

# DSC 40B

## Theoretical Foundations II

### Graph Search Strategies

## **How do we:**

- ▶ determine if there is a path between two nodes?
- ▶ check if graph is connected?
- ▶ count connected components?

# Search Strategies

- ▶ A **search strategy** is a procedure for exploring a graph.
- ▶ Different strategies have different properties.

# Node Statuses

At any point during a search, a node is in exactly one of three states:

- ▶ **visited**
- ▶ **pending** (discovered, but not yet visited)
- ▶ **undiscovered**

# Rules

- ▶ At every step, next visited node chosen from among **pending** nodes.
- ▶ When a node is marked as **visited**, all of its neighbors have been marked as **pending**.

# Choosing the next Node

How to choose among pending nodes?

- ▶ Visit **newest** pending (**depth-first** search).
- ▶ Visit **oldest** pending (**breadth-first** search).

# Breadth-First Search

At every step:

1. Visit oldest pending node.
2. Mark its undiscovered neighbors as pending.  
To store pending nodes, use a FIFO **queue**.

# Queues in Python

- ▶ Want  $\Theta(1)$  time pops/appends on either side.
- ▶ `from collections import deque ("deck")`.
  - ▶ `.popleft()` and `.pop()`
  - ▶ `list` doesn't have right time complexity!
  - ▶ `import queue` isn't what you want!
- ▶ Keep track of node status attribute using dictionary.

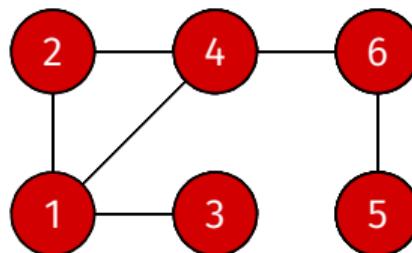
# BFS

```
from collections import deque

def bfs(graph, source):
    """Start a BFS at `source`."""
    status = {node: 'undiscovered' for node in graph.nodes}
    status[source] = 'pending'
    pending = deque([source])

    # while there are still pending nodes
    while pending:
        u = pending.popleft()
        for v in graph.neighbors(u):
            # explore edge (u,v)
            if status[v] == 'undiscovered':
                status[v] = 'pending'
                # append to right
                pending.append(v)
        status[u] = 'visited'
```

# Example



pending = [1, 3, 4]

Before iterating.  
After 1st iteration.  
After 2nd iteration.  
After 3rd iteration.  
After 4th iteration.  
After 5th iteration.  
After 6th iteration.

# Note

- ▶ BFS works just as well for directed graphs.

# Claim

- ▶ bfs with source  $u$  will visit all nodes reachable from  $u$  (and only those nodes).
- ▶ Useful!
  - ▶ Is there a path between  $u$  and  $v$ ?
  - ▶ Is graph connected?

# **DSC 40B**

*Theoretical Foundations II*

**Analysis of BFS**

# Exploring with BFS

- ▶ BFS will visit all nodes reachable from source.
- ▶ If **disconnected**, BFS will not visit all nodes.
- ▶ We can do so with a **full BFS**.
  - ▶ Idea: “re-start” BFS on undiscovered node.
  - ▶ Must pass statuses between calls.

# Making Full BFS

Modify bfs to accept statuses:

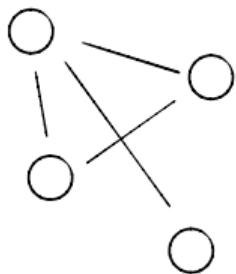
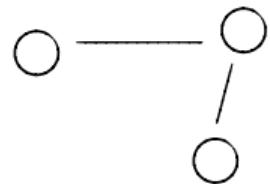
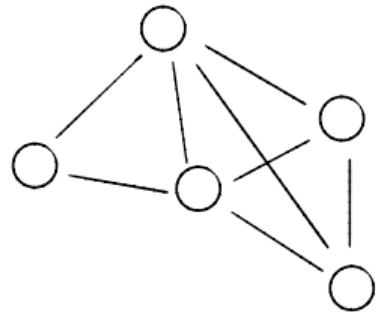
```
def bfs(graph, source, status=None):
    """Start a BFS at `source`."""
    if status is None:
        status = {node: 'undiscovered' for node in graph.nodes}
    # ...
```

# Making Full BFS

Call bfs multiple times:

```
def full_bfs(graph):
    status = {node: 'undiscovered' for node in graph.nodes}
    for node in graph.nodes:
        if status[node] == 'undiscovered'
            bfs(graph, node, status)
```

# Example



# Observation

- ▶ If there are  $k$  connected components, bfs in line 5 is called exactly  $k$  times.

```
1 def full_bfs(graph):
2     status = {node: 'undiscovered' for node in graph.nodes}
3     for node in graph.nodes:
4         if status[node] == 'undiscovered'
5             bfs(graph, node, status)
```

# Key Properties of full\_bfs

- ▶ Each node added to queue **exactly once**.
- ▶ Each edge is explored **exactly**:
  - ▶ **once** if graph is **directed**.
  - ▶ **twice** if graph is **undirected**.

# Time Complexity of full\_bfs

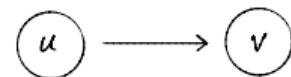
- ▶ Analyzing full\_bfs is easier than analyzing bfs.
  - ▶ full\_bfs visits all nodes, no matter the graph.
- ▶ Result will be **upper bound** on time complexity of bfs.
- ▶ We'll use a **aggregate analysis**.

# BFS

```
from collections import deque

def bfs(graph, source):
    """Start a BFS at `source`."""
    status = {node: 'undiscovered' for node in graph.nodes}
    status[source] = 'pending'
    pending = deque([source])

    # while there are still pending nodes
    while pending:
        u = pending.popleft()
        for v in graph.neighbors(u):
            # explore edge (u,v)
            if status[v] == 'undiscovered':
                status[v] = 'pending'
                # append to right
                pending.append(v)
    status[u] = 'visited'
```



# Time Complexity

```
def full_bfs(graph):
    status = {node: 'undiscovered' for node in graph.nodes}
    for node in graph.nodes:
        if status[node] == 'undiscovered'
            bfs(graph, node, status)
```

# Time Complexity of Full BFS

- ▶  $\Theta(V + E)$
- ▶ If  $|V| > |E|$ :  $\Theta(V)$
- ▶ If  $|V| < |E|$ :  $\Theta(E)$
- ▶ Namely, if graph is **complete**:  $\Theta(V^2)$ .
- ▶ Namely, if graph is **very sparse**:  $\Theta(V)$ .

# Next Time

- ▶ Finding **shortest paths** using BFS.