
DSC 40A - Homework 07

Due: Friday, February 28, 2020

Write your solutions to the following problems by either typing them up or handwriting them on another piece of paper. Unless otherwise noted by the problem's instructions, show your work or provide some justification for your answer. Homeworks are due via Gradescope on Friday afternoon at 5:00 p.m.

Problem 1.

Let $0 < k < n$. Prove that

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}.$$

Problem 2.

At UC San Diego, incoming students are each randomly assigned to join one of the six colleges: Revelle, Muir, Marshall, Warren, Roosevelt, and Sixth. Suppose there are 24 incoming students, including Winona, Xanthippe, and Zelda. It cannot be assumed that all colleges will be assigned four students; since each student's assignment is random, some colleges may be assigned more students than others.

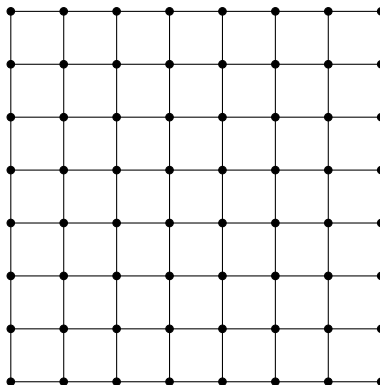
Please leave your answers as unsimplified expressions with factorials, exponents, permutations, combinations, etc.

- a) All 24 incoming students get called up one at a time to be assigned to a college. How many possible orders of all 24 students have Winona as the first student called?
- b) How many possible orders of all 24 students have Zelda, Xanthippe, and Winona (in any order) as the first three students called up to wear the sorting hat?
- c) How many ways are there to assign all 24 students to colleges such that Zelda, Xanthippe, and Winona all get assigned to Muir? Here, and in what follows, the order in which the students are assigned does not matter.
- d) How many ways are there to assign all 24 students to colleges such that Zelda, Xanthippe, and Winona all get assigned to the same college?
- e) How many ways are there to assign all 24 students to colleges such that Zelda, Xanthippe, and Winona are assigned to different colleges?
- f) How many ways are there to assign all 24 students to colleges such that exactly 4 students get assigned to Warren?
- g) How many ways are there to assign all 24 students to colleges if each of the six colleges has room for only four incoming students?
- h) How many ways are there to assign all 24 students to colleges if nobody is assigned to the same colleges as the person called up just before them?
- i) What is the probability that Winona, Xanthippe, and Zelda are all assigned to Muir?
- j) Suppose again that the students are called up one at a time. In how many possible orders of all 24 students does Zelda get called up some time before Xanthippe? Simplify your answer as much as

possible, but let factorials remain unsimplified.

Problem 3.

Four different points are chosen at random from a grid which is 7 units wide and 7 units tall:



What is the probability that the four points form a rectangle (with vertical and horizontal sides)?

For the purposes of this problem, the width of each of the rectangle's sides must be nonzero.

Problem 4.

One hundred Californians from four different cities (SD, LA, SF, and SJ) were asked whether they like or dislike Los Angeles. The results were distributed as follows:

City	# of People	# of Likes
SD	30	5
LA	40	21
SF	10	4
SJ	20	7

In what follows, suppose that the above data are stored in a table with one hundred rows (one row per person).

You may leave your answers unsimplified, but they should not contain \sum or \dots

- a) How many different ways are there to order the table's rows?
- b) How many different ways are there to order the table's rows in which all rows from the same city are grouped together?
- c) How many different subsets of size 10 contain 5 people from LA and 5 people from SD?
- d) How many different subsets of size 10 contain exactly 2 people from SF and exactly 5 people from SJ?
- e) What is the probability that a person selected at random is from LA and does not like LA?
- f) What is the probability that a person selected at random is from SD or likes LA?
- g) What is the probability that a person selected at random is from southern California (LA or SD)?
- h) What is the probability that, if a group of five people is randomly selected, exactly one will be from SD and exactly three will like LA?

Problem 5.

You arrive to campus in one of three ways: on the bus, in your car, or on foot.

The buses often run late and have long lines, so when you take the bus, you have a 50% chance of being late for class. When you drive, you sometimes hit traffic or have trouble finding a parking spot, so you have a 30% chance of being late for class. When you walk to campus, you only have a 5% of arriving late. One day, you arrive late for your midterm, and your professor wonders how you got to school that day.

- a) If your professor assumes that you are equally likely to use all three modes of transportation, what will the professor calculate for the probability that you took the bus on the day of your midterm?
- b) If your professor happens to know that you take the bus 20% of the time, drive 20% of the time, and walk 60% of the time, what will the professor calculate for the probability that you took the bus on the day of your midterm?