**Answers to Small Project 3**

1.

(i) Holding other factors fixed,

$$\Delta voteA = \beta_1 \Delta \log(expendA) = (\beta_1/100)[100 \cdot \Delta \log(expendA)]$$
$$\approx (\beta_1/100)(\%\Delta expendA),$$

where we use the fact that $100 \cdot \Delta \log(expendA) \approx \%\Delta expendA$. So $\beta_1/100$ is the (ceteris paribus) percentage point change in *voteA* when *expendA* increases by one percent.

(ii) The null hypothesis is $H_0$: $\beta_2 = -\beta_1$, which means a z% increase in expenditure by A and a z% increase in expenditure by B leaves *voteA* unchanged. We can equivalently write $H_0$: $\beta_1 + \beta_2 = 0$.

(iii) The estimated equation (with standard errors in parentheses below estimates) is

$$\underset{}{\widehat{voteA}} = \underset{(3.93)}{45.08} + \underset{(0.382)}{6.083} \log(expendA) - \underset{(0.379)}{6.615} \log(expendB) + \underset{(0.062)}{.152}\, prtystrA$$

$$n = 173, \quad R^2 = 0.793.$$

The coefficient on log(*expendA*) is very significant ($t$ statistic $\approx$ 15.92), as is the coefficient on log(*expendB*) ($t$ statistic $\approx$ –17.45). The estimates imply that a 10% ceteris paribus increase in spending by candidate A increases the predicted share of the vote going to A by about 0.61 percentage points. [Recall that, holding other factors fixed, $\Delta\widehat{voteA} \approx$ (6.083/100)%Δ*expendA*).]
Similarly, a 10% ceteris paribus increase in spending by B reduces $\Delta\widehat{voteA}$ by about 0.66 percentage points. These effects certainly cannot be ignored.

While the coefficients on log(*expendA*) and log(*expendB*) are of similar magnitudes (and opposite in sign, as we expect), we do not have the standard error of $\hat{\beta}_1 + \hat{\beta}_2$, which is what we would need to test the hypothesis from part (ii).

(iv) Write $\theta_1 = \beta_1 + \beta_2$, or $\beta_1 = \theta_1 - \beta_2$. Plugging this into the original equation, and rearranging, gives

$$\widehat{voteA} = \beta_0 + \theta_1 \log(expendA) + \beta_2[\log(expendB) - \log(expendA)] + \beta_3 prtystrA + u,$$

When we estimate this equation, we obtain $\hat{\theta}_1 \approx -0.532$ and se($\hat{\theta}_1$) $\approx 0.533$. The $t$ statistic for the hypothesis in part (ii) is $-0.532/0.533 \approx -1$. Therefore, we fail to reject $H_0$: $\beta_2 = -\beta_1$.

2.

(i) Holding *exper* (and the elements in $u$) fixed, we have

$$\Delta\log(wage) = \beta_1\Delta educ + \beta_3(\Delta educ)exper = (\beta_1 + \beta_3 exper)\Delta educ,$$

or

$$\frac{\Delta\log(wage)}{\Delta educ} = (\beta_1 + \beta_3 exper).$$

This is the approximate proportionate change in *wage* given one more year of education.

(ii) $H_0$: $\beta_3 = 0$. If we think that education and experience interact positively – so that people with more experience are more productive when given another year of education – then $\beta_3 >$ 0 is the appropriate alternative.

(iii) The estimated equation is

$$\begin{aligned}\log\widehat{(wage)} = \underset{(0.24)}{5.95} + \underset{(.0174)}{.0440}\, educ - \underset{(.0200)}{.0215}\, exper + \underset{(.00153)}{.00320}\, educ \cdot exper\end{aligned}$$

$$n = 935, \quad R^2 = 0.135, \quad \bar{R}^2 = 0.132.$$

The $t$ statistic on the interaction term is about 2.09, which gives a $p$-value below 0.04 against $H_1$: $\beta_3 > 0$. Therefore, we reject $H_0$: $\beta_3 = 0$ against $H_1$: $\beta_3 > 0$ at the 4% level.

(iv) We rewrite the equation as

$$\log(wage) = \beta_0 + \theta_1 educ + \beta_2 exper + \beta_3 educ(exper - 10) + u$$

and run the regression log(*wage*) on *educ*, *exper*, and *educ*(*exper* – 10). We want the coefficient on *educ*. We obtain $\hat{\theta}_1 \approx .0761$ and $se(\hat{\theta}_1) \approx .0066$. The 95% CI for $\theta_1$ is about 0.063 to 0.089.

3.

(i) The estimated equation is

$$
\begin{aligned}
\widehat{\log(wage)} = {} & \underset{(0.11)}{5.40} + \underset{(.0063)}{.0654}\, educ + \underset{(.0032)}{.0140}\, exper + \underset{(.0025)}{.0117}\, tenure \\
& + \underset{(.039)}{.199}\, married - \underset{(.038)}{.188}\, black - \underset{(.026)}{.091}\, south + \underset{(.027)}{.184}\, urban
\end{aligned}
$$

$n = 935,\ R^2 = 0.253.$

The coefficient on *black* implies that, at given levels of the other explanatory variables, black men earn about 18.8% less than nonblack men. The $t$ statistic is about –4.95, and so it is very statistically significant.

(ii) The $F$ statistic for joint significance of $exper^2$ and $tenure^2$, with 2 and 925 $df$, is about 1.49 with $p$-value $\approx$ 0.226. Because the $p$-value is above 0.20, these quadratics are jointly insignificant at the 20% level.

(iii) We add the interaction $black \cdot educ$ to the equation in part (i). The coefficient on the interaction is about −0.0226 (se $\approx$ 0.0202). Therefore, the point estimate is that the return to another year of education is about 2.3 percentage points lower for black men than nonblack men. (The estimated return for nonblack men is about 6.7%.) This is nontrivial if it really reflects differences in the population. But the $t$ statistic is only about 1.12 in absolute value, which is not enough to reject the null hypothesis that the return to education does not depend on race.

(iv) We choose the base group to be single, nonblack. Then we add dummy variables *marrnonblck*, *singblck*, and *marrblck* for the other three groups. The result is

$$
\begin{aligned}
\widehat{\log(wage)} = {} & \underset{(0.11)}{5.40} + \underset{(.0063)}{.0655}\, educ + \underset{(.0032)}{.0141}\, exper + \underset{(.0025)}{.0117}\, tenure \\
& - \underset{(.026)}{.092}\, south + \underset{(.027)}{.184}\, urban + \underset{(.043)}{.189}\, marrnonblck \\
& - \underset{(.096)}{.241}\, singblck + \underset{(.0560)}{.0094}\, marrblck
\end{aligned}
$$

$n = 935,\ R^2 = 0.253.$

We obtain the ceteris paribus differential between married blacks and married nonblacks by taking the difference of their coefficients: 0.0094 − 0.189 = −0.1796, or about −0.18. That is, a married black man earns about 18% less than a comparable, married nonblack man.