Multivariate Data Analysis

(MGT513, BAT531, TIM711)

Lecture 9

Structural Equation Models 1: Introduction

References

- KEYNOTE: Structural Equation Modeling: models, software and stories by *Yves Rosseel*
  - https://users.ugent.be/~yrosseel/lavaan/rosseel_user2017.pdf
- Structural Equation Modelling by *Gillian Smith*
  - https://www.academia.edu/1672242/An_introduction_to_Structural_Equation_Modelling
- Multivariate Data Analysis by *Joseph Hair, B. Babin, R. Anderson, and W. Black*: Ch.9 ~ Ch.13
- LCG (textbook): Ch.10

What is Structural Equation Modeling?

SEM = **S**tructural **E**quation **M**odeling

- SEM is a multivariate statistical modeling technique
- SEM allows us to test a hypothesis/model about the data
  - We postulate a data-generating model
  - This model may or may not fit the data
- What is so special about SEM?
  - The model can contain latent variables
    - Latent variables can be hypothetical “constructs” measured by a set of indicator variables
    - Latent variables can be random effects (e.g. random intercepts)
  - SEM allows for indirect effects (mediation), reciprocal effects
  - The model is depicted as diagram

What is Structural Equation Modeling?

Assumptions (same as linear regression)

1. No outliers
2. Linearity
3. Normality (but can be ‘handled’ e.g. robust estimation techniques)
4. Identification (the model must receive as many or more inputs than requested parameters)

What is Structural Equation Modeling?

- Structural equation modeling (SEM) is a family of statistical models that seeks to *explain* relationships among multiple variables. In doing so, SEM examines the structure of interrelationships expressed in a series of equations.
- SEM involves both interdependence and dependence.
  - Multiple equations represent the theoretical structure.
  - Theory determines what things are connected.
  - Just as importantly, theory determines what things are NOT connected.
- SEM combines two multivariate procedures into one:
  - Factor Analysis – assesses fit of the theoretical measurement model connecting latent constructs and measured variables
  - Regression Analysis – assesses fit of the structural theory connecting latent constructs to each other

SEM in a nutshell

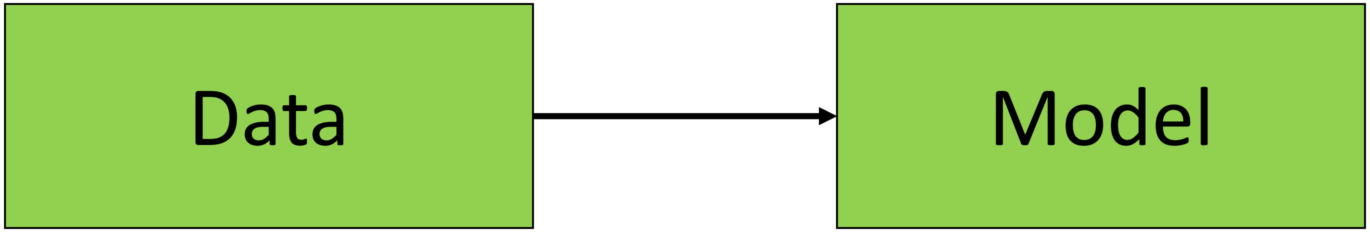
It is the combination of

- **Measurement model**
  - The part of the model that relates the latent variables to the observed variables
  - Factor analytic part of SEM

- **Path analysis (Structure)**
  - Regression between variables
  - No latent factors? Only path model exists between the measured variables

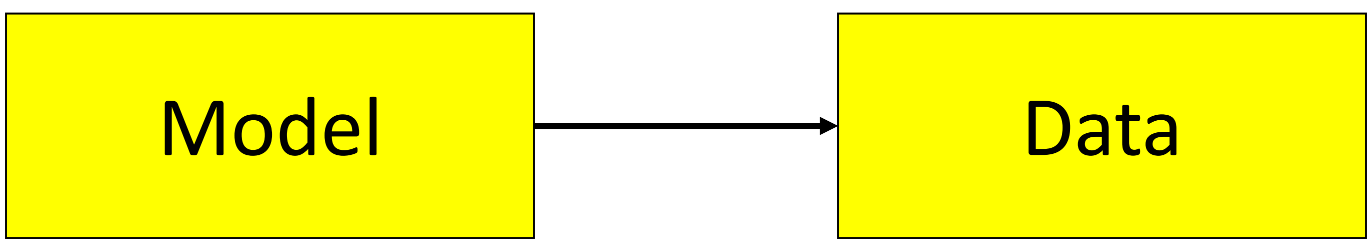
Typical Statistical Modelling

- Modelling process
  - What is the best model to describe a set of data?
  - Take the data and develop a model from that data
    - Take “Model” to mean, SD, median, correlation, F-ratio, t-value



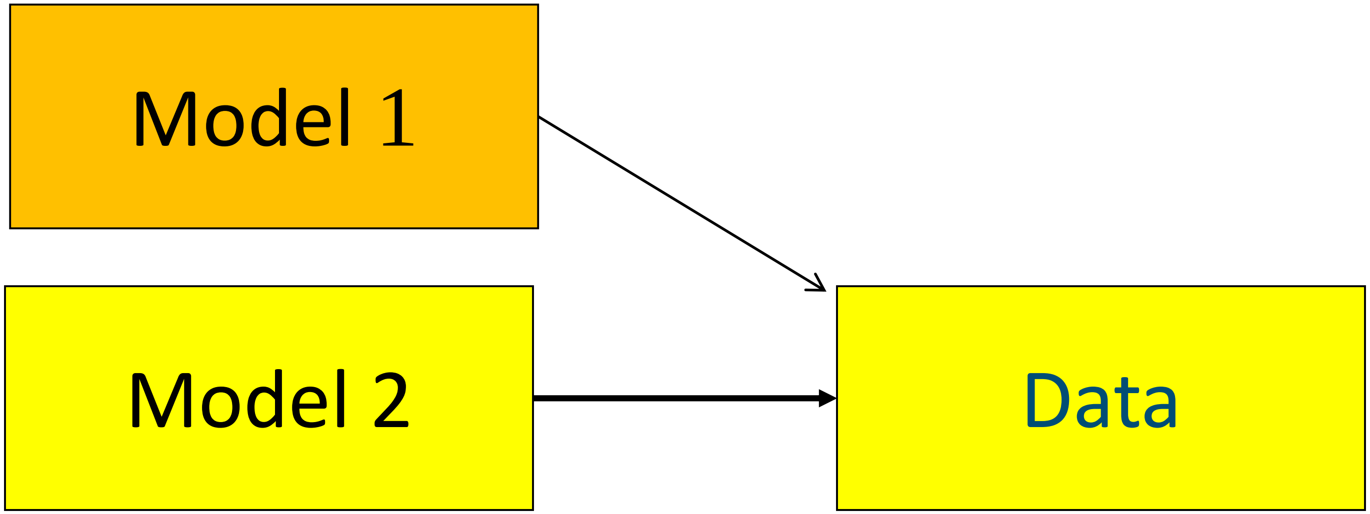
SEM – Theory driven

- Modelling process
  - Could this model have led to the data that I have?
  - Using theory, we hypothesise that there are some relationships between variables
  - Do these hypotheses fit the associations in the data we have found?
  - Test a model of the data
    - We propose a data-generating model
    - The model may or may not fit the data



SEM – Theory driven

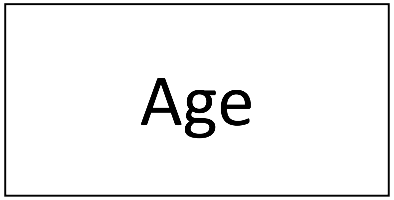
- Modelling process
  - We can test competing theories too!
  - Which one best represents relationships in the data?



Notes on the Structure in SEM

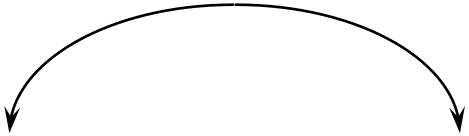
- **Structure Represents Theory**
- The **user** specifies a structural model that includes a limited set of relationships. Thus, SEM is distinct from other multivariate procedures:
  - Structural analysis is **not** exploratory
  - SEM **tests** the user's theory and does not explore relationships or reveal a model
  - SEM provides more accurate estimates of parameters
    - Corrects for error attenuation
  - Structural equations only include the relationships necessary to represent the model.
    - All other possible connections are assumed to be 0 (do not exist)
- Structural equations **are contrasted with reduced form equations**:
  - A reduced form equation solves for a single endogenous construct (or dependent variable) in a single equation with all and only exogenous constructs (independent variables) employed as predictors

SEM Diagrams: Representing your SEM model



Observed variables are in boxes

Causal arrow represents a causal association between two variables

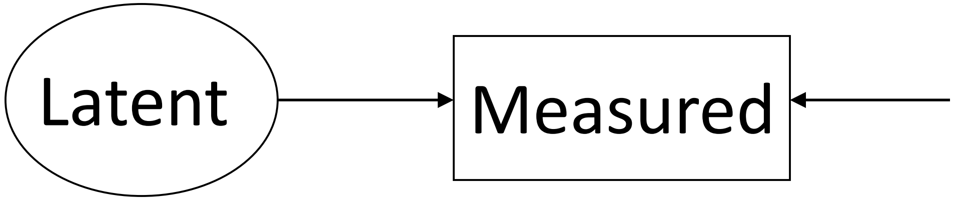


Correlational relationships with no causal direction

SEM Diagrams: Measuring a Latent Variable

- Latent variables are drawn as ellipses/circles
  - Usually seen as causing the measured variables (as not directly assessed)

- Measured variable has two separate causes
  - latent variable
  - "other stuff" (error / variance from other sources that we are uninterested in)



Symbols in SEM

Name	Symbol	Explanation
(latent construct)		η_i for dependent variables and ξ_i for independent variables
Indicator Variables		y_i for dependent variable and x_i for independent variable
regression coefficient	$\longrightarrow$	-γ_i for the impact of relationship between two indicators -β_i for the impact of relationship between two latent construct and for the impact of relationship between an indicator and latent construct
correlation/covariance relationship	$\longleftrightarrow$	φ_i for structural model and Φ_i for measurement model
error estimation for indicator	$\rightarrow$	δ_i for measurement model and ϵ_i for structural model
error estimation for latent construct	$\searrow$	θ_i (only used for endogenous factor)

Source:
https://www.researchgate.net/publication/307597767_Structural_Equation_Modelling_Confirmatory_Factor_Analysis_To_Construct_Measurement_Model_Mediator_Check_Among_Formed_Factors

Latent Constructs are Hypothetical, Unobservable Variables

Exogenous Constructs

Exogenous constructs are the *latent*, multi-item equivalent of *independent* variables. They use a variate (linear combination) of measures to represent the construct, which acts as an *independent* variable in the model.

Multiple measured (sometimes called manifest) variables (x) represent the exogenous constructs.

