DSC 40A - Final Exam

December 11, 2019

Name:

PID:

Version: A

By signing below, you are agreeing that you will behave honestly and fairly during and after this exam. You should not discuss any part of this exam with anyone enrolled in the course who has not yet taken the exam (this includes posting questions about the exam on Piazza!)

Signature:

Name of student to your **left**:

Name of student to your **right**:

Exam version of student to your **left**:

Exam version of student to your **right**:

(Write “N/A” if a wall/aisle is to your left/right.) Your version should be different than either of the students to your left/right.

Instructions:

- Write your solutions to the following problems in the boxes provided.
- Scratch paper is provided at the end of the exam.
- No calculators are permitted, but a cheat sheet is.
- Write your name or PID at the top of each sheet in the space provided.

(Please do not open your exam until instructed to do so.)

Problem 1.

Determine the single best answer for each question. You are not penalized for guessing.

a) Let $L(h)$ be a loss function and let c be a constant. Define a new loss function $L'(h) = L(h) + c$. Then if h^* minimizes L, it also minimizes L'.

◯ True ◯ False

b) Let A, B, and C be events. If A and B are independent, they must also be conditionally independent given C.

◯ True ◯ False

c) Let $x_1, \ldots, x_n$ and $y_1, \ldots, y_n$ be real numbers. Then

$$\left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right) = \sum_{i=1}^{n} x_i y_i.$$

◯ True ◯ False

d) Suppose that when a straight line is fit to a data set using least squares regression the sum of squared errors is S_1. A new point is added to the data set and a new straight line is fit; let S_2 be the resulting sum of squared errors. It is possible for $S_2 < S_1$.

◯ True ◯ False

e) Let $x_1, \ldots, x_n$ be a data set of real numbers. Then $L(h) = \sum_{i=1}^{n} (x_i - h)^2$ is minimized by the:

◯ mode ◯ midpoint ◯ mean ◯ median

f) Let $x_1, \ldots, x_n$ be a data set of real numbers. Then

$$L(h) = \max\{|x_1 - h|, |x_2 - h|, \ldots, |x_n - h|\}$$

is minimized by the:

◯ mode ◯ midpoint ◯ mean ◯ median

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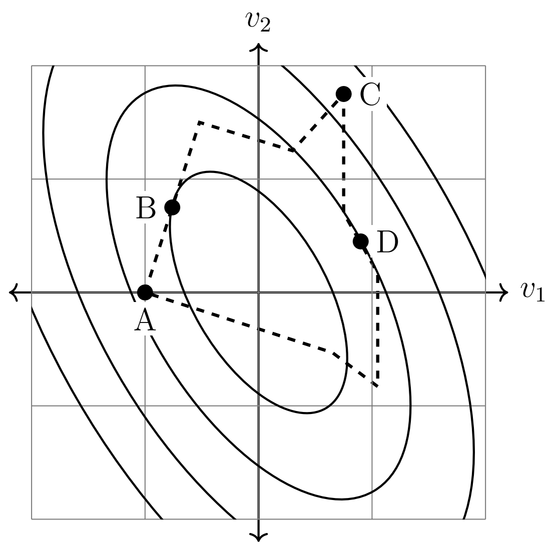
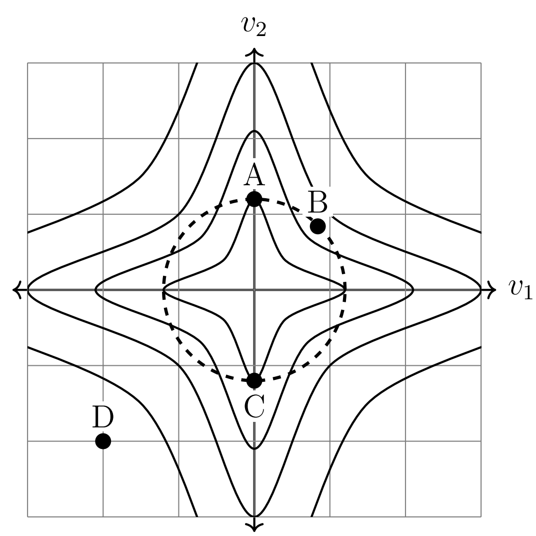
g) Let $f(v_1, v_2)$ be a function whose value increases away from the origin. Suppose that the contour lines of f are shown below as solid lines, and suppose that the dashed lines represent a constraint. Which of the points shown **maximizes** the function f subject to the constraint?



◯ Point A ◯ Point B ◯ Point C ◯ Point D

h) Let $f(v_1, v_2)$ again be a function whose value increases away from the origin. Suppose that the contour lines of f are shown below as solid lines, and suppose that the dashed lines represent a constraint. Which of the points shown **maximizes** the function f subject to the constraint?



◯ Point A ◯ Point B ◯ Point C ◯ Point D

Problem 2.

Using least squares regression, fit a **quadratic** function of the form $y \approx c_0 + c_1 x^2$ to the data below:

x	y
-2	5
1	2
2	3
3	6

For your reference, recall that the least squares solutions for the slope b_1 and intercept b_0 of a linear fit to the data are:

$$b_1 = \frac{\sum_{i=1}^{n} x_i y_i - \frac{1}{n}\left(\sum_{i=1}^{n} y_i\right)\left(\sum_{i=1}^{n} x_i\right)}{\sum_{i=1}^{n} x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2}$$

$$b_0 = \frac{1}{n}\left(\sum_{i=1}^{n} y_i - b_1 \sum_{i=1}^{n} x_i\right)$$

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$c_0 = \boxed{}$ $c_1 = \boxed{}$

Show your work here:

Problem 3.

