatory Factor Analysis (CFA)

- used to confirm the fit of the hypothesized factor structure to the observed (sample) data. It is theory driven.

Objectives

Data Summarization versus Data Reduction

- Data summarization
  - Defining structure through underlying dimensions that, when interpreted and understood, describe the data in a much smaller number of concepts than the original individual variables. The basis for scale development.
- Data reduction
  - Extends the process of data summarization by deriving an empirical value (factor score or summated scale) for each dimension (factor) and then substituting this value for the original variable values in subsequent analysis.

Example 1: Brand Personality

TABLE 5.1 Traits associated with different dimensions of brand personality

Sincerity	Excitement	Competence	Sophistication	Ruggedness
Honest	Daring	Reliable	Glamorous	Tough
Genuine	Spirited	Responsible	Pretentious	Strong
Cheerful	Imaginative	Dependable	Charming	Outdoorsy
Down-to-earth	Up-to-date	Efficient	Romantic	Masculine
Friendly	Cool	Intelligent	Upper class	
		Successful	Smooth	

114 personal traits

Five-factor solution:

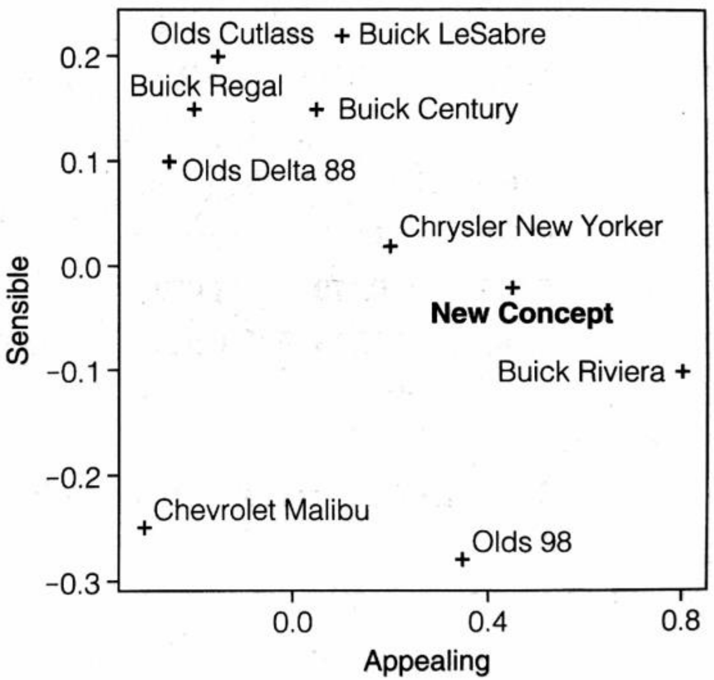
1. Sincerity
2. Excitement
3. Competence
4. Sophistication
5. Ruggedness

Example 2: New Luxury Car Concept

TABLE 5.2 Two-factor solution for Roberts's data: Factor loadings matrix

	Appealing	Sensible
Luxury	0.884	−0.051
Style	0.748	0.153
Reliability	0.396	0.691
Fuel Economy	−0.202	0.786
Safety	0.720	0.172
Maintenance	0.149	0.756
Quality	0.501	0.650
Durable	0.386	0.677
Performance	0.686	0.391

FIGURE 5.1 Map of factor scores of existing automobiles and new car concept (Source: Roberts, 1984)



Simple illustration

- Holzinger and Swineford (1939)
  - Psychological Testing of Children
  - Five tests with 7th & 8th-graded children(n=145)
  - $X1$ = Paragraph Comprehension (PARA)
  - $X2$ = Sentence Completion (SENT)
  - $X3$ = Word Meaning (WORD)
  - $X4$ = Addition (ADD)
  - $X5$ = Counting Dots (DOTS)