Endogenous Constructs

Endogenous constructs are the *latent*, multi-item equivalent to *dependent* variables. These constructs are theoretically determined by factors within the model.

Multiple measured (sometimes called manifest) variables (y) represent the endogenous constructs.

Depicting Theoretical Relationships in SEM

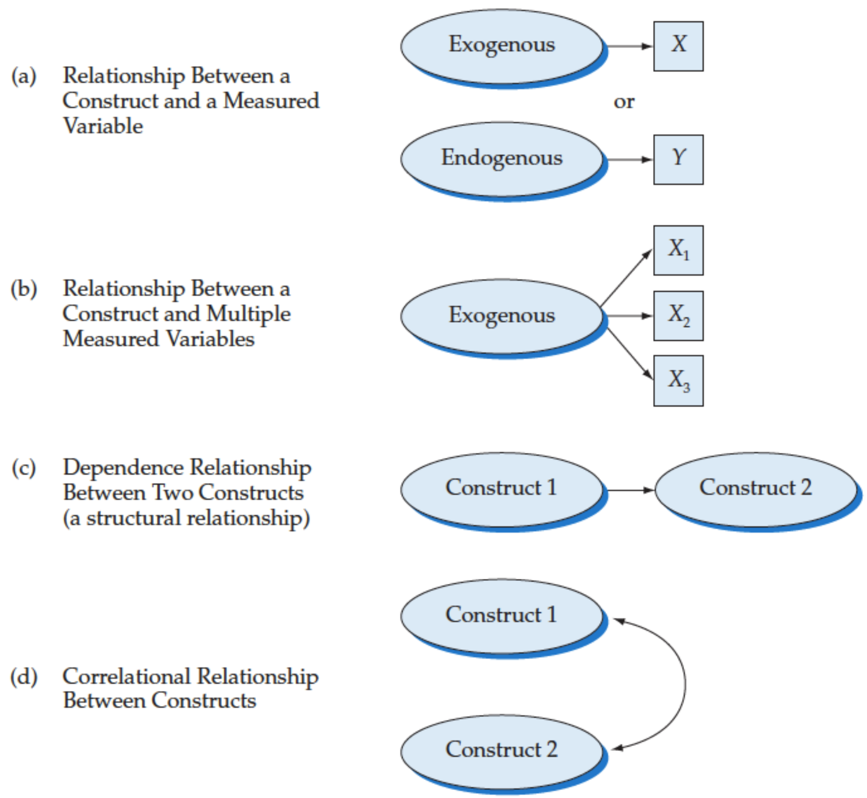


Figure 9.1
Common Type of Theoretical Relationships in an SEM Model

What do the Theoretical Models Look Like?

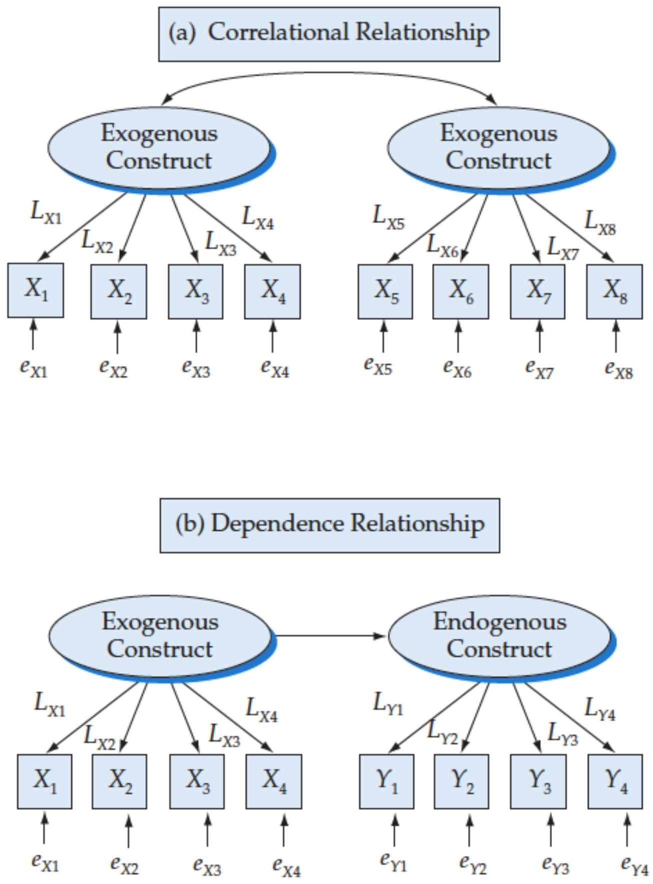


Figure 9.2
Visual Representation Measurement and Structural Model Relationships in a Simple SEM Model

SEM Implies Causality – "Causal Modeling"

Evidences for causality

1. Temporal sequences
   The cause must occur before the effect
2. Concomitant variation
   When a change in the cause occurs, a change in outcome is also observed
3. Nonspurious association
   Any covariation between a cause and an effect is true and not simply due to some other variable

Nonspurious association

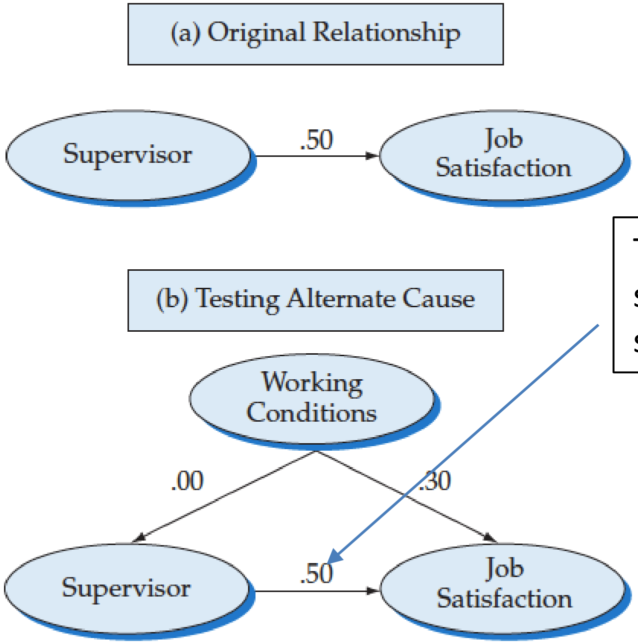


Figure 9.4
Testing for a Nonspurious Relationship Between Constructs

To the extent that this relationship changes, the supervisor – job satisfaction relationship is spurious

Example Structural Model

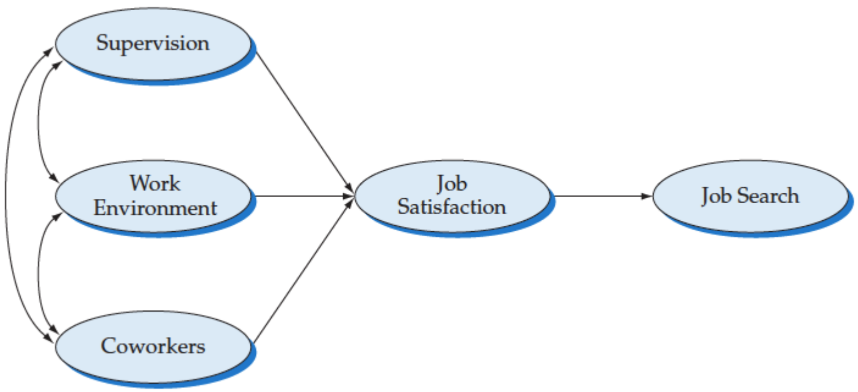
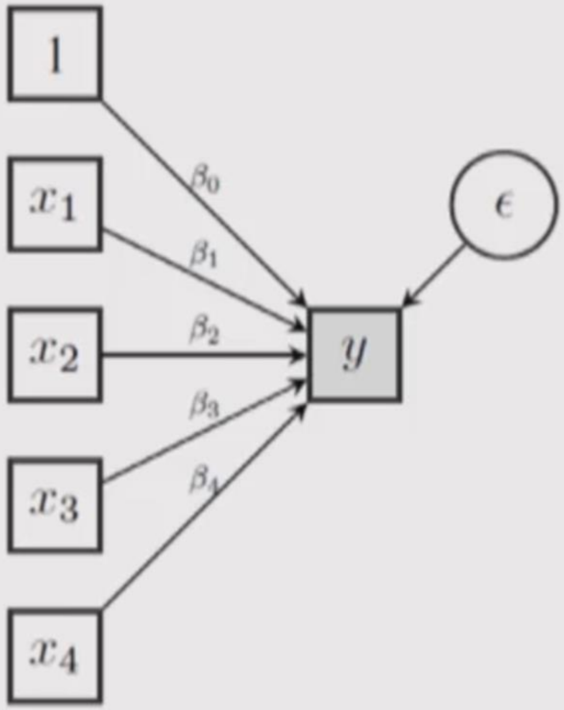


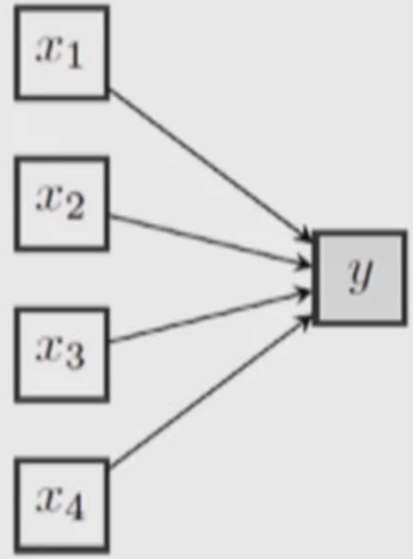
Figure 9.5
Path Diagram of a Simple Structural Model

- A structural model showing only the latent constructs
- 5 latent constructs
- Arrows depict relationships
  - 7 relationships specified
  - 3 possible relationships NOT specified, meaning theory says they do not exist

Univariate Linear Regression

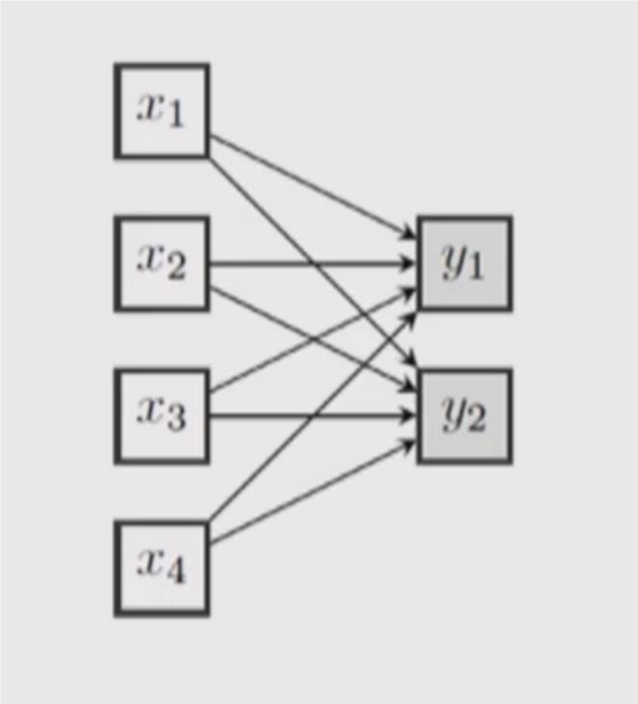


<u>Simplified version</u>



$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4} + \epsilon_i \quad (i = 1, 2, \ldots, n)$$