Recall that an $n \times n$ matrix A is said to be *orthogonal* if $A^{\intercal}A = I$, where I is the $n \times n$ identity matrix. Show that if $\vec{u}$ is an eigenvector of A, and A is an orthogonal matrix, then $\vec{u}$ is also an eigenvector of $A^{\intercal}$.

Problem 4.

Let $\vec{x}^{(1)}, \ldots, \vec{x}^{(n)}$ be vectors in $\mathbb{R}^2$. Suppose that W is the matrix:

$$W = \begin{pmatrix} w_1 & 0 \\ 0 & w_2 \end{pmatrix}.$$

Let $L : \mathbb{R}^2 \to \mathbb{R}$ be the loss function defined by:

$$L(\vec{h}) = \sum_{i=1}^{n} \left\| W \left(\vec{x}^{(i)} - \vec{h} \right) \right\|^2.$$

Find the vector $\vec{h}^* = (h_1^*, h_2^*)^\intercal$ which minimizes L.

Problem 5.

The holiday season is the busiest time of year for air travel. Flying anywhere at this time of year can be chaotic: tickets are oversold, flights are delayed, and luggage is lost. We can better understand each of these events using the tools of probability and combinatorics.

In what follows, assume that a particular airliner has 246 seats.

a) How many ways are there for the airline to assign seats if the flight is full (that is, 246 passengers must be assigned to 246 seats)?

b) Suppose the flight is not full; it has only 150 passengers. How many ways are there for the airline to assign 150 passengers to 246 seats?

c) The airline oversold the flight, and 300 people showed up – but there are only 246 seats. How many ways are there for the airline to choose the people will board the plane?

d) Assume once again that 300 passengers arrive for a flight with only 246 seats. How many ways are there for the airline to assign seats? (Not everyone will get a seat).

e) You and your friends are 6 of the 300 people who arrived for the flight. Suppose that the airline chooses people at random to board the plane. What is the probability that all six of you are able to board?

f) When doing their calculations, the airline assumes that an arbitrary person has a 98% chance of actually showing up for their flight. Assuming that this is true, what is the probability that exactly 300 people show up?

g) Out of the 246 seats on the plane, 82 are window seats, 82 are middle seats, and 82 are aisle seats. How many ways are there of choosing who sits next to a window, who sits in the middle, and who sits next to the aisle if 246 passengers board the plane?

Airline	% of flights on time	% of all flights
Northeast	7/8	1/6
Epsilon	3/4	1/2
Divided	4/5	1/3

The table above shows the fraction of flights which are on time for three airlines, along with the fraction of all flights that the airline is responsible for. For instance, Northeast Airlines operates 1/6 of all flights, and 7/8 of their flights are on time.

h) Suppose that the probability that a flight encounters a storm is 1/5, independent of which airline operates the flight. What is the probability that a randomly-selected flight is operated by Epsilon **or** it encounters a storm?

i) Let E_1 be the event that a flight arrives on time, and let E_2 be the event that the pilot forgot the keys to the airplane. Are these two events independent?

◯ Yes ◯ No

j) A randomly-selected flight arrived on time. What is the probability that the flight was operated by Northeast?

Problem 6.

In parts of the world other than San Diego, the weather changes from day to day. In these places, people try to guess tomorrow's weather using the current conditions.

Weather data for 20 random days in Columbus, Ohio are recorded below, along with the next day's weather (rainy, cloudy, or sunny).

Suppose that today's humidity is $> 50\%$, the temperature is hot, and the air pressure is low. Use naïve Bayes to predict whether tomorrow will be rainy, cloudy, or sunny. Show your work.

Next Day's Weather	**Humidity**	**Temperature**	**Air Pressure**
Rainy	$> 50\%$	Cool	Low
Rainy	$> 50\%$	Hot	Low
Rainy	$> 50\%$	Cool	Low
Rainy	25%-50%	Hot	High
Rainy	25%-50%	Hot	Low
Rainy	25%-50%	Cool	Low
Rainy	25%-50%	Cool	Low
Rainy	$< 25\%$	Cool	Low
Rainy	$< 25\%$	Hot	Low
Rainy	$< 25\%$	Hot	High
Cloudy	$> 50\%$	Cool	Low
Cloudy	$> 50\%$	Cool	Low
Cloudy	25%-50%	Hot	High
Cloudy	$< 25\%$	Cool	High
Cloudy	$< 25\%$	Cool	Low
Sunny	$> 50\%$	Cool	Low
Sunny	$> 50\%$	Hot	High
Sunny	$> 50\%$	Cool	High
Sunny	25%-50%	Hot	High
Sunny	$< 25\%$	Hot	High

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Prediction: tommorow will be ◯ rainy ◯ cloudy ◯ sunny

Show your work here:

Problem 7. (Extra Credit)

Draw a picture that conveys your feelings about the fact that winter break is (almost) here.

Before turning in your exam, please check that your name is on every page.
After turning in your exam, have a good break!

(This is scratch paper. You may remove it from the exam; it does not need to be turned in.)

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