CSE 151A - Homework 09

Due: Wednesday, June 10, 2020

These are extra plus problems that you can do to boost your homework score. They are entirely optional!

Plus Problem 1. (4 plus points)

Suppose we want to maximize some function $f(\vec{v})$ subject to $\|\vec{v}\| = 1$. We know that the constrained maximum is either $\vec{v}^{(1)} = (-0.28, 0.96)^\intercal$ or $\vec{v}^{(2)} = (0.6, -0.8)^\intercal$, but we don't know what the function looks like so we can't directly evaluate our function at either of these vectors. Instead, we only know that the gradient is $\nabla f(\vec{v}) = 2C\vec{v}$, where

$$C = \begin{bmatrix} 0.46 & -0.24 \\ -0.24 & 0.6 \end{bmatrix}$$

Which choice of vector corresponds to the constrained local maximum of $f(\vec{v})$? Show your work.

Plus Problem 2. (5 plus points)

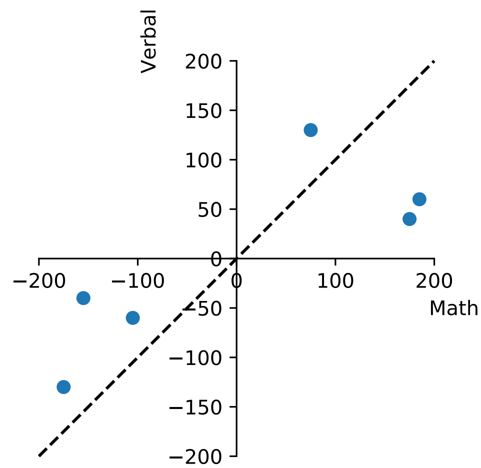
SAT scores for six students are shown in the table below:

Math	Verbal
780	700
500	600
790	720
450	620
680	790
430	530

In this problem, we will use PCA to produce a single score for each student which combines their verbal and math scores.

a) Center the data by subtracting the mean math score from each math score, and the mean verbal score from each verbal score. Write your results in the form of a table like that above. The mean of each column should be zero.

b) When plotted, the centered data looks as follows:



The dashed line is the line of 45°. Suppose $\hat{v} = (v_1, v_2)^T$ is the unit vector which lies in the direction of maximum variance. Which is larger: $|v_1|$ or $|v_2|$? Provide reasoning using the plot above.

c) Find the $\hat{v}$ which lies in the direction of maximum variance (of the centered data). Show your work. You may use `numpy`, but you should paste your code and describe the steps taken.

d) Use PCA to map each data vector to a single number. That is, using your solution to the previous part, compute the new variable $z^{(i)} = \vec{x}^{(i)} \cdot \hat{v}$.

Plus Problem 3. (7 plus points)

Let A be a $d \times n$ matrix and let $f : \mathbb{R}^d \to \mathbb{R}$ be the function defined by $f(\vec{x}) = \vec{x}^T A A^T \vec{x}$. Show that $\nabla f(\vec{b}) = 2AA^T\vec{b}$.

Hint: recall that $\nabla f(\vec{b})$ denotes the gradient of f at $\vec{b}$. Is is defined to be:

$$\nabla f(\vec{b}) = \begin{pmatrix} \frac{\partial f}{\partial x_1}(\vec{b}) \\ \frac{\partial f}{\partial x_2}(\vec{b}) \\ \vdots \\ \frac{\partial f}{\partial x_d}(\vec{b}) \end{pmatrix}$$