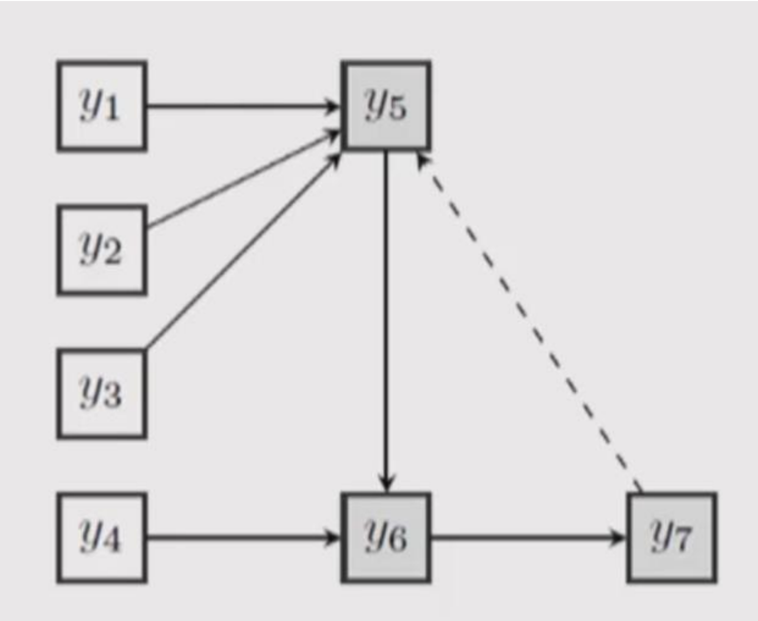
Multivariate Regression



Strict distinction between ‘independent’ variables and ‘dependent’ variables

SEM: Path Analysis

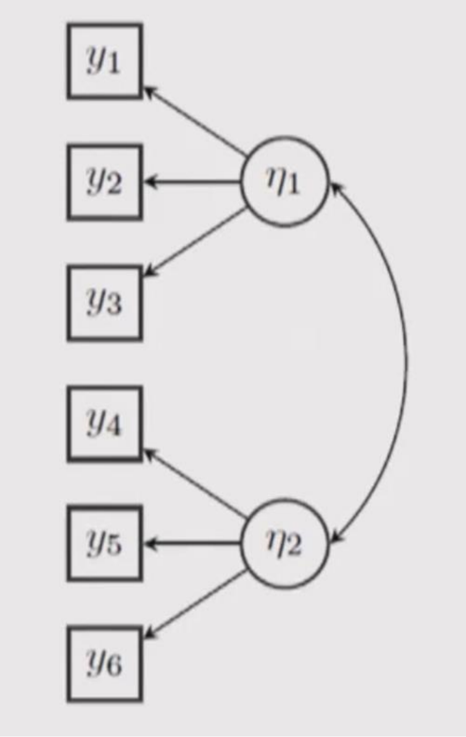
- All variables are observed (manifest)
- We allow for indirect effects (y_5, via y_6 on y_7)
- We allow cycles: y_7 *could* influence y_5



y_1 = read for school
y_2 = read with parent
y_3 = access to other media
y_4 = access to books
y_5 = reading motivation
y_6 = reading frequency
y_7 = reading ability

SEM: Confirmatory Factor Analysis (CFA)

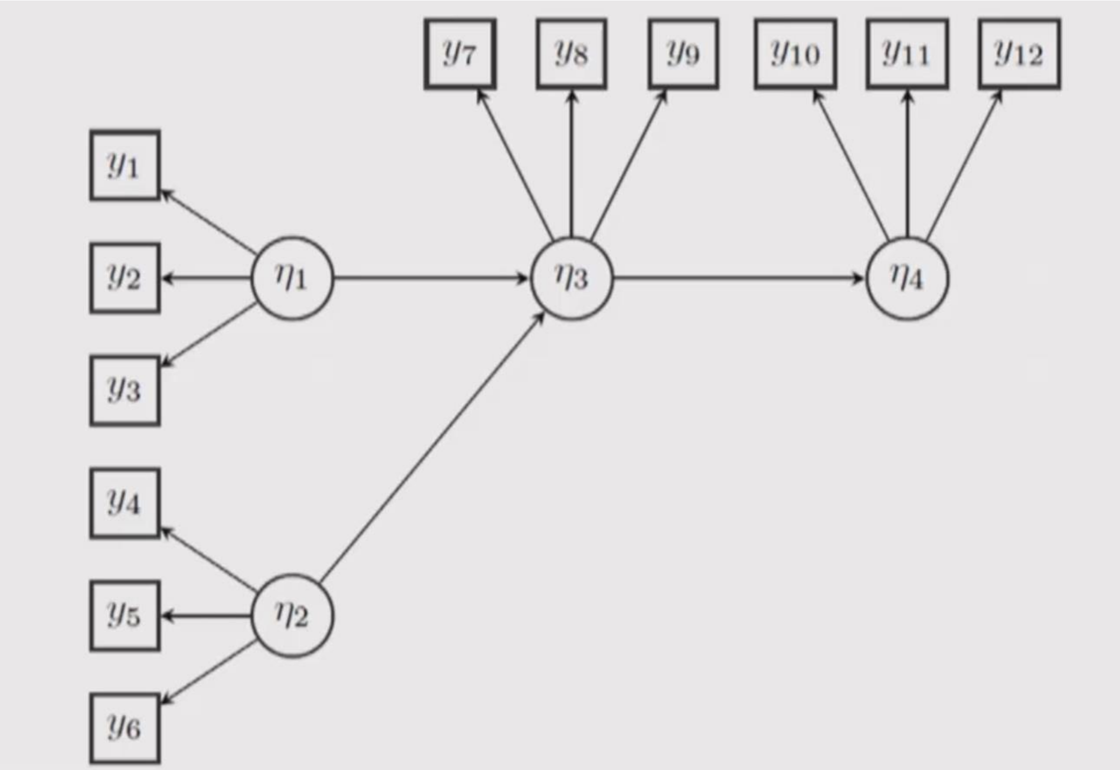
- Measurement model: representing the relationship between one or more latent variables and their (observed) indicators



η_1 = depression
η_2 = neuroticism

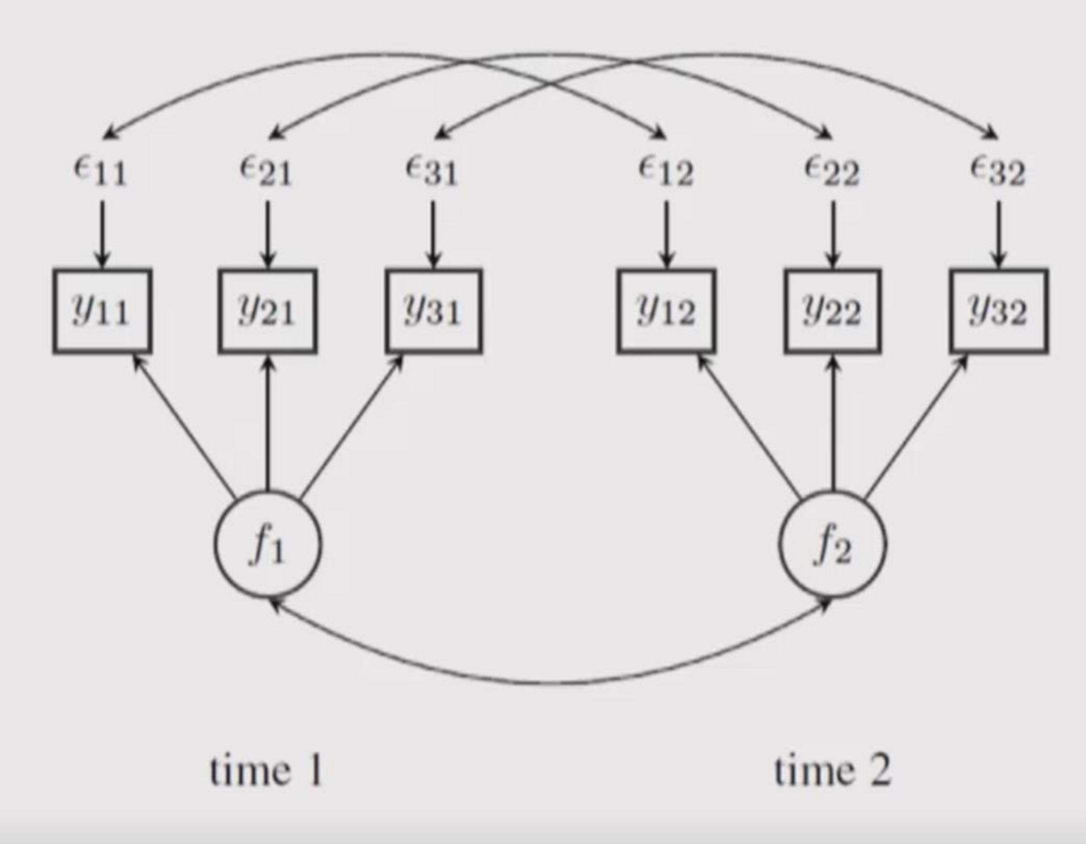
SEM: Measurement Model + Structural Part

- Full SEM = Path Analysis with Latent Variables



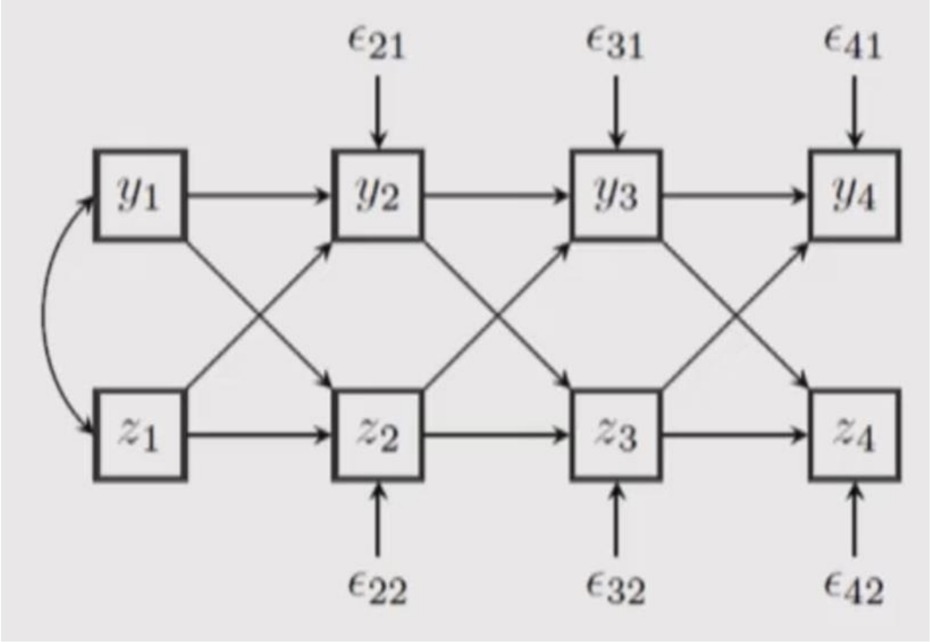
Example: Paired t-test using latent variables

