DSC 40A - Discussion 05 - Perceptrons and Probability

February 18, 2020

Problem 1.

Let $\vec{x}^{(1)}, \ldots, \vec{x}^{(n)}$ be a set of n data points with associated labels $y_1, \ldots, y_n \in \{-1, 1\}$. Recall from lecture that the perceptron update rule is

$$\vec{w}^{(t)} = \vec{w}^{(t-1)} - \frac{\alpha}{n} \sum_{i \in M} \begin{cases} \text{Aug}(\vec{x}^{(i)}), & \text{Aug}(\vec{x}^{(i)}) \cdot \vec{w}^{(t-1)} \geq 0 \\ -\text{Aug}(\vec{x}^{(i)}), & \text{Aug}(\vec{x}^{(i)}) \cdot \vec{w}^{(t-1)} < 0 \end{cases}$$

where M is the set of data points which are currently misclassified by $\vec{w}^{(t-1)}$.

Suppose we gather the following data:

$$\begin{array}{ll} x_1 = (1, 0)^T & y_1 = 1 \\ x_2 = (0, 1)^T & y_2 = 1 \\ x_3 = (-1, 1)^T & y_3 = 1 \\ x_4 = (1, 2)^T & y_4 = -1 \\ x_5 = (2, 2)^T & y_5 = -1 \\ x_6 = (2, 1)^T & y_6 = -1 \end{array}$$

Run the perceptron algorithm by hand with w initialized to $(1, 1, 1)^T$, and with a step size of $\alpha = 3$. Stop the algorithm when all points are classified correctly. What is the weight vector w at each step? How many training examples are misclassified at each step? For the purpose of this problem, a training example which lies on the decision boundary is considered to be misclassified.

Problem 2.

I have 10 shirts, 6 pairs of pants, and 3 jackets. Every day I dress at random, picking one of each category. What is the probability that today I am wearing at least one garment I was wearing yesterday?

Problem 3.

Suppose that bitstrings (sequences of 0s and 1s) of length 4 are generated randomly with equal probability. Assume that positions are numbered $1, 2, 3, 4$ reading from left to right.

For each question below, find the probability. You do not need to simplify your answers at all. For example, you can leave answers like $2/4$ or $3/18 + 1/3$.

a) What is the probability that a bitstring starts with a one?

b) What is the probability that a bitstring has a 1 in position 2 and a 0 in position 3?

c) What is the probability that a bitstring has the same first and last bits (the same bits in positions 1 and 4)?

d) What is the probability that a bitstring has all four bits the same?

e) What is the probability that a bitstring has more 0s than 1s?

Problem 4.

There are 5 teams in a sports league. Over the course of a season, each team plays every other team exactly once. If the outcome of each game is a fair random coin flip, and there are no ties, what is the probability that some team wins all of its games?

Problem 5.

A standard deck of cards contains 13 cards (2 – 10, J, Q, K, A) in 4 suits (spades, clubs, diamonds, hearts) for a total of 52 cards. Hearts and diamonds are red; clubs and spades are black.

Suppose one card is drawn. What is the probability of the card being:

a) the ace of spades?

b) a diamond?

c) a heart or a diamond?

d) an ace?

e) a face card (jack, queen, or king)?

f) a red face card?

g) even-numbered?

h) a club and ≤ 10?

i) red and even-numbered and ≤ 6?

Now a card is drawn, placed back into the deck; then a second card is drawn. What is the probability that:

j) the ace of spades is drawn, then the king of hearts?

k) aces are drawn both times?

l) a red ace is drawn and a black ace is drawn?

m) the first card is red and the second card is black?

n) at least one of the draws is a heart?