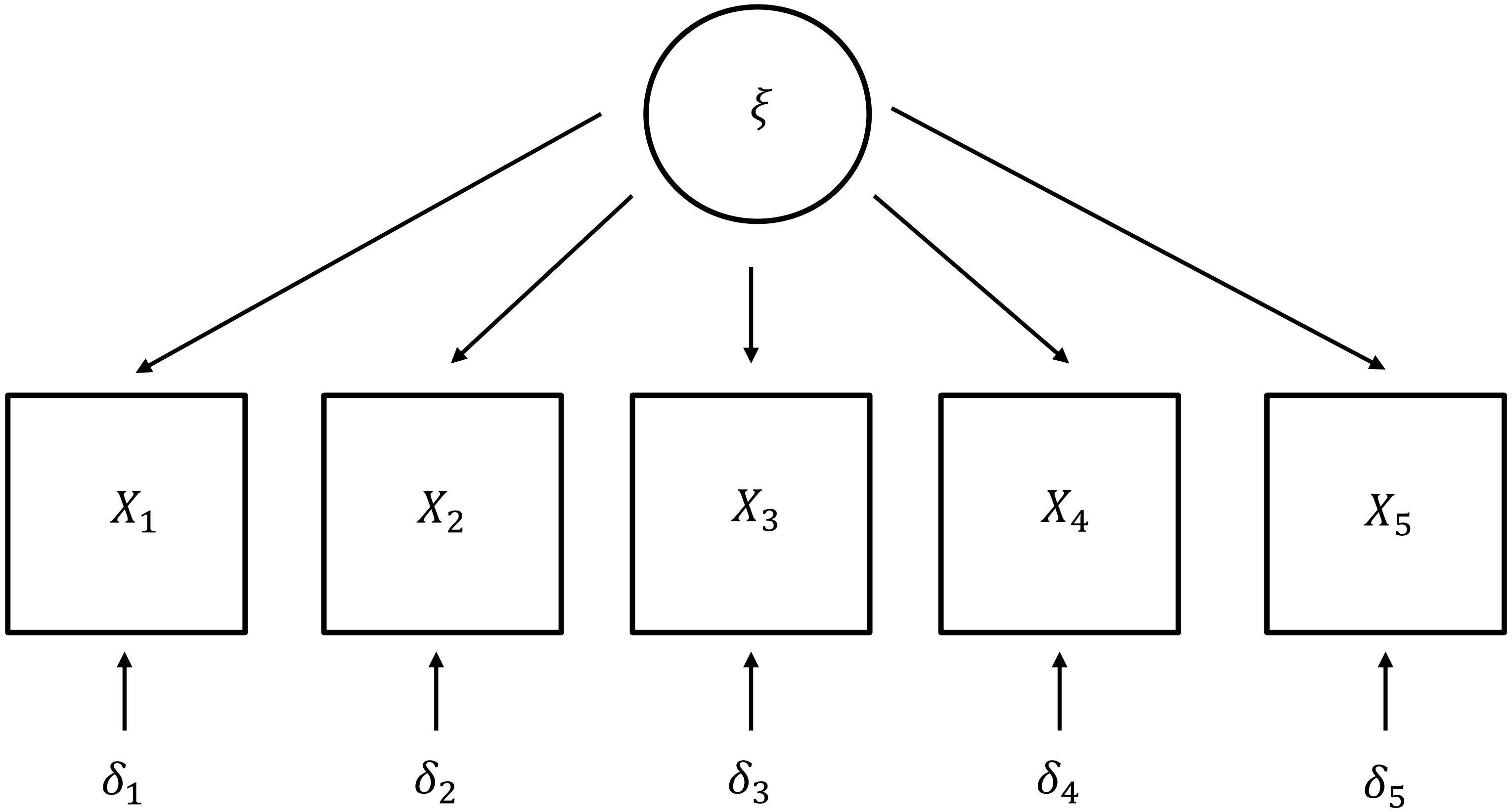
Simple illustration

- Correlation Matrix

	PARA	SENT	WORD	ADD	DOTS
PARA	1.000				
SENT	0.722	1.000			
WORD	0.714	0.685	1.000		
ADD	0.203	0.246	0.170	1.000	
DOTS	0.095	0.181	0.113	0.585	1.000

Simple illustration: One-Factor Model

- One-factor model with five variables



Simple illustration : One-Factor Model

- Let ξ = common factor and δ_i= a specific factor
- One-factor model with five variables

$$X_i = \lambda_i \xi + \delta_i, \qquad\qquad i = 1,2,3,4,5$$

where $\operatorname{cor}(\delta_i, \delta_j) = 0, i \neq j$ and $\operatorname{cor}(\delta_i, \xi) = 0$

- Assume X and ξ are standardized variables. Then

$$\operatorname{Var}(X_i) = \operatorname{Var}(\lambda_i \xi + \delta_i) = {\lambda_i}^2 + Var(\delta_i) = 1$$

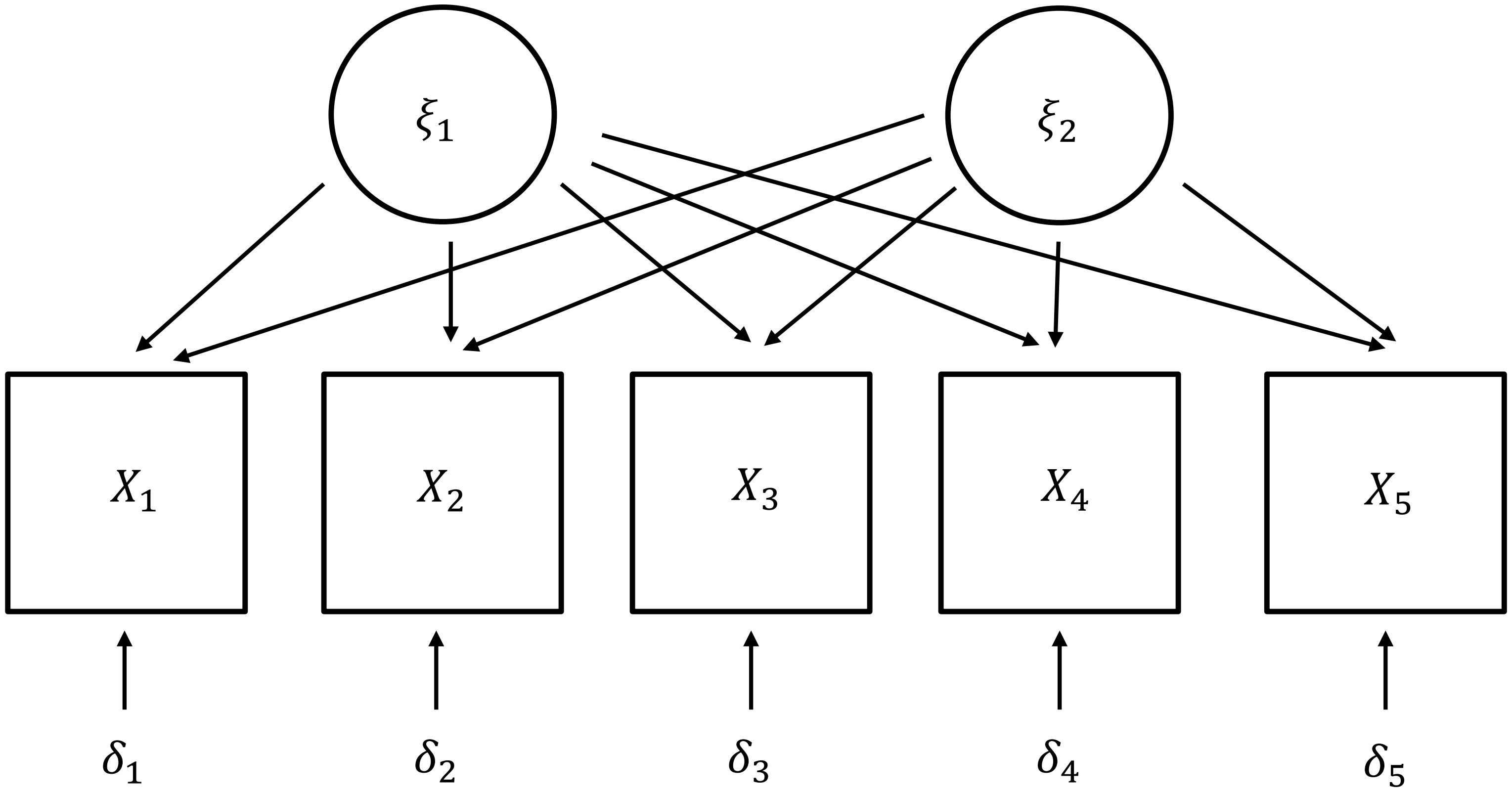
$$\begin{aligned} {\lambda_i}^2 &= \text{communality of } X_i \\ &= \text{the proportion of the variation in } X_i \text{ explained by } \xi \\ &= 1 - \operatorname{Var}(\delta_i) = 1 - \theta_{ii}^2 \end{aligned}$$

where θ_{ii}^2 = $\operatorname{Var}(\delta_i)$ = variance of specific factor X_i

- As ${\lambda_i}^2 \to 1 \left(\theta_{ii}^2 \to 0\right)$, X_i is a nearly perfect measure of ξ
- As ${\lambda_i}^2 \to 0 \left(\theta_{ii}^2 \to 1\right)$, X_i is not explained by ξ

What is Exploratory Factor Analysis?

