DSC 40B - Midterm 02 Review

Problem 1.

The goal of contact tracing is to determine how the spread of a virus occurs. Which type of graph would be best for modelling the spread of a virus?

- ☐ Directed graph
- ☐ Undirected graph

Problem 2.

A directed graph has 7 nodes. What is the maximum number of edges it can have?

Problem 3.

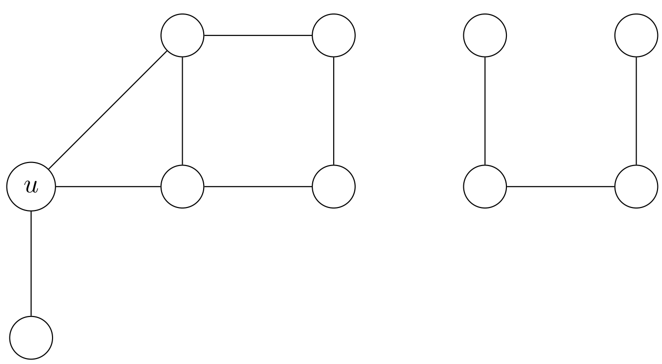
An undirected graph has 12 nodes. What is the maximum number of connected components it can have?

Problem 4.

A directed graph has 5 nodes. What is the largest degree that a node in the graph can possibly have?

Problem 5.

How many nodes are reachable from node u in the graph?



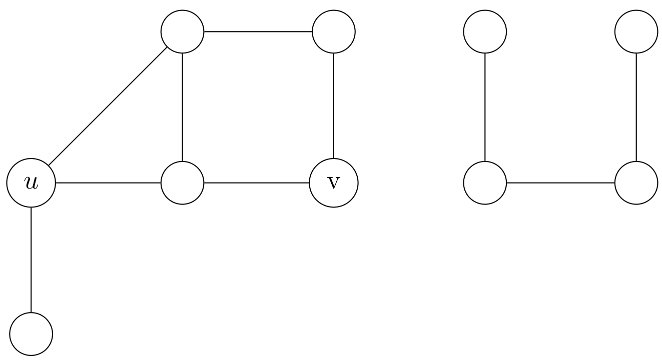
Problem 6.

Both BFS and DFS can be used to count the number of connected components in an undirected graph.

- ☐ True
- ☐ False

Problem 7.

How many paths are there from node u to node v in the graph below?



- ☐ Infinitely many
- ☐ 4
- ☐ 3
- ☐ 5

Problem 8.

In an unweighted graph, there is at most one shortest path between any pair of given nodes.

- ☐ True
- ☐ False

Problem 9.

An undirected graph has 5 nodes. What is the smallest number of connected components it can have?

Problem 10.

In a full BFS of a graph G=(V, E), the number of times that something is popped form the queue is 2V if the graph is undirected and V if the graph is directed.

☐ True

☐ False

Problem 11.

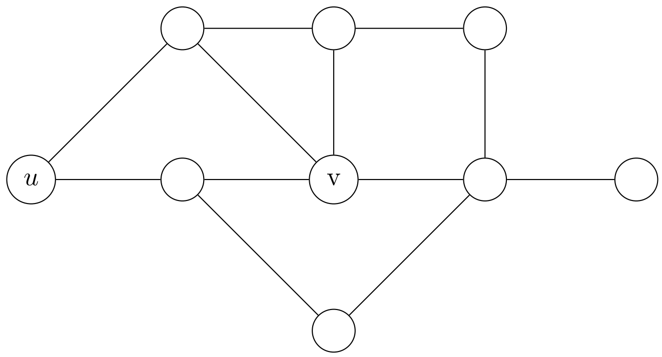
In BFS it is possible for the queue to simultaneously contain a node whose distance from the source is 3 and node whose distance from the source is 5.

☐ True

☐ False

Problem 12.

Suppose a BFS is run on the graph below with u as the source.



Of course, u is the first node to be popped of the queue. Suppose that node v is the kth node popped from the queue.

a) What is the smallest that k can possibly be?

b) What is the largest that k can possibly be?

Problem 13.