- Example with 2 time points (longitudinal analysis):



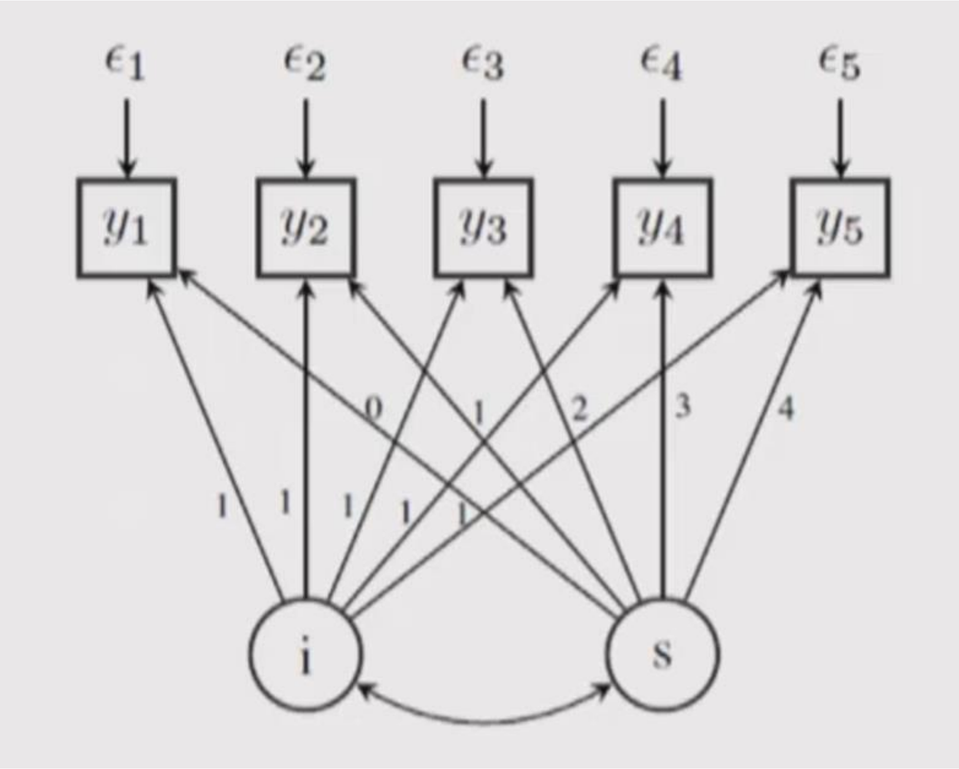
Example: Panel Model with Cross-lagged effects

- What is the directional effect of one variable on the other (longitudinal analysis)?
  - Do the two variables develop independently of each other?
  - Or does *Y* exert a greater influence on *Z*, or vice versa?



Example: Growth Curve Model

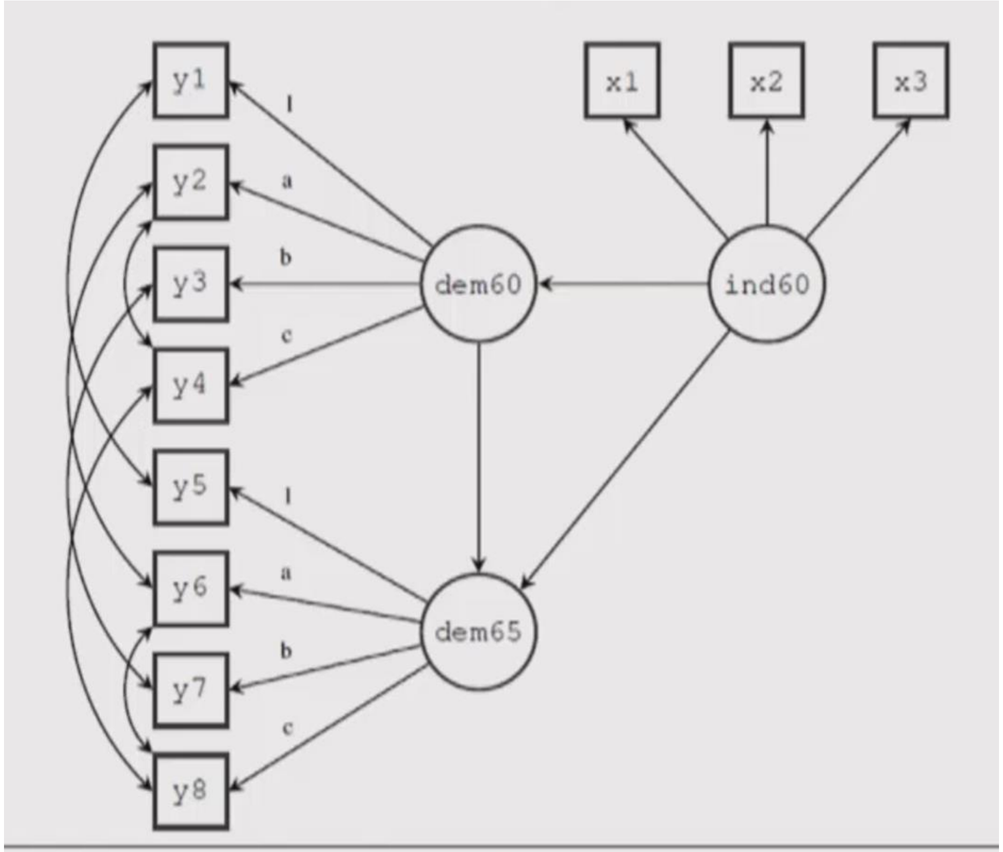
- Random Intercept and Random Slope (mixture model framework)



$$y_t = \text{intercept} + \text{slope} \times \text{time} + \text{error}$$

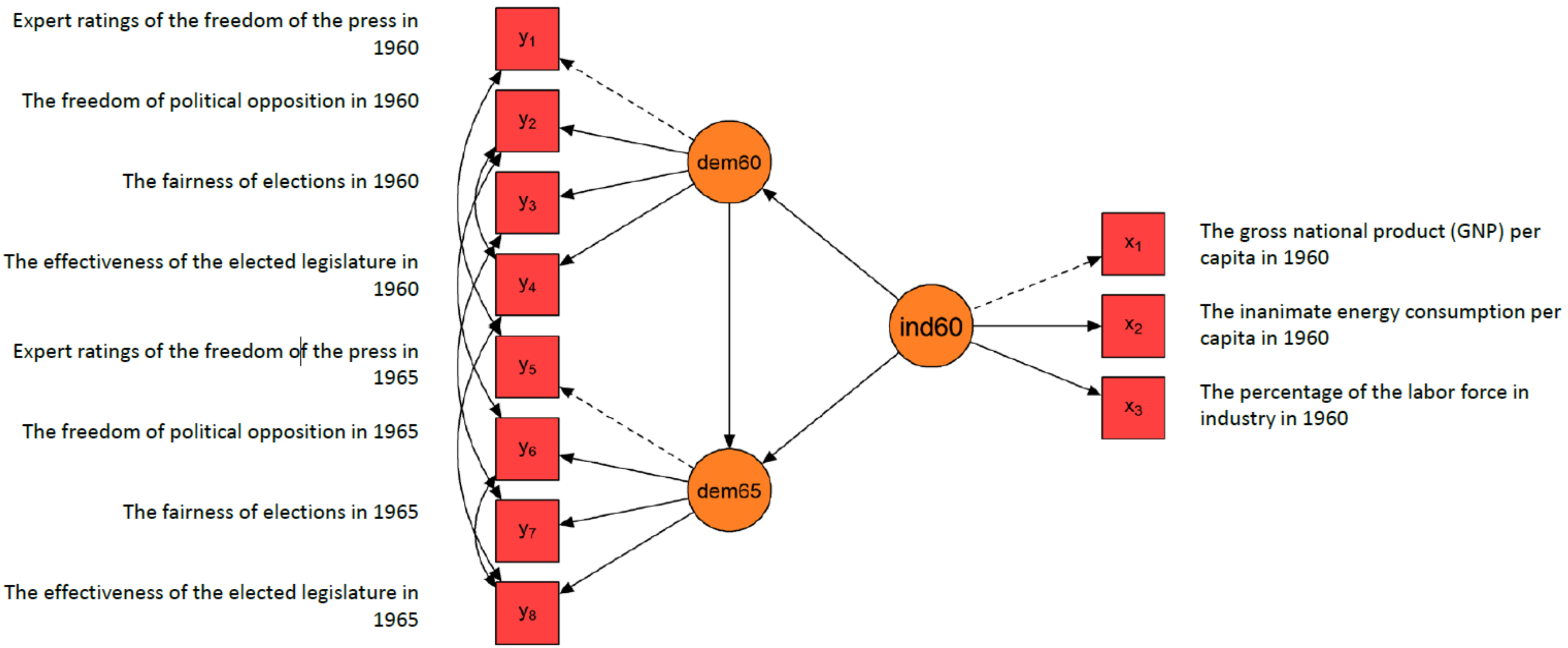
Example: Industrialization and Political Democracy (Bollen, 1989)

- Influence of industrialization ('60) on political democracy ('60 and '65)
  - Direct effect and Indirect effect of industrialization ('60) on political democracy ('65)



Bollen (1989), "A panel model of political democracy and industrialization for developing countries"

Example: Industrialization and Political Democracy (Bollen, 1989) - cont



Bollen (1989), "A panel model of political democracy and industrialization for developing countries"

Source: Structural Equation Modeling by Bowen and Guo (https://global.oup.com/us/companion.websites/9780195367621/examples/)

