DSC 190 - Homework 09

Due: Wednesday, March 9

Write your solutions to the following problems by either typing them up or handwriting them on another piece of paper. Unless otherwise noted by the problem's instructions, show your work or provide some justification for your answer. Homeworks are due via Gradescope at 11:59 PM.

Problem 1.

Suppose a Bloom filter is designed using a bit array with $c = 15$ buckets and $k = 3$ hash functions. The objects below are inserted into the filter:

object	`hash_1(x)`	`hash_2(x)`	`hash_3(x)`
x_1	12	2	14
x_2	1	5	4
x_3	7	10	3
x_4	5	12	0
x_5	13	6	7
x_6	4	10	4

a) Draw the Bloom filter by specifying the content of each bucket.

Solution:

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1	1	1	1	1	1	1	0	0	1	0	1	1	1

b) Suppose a new element has the following hashes:

`hash_1(x)`: 5
`hash_2(x)`: 3
`hash_3(x)`: 7

When the Bloom filter is asked if x is an element, what does it return?

Solution: The values at indices 5, 3, 7 are all 1, therefore the bloom filter would return true. This is a false positive.

Problem 2.

Suppose a count-min sketch with $k = 3$ rows and $c = 5$ buckets in each row is designed. Assume that the following sequence of objects is inserted:

$$x_1, x_2, x_1, x_1, x_3, x_4, x_3, x_1, x_4, x_1, x_2$$

The values of each of the k hash functions on the objects is shown below.

object	`hash_1(x)`	`hash_2(x)`	`hash_3(x)`
x_1	1	2	3
x_2	4	2	0
x_3	4	1	3
x_4	3	3	4

a) Draw the count-min sketch table by writing the count that is contained within each bucket after the above items are inserted.

Solution: Each time we add an object to the count-min sketch, we increment the value at indices given by the hashes.

0	1	2	3	4
0	5	0	2	4
0	2	7	2	0
2	0	0	7	2

b) Suppose the count of item x_3 is queried. What is returned?

Solution: The count of x_3 is given by $min(4, 2, 7) = 2$