DSC 40B

Theoretical Foundations II

Graph Search Strategies

How do we:

- determine if there is a path between two nodes?
- check if graph is connected?
- count connected components?

Search Stategies

- A **search strategy** is a procedure for exploring a graph.
- Different strategies have different properties.

Node Statuses

At any point during a search, a node is in exactly one of three states:

- **visited**
- **pending** (discovered, but not yet visited)
- **undiscovered**

Rules

- At every step, next visited node chosen from among **pending** nodes.
- When a node is marked as **visited**, all of its neighbors have been marked as **pending**.

Choosing the next Node

How to choose among pending nodes?

- Visit **newest** pending (**depth-first** search).

- Visit **oldest** pending (**breadth-first** search).

Breadth-First Search

At every step:

1. Visit oldest pending node.

2. Mark its undiscovered neighbors as pending.

To store pending nodes, use a FIFO **queue**.

Queues in Python

- Want Θ(1) time pops/appends on either side.
- `from collections import deque` (“deck”).
  - `.popleft()` and `.pop()`
  - `list` doesn’t have right time complexity!
  - `import queue` isn’t what you want!
- Keep track of node status attribute using dictionary.

BFS

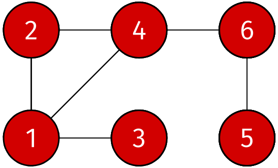
```python
from collections import deque

def bfs(graph, source):
    """Start a BFS at `source`."""
    status = {node: 'undiscovered' for node in graph.nodes}

    status[source] = 'pending'
    pending = deque([source])

    # while there are still pending nodes
    while pending:
        u = pending.popleft()
        for v in graph.neighbors(u):
            # explore edge (u,v)
            if status[v] == 'undiscovered':
                status[v] = 'pending'
                # append to right
                pending.append(v)
        status[u] = 'visited'
```

Example



`pending = [1,3,4]`

Before iterating.After 1st iteration.After 2nd iteration.After 3rd iteration.After 4th iteration.After 5th iteration.After 6th iteration.

Note

- BFS works just as well for directed graphs.

Claim

- `bfs` with source u will visit all nodes reachable from u (and only those nodes).
- Useful!
  - Is there a path between u and v?
  - Is graph connected?

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Analysis of BFS

Exploring with BFS

- BFS will visit all nodes reachable from source.
- If **disconnected**, BFS will not visit all nodes.
- We can do so with a **full BFS**.
  - Idea: “re-start” BFS on undiscovered node.
  - Must pass statuses between calls.

Making Full BFS

Modify `bfs` to accept statuses:

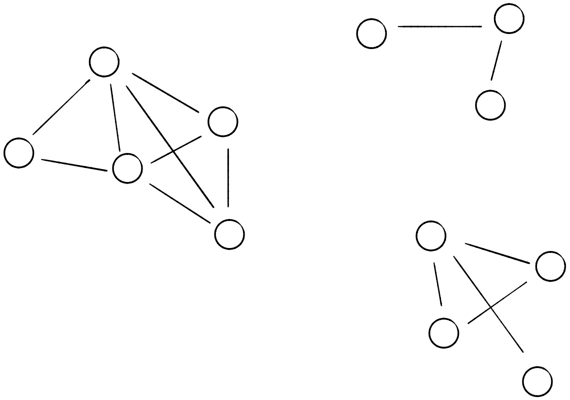
```
def bfs(graph, source, status=None):
    """Start a BFS at `source`."""
    if status is None:
        status = {node: 'undiscovered' for node in graph.nodes}
    # ...
```

Making Full BFS

Call `bfs` multiple times:

```
def full_bfs(graph):
    status = {node: 'undiscovered' for node in graph.nodes}
    for node in graph.nodes:
        if status[node] == 'undiscovered'
           bfs(graph, node, status)
```

Example



Observation

- If there are k connected components, `bfs` in line 5 is called exactly k times.

```
1 def full_bfs(graph):
2     status = {node: 'undiscovered' for node in graph.nodes}
3     for node in graph.nodes:
4         if status[node] == 'undiscovered'
5             bfs(graph, node, status)
```

Key Properties of `full_bfs`

- Each node added to queue **exactly once**.

- Each edge is explored **exactly**:
    - **once** if graph is **directed**.
    - **twice** if graph is **undirected**.

Time Complexity of `full_bfs`

- Analyzing `full_bfs` is easier than analyzing `bfs`.
  - `full_bfs` visits all nodes, no matter the graph.
- Result will be **upper bound** on time complexity of `bfs`.
- We'll use a **aggregate analysis**.

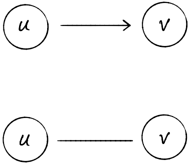
BFS

```python
from collections import deque

def bfs(graph, source):
    """Start a BFS at `source`."""
    status = {node: 'undiscovered' for node in graph.nodes}

    status[source] = 'pending'
    pending = deque([source])

    # while there are still pending nodes
    while pending:
        u = pending.popleft()
        for v in graph.neighbors(u):
            # explore edge (u,v)
            if status[v] == 'undiscovered':
                status[v] = 'pending'
                # append to right
                pending.append(v)
        status[u] = 'visited'
```



Time Complexity

```python
def full_bfs(graph):
    status = {node: 'undiscovered' for node in graph.nodes}
    for node in graph.nodes:
        if status[node] == 'undiscovered'
            bfs(graph, node, status)


def bfs(graph, source):
    """Start a BFS at `source`."""
    status = {node: 'undiscovered' for node in graph.nodes}

    status[source] = 'pending'
    pending = deque([source])

    # while there are still pending nodes
    while pending:
        u = pending.popleft()
        for v in graph.neighbors(u):
            # explore edge (u,v)
            if status[v] == 'undiscovered':
                status[v] = 'pending'
                # append to right
                pending.append(v)
        status[u] = 'visited'
```

Time Complexity of Full BFS

- $\Theta(V + E)$
- If $|V| > |E|$: $\Theta(V)$
- If $|V| < |E|$: $\Theta(E)$
- Namely, if graph is **complete**: $\Theta(V^2)$.
- Namely, if graph is **very sparse**: $\Theta(V)$.

Next Time

- Finding **shortest paths** using BFS.