Example: Industrialization and Political Democracy (Bollen, 1989) - cont

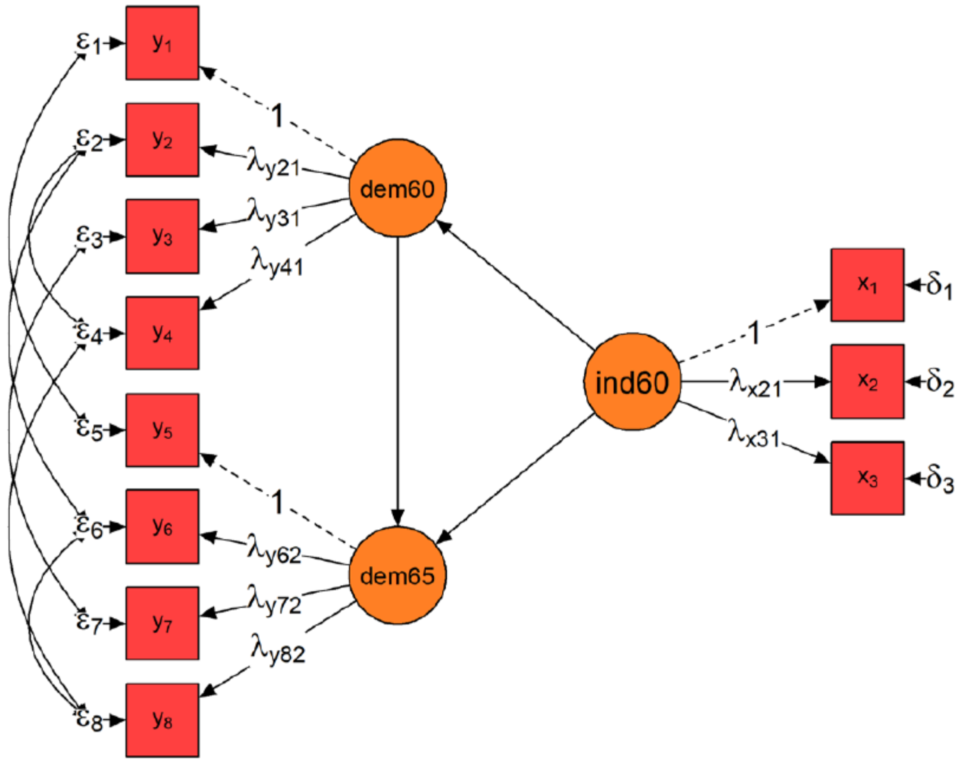
Measurement Model

$\mathbf{x} = \mathbf{\Lambda}_x \boldsymbol{\xi} + \boldsymbol{\delta}$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ \lambda_{x21} \\ \lambda_{x31} \end{bmatrix} [ind60] + \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix}$$

$\mathbf{y} = \mathbf{\Lambda}_y \boldsymbol{\xi} + \boldsymbol{\epsilon}$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \lambda_{y21} & 0 \\ \lambda_{y31} & 0 \\ \lambda_{y41} & 0 \\ 0 & 1 \\ 0 & \lambda_{y62} \\ 0 & \lambda_{y72} \\ 0 & \lambda_{y82} \end{bmatrix} \begin{bmatrix} dem60 \\ dem65 \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \\ \epsilon_7 \\ \epsilon_8 \end{bmatrix}$$



Bollen (1989), "A panel model of political democracy and industrialization for developing countries"

Example: Industrialization and Political Democracy (Bollen, 1989) - cont

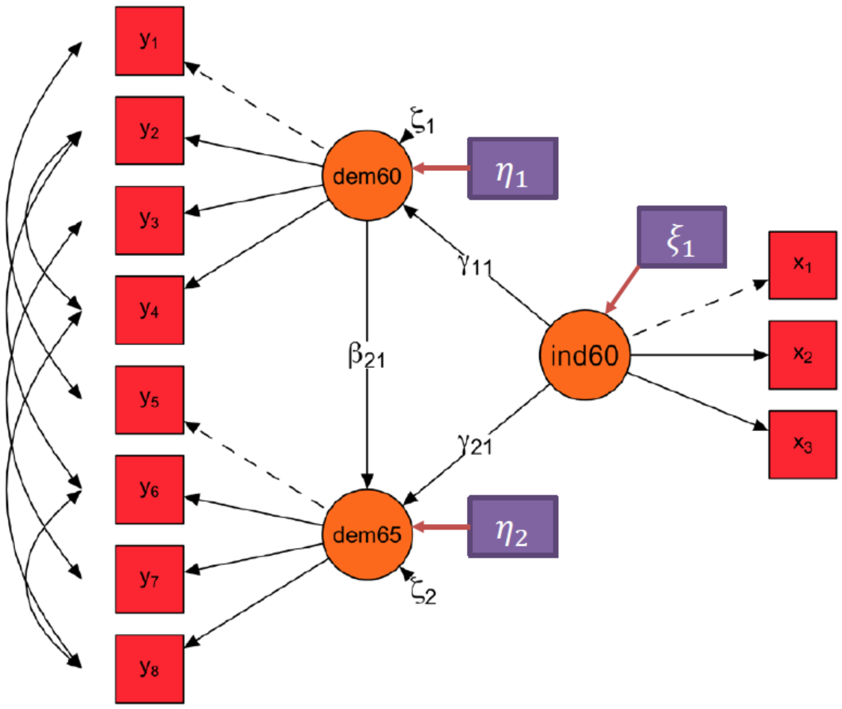
Latent Variable Model

Vector of endogenous latent variables	eta	$\boldsymbol{\eta}$
Matrix of coefficients	beta	$\boldsymbol{\beta}$
Matrix of coefficients of effect of exogenous latent variables on endogenous latent variables	gamma	$\boldsymbol{\Gamma}$
Vector of exogenous latent variables	xi	$\boldsymbol{\xi}$
Vector of errors	zeta	$\boldsymbol{\zeta}$
Covariance Matrix of latent exogenous variables	phi	$\boldsymbol{\Phi}$
Covariance Matrix of equation errors	psi	$\boldsymbol{\Psi}$

Structural Model

$$\boldsymbol{\eta} = \boldsymbol{\beta}\,\boldsymbol{\eta} + \boldsymbol{\Gamma}\,\boldsymbol{\xi} + \boldsymbol{\zeta}$$

$$\begin{bmatrix} dem60 \\ dem65 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \beta_{21} & 0 \end{bmatrix} \begin{bmatrix} dem60 \\ dem65 \end{bmatrix} + \begin{bmatrix} \gamma_{11} \\ \gamma_{21} \end{bmatrix} [ind60] + \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix}$$



Bollen (1989), "A panel model of political democracy and industrialization for developing countries"

Source: Structural Equation Modeling by Bowen and Guo (https://global.oup.com/us/companion.websites/9780195367621/examples/)

Basics of SEM Estimation

SEM, as Analysis of Covariance, explains the observed covariance among a set of measured variables:

It does so by estimating the observed covariance matrix with an estimated covariance matrix constructed based on the estimated relationships among variables.

Data collected externally	Calculated based on user's theory
Observed Covariance Matrix	Estimated Covariance Matrix
$\lvert \mathbf{S} \rvert$	$\lvert \boldsymbol{\Sigma}_k \rvert$

The closer these are, the better the fit, the more accurate the theory, and the better the explanation. When they are equal, the fit is perfect.

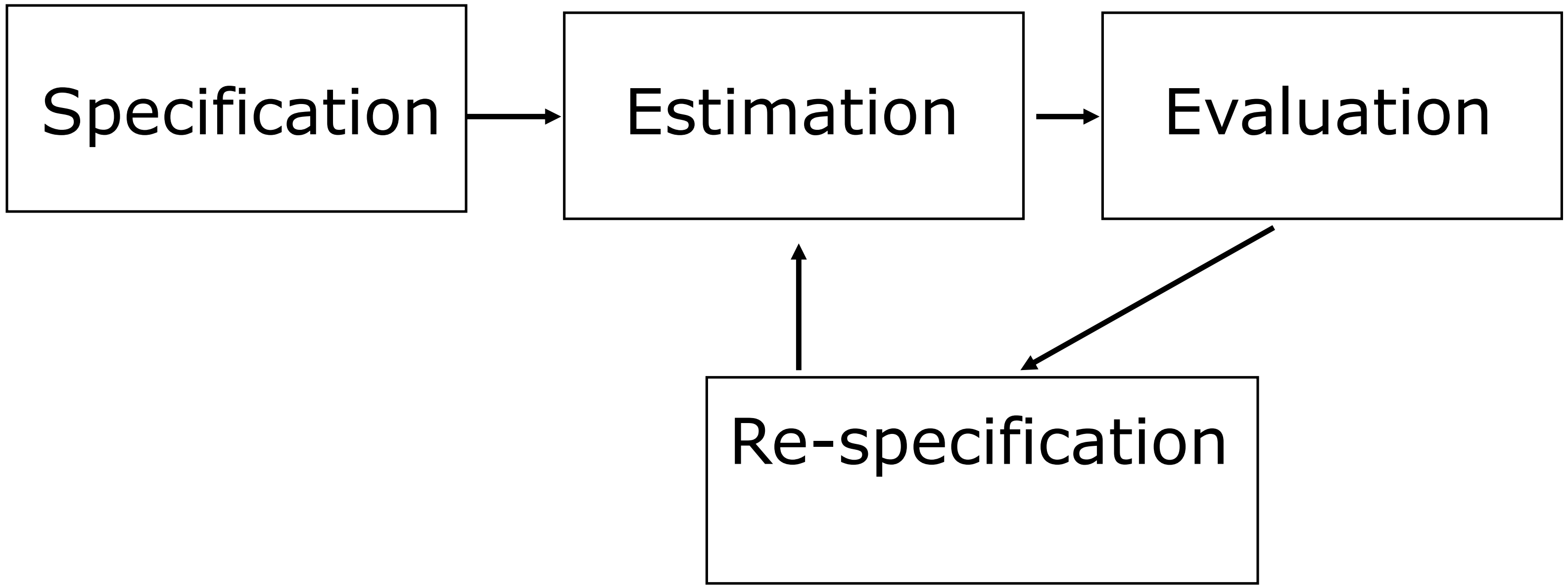
Basics of SEM Estimation

Observed, Estimated, and Residual Covariance Matrices

	Supervision	Work Environment	Coworkers	Job Satisfaction	Job Search
(A) Observed Covariance Matrix: (S)	*Var (SP)*	—	—	—	—
	.20	*Var (WE)*	—	—	—
	.20	.15	*Var (CW)*	—	—
	.20	.30	.50	*Var (JS)*	—
	−.05	.25	.40	.50	*Var(JS)*
(B) Estimated Covariance Matrix: (Σ)	—	—	—	—	—
	.20	—	—	—	—
	.20	.15	—	—	—
	.20	.30	.50	—	—
	.10	.15	.25	.50	—
(C) Residuals: Observed Minus Estimated Covariances					
Supervision	—	—	—	—	—
Work Environment	.00	—	—	—	—
Coworkers	.00	.00	—	—	—
Job Satisfaction	.00	.00	.00	—	—
Job Search	−.15	.10	.15	.00	—

Relatively small residuals signal good fit!