- Two-factor model with five variables



Simple illustration : Two-Factor Model

- Two-factor model with five variables

$$X_i = \lambda_{i1}\xi_1 + \lambda_{i2}\xi_2 + \delta_i, \qquad i = 1,2,3,4,5$$

where $\operatorname{cor}(\delta_i, \delta_j) = 0, i \neq j$ and $\operatorname{cor}(\delta_i, \xi_k) = 0$, $k = 1,2$

where ξ_1 = Verbal Aptitude Factor

ξ_2 = Quantitative Aptitude Factor

- Assume X and ξ are standardized variables. Then

$$\operatorname{Var}(X_i) = \operatorname{Var}(\lambda_{i1}\xi_1 + \lambda_{i2}\xi_2 + \delta_i) = {\lambda_{i1}}^2 + {\lambda_{i2}}^2 + Var(\delta_i)$$
$$= {\lambda_{i1}}^2 + {\lambda_{i2}}^2 + \theta_{ii}^2 = 1$$

$${\lambda_{i1}}^2 + {\lambda_{i2}}^2 = \text{communality of } X_i = 1 - \theta_{ii}^2$$

- Consider a student with high ξ_1 and low ξ_2. We expect the student to perform well on those tests requiring more verbal than quantitative ability. That is, If the student's performance in a task of sentence completion (measured by X_1), we should expect a value of λ_{11} near 1 and a value for λ_{12} closer to 0.

Exploratory Factor Analysis with *c* Common Factors

$$
\begin{gathered}
X_1 = \lambda_{11}\xi_1 + \lambda_{12}\xi_2 + \cdots + \lambda_{1c}\xi_c + \delta_1 \\
X_2 = \lambda_{21}\xi_1 + \lambda_{22}\xi_2 + \cdots + \lambda_{2c}\xi_c + \delta_2 \\
\vdots \\
X_p = \lambda_{p1}\xi_1 + \lambda_{p2}\xi_2 + \cdots + \lambda_{pc}\xi_c + \delta_p
\end{gathered}
$$

In matrix notation,

$$\boldsymbol{X} = \boldsymbol{\Xi}\boldsymbol{\Lambda}_c^T + \boldsymbol{\Delta}$$

where $\boldsymbol{\Xi} = [\xi_1, \xi_2, \cdots, \xi_c]$

$\boldsymbol{\Delta} = [\delta_1, \delta_2, \cdots, \delta_p]$

$\boldsymbol{\Lambda}_c = p \times c$ matrix of coefficients

Exploratory Factor Analysis with *c* Common Factors

- Assumptions For Common Factor Models

1. Common factors ξ_i's are mutually uncorrelated and have a unit variance:

$$\frac{1}{n-1}\boldsymbol{\Xi}^T\boldsymbol{\Xi} = I$$

2. Specific factors δ_i's are mutually uncorrelated and have a diagonal covariance matrix variance :

$$\boldsymbol{\Theta} = \frac{1}{n-1}\boldsymbol{\Delta}^T\boldsymbol{\Delta} = diag(\theta_{11}^2, \theta_{22}^2, \cdots, \theta_{pp}^2)$$

3. Common factors ξ_k's and specific factors δ_i's are mutually uncorrelated:

$$\boldsymbol{\Xi}^T\boldsymbol{\Delta} = \mathbf{0}$$

Exploratory Factor Analysis with *c* Common Factors

- Assumptions for common factor models produce the correlation of $\boldsymbol{X}$

$$
\begin{aligned}
\boldsymbol{R} = \mathrm{Var}(\boldsymbol{X}) = \mathrm{E}[\boldsymbol{X}^{\boldsymbol{T}}\boldsymbol{X}] = \mathrm{E}[(\boldsymbol{\Xi}\boldsymbol{\Lambda}_c^T + \boldsymbol{\Delta})^{\boldsymbol{T}}(\boldsymbol{\Xi}\boldsymbol{\Lambda}_c^T + \boldsymbol{\Delta})]\\
= \mathrm{E}[\boldsymbol{\Lambda}_c\boldsymbol{\Xi}^T\boldsymbol{\Xi}\boldsymbol{\Lambda}_c^T] + \mathrm{E}[\boldsymbol{\Lambda}_c\boldsymbol{\Xi}^T\boldsymbol{\Delta}] + \mathrm{E}[\boldsymbol{\Delta}^T\boldsymbol{\Xi}\boldsymbol{\Lambda}_c^T] + \mathrm{E}[\boldsymbol{\Delta}^T\boldsymbol{\Delta}]\\
= \boldsymbol{\Lambda}_c\mathrm{E}[\boldsymbol{\Xi}^T\boldsymbol{\Xi}]\boldsymbol{\Lambda}_c^T + \boldsymbol{\Lambda}_c\mathrm{E}[\boldsymbol{\Xi}^T\boldsymbol{\Delta}] + \mathrm{E}[\boldsymbol{\Delta}^T\boldsymbol{\Xi}]\boldsymbol{\Lambda}_c^T + \mathrm{E}[\boldsymbol{\Delta}^T\boldsymbol{\Delta}]\\
= \boldsymbol{\Lambda}_c\ \boldsymbol{\Lambda}_c^T + \boldsymbol{\Theta}
\end{aligned}
$$

- The correlation between $\boldsymbol{X}$ and $\boldsymbol{\Xi}$ (factor loading) is

$$
\mathrm{Cor}(\boldsymbol{X}, \boldsymbol{\Xi}) = \mathrm{E}[\boldsymbol{X}^{\boldsymbol{T}}\boldsymbol{\Xi}] = \mathrm{E}[(\boldsymbol{\Xi}\boldsymbol{\Lambda}_c^T + \boldsymbol{\Delta})^{\boldsymbol{T}}\boldsymbol{\Xi}] = \boldsymbol{\Lambda}_c^T
$$

Exploratory Factor Analysis with *c* Common Factors

- Common Factor Model:

$$\boldsymbol{R} = \boldsymbol{\Lambda}_c\ \boldsymbol{\Lambda}_c^T + \boldsymbol{\Theta}$$

$$\boldsymbol{R}\ - \boldsymbol{\Theta} = \boldsymbol{\Lambda}_c\ \boldsymbol{\Lambda}_c^T$$

