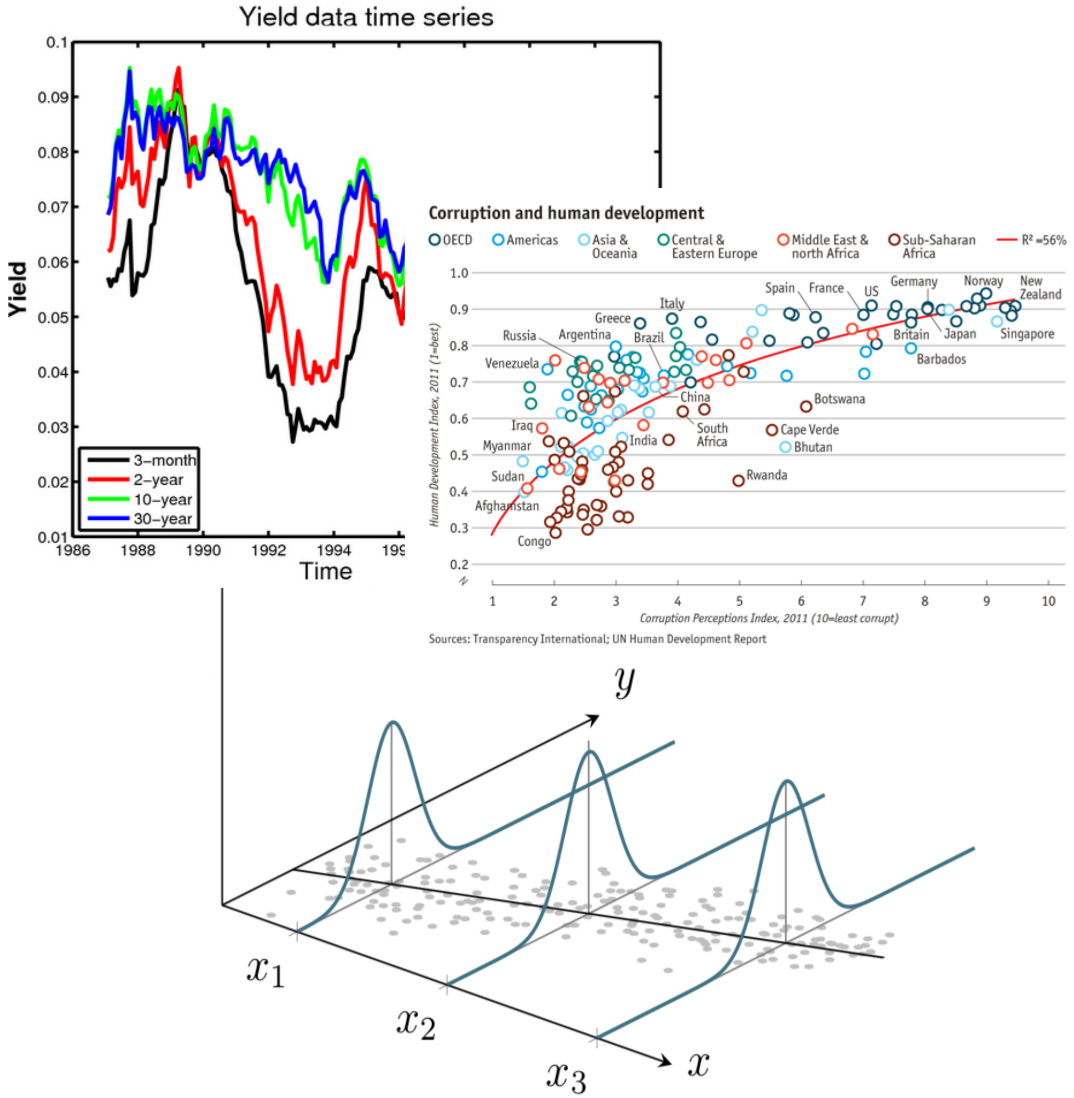
Chapter 14

Advanced Panel Data Methods



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- **Fixed effects estimation**

