Linear Algebra and Calculus

Unit-I: Matrices

- Introduction of Matrix
- Rank of a Matrix
- Canonical Form of a Matrix (Echelon Form)
- **Normal Form of a Matrix**
- System of Linear Equations
- Orthogonal Transformations
- Eigen Values and Eigen Vectors
- Diagonalization of Matrices
- Cayley Hamilton Theorem
- Applications to problems in Engineering

Normal Form of a Matrix

The following forms are called the normal forms of the matrix.

$$[I_r], \qquad [I_r \quad O], \qquad \begin{bmatrix} I_r \\ O \end{bmatrix}, \qquad \begin{bmatrix} I_r & O \\ O & O \end{bmatrix}$$

where I_r is the identity matrix of order ‘r’ and O is null matrix of suitable order. If any matrix is reduced to its normal form, then the rank of matrix will be equal to the order of $\mathrm{I_r}$.

Example: *Following matrices are in the Normal form*

$$(i)\ \ A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} (ii) \qquad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} (iii)\ \ C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Here the rank of the matrices are $\rho(A) = \rho(B) = \rho(C) = 3$.

Procedure to convert Normal Form of a Matrix

1. *We have to first check that* a_{11} *is one(1) or not?*

2. *If it is not 1, then by using elementary transformation we should make it 1.*

Example: *Following matrices are in the Normal form*

$$(i)\ \ A=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\ (ii)\qquad B=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\\0&0&0\end{bmatrix}\ (iii)\ \ C=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\end{bmatrix}$$

Here the rank of the matrices are $\rho(A)=\rho(B)=\rho(C)=3.$

- ***Example 1:*** *Reduce the matrix* $A = \begin{bmatrix} 3 & 2 & 5 & 2 & 7 \\ 1 & 1 & 2 & 3 & 5 \\ 3 & 3 & 6 & 9 & 15 \end{bmatrix}$ *to its normal form and hence find its rank.*
- **Solution:** Let us define the following terms first.
- Normal Form: *The following forms are called the normal forms of the matrix.*

$$[I_r], \qquad [I_r \quad O], \qquad \begin{bmatrix} I_r \\ O \end{bmatrix}, \qquad \begin{bmatrix} I_r & O \\ O & O \end{bmatrix}$$

where I_r is the identity matrix of order ‘r’ and O is null matrix of suitable order.

Rank of a Matrix: *Rank of a matrix A is the order of highest order non-vanishing (nonzero) minor of it. If the matrix is reduced to its normal form, then the rank of matrix will be equal to the order of* I_r.

- We have to reduce the following matrix to its normal form and also find the rank.

$$A = \begin{bmatrix} 3 & 2 & 5 & 2 & 7 \\ 1 & 1 & 2 & 3 & 5 \\ 3 & 3 & 6 & 9 & 15 \end{bmatrix} \quad \text{---------} \qquad (1)$$

We have

- