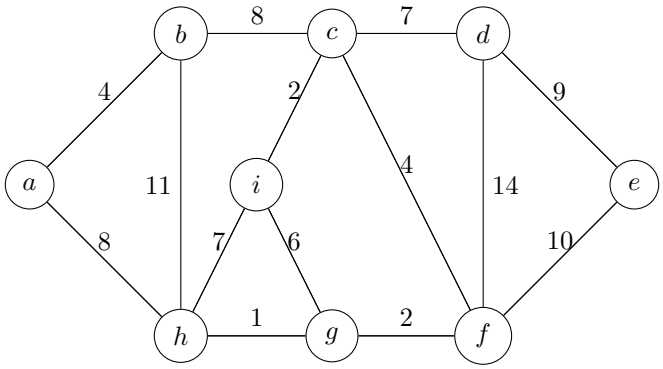
DSC 40B - Discussion 10

Problem 1.

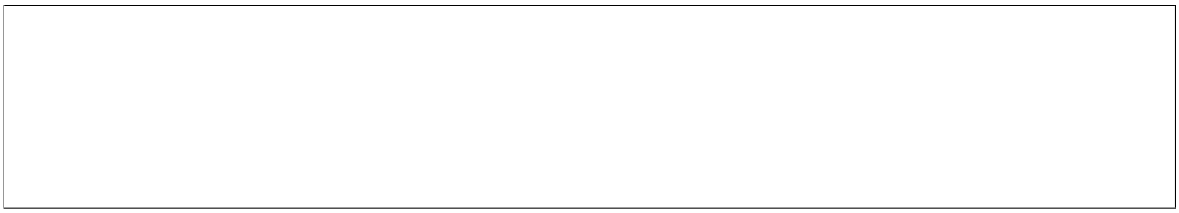
Compute the minimum spanning tree for the following graph using Kruskal's algorithm. (Also compute the MST using Prim's algorithm and compare the results.)



Problem 2.

Suppose we are given both an undirected graph G with weighted edges and a minimum spanning tree T of G.

a) Describe an efficient algorithm to update the minimum spanning tree when the weight of one edge e in T is decreased.



b) Describe an efficient algorithm to update the minimum spanning tree when the weight of one edge e not in T is increased.

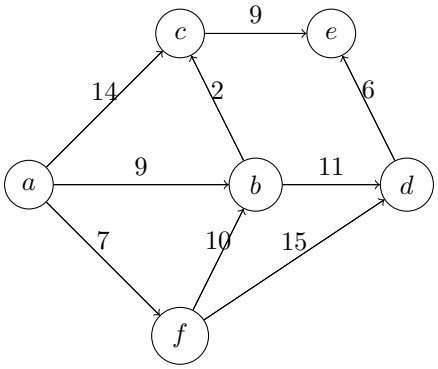
c) Describe an efficient algorithm to update the minimum spanning tree when the weight of one edge e in T is increased.

d) Describe an efficient algorithm to update the minimum spanning tree when the weight of one edge e not in T is decreased.

Extra Practice

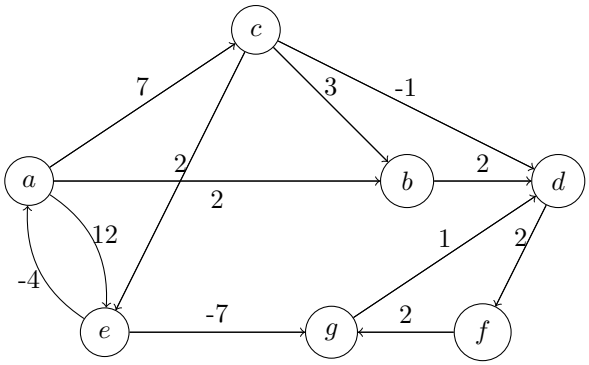
Problem 3.

Run Dijkstra's Algorithm on the following graph using node a as the source. Below each node u, write the shortest path length from a to u. Mark the predecessor of u by highlighting it or making a bold arrow.



Problem 4.

a) Run Dijkstra's Algorithm on the following graph using node a as the source. Below each node u, write the shortest path length from a to u. Mark the predecessor of u by highlighting it or making a bold arrow.



b) Dijkstra's algorithm found the wrong path to some of the vertices. For just the vertices wherethe wrong path was computed, indicate both the path that was computed and the correct path.

c) What single edge could be removed from the graph such that Dijkstra's algorithm would happen to compute correct answers for all vertices in the remaining graph?

Problem 5.

True or False. Suppose $G = (V, E, \omega)$ is a weighted graph for which all edges are positive, except for those edges of a node s which may or may not be negative. If Dijkstra's algorithm is run on G with s as the source, the correct shortest paths will be found. Assume that the graph does not have any negative loops. Justify your answer.