- PCA revisited:
  - Singular Value Decomposition of $\mathbf{X}$

$$\boldsymbol{X} = \boldsymbol{Z}_s \boldsymbol{D}^{1/2} \boldsymbol{U}^T$$

$$\boldsymbol{R} = \mathrm{E}[\boldsymbol{X}^T \boldsymbol{X}] = \mathrm{E}\left[\boldsymbol{U}\boldsymbol{D}^{1/2}(\boldsymbol{Z}_s{}^T \boldsymbol{Z}_s)\boldsymbol{D}^{1/2}\boldsymbol{U}^T\right] = (\boldsymbol{U}\boldsymbol{D}^{\frac{1}{2}})(\boldsymbol{U}\boldsymbol{D}^{\frac{1}{2}})^T = \boldsymbol{F}\boldsymbol{F}^T$$

$$\boldsymbol{R} \approx \boldsymbol{F}_c \boldsymbol{F}_c^T$$

For dimension reduction, we try to extract some subset of *c* components that closely approximates $\boldsymbol{R}$: $\boldsymbol{F}_c$ is the first c columns of factor loading matrix ***F*** whose elements are interpretable as the correlations between original variables ***X*** and *c* extracted common factors

Solution Procedure

- Principal factor method (Principal axis factoring)
  - Main idea: Replace the diagonal elements of $\boldsymbol{R}$ with communality values and conduct PCA
  - No prior information about measurement error δ
  - Initial estimate of communalities: use Squared Multiple Correlation (SMC)
    - As a initial value, use R_i^2 from the regression of X_i on the remaining X_j 's
  - Iterate until the result converges

Solution Procedure

One iteration

TABLE 5.4 Matrix of factor loadings for two-factor model (with approximate communalities)

	Factor 1	Factor 2
PARA	0.7722	−0.2351
SENT	0.7838	−0.1576
WORD	0.7562	−0.2372
ADD	0.4293	0.6017
DOTS	0.3476	0.6506

Final result

TABLE 5.5 Factor analysis with SMCs as initial estimates

Prior Communality Estimates: SMC

PARA	*SENT*	*WORD*	*ADD*	*DOTS*
0.6158	0.5914	0.5701	0.3672	0.3493

Final Eigenvalues

	1	2	3	4	5
Eigenvalue	2.2826	1.0273	0.0252	−0.0010	−0.0247

Factor Pattern

	Factor 1	Factor 2
PARA	0.8349	−0.2418
SENT	0.8253	−0.1398
WORD	0.7898	−0.2274
ADD	0.4146	0.6503
DOTS	0.3297	0.6890
Variance Explained by Each Factor		
	2.2826	1.0273

Rotation Indeterminancy

- Infinitely number of solutions: Infinitely number of bases (column vectors) for $\boldsymbol{R} - \boldsymbol{\Theta}$

$$\boldsymbol{T} = c \times c \text{ orthogonal transformation matrix } (\boldsymbol{T}^T\boldsymbol{T} = \boldsymbol{T}\boldsymbol{T}^T = \boldsymbol{I})$$

e.g. Two-dimensional orthogonal rotation:

$$\boldsymbol{T} = \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{pmatrix}$$

-30 degree rotation:

$$\boldsymbol{T} = \begin{pmatrix} \cos(-30) & -\sin(-30) \\ \sin(-30) & \cos(-30) \end{pmatrix} = \begin{pmatrix} 0.866 & 0.500 \\ -0.500 & 0.866 \end{pmatrix}$$

- The correlations between the rotated factors ($\boldsymbol{\Xi}^T\boldsymbol{T}$) and the original variables $\boldsymbol{X} = \boldsymbol{\Xi}\boldsymbol{\Lambda}_c^T + \boldsymbol{\Delta}$ is

$$\boldsymbol{\Lambda}_c^* = \frac{1}{n-1}(\Xi\boldsymbol{\Lambda}_c^T + \Delta)^T\boldsymbol{\Xi}^T\boldsymbol{T} = \boldsymbol{\Lambda}_c\,\boldsymbol{T}$$

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Rotation Indeterminancy

- Communalities are unchanged by rotation

$$\boldsymbol{R} - \boldsymbol{\Theta} = \boldsymbol{\Lambda}_c^* \boldsymbol{\Lambda}_c^{*T} = (\boldsymbol{\Lambda}_c \boldsymbol{T})(\boldsymbol{\Lambda}_c \boldsymbol{T})^T = \boldsymbol{\Lambda}_c \boldsymbol{T}\boldsymbol{T}^T \boldsymbol{\Lambda}_c^T = \boldsymbol{\Lambda}_c \boldsymbol{\Lambda}_c^T$$

Total variance explained by **Unrotated** factors

= Total variance explained by **Rotated** factors

- Type of rotations
  1. Orthogonal rotation: varimax or quartimax
  2. Non-orthogonal (Oblique) rotation: promax or direct oblimin

Factor Rotation

- Desirable factor loading pattern (Comrey, 1973)
  1. Most of the loadings on any specific factor (column) should be small (as close to zero as possible), and only a few loadings should be large in absolute value.
  2. A specific row of the loadings matrix, containing the loadings of a given variable with each factor, should display nonzero loadings on only one or no more than a few factors.
  3. Any pair of factors (columns) should exhibit different patterns of loadings. Otherwise, one could not distinguish the two factors represented by these columns.

Factor Rotation: Orthogonal Rotation Example

TABLE 5.6 Factor loadings for pain relievers

	Unrotated Solution		Rotated Solution	
Attribute	Factor 1	Factor 2	Factor 1	Factor 2
No upset stomach	0.579	−0.452	0.139	0.721
No bad side effects	0.522	−0.572	0.017	0.774
Stops the pain	0.645	0.436	0.772	0.097
Works quickly	0.542	0.542	0.764	−0.051
Keeps me awake	−0.476	0.596	−0.034	−0.762
Limited relief	−0.613	−0.439	−0.750	−0.074
	Variance accounted for		Variance accounted for	
	1.921	1.562	1.765	1.718

