CSE 151A - Discussion 10

REVIEW : TBD

Problem 1.

What are the eigenvectors and eigenvalues of $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$?

Solution: A is a diagonal matrix, which tells us that the eigenvalues are simply the diagonal entries themselves (see 'spectral theorem' and 'eigendecomposition'). So, $\lambda_1 = 2$, $\lambda_2 = -1$, and $\lambda_3 = 3$. Additionally, this indicates that the eigenvectors are the unit vectors $u_1 = e_1 = (1,0,0)^T$, $u_2 = e_2 = (0,1,0)^T$, and $u_3 = e_3 = (0,0,1)^T$. We can confirm this by checking if $Au = \lambda u$ for all three eigenvector-eigenvalue pairs, which it indeed does.

Problem 2.

Given $A = \begin{pmatrix} 1 & -2 \\ -2 & 1 \end{pmatrix}$ and eigenvectors $u_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $u_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$, first confirm that the eigenvectors are orthonormal, then compute the corresponding eigenvalues.

Solution: To show that two vectors u_1 and u_2 are orthonormal, we must show that $||u_1|| = ||u_2|| = 1$ and that $u_1 \cdot u_2 = 0$.
$||u_1|| = \frac{1}{\sqrt{2}}\sqrt{1^2 + 1^2} = \frac{1}{\sqrt{2}}\sqrt{2} = 1.$
$||u_2|| = \frac{1}{\sqrt{2}}\sqrt{(-1)^2 + 1^2} = \frac{1}{\sqrt{2}}\sqrt{2} = 1.$
$u_1 \cdot u_2 = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}((1)(-1) + (1)(1)) = 0.$
To solve for the corresponding eigenvalues, we must solve $Au_i = \lambda_i u_i$ for both λ_1 and λ_2.
$Au_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ -1 \end{pmatrix} = (-1)\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = -1u_1 \longrightarrow \lambda_1 = -1$
$Au_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} -3 \\ 3 \end{pmatrix} = (3)\frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \end{pmatrix} = 3u_2 \longrightarrow \lambda_2 = 3$