DSC 40A - Discussion 06 - Combinatorics and Conditional Probability

February 25, 2020

Problem 1.

You want to plant an herb garden, so you go to a garden store that has 50 different herbs: 28 are culinary herbs, 12 are medicinal herbs, and 10 are aromatic herbs. You select 5 herbs for your herb garden by taking a random sample **without replacement** from the 50 available herbs.

a) If you consider the herbs you select as a sequence where the order in which you select each herb matters, how many sequences of 5 herbs are possible?

b) If you consider the herbs you select as a sequence where the order in which you select each herb matters, how many sequences of 5 herbs include 2 culinary herbs and 3 aromatic herbs?

c) If you consider the herbs you select as a set where the order in which you select each herb does not matter, how many sets of 5 herbs are possible?

d) If you consider the herbs you select as a set where the order in which you select each herb does not matter, how many sets of 5 herbs include 2 culinary herbs and 3 aromatic herbs?

e) What is the probability that you choose 2 culinary herbs and 3 aromatic herbs for your garden?

Problem 2.

Suppose we throw a fair 6-sided die. If we get i after throwing the die, we throw coin C_i, which has $P(C_i = H) = \frac{i}{10}$ for $i \in \{1, 2, 3, 4, 5, 6\}$.

a) What is the probability that after repeating the above procedure for 3 times,

- we get all heads?
- we get all tails?
- we get exactly two heads?

b) Suppose now that in the above procedure, whenever we throw a coin, if the outcome is tails, we replace the coin with a fair coin. What is the probability that after repeating it 2 times,

- we get all heads?
- **challenge problem (beyond the scope of this course):** we get all tails?

Problem 3.

There are two boxes. Box 1 contains three red and five white balls and box 2 contains two red and five white balls. A box is chosen at random P(box = 1) = P(box = 2) = 0.5 and a ball chosen at random from this box turns out to be red. What is the probability that the red ball came from box 1?

Problem 4.

Challenge follow-up: Two balls are placed in a box as follows: A fair coin is tossed and a white ball is placed in the box if a head occurs, otherwise a red ball is placed in the box. The coin is tossed again and a red ball is placed in the box if a tail occurs, otherwise a white ball is placed in the box. Balls are drawn from the box three times in succession (always with replacing the drawn ball back in the box). It is found that on all three occasions a red ball is drawn. What is the probability that both balls in the box are red?