Factor Rotation: Orthogonal Rotation Example

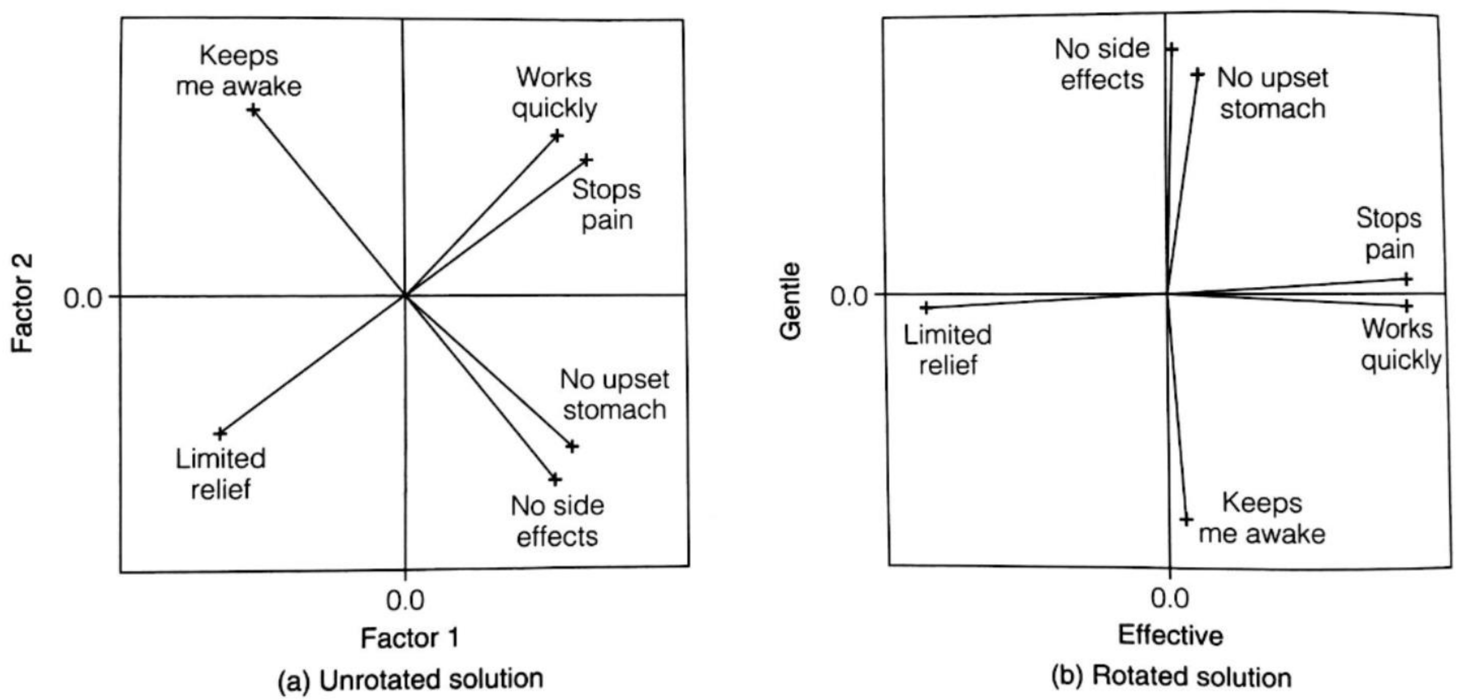


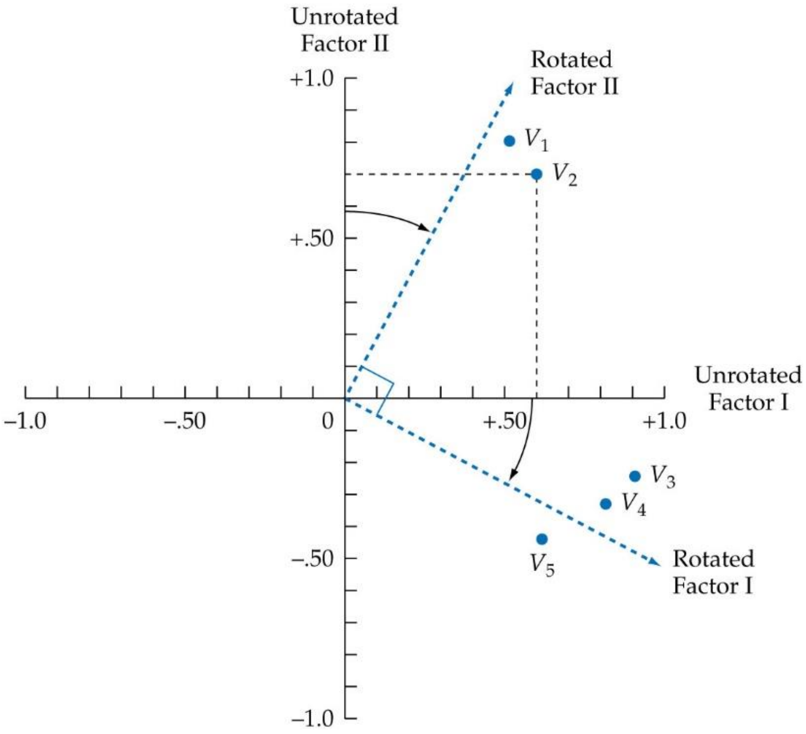
FIGURE 5.4 Plot of factor loadings for attributes of pain relievers: Unrotated and rotated

Factor Rotation

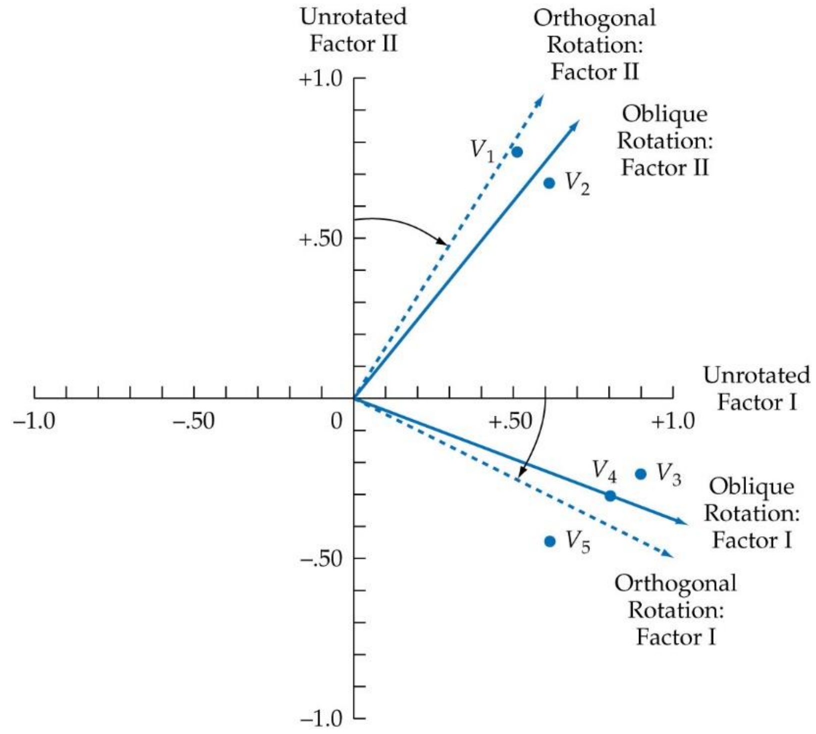
- The factor solution is reoriented through a process called *rotation.* Two types of rotation:
  - ***Orthogonal rotation*** preserves the perpendicularity of the axes (i.e., the rotated factors remain uncorrelated). Two widely used methods for orthogonal rotation are *varimax rotation* (which tries to achieve simple structure by focusing on the columns of the factor loadings matrix) and *quartimax rotation* (which focuses on the rows).
  - ***Oblique rotation*** allows for correlation between the rotated factors. One approach involves rotating the solution to match a target matrix exhibiting simple structure. Some cautions about the properties of oblique factor solutions:
    - Because the common factors are now correlated, the communalities are no longer given by the sum of the squared factor loadings.
    - In an oblique solution, there is a distinction between a *structure loading* (i.e., the correlation between a variable and a factor) and a *pattern loading* (i.e., the partial correlation between a variable and a common factor controlling for other common factors).