The Process of SEM



The Stages in Conducting SEM

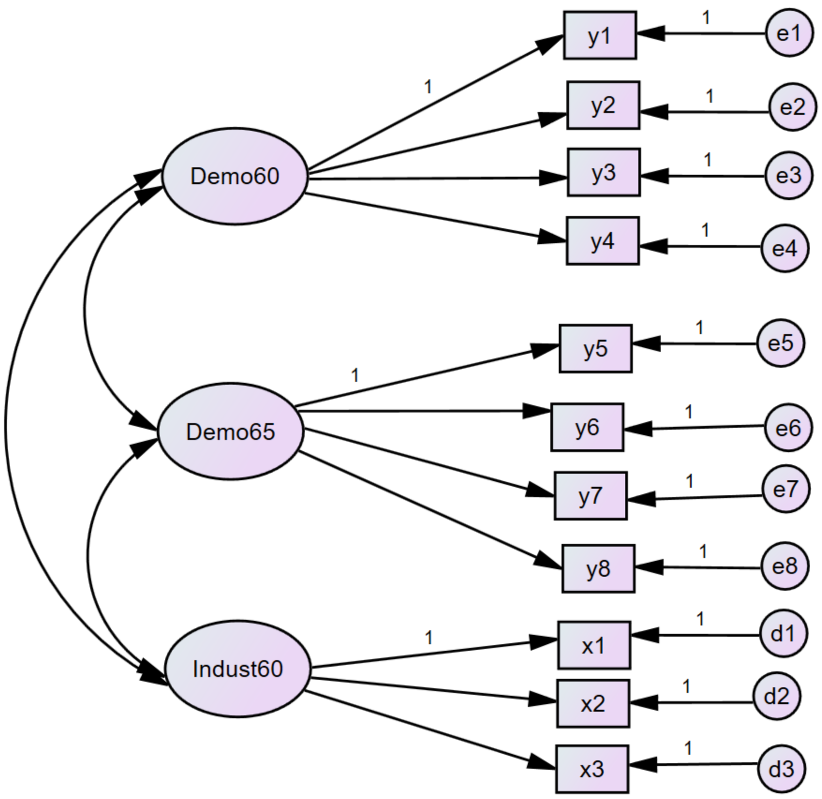
1. Defining Individual Constructs
2. Developing the Overall Measurement Model
3. Designing a Study to Produce Empirical Results
4. Assessing the Measurement Model Validity
5. Specifying the Structural Model
6. Assessing Structural Model Validity

Stage 1 - Stage 4: Input Data– Covariance Matrix

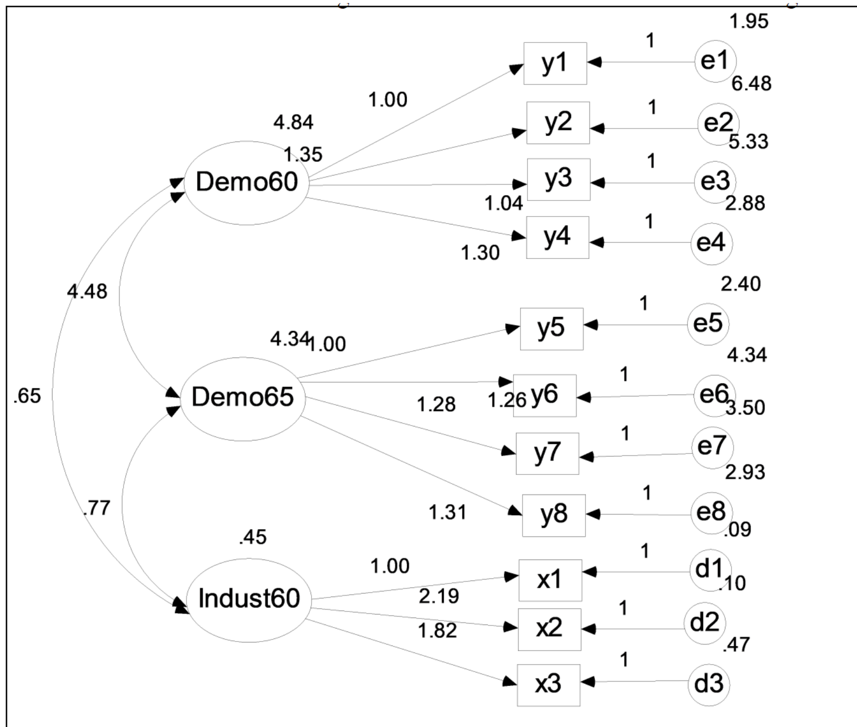
rowtype_	varname_	y1	y2	y3	y4	y5	y6	y7	y8	x1	x2	x3
n		75	75	75	75	75	75	75	75	75	75	75
cov	y1	6.89										
cov	y2	6.25	15.58									
cov	y3	5.84	5.84	10.76								
cov	y4	6.09	9.51	6.69	11.22							
cov	y5	5.06	5.6	4.94	5.7	6.83						
cov	y6	5.75	9.39	4.73	7.44	4.98	11.38					
cov	y7	5.81	7.54	7.01	7.49	5.82	6.75	10.8				
cov	y8	5.67	7.76	5.64	8.01	5.34	8.25	7.59	10.53			
cov	x1	0.73	0.62	0.79	1.15	1.08	0.85	0.94	1.1	0.54		
cov	x2	1.27	1.49	1.55	2.24	2.06	1.81	2	2.23	0.99	2.28	
cov	x3	0.91	1.17	1.04	1.84	1.58	1.57	1.63	1.69	0.82	1.81	1.98

Source: Structural Equation Modeling by Bowen and Guo (https://global.oup.com/us/companion.websites/9780195367621/examples/)

Stage 1 - Stage 4: Measurement Model 1



BM1: CFA 1 of Bollen Mode(1989) p.324

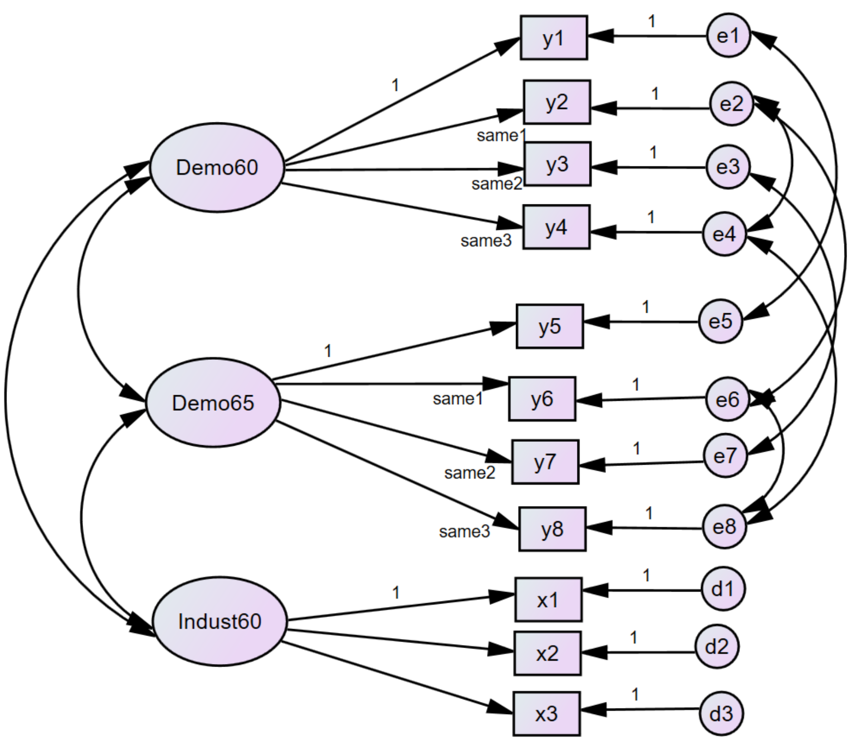


BM1: CFA 1 of Bollen Mode(1989) p.324

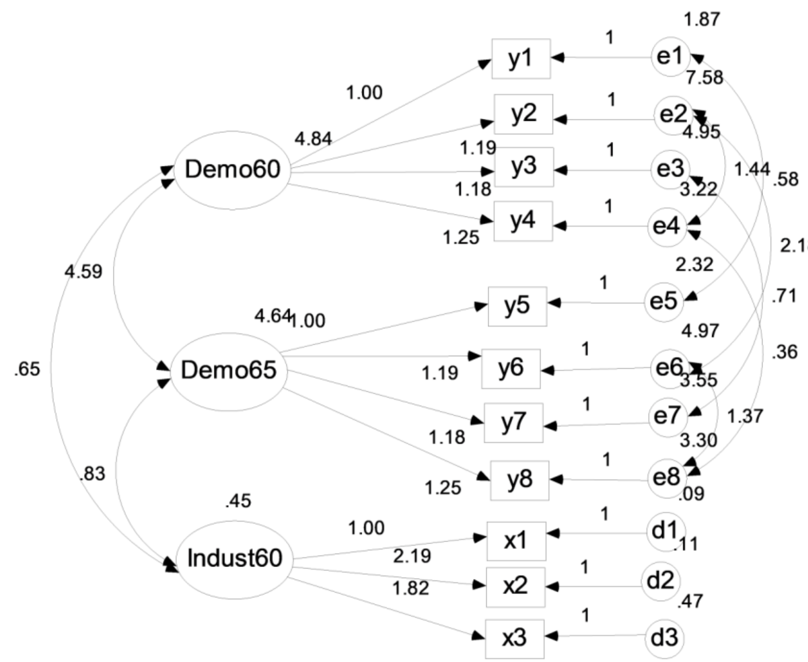
Chi-square = 70.575 (41 df), p=.003
RMSEA=.099
GFI=.856, CFI=.955

Source: Structural Equation Modeling by Bowen and Guo (https://global.oup.com/us/companion.websites/9780195367621/examples/)

