DSC 40B - Discussion 08

Problem 1.

Two containers A and B have capacities of 3 liters and 5 liters, respectively. Describe how you can measure exactly one liter of water using only these two containers. You may 1) fill each container to the top using a faucet; 2) completely empty either container into a drain; and 3) pour one container into the other until it is empty or the other container is full. Both containers are empty to start.

Problem 2.

What is the result of updating the edge (u,v) when the est[u], est[v] and weight(u,v) are given as follows?

Figure 1: Bellman Ford update subroutine

```
def update(u, v, weights, est, predecessor):
    if est[v] > est[u] + weights(u,v):
       est[v]=est[u]+weights(u,v)
       predecessor[v]=u
       return True
    else:
       return False
```

a) est[u] = 7, est[v] = 11, weight(u,v) = 3

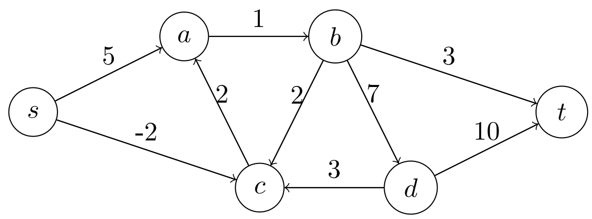
b) est[u] = 15, est[v] = 12, weight(u,v) = -3

c) est[u] = 12, est[v] = 14, weight(u,v) = 3

Problem 3.

Run Bellman-Ford on the following graph using node s as the source. Below each node u, write the shortest path length from s to u. Mark the predecessor of u by highlighting it or making a bold arrow.

```
def bellman_ford(graph, weights, source):
    est={node:float('inf') for node in graph.nodes}
    est[source]=0
    predecessor={node: None for node in graph.nodes}
    for i in range(len(graph.nodes)-1):
        for(u, v) in graph.edges:
            update(u, v, weights, est, predecessor)
    return est, predecessor
```



s
a
b
c
d
t
5
-2
1
2
2
7
3
3
10

Problem 4.

State TRUE or FALSE for the following statements:

a) If (s, v_1, v_2, v_3, v_4) is a shortest path from s to v_4 in a weighted graph, then (s, v_1, v_2, v_3) is a shortest path from s to v_3

b) Let P be a shortest path from some vertex s to some other vertex t in a directed graph. If the weight of each edge in the graph is increased by one, P will still be a shortest path from s to t.

c) Suppose the update function is modified such the est[v] is updated when est[v] $\geq$ est[u] + weight(u,v) instead of strictly greater than. The est values of all nodes at the end of the algorithm would still give the shortest distance from the source.

d) Suppose the update function is modified such the est[v] is updated when est[v] $\geq$ est[u] + weight(u,v) instead of strictly greater than. We can still find the shortest path from the source to any node using the predecessors using the new algorithm.