Fixed effect, potentially correlated with explanatory variables

$$y_{it} = \beta_1 x_{it1} + \ldots + \beta_k x_{itk} + a_i + u_{it}, \; i = 1, \ldots, N, t = 1, \ldots, T$$

$$\bar{y}_i = \beta_1 \bar{x}_{i1} + \ldots + \beta_k \bar{x}_{ik} + \bar{a}_i + \bar{u}_i$$

Form time-averages for each individual

$$\Rightarrow \; [y_{it} - \bar{y}_i] = \beta_1 [x_{it1} - \bar{x}_{i1}] + \ldots + \beta_k [x_{itk} - \bar{x}_{ik}] + [u_{it} - \bar{u}_i]$$

Because $a_i - \bar{a}_i = 0$ (the fixed effect is removed)

- Estimate time-demeaned equation by OLS
  - Uses time variation within cross-sectional units (= within estimator)

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- **Example: Effect of training grants on firm scrap rate**

$$scrap_{it} = \beta_1 d88_{it} + \beta_2 d89_{it} + \beta_3 grant_{it} + \beta_4 grant_{it-1} + a_i + u_{it}$$

Time-invariant reasons why one firm is more productive than another are controlled for. The important point is that these may be correlated with the other explanatory variables.

- Fixed-effects estimation using the years 1987, 1988, and 1989:

$$\widehat{scrap}^*_{it} = -\underset{(.109)}{.080}\ d88^*_{it} - \underset{(.133)}{.247}\ d89^*_{it} - \underset{(.151)}{.252}\ grant^*_{it} - \underset{(.210)}{.422}\ grant^*_{it-1}$$

Stars denote time-demeaning

$$n = 162, R^2 = .201$$

Training grants significantly improve productivity (with a time lag)

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- **Discussion of fixed effects estimator**
  - Strict exogeneity in the original model has to be assumed.
  - The R-squared of the demeaned equation is inappropriate.
  - The effect of time-invariant variables cannot be estimated.
  - The effect of interactions with time-invariant variables can be estimated (e.g. the interaction of education with time dummies).
  - If a full set of time dummies are included, the effect of variables whose change over time is constant cannot be estimated (e.g. experience).
  - Degrees of freedom have to be adjusted because the N time averages are estimated in addition (resulting degrees of freedom = NT-N-k).

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- **Interpretation of fixed effects as dummy variable regression**
  - The fixed effects estimator is equivalent to introducing a dummy for each individual in the original regression and using pooled OLS:

$$y_{it} = a_1 ind1_{it} + a_2 ind2_{it} + \ldots + a_N indN_{it} + \beta_1 x_{it1} + \ldots + \beta_k x_{itk} + u_{it}$$

For example, = 1 if the observation stems from individual N, = 0 otherwise

- After fixed effects estimation, the fixed effects can be estimated as:

$$\hat{a}_i = \bar{y}_i - \hat{\beta}_1 \bar{x}_{i1} - \ldots - \hat{\beta}_k \bar{x}_{ik}, \; i = 1, \ldots, N$$

Estimated individual effect for individual i

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- **Fixed effects or first differencing?**
  - Remember that first differencing can also be used if $T > 2$.
  - In the case $T = 2$, fixed effects and first differencing are identical.
  - For $T > 2$, fixed effects is more efficient if classical assumptions hold.
  - First differencing may be better in the case of severe serial correlation in the errors, for example if the errors follow a random walk.
  - If T is very large (and N not so large), the panel has a pronounced time series character and problems such as strong dependence arise.
  - In these cases, it is probably better to use first differencing.
  - Otherwise, it is a good idea to compute both and check robustness.

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- **Unbalanced panels**
  - An unbalanced panel is when not all cross-sectional units have the same number of observations.
    - Dropping units with only one time period does not cause bias or inconsistency.