Factor Rotation

Orthogonal Rotation



Oblique rotation



Factor Rotation

Orthogonal rotation methods

- are the most widely used rotational methods.
- are the preferred method when the research goal is data reduction to either a smaller number of variables or a set of uncorrelated measures for subsequent use in other multivariate techniques.

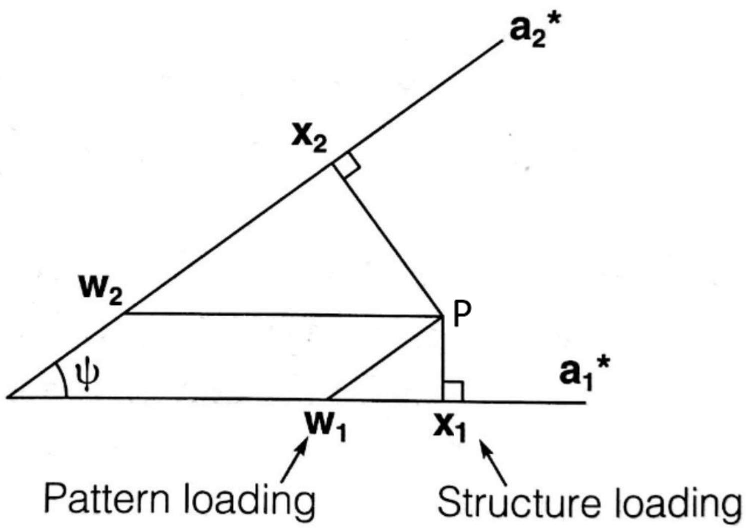
Oblique rotation methods

- best suited to the goal of obtaining several theoretically meaningful factors or constructs because, realistically, very few constructs in the "real world" are uncorrelated.

Factor Rotation: Oblique Rotation

- Pattern Loading vs. Structure Loading

FIGURE 5.7 Diagram depicting difference between structure loadings and pattern loadings



Factor Rotation: Oblique Rotation

TABLE 5.12 Results from oblique rotation of psychological test data

	Pattern Loadings		Structure Loadings	
	Factor 1	Factor 2	Factor 1	Factor 2
PARA	0.8480	−0.0300	0.8687	0.1906
SENT	0.7905	0.0666	0.8344	0.2753
WORD	0.8016	−0.0271	0.8215	0.1815
ADD	0.0499	0.7312	0.2428	0.7696
DOTS	−0.0432	0.7488	0.1510	0.7626

	Interfactor Correlation
	Factor 2
Factor 1	0.2528

Factor Scores

- Principal Component Score

$$\boldsymbol{z_i} = \boldsymbol{X_s u_i}$$

- Consider approximating $\boldsymbol{\Xi}$ as

$$\boldsymbol{\Xi} = \boldsymbol{X_s B}$$

where $\boldsymbol{B}$ is a matrix of factor score coefficients

- $\boldsymbol{\Xi}$ is unobservable and so we cannot choose $\boldsymbol{B}$ using OLS
- Let's premultiply each side of the above by $\frac{1}{n-1}\boldsymbol{X_s^T}$

$$\frac{1}{n-1}\boldsymbol{X_s^T \Xi} = \frac{1}{n-1}\boldsymbol{X_s^T X_s B} \Leftrightarrow \boldsymbol{\Lambda_c} = \boldsymbol{RB}$$

which reduces to $\boldsymbol{B} = \boldsymbol{R^{-1}\Lambda_c}$

- Hence, estimated factor scores are given by

$$\boldsymbol{\Xi} = \boldsymbol{X_s R^{-1} \Lambda_c}$$

which is not unique because of rotational indeterminacy.

Sample Problem: Perception of Ready-To-Eat Cereals

- To characterize the consideration behavior of cereal consumers (i.e., to explain which brands the consumer is willing to purchase) as a function of the underlying characteristics of 12 different types of brands

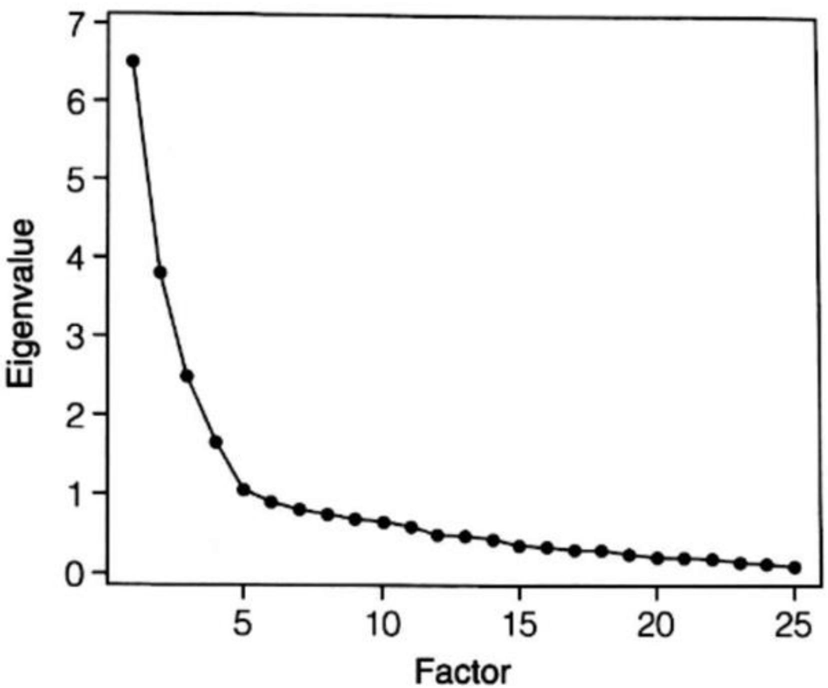
TABLE 5.8 List of 25 RTE cereal attributes and 12 RTE cereal brands