Consider the modified mergesort given below:

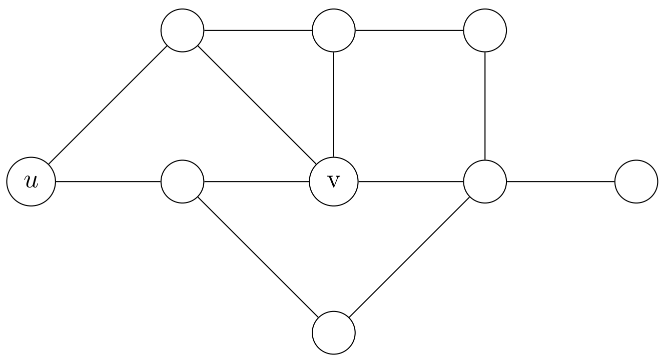
```
def bfs(graph, source, status=None):
    if status is None:
        status = {node: 'undiscovered' for node in graph.nodes}

    status[source] = 'pending'
    pending = deque([source])

    # while there are still pending nodes
    while pending:
        u = pending.popleft()
        for v in graph.neighbors(u):
            # explore edge (u,v)
            if status[v] == 'undiscovered':
```

```
            print ("Hey")
            status[v] = 'pending'
            # append to right
            pending.append(v)
    status[u] = 'visited'
```

Suppose this code is run on a connected undirected graph with 12 nodes. Exactly how many times will 'Hey' be printed?

Problem 14.

Suppose a DFS is run on the graph below with u as the source.



Node u will be the first node marked pending. Suppose that node v is the kth node marked pending.

a) What is the smallest that k can possibly be?

b) What is the largest that k can possibly be?

Problem 15.

If DFS is called on a complete graph, the time complexity is $\theta(V^2)$

☐ True
☐ False

Problem 16.
What is the result of updating the edge (u,v) when the est[u], est[v] and weight(u,v) are given as follows?

Figure 1: Bellman Ford update subroutine

```
def update(u, v, weights, est, predecessor):
    if est[v] > est[u] + weights(u,v):
       est[v]=est[u]+weights(u,v)
       predecessor[v]=u
       return True
    else:
       return False
```

a) est[u] = 7, est[v] = 11, weight(u,v) = 3

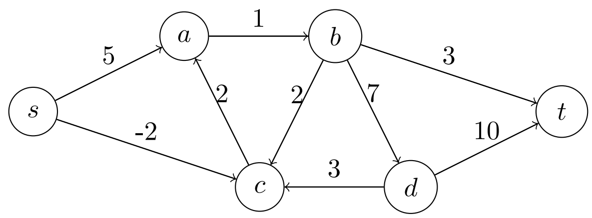
b) est[u] = 15, est[v] = 12, weight(u,v) = -3

c) est[u] = 12, est[v] = 14, weight(u,v) = 3

Problem 17.

Run Bellman-Ford on the following graph using node s as the source. Below each node u, write the shortest path length from s to u. Mark the predecessor of u by highlighting it or making a bold arrow.

```
def bellman_ford(graph, weights, source):
    est={node:float('inf') for node in graph.nodes}
    est[source]=0
    predecessor={node: None for node in graph.nodes}
    for i in range(len(graph.nodes)-1):
        for(u, v) in graph.edges:
            update(u, v, weights, est, predecessor)
    return est, predecessor
```



Problem 18.

State TRUE or FALSE for the following statements:

a) If (s, v_1, v_2, v_3, v_4) is a shortest path from s to v_4 in a weighted graph, then (s, v_1, v_2, v_3) is a shortest path from s to v_3

b) Let P be a shortest path from some vertex s to some other vertex t in a directed graph. If the weight of each edge in the graph is increased by one, P will still be a shortest path from s to t.

c) Suppose the update function is modified such the est[v] is updated when est[v] $\geq$ est[u] + weight(u,v) instead of strictly greater than. The est values of all nodes at the end of the algorithm would still give the shortest distance from the source.

d) Suppose the update function is modified such the est[v] is updated when est[v] $\geq$ est[u] + weight(u,v) instead of strictly greater than. We can still find the shortest path from the source to any node using the predecessors using the new algorithm.