- Fixed effects (FE) or First Differencing (FD) with unbalanced panels
  - FE will preserve more data than FD when we have unbalanced panels, since FD requires that each observation have data available for both t and t-1.
  - For example, consider a scenario in which we have seven years of data, but data is missing for all even numbered years. Thus, we observe t=1,3,5,7.
    - FE will use time periods 1,3,5,7
    - FD will lose all observations.

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- **Random effects (RE) models**

$$y_{it} = \beta_0 + \beta_1 x_{it1} + \cdots + \beta_k x_{itk} + a_i + u_{it}$$

The individual effect is assumed to be "random" i.e. completely unrelated to explanatory variables

<u>Random effects assumption</u>: $Cov(x_{itj}, a_i) = 0, j = 1,2, \dots, k$

- The composite error $a_i + u_{it}$ is uncorrelated with the explanatory variables but it is serially correlated for observations coming from the same i:

$$Cov(a_i + u_{it}, a_i + u_{is}) = Cov(a_i, a_i) = \sigma_a^2$$

Under the assumption that idiosyncratic errors are serially uncorrelated

For example, in a wage equation, for a given individual the same unobserved ability appears in the error term of each period. Error terms are thus correlated across periods for this individual.

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- **Estimation in the random effects model**
  - Under the random effects assumptions explanatory variables are exogenous so that pooled OLS provides consistent estimates.
  - If OLS is used, standard errors have to be adjusted for the fact that errors are correlated over time for given i (= clustered standard errors).
  - But, because of the serial correlation, OLS is not efficient.
  - One can transform the model so that it satisfies the GM-assumptions:

$$[y_{it} - \lambda \bar{y}_i] = \beta_1 [x_{it1} - \lambda \bar{x}_{i1}] + \ldots + \beta_k [x_{itk} - \lambda \bar{x}_{ik}]$$ ← Quasi-demeaned data

$$+ [a_i - \lambda \bar{a}_i + u_{it} - \lambda \bar{u}_i]$$ ← Error can be shown to satisfy GM-assumptions

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- **Estimation in the random effects model (cont.)**

$$\lambda = 1 - \left[\sigma_u^2/(\sigma_u^2 + T\sigma_a^2)\right]^{1/2}, \ 0 \leq \lambda \leq 1$$

  - The quasi-demeaning parameter is unknown but it can be estimated.
  - FGLS using the estimated λ is called random effects estimation.
  - If the random effect is relatively unimportant compared to the idosyncratic error, FGLS will be close to pooled OLS (because λ goes to 0).
  - If the random effect is relatively important compared to the idiosyn-cratic term, FGLS will be similar to fixed effects (because λ goes to 1).
  - Random effects estimation can be used to estimate the effect of time-invariant variables.

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- **Example: Wage equation using panel data**

$$\widehat{\log}(wage_{it}) = \underset{(.011)}{.092}\ educ_{it} - \underset{(.048)}{.139}\ black_{it} + \underset{(.043)}{.022}\ hispan_{it}$$

$$+ \underset{(.015)}{.106}\ exper_{it} - \underset{(.0007)}{.0047}\ exper_{it}^2 + \underset{(.017)}{.064}\ married_{it}$$

$$+ \underset{(.018)}{.106}\ union_{it} + time\ dummies$$

Random effects is used because many of the variables are time-invariant. But is the random effects assumption realistic?

- **Random effects or fixed effects?**
  - In economics, unobserved individual effects are seldomly uncorrelated with explanatory variables so that fixed effects is more convincing.