Cereals	Attributes	
1. All Bran	Filling	Family
2. Cerola Muesli	Natural	Calories
3. Just Right	Fibre	Plain
4. Kellogg's Corn Flakes	Sweet	Crisp
5. Komplete	Easy	Regular
6. NutriGrain	Salt	Sugar
7. Purina Muesli	Satisfying	Fruit
8. Rice Bubbles	Energy	Process
9. Special K	Fun	Quality
10. Sustain	Kids	Treat
11. Vitabrit	Soggy	Boring
12. Weetbix	Economical	Nutritious
	Health	

Sample Problem: Perception of Ready-To-Eat Cereals

- Number of factors: 4 factors?

FIGURE 5.6
Scree plot for RTE cereal data



Sample Problem: Perception of Ready-To-Eat Cereals

- SMCs for each of 25 RTE cereal attributes; explaining at least 52%=13/25

TABLE 5.9 SMCs for each of 25 RTE cereal attributes

Filling 0.6333	Natural 0.6359	Fibre 0.7035	Sweet 0.6116	Easy 0.2304	Salt 0.4481	Satisfying 0.6150
Energy 0.5711	Fun 0.4833	Kids 0.6360	Soggy 0.3143	Economical 0.3722	Health 0.7500	Family 0.5989
Calories 0.4168	Plain 0.4083	Crisp 0.4105	Regular 0.5211	Sugar 0.6642	Fruit 0.4877	Process 0.2975
Quality 0.6144	Treat 0.5607	Boring 0.3340	Nutritious 0.7107			

Sample Problem: Perception of Ready-To-Eat Cereals

TABLE 5.10 Unrotated factor solution for RTE cereal data

	Factor 1	Factor 2	Factor 3	Factor 4
Filling	0.7276	0.1030	−0.0674	0.1960
Natural	0.7323	−0.2422	−0.1077	0.1052
Fibre	0.7226	−0.2385	−0.3098	0.1607
Sweet	0.0850	0.7407	−0.2053	0.1510
Easy	0.3181	0.1454	0.2064	0.1074
Salt	−0.2062	0.4995	−0.1385	0.3971
Satisfying	0.7207	0.1809	0.1627	0.1704
Energy	0.7022	0.1328	−0.0657	0.1264
Fun	0.3886	0.4959	0.2101	−0.1579
Kids	0.2133	0.2759	0.7335	0.1118
Soggy	−0.0984	−0.2319	0.1515	0.4257
Economical	0.1508	−0.2361	0.4890	0.1002
Health	0.8081	−0.3075	−0.1068	0.0713
Family	0.3074	0.2179	0.6731	0.0271
Calories	−0.1603	0.5663	−0.1721	0.2151
Plain	−0.3055	−0.3560	0.2178	0.4086
Crisp	0.2888	0.4544	0.2143	−0.1927
Regular	0.5943	−0.1354	−0.1927	0.0682
Sugar	−0.2476	0.7213	−0.2467	0.2416
Fruit	0.3733	0.2534	−0.4802	−0.1569
Process	−0.3033	0.2721	0.0030	0.2470
Quality	0.7324	−0.1394	0.0515	−0.0328
Treat	0.4648	0.5632	0.0758	−0.2018
Boring	−0.3789	−0.2619	−0.0988	0.3392
Nutritious	0.7978	−0.2176	−0.1422	0.1252
Variance explained by each factor				
	6.1085	3.3528	2.0456	1.1184

Final Communality Estimates

Filling 0.5829	Natural 0.6176	Fibre 0.7008	Sweet 0.6208	Easy 0.1765	Salt 0.4689	Satisfying 0.6077
Energy 0.5311	Fun 0.4660	Kids 0.6722	Soggy 0.2676	Economical 0.3277	Health 0.7641	Family 0.5958
Calories 0.4222	Plain 0.4344	Crisp 0.3730	Regular 0.4133	Sugar 0.7008	Fruit 0.4588	Process 0.2270
Quality 0.5596	Treat 0.5798	Boring 0.3370	Nutritious 0.7197			

Sample Problem: Perception of Ready-To-Eat Cereals

TABLE 5.11 Varimax rotation of factor solution for RTE cereal data

	Factor 1	Factor 2	Factor 3	Factor 4
Filling	0.7057	0.0885	0.1998	0.1512
Natural	0.7529	−0.2084	0.0555	0.0366
Fibre	0.8214	−0.1156	−0.1200	0.0211
Sweet	0.0684	0.7020	0.0744	0.3469
Easy	0.2382	0.0634	0.3252	0.0643
Salt	−0.0924	0.6855	0.0161	−0.0836
Satisfying	0.6254	0.0770	0.4240	0.1703
Energy	0.6594	0.0786	0.1930	0.2098
Fun	0.1629	0.1765	0.4175	0.4781
Kids	−0.0252	0.0340	0.8502	0.0102
Soggy	0.0330	0.0143	0.0942	−0.4806
Economical	0.0686	−0.2809	0.4149	−0.2289
Health	0.8287	−0.2877	0.0519	0.0460
Family	0.0616	−0.0554	0.7612	0.0899
Calories	−0.1141	0.6267	−0.0072	0.1203
Plain	−0.1466	−0.0622	0.0680	−0.6566
Crisp	0.0734	0.1458	0.3732	0.4362
Regular	0.6132	−0.0996	−0.0270	0.0889
Sugar	−0.1844	0.8166	−0.0522	0.1651
Fruit	0.3764	0.1874	−0.2669	0.4429
Process	−0.2363	0.3737	0.0262	−0.1259
Quality	0.6466	−0.2444	0.2048	0.1704
Treat	0.2444	0.2335	0.3368	0.6019
Boring	−0.1646	0.0668	−0.2255	−0.5048
Nutritious	0.8315	−0.1764	0.0517	0.0558
	Variance Explained by Each Factor			
	5.2026	2.6598	2.4747	2.2837

Factor 1: *Healthful*
Factor 2: *Artificial*
Factor 3: *Non-Adult*
Factor 4: *Interesting*

