Q1

1 Point

Suppose $C = \begin{pmatrix} 4 & 2 \\ 2 & 6 \end{pmatrix}$ is the empirical covariance matrix for a data set of points. What is the variance in the direction given by the unit vector $\vec{u} = (-1, 0)^T$?

8

3

4

6

Explanation

Variance in the direction of $\vec{u} = \vec{u}^T C \vec{u}$. And, $\vec{u}^T C \vec{u} =$
$\begin{pmatrix} -1 & 0 \end{pmatrix} \begin{pmatrix} 4 & 2 \\ 2 & 6 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \end{pmatrix} \begin{pmatrix} -4 \\ -2 \end{pmatrix} = 4$

Q2

1 Point

Suppose a 4×4 covariance matrix C has eigenvalues of 5, 3, 2, and 0. PCA is performed by projecting the data onto the first three eigenvectors. What will be the reconstruction error?

=0+-0

Explanation

The reconstruction error will be zero because in the direction of the fourth eigenvector, the data exhibits zero variance. That is, the data really lies on the three dimensional linear subspace spanned by the first three eigenvectors. Therefore, when we project the data onto this subspace, each point stays put -- there is no difference between the point before and after projection, and so the reconstruction error is zero.

Q3
1 Point

Given an input data point $\vec{x} = (2, \sqrt{2}, -2) \in \mathcal{R}^3$, you wish to perform PCA on this data point to reduce its dimensionality to 2 features.

The top two principal components are given by $\vec{u}^1 = \left(\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}\right)^T$ and $\vec{u}^2 = \left(\frac{1}{2}, \frac{-1}{\sqrt{2}}, \frac{1}{2}\right)^T$.

What is the representation $\vec{z}$ of this data point in $\mathcal{R}^2$?

- $(1, \sqrt{2})^T$
- $(1, -1)^T$
- $(1, 1)^T$
- $(\sqrt{2}, 1)^T$

Explanation

$\vec{z}_1 = \vec{x} \cdot \vec{u}^1 = 1 + 1 - 1 = 1$. Similarly, $\vec{z}_2 = \vec{x} \cdot \vec{u}^2 = 1 - 1 - 1 = -1$. Hence, $\vec{z} = (1, -1)$

Q4
1 Point

Consider the same data as given in the problem above. Now you need to reconstruct the point back into $\mathcal{R}^3$ (the original space). What will be the reconstructed $\vec{x}$?

- $(2, 0, \sqrt{2})^T$
- $(1, 0, 0)^T$
- $(\sqrt{2}, \sqrt{2}, 0)^T$
- $(0, \sqrt{2}, 0)^T$

Explanation

Reconstruction of $\vec{x} = z_1\vec{u}^1 + z_2\vec{u}^2$. Use the values of z_1 and z_2 from the previous part to perform the calculation. Hence, $\vec{x} = 1 * \left(\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}\right)^T + (-1) * \left(\frac{1}{2}, \frac{-1}{\sqrt{2}}, \frac{1}{2}\right)^T = (0, \sqrt{2}, 0)^T$

Q5

1 Point

Consider the same data as given in the problem above. What will be the reconstruction error for the data point $\vec{x}$?

(Note that we typically talk about the reconstruction error of an entire data set; here we are concerned with the reconstruction error of a single point.)

8

16

9

32

Explanation

Reconstruction error is the square of euclidean distance between the original point and the reconstructed point. Hence, error = $(2-0)^2 + (\sqrt{2} - \sqrt{2}) + (-2-0)^2 = 8$

Q6

1 Point

Consider a bunch of data points in $\mathcal{R}^3$. The top 3 eigenvectors of its covariance matrix (C) have eigenvalues 12, 8, 7.

What is the *total variance*?

=27+-0

Explanation

The eigenvalues of a covariance matrix tell us the variance in the direction of its eigenvectors. Hence, total variance explained by these 3 components is 12 + 8 + 7 = 27

Q7

1 Point

Suppose you perform PCA on data points from $\mathcal{R}^d \to \mathcal{R}^k$. Each data point gets projected into a new feature space of dimension k.

What percentage of the total variance in the original dataset is captured by PCA if $d = k$?

- 0%
- 100%
- 50%
- Depends on value of k

Explanation

If $d = k$, the reconstruction error is 0. Since the points are projected into the same space, the total variance gets captured.

Q8

1 Point

Let X be a data matrix, and suppose its 4×4 covariance matrix has eigenvalues of 8, 6, 5, and 2.

Now suppose PCA is performed without reducing the dimensionality; that is, the data in X are projected onto the eigenbasis made up of all d eigenvectors of the covariance matrix. Let C_Z be the covariance matrix of this new data set.

True or False: the largest entry along the diagonal of C_Z will be 8.

True

False

Explanation

Remember that PCA "decorrelates" the features; that is, it find a basis in which the covariance matrix is diagonal. So C_Z will be a diagonal matrix, and its diagonal entries will be 8, 6, 5, and 2, exactly.