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- **Correlated Random Effects (CRE)**
  - When using CRE to choose between FE and RE, we must include any time-constant variables that appear in RE estimation:

$$y_{it} = \alpha_1 + \alpha_2 d2_t + \cdots + \alpha_T dT_t + \beta_1 x_{it1} + \cdots + \beta_k x_{itk}$$
$$+\gamma_1 \bar{x}_{i1} + \cdots + \gamma_k \bar{x}_{ik} + \delta_1 z_{i1} + \cdots + \delta_m z_{im} + r_i + u_{it}$$

- Estimating this equation by RE (or even just pooled OLS) yields:

$$\hat{\beta}_{CRE,j} = \hat{\beta}_{FE,j};\ j = 1, \dots, k \longleftarrow$$ Time varying estimates will be the same as in FE
$$\hat{\alpha}_{CRE,t} = \hat{\alpha}_{FE,t};\ t = 1, \dots, T$$

$$H_0: \gamma_1 = \gamma_2 = \cdots = \gamma_k = 0 \longleftarrow$$ Under the null, RE is sufficient. If we reject the null, then FE is preferred.

  - An advantage of CRE is that it allows for estimation of the effects of time-constant explanatory variables, not possible using FE.

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- **General policy analysis with panel data**
  - The two-period, before-after setting is a special case of a more general policy analysis framework when T ≥ 2.

$$y_{it} = \eta_1 + \alpha_2 d2_t + \cdots + \alpha_T dT_t + \beta w_{it} + \boldsymbol{x_{it}\psi} + a_i + u_{it}$$

w_{it} is the binary policy variable and β estimates the average treatment effect of the policy

  - To allow w_{it} to be systematically related to the unobserved fixed effect a_i, we estimate the regression with either FD or FE, using cluster-robust standard errors.
  - We can also include lags of the policy intervention: w_{it-1}, w_{it-2},…

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- **Testing for feedback from the error term to the policy variable**
- We need to be careful if the policy variable w_{it} it reacts to past shocks.
  - Example: y_{it} is the poverty rate and w_{it} is some measure of government assistance.
  - A large shock to the poverty rate in year t could prompt an increase in government assistance the following year.

- If we have at least three time periods, we can test for feedback

$$y_{it} = \eta_1 + \alpha_2 d2_t + \cdots + \alpha_{T-1} dT - 1_t + \beta w_{it} + \delta w_{it+1} + \boldsymbol{x_{it}\psi} + a_i + u_{it}$$

Estimate with FE and compute a cluster robust t-statistic for $\hat{\delta}$

  - This is known as a “falsification test.”
    - If the forward policy variable is statistically significant, there is potential feedback from the error term to the policy variable.

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- **The heterogeneous trend model**
- What if time trends are unique across individuals?

$$y_{it} = \eta_1 + \alpha_2 d2_t + \cdots + \alpha_T dT_t + \beta w_{it} + \boldsymbol{x}_{it}\boldsymbol{\psi} + a_i + g_i t + u_{it}$$

The new term $g_i t$ is a unit-specific time trend.

- This allows the policy intervention to not only be correlated with level differences among units (captured by a_i), but also by trend differences.
- We can estimate this model by taking first differences:

$$\Delta y_{it} = \alpha_2 \Delta d2_t + \cdots + \alpha_T \Delta dT_t + \beta \Delta w_{it} + \boldsymbol{\Delta x}_{it}\boldsymbol{\psi} + g_i + \Delta u_{it}$$

Estimate by FE., though we need to ensure we have T ≥ 3

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- **Applying panel data methods to other data structures**
  - Panel data methods can be used in other contexts where constant unobserved effects have to be removed.
- Example: Wage equations for twins

Unobserved genetic and family characteristics that do not vary across twins

$$\log(wage_{i1}) = \beta_0 + \beta_1 educ_{i1} + \ldots + a_i + u_{i1}$$

Equation for twin 1 in family i

$$\log(wage_{i2}) = \beta_0 + \beta_1 educ_{i2} + \ldots + a_i + u_{i2}$$

Equation for twin 2 in family i

$$\Rightarrow \quad \Delta \log(wage_i) = \beta_1 \Delta educ_i + \ldots + \Delta u_i$$

Estimate differenced equation by OLS