Stage 1 - Stage 4: Measurement Model 2



BM2: CFA 2 of Bollen Mode(1989) p.324 - Respedification

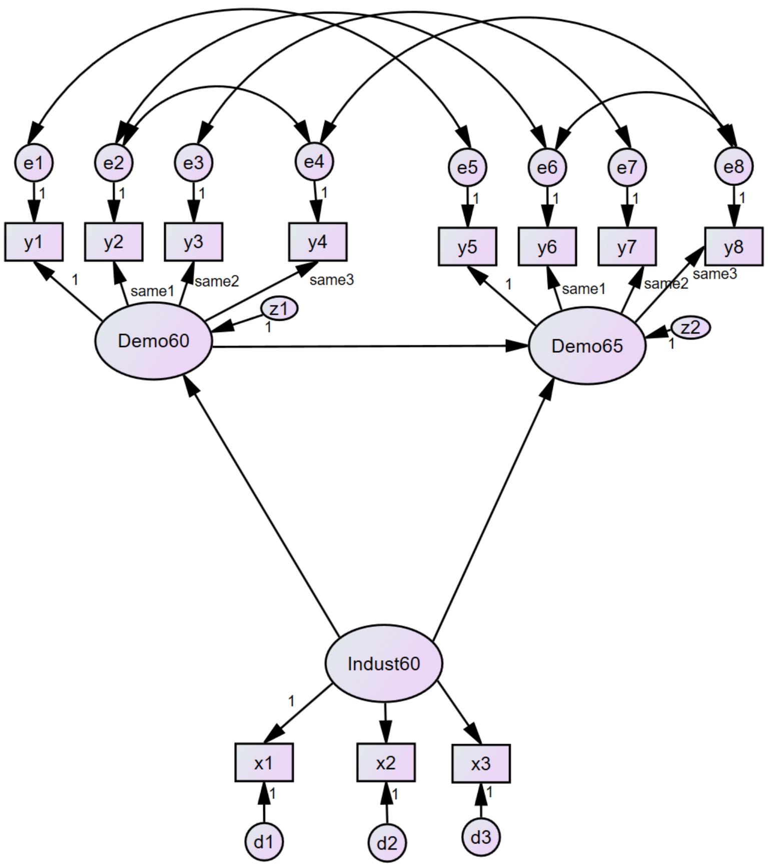


BM2: CFA 2 of Bollen Mode(1989) p.324 - Respedification

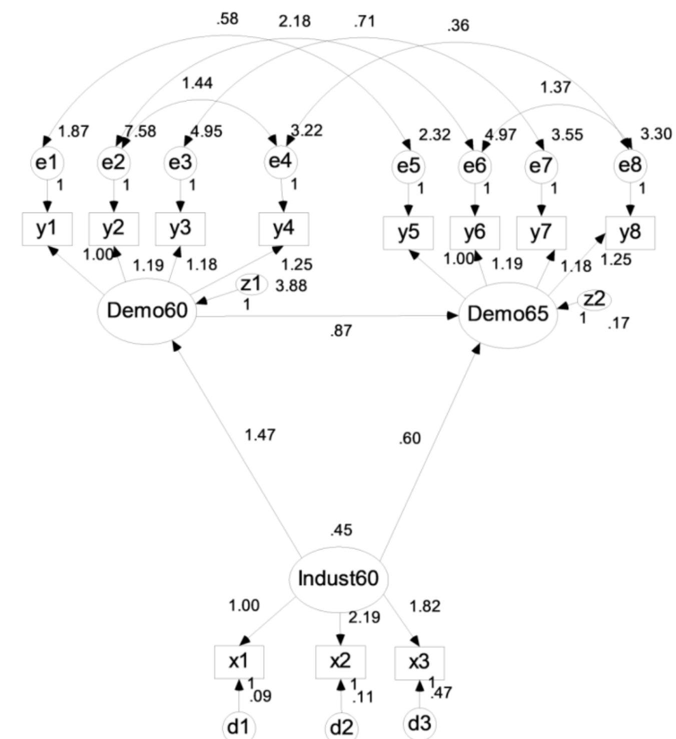
Chi-square = 38.767 (38 df), p=.435
RMSEA=.017
GFI=.921, CFI=.999

Source: Structural Equation Modeling by Bowen and Guo (https://global.oup.com/us/companion.websites/9780195367621/examples/)

Stage 5 – Stage 6: Structural Model



BM3: Structural Regression Model (A Panel Model) Bollen (1989) p.324



BM3: Structural Regression Model (A Panel Model) Bollen (1989) p.324

Chi-square = 38.767 (38 df), p=.435
RMSEA=.017
GFI=.921, CFI=.999

Source: Structural Equation Modeling by Bowen and Guo (https://global.oup.com/us/companion.websites/9780195367621/examples/)

Stage 3

Missing Data Options

- Is the missing data percentage high and nonrandom so as to cause problems in estimation or interpretation?
  - If small and random, then any treatment is adequate.
  - If more than 10% and/or nonrandom, missing data must be remedied.
- What is the best approach?
    - All Available Complete Case or List-wise Deletion
    - or Pairwise Deletion
    - Model-Based (EM)
    - Modeled FIML

Stage 3

Sample Size

- SEM is often thought to require a larger sample relative to other multivariate approaches.
  - Most importantly, the sample size required for any given statistic is a question secondary to the sample size required to generalize from a sample to a population. In almost all instances, the sample size requirement to infer to the population exceeds that for a specific statistic, including any SEM approach.
  - Five considerations affecting the required sample size for SEM include the following: (1) multivariate normality of the data, (2) estimation technique, (3) model complexity, (4) the amount of missing data, and (5) the average error variance among the reflective indicators.
- More observations than variables required as bottom-line.
- With good measurement characteristics, samples approaching 100 produce stable results.

Stage 4 and Stage 6: Model fits

- Absolute fit indices
  - Always report chi-square (χ^2) and degrees of freedom (df)
  - Goodness of fit index (GFI) – illustrates goodness of fit index
  - Root mean squared error of approximation (RMSEA) – illustrates badness of fit index
- Incremental fit indices
  - Relative fit
  - Comparative Fit Index (CFI) – incremental, goodness of fit index, which shows improvement in fit over null model fit
- Parsimony fit indices
  - Parsimony Fit Index (PNFI) – assessing relative fit of models

Stage 4 and Stage 6 : Model fits

Table 9.4 Characteristics of Different Fit Indices Demonstrating Goodness-of-Fit Across Different Model Situations

No. of Stat. vars. (m)	$N < 250$			$N > 250$		
	$m \leq 12$	$12 < m < 30$	$m \geq 30$	$m < 12$	$12 < m < 30$	$m \geq 30$
χ^2	Insignificant *p*-values expected	Significant *p*-values even with good fit	Significant *p*-values expected	Insignificant *p*-values even with good fit	Significant *p*-values expected	Significant *p*-values expected
CFI or TLI	.99 or better	.97 or better	Above .93	.96 or better	Above .94	Above .92
RNI	May not diagnose misspecification well	.97 or better	Above .93	.96 or better, not used with $N > 1{,}000$	Above .94, not used with $N > 1{,}000$	Above .92, not used with $N > 1{,}000$
SRMR	Biased upward, use other indices	.08 or less (with CFI of .95 or higher)	Less than .09 (with CFI above .93)	Biased upward; use other indices	.08 or less (with CFI above .94)	.08 or less (with CFI above .92)
RMSEA	Values < .08 with CFI of = .99 or higher	Values < .08 with CFI of .97 or higher	Values < .08 with CFI above .93	Values < .07 with CFI of .96 or higher	Values < .07 with CFI of .94 or higher	Values < .07 with CFI of .92 or higher

Note: m = number of observed variables; N applies to number of observations per group when applying CFA to multiple groups at the same time.

Stage 4 and Stage 6 : Model fits

There's different types of fit statistic, but there are recommended cut off points

Fit statistic	Cut off point
Chi-square	Non-significant
TLI	.95 or above
CFI	.95 or above
SRMR	.08 or below
AIC, BIC, SSABIC	Lowest value
RMSEA (90% CI)	**.05 close fit** **Up to .08 reasonable**

Stage 4 and Stage 6 : Model fits

Table 1: The three categories of model fit and their level of acceptance

Name of category	Name of index	Level of acceptance
1. Absolute fit	Chi-Square	P-value > 0.05
	RMSEA	RMSEA < 0.08
	GFI	GFI > 0.90
2. Incremental fit	AGFI	AGFI > 0.90
	CFI	CFI > 0.90
	TLI	TLI > 0.90
	NFI	NFI > 0.90
3. Parsimonious fit	**Chisq/df**	Chi-Suare/ df < 3.0

***The indexes in bold are recommended since they are frequently reported in literatures

Source: A Handbook on SEM 2nd Edition by Zainudin Awang

Stage 4 and Stage 6 : Model fits

Table 2: The literature support for the respective fitness index

Name of category	Name of index	Index full name	Literature
1. Absolute fit	Chi-Square	Discrepancy Chi Square	Wheaton et al. (1977)
	RMSEA	Root Mean Square of Error Approximation	Browne and Cudeck (1993)
	GFI	Goodness of Fit Index	Joreskog and Sorbom (1984)
2. Incremental fit	AGFI	Adjusted Goodness of Fit	Tanaka and Huba (1985)
	CFI	Comparative Fit Index	Bentler (1990)
	TLI	Tucker-Lewis Index	Bentler and Bonett (1980)
	NFI	Normed Fit Index	Bollen (1989b)
3. Parsimonious fit	Chisq/df	Chi Square/Degrees of Freedom	Marsh and Hocevar (1985)

*** One could ignore the absolute fit index of minimum discrepancy chi-square if the sample size obtained for the study is greater than 200 (Hair et al., 1996; Joreskog and Sorbom, 1996).

Source: A Handbook on SEM 2nd Edition by Zainudin Awang

Stage 4 and Stage 6 : Model fits

Measure	Name	Description	Cut-off for good fit
X^2	Model Chi-Square	Assess overall fit and the discrepancy between the sample and fitted covariance matrices. Sensitive to sample size. H_0: The model fits perfectly.	p-value> 0.05
(A)GFI	(Adjusted) Goodness of Fit	GFI is the proportion of variance accounted for by the estimated population covariance. Analogous to R^2. AGFI favors parsimony.	GFI ≥ 0.95 AGFI ≥0.90
(N)NFI TLI	(Non) Normed-Fit Index Tucker Lewis index	An NFI of .95, indicates the model of interest improves the fit by 95% relative to the null model. NNFI is preferable for smaller samples. Sometimes the NNFI is called the Tucker Lewis index (TLI)	NFI ≥ 0.95 NNFI ≥ 0.95
CFI	Comparative Fit Index	A revised form of NFI. Not very sensitive to sample size. Compares the fit of a target model to the fit of an independent, or null, model.	CFI ≥.90
RMSEA	Root Mean Square Error of Approximation	A parsimony-adjusted index. Values closer to 0 represent a good fit.	RMSEA < 0.08
(S)RMR	(Standardized) Root Mean Square Residual	The square-root of the difference between the residuals of the sample covariance matrix and the hypothesized model. If items vary in range (i.e. some items are 1-5, others 1-7) then RMR is hard to interpret, better to use SRMR.	SRMR <0.08
AVE (CFA only)	Average Value Explained	The average of the R^2s for items within a factor	AVE >.5

Source: https://www.cscu.cornell.edu/news/Handouts/SEM_fit.pdf

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Aadvantages of SEM

- Confirmatory approach: test your theory
- Goodness-of-fit measures
- Flexible statistical modeling approach
  - Missing data
  - (in)equality constraints
  - Allows for indirect effects (mediation), reciprocal effects
  - Categorical data
  - Discrete and continuous latent variables
  - Longitudinal analysis
  - Multilevel data
- Many other approaches turn out to be special cases

