DSC 40B - Discussion 01

Problem 1.

Let A be a *symmetric* $n \times n$ matrix, with entries:

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}.$$

Suppose $\vec{x} \in \mathbb{R}^n$.

a) Show that $\vec{x}^T A \vec{x} = \displaystyle\sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j a_{ij}$.

b) Show that

$$\frac{\partial}{\partial x_1} \left(\vec{x}^T A \vec{x} \right) = \sum_{j=1}^{n} x_j a_{1j}$$

Problem 2.

Consider the **absolute loss**:

$$L_{\text{abs}}(H(\vec{x}), y) = |H(\vec{x}) - y|.$$

For a linear prediction rule $H(\vec{x}; \vec{w}) = \text{Aug}(\vec{x}) \cdot \vec{w}$, this takes the form:

$$L_{\text{abs}}(\vec{w}, \vec{x}, y) = |\,\text{Aug}(\vec{x}) \cdot \vec{w} - y|.$$

Show that a subgradient of L_{abs} with respect to $\vec{w}$ is:

$$\begin{cases} \text{Aug}(\vec{x}), & \text{if } \text{Aug}(\vec{x}) \cdot \vec{w} - y > 0 \\ -\text{Aug}(\vec{x}), & \text{if } \text{Aug}(\vec{x}) \cdot \vec{w} - y < 0 \\ \vec{0}, & \text{if } \text{Aug}(\vec{x}) \cdot \vec{w} - y = 0 \end{cases}$$