RTE Cereal: Factor Scores

TABLE 5.13 Factor score coefficients for RTE cereal data

	Factor Score Coefficients			
	Factor 1	Factor 2	Factor 3	Factor 4
Filling	0.1456	0.0915	0.0228	−0.0420
Natural	0.1370	−0.0142	−0.0112	−0.0417
Fibre	0.2160	0.0542	−0.1280	−0.0746
Sweet	0.0425	0.2261	−0.0030	0.0745
Easy	0.0088	0.0176	0.0663	−0.0115
Salt	0.0301	0.2010	0.0158	−0.1196
Satisfying	0.1101	0.0731	0.1540	−0.0361
Energy	0.0918	0.0752	0.0266	0.0126
Fun	−0.0024	0.0017	0.1017	0.1316
Kids	−0.0433	0.0151	0.3735	−0.0789
Soggy	0.0390	0.0646	0.0402	−0.1995
Economical	−0.0043	−0.0453	0.1190	−0.0855
Health	0.2155	−0.0792	−0.0091	−0.0590
Family	−0.0294	−0.0453	0.2700	−0.0043
Calories	0.0259	0.1567	−0.0080	−0.0253
Plain	0.0319	0.0699	0.0740	−0.2534
Crisp	−0.0372	−0.0136	0.0864	0.1507
Regular	0.0672	−0.0267	−0.0316	0.0129
Sugar	0.0444	0.3824	−0.0310	−0.0384
Fruit	0.0376	0.0321	−0.1393	0.1689
Process	0.0091	0.1020	0.0196	−0.0824
Quality	0.0576	−0.0674	0.0531	0.0527
Treat	−0.0279	0.0329	0.0654	0.2471
Boring	0.0327	0.0680	−0.0178	−0.1752
Nutritious	0.2004	0.0045	−0.0229	−0.0711

RTE Cereal: Factor Scores

TABLE 5.14 Average factor scores for 12 RTE cereals

Brand	Number of Obs	Factor 1	Factor 2	Factor 3	Factor 4
All Bran	15	0.3490	−0.3185	−0.8861	−0.3754
Cerola	13	0.5174	0.4994	−0.2332	0.6582
Corn Flakes	27	−0.5541	0.1165	0.5738	0.0241
Just Right	16	−0.0133	0.2278	−0.4196	0.4753
Komplete	14	0.5464	0.2338	−1.0084	0.6024
NutriGrain	24	−0.4255	0.8086	0.5482	0.2852
Purina	18	0.6226	0.6838	−0.4095	0.5754
Rice Bubbles	21	−1.1650	−0.4295	0.6041	0.0627
Special K	23	−0.3058	−0.2142	0.1541	−0.0549
Sustain	12	0.6772	−0.3021	−0.3234	0.8566
Vitabrits	25	0.3649	−0.5921	0.2180	−0.9316
Weetbix	27	0.3266	−0.4120	−0.0713	−0.8837

RTE Cereal: Factor Scores

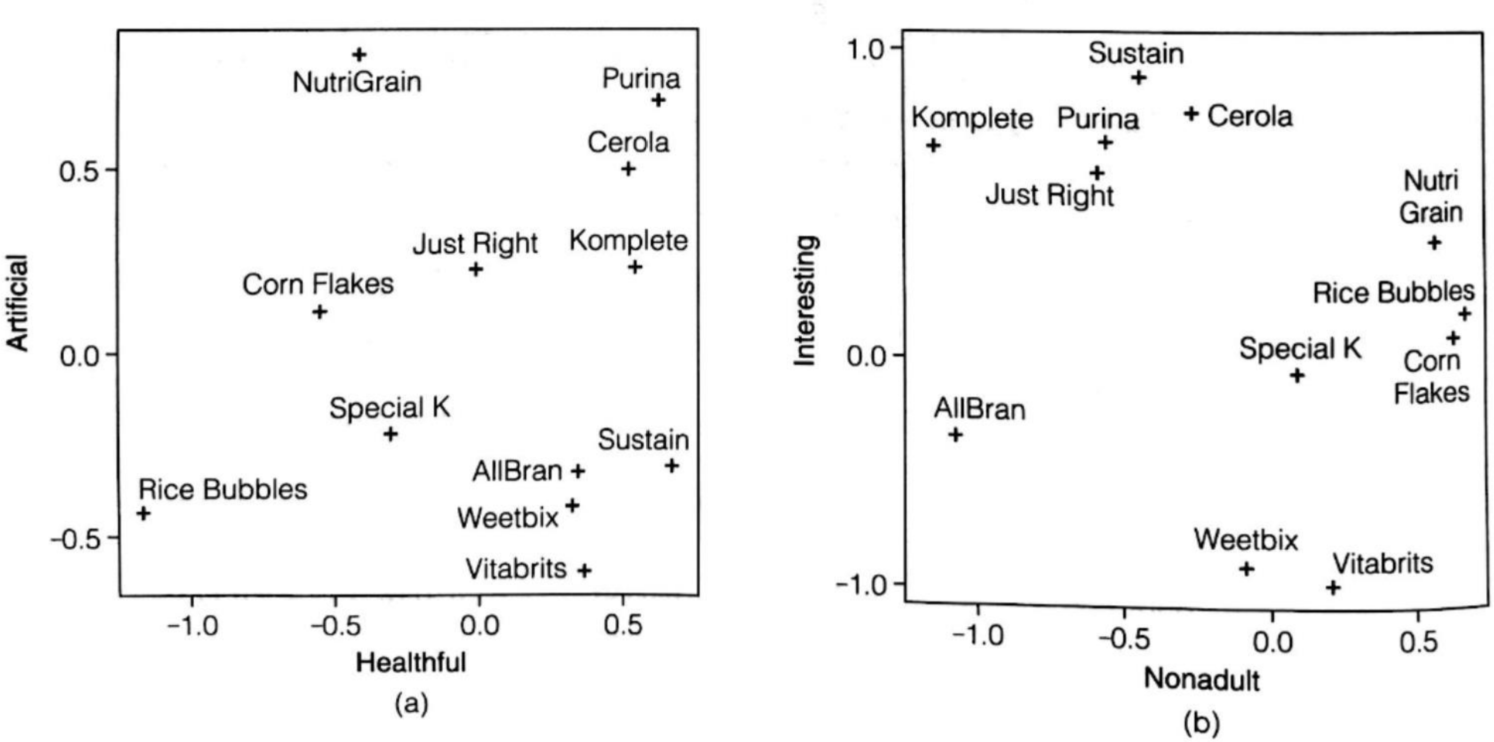


FIGURE 5.8 Scatter plots of factor scores for RTE cereals