Main advantages of SEM

1. Flexibility
2. Latent variable modelling
3. Dealing with error
4. Testing models and theory

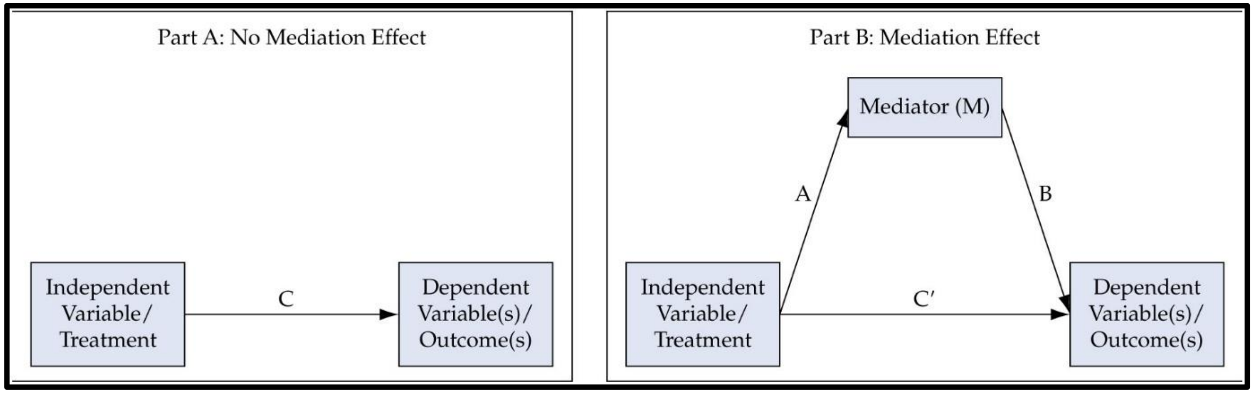
1. Flexibility

- SEMs allow the researcher to test complex hypotheses that may include
  - Direct and Indirect effects (Mediation)
  - Interactions (Moderation)
  - Reciprocal relations
- Testing relationships between one or more IV's and one or more DV's
  - Variables can be observed or latent

1. Flexibility: Mediation

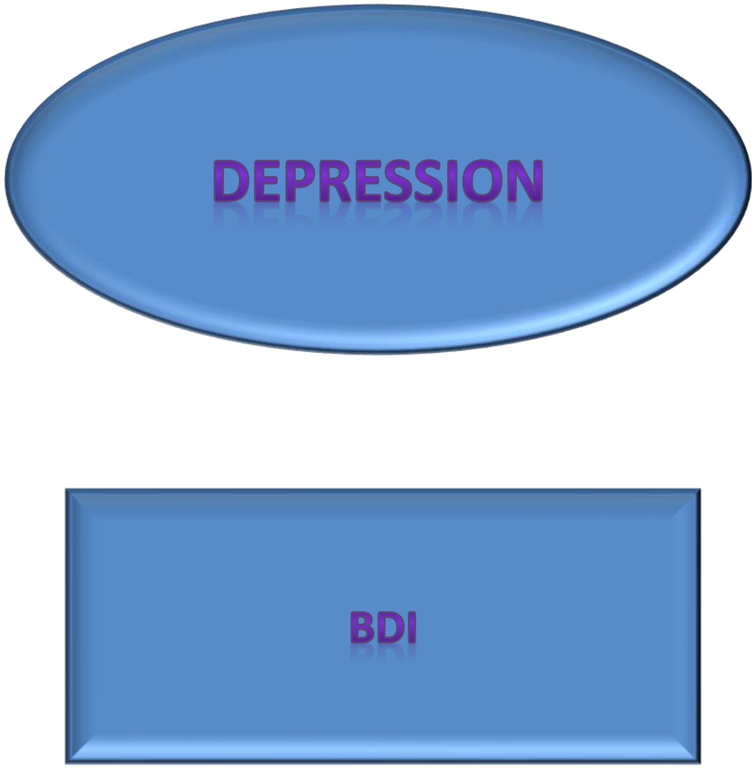
Involves two additional relationships (A and B) which create the indirect effect (A x B)

- Introducing this indirect effect into the analysis allows for an "alternative" effect to supplant the direct effect (C).
- The result is **two** types of mediation:
  - Full (Complete) mediation -- C' becomes non-significant
  - Partial mediation – C' is still significant while the indirect effect is also significant



- Statistical significance of the indirect effect tested by *Sobel test* or *bootstrapping*

2. Latent Variables



Unobservable

Observable

Beck's Depression Inventory
https://www.ismanet.org/doctoryourspirit/pdfs/Beck-Depression-Inventory-BDI.pdf

2. Latent Variables

Beck's Depression Inventory

This depression inventory can be self-scored. The scoring scale is at the end of the questionnaire.

1.
- 0 I do not feel sad.
- 1 I feel sad
- 2 I am sad all the time and I can't snap out of it.
- 3 I am so sad and unhappy that I can't stand it.

2.
- 0 I am not particularly discouraged about the future.
- 1 I feel discouraged about the future.
- 2 I feel I have nothing to look forward to.
- 3 I feel the future is hopeless and that things cannot improve.

3.
- 0 I do not feel like a failure.
- 1 I feel I have failed more than the average person.
- 2 As I look back on my life, all I can see is a lot of failures.
- 3 I feel I am a complete failure as a person.

4.
- 0 I get as much satisfaction out of things as I used to.
- 1 I don't enjoy things the way I used to.
- 2 I don't get real satisfaction out of anything anymore.
- 3 I am dissatisfied or bored with everything.

5.
- 0 I don't feel particularly guilty
- 1 I feel guilty a good part of the time.
- 2 I feel quite guilty most of the time.
- 3 I feel guilty all of the time.

6.
- 0 I don't feel I am being punished.
- 1 I feel I may be punished.
- 2 I expect to be punished.
- 3 I feel I am being punished.

7.
- 0 I don't feel disappointed in myself.
- 1 I am disappointed in myself.
- 2 I am disgusted with myself.
- 3 I hate myself.

8.
- 0 I don't feel I am any worse than anybody else.
- 1 I am critical of myself for my weaknesses or mistakes.
- 2 I blame myself all the time for my faults.
- 3 I blame myself for everything bad that happens.

9.
- 0 I don't have any thoughts of killing myself.
- 1 I have thoughts of killing myself, but I would not carry them out.
- 2 I would like to kill myself.
- 3 I would kill myself if I had the chance.

10.
- 0 I don't cry any more than usual.
- 1 I cry more now than I used to.
- 2 I cry all the time now.
- 3 I used to be able to cry, but now I can't cry even though I want to.

20.
- 0 I am no more worried about my health than usual.
- 1 I am worried about physical problems like aches, pains, upset stomach, or constipation.
- 2 I am very worried about physical problems and it's hard to think of much else.
- 3 I am so worried about my physical problems that I cannot think of anything else.

21.
- 0 I have not noticed any recent change in my interest in sex.
- 1 I am less interested in sex than I used to be.
- 2 I have almost no interest in sex.
- 3 I have lost interest in sex completely.

INTERPRETING THE BECK DEPRESSION INVENTORY

Now that you have completed the questionnaire, add up the score for each of the twenty-one questions by counting the number to the right of each question you marked. The highest possible total for the whole test would be sixty-three. This would mean you circled number three on all twenty-one questions. Since the lowest possible score for each question is zero, the lowest possible score for the test would be zero. This would mean you circles zero on each question. You can evaluate your depression according to the Table below.

Total Score	Levels of Depression
1-10	These ups and downs are considered normal
11-16	Mild mood disturbance
17-20	Borderline clinical depression
21-30	Moderate depression
31-40	Severe depression
over 40	Extreme depression

http://www.med.navy.mil/sites/NMCP2/PatientServices/SleepClinicLab/Documents/Beck_Depression_Inventory.pdf

2. Latent Variables

- Latent variables are non-observed
- In the case before we had depression
  - We can't observe depression
  - But we know a little about the symptoms of depression which might indicate someone's overall depression
- Could be lots of other things, verbal IQ, alcohol or drug consumption, etc.
- Two ways of bagging yourself a latent variable
  - Exploratory factor analysis
  - Confirmatory Factor Analysis

2. Latent Variables
EFA / CFA

- Exploratory factor analysis (EFA)
  - Summarizing data by grouping correlated variables
  - Investigating sets of measured variables for underlying constructs (data reduction)

- Confirmatory Factor Analysis (CFA: typical of SEM)
  - Tests theoretical relationships between variables when factor structure is known or theorized
  - Use this first if there is a known structure in the questionnaire

2. Latent Variables

- The true power of SEM comes from *latent variable modelling*

- Variables in psychology are rarely (never?) measured directly
  - Different to other sciences
  - Weight, height, temperature are measured with little error (there is little difference between measurement and construct of interest)
  - However, depression, anxiety, etc can only be measured in how it is manifested e.g. loss of sleep, appetite, low mood, etc

2. Latent Variables

Why Latent Variable?

- Unidimensional scale construction
  - If you sum all the items in a factor, you need to know that all those variables are measuring the same underlying construct
- Reliability
  - Correction for error attenuation

3. Dealing with error

- Probably the most useful aspect of SEM is the ability to correct for measurement error.
  - Structural equation models estimate the relationship between the latent, not the observed variables, thereby correcting for the effects of measurement error.
- McNemar (1956) noted that "...all measurement is befuddled with error".
- This is particularly relevant to social scientists who are not generally interested in manifest, tangible variables but unobservable, or latent, variables such as anxiety, stress or depression.

4. Model Testing

- The final advantage of the structural equation model is the ability to test a model, as an explanation of the underlying processes which have given rise to the data.
- A model is a simple mathematical representation of a particular psychological structure (eg., IQ) or process.
- ANOVA and exploratory factor analysis are generally not guided by substantive theory, whereas SEM is driven by theory (Hoyle, 1995)
- If a proposed model can explain the observed patterns of data this represents evidence that the model is an acceptable description of the proposed structure or process.
- SEM allows a statistical test of goodness-of-fit.

Disadvantages of SEM

- The modeling flexibility can be overwhelming
- You need dedicated software (LISREL, AMOS, Mplus, STATA)
  - Now “Lavaan” in ***R*** *is* usable!
  - “semopy” package in ***Python***
- ‘Specifying’ your model (using software) can be challenging
  - AMOS is relatively easy
- Challenges for SEM as a statistical field:
  - Better inference for small samples
  - Outlier-robust methods are not part of the standard SEM toolbox

SEM limitations

- SEM is a confirmatory approach
  - What is the established theory about the relationships
  - Exploratory methods (e.g. model modification) can be used on top of the original theory- but be careful
  - SEM itself is not causal; experimental design = cause
- SEM typically correlational but can be used with experimental data
  - Mediation and manipulation can be tested

SEM limitations

- SEM is a super cool technique but it does not make up for a bad methods
- More complex does not always mean better
- Biggest limitation is sample size
  - It needs to be large to get stable estimates of the covariances (n>200 subjects for small to medium sized model)
  - A minimum of 10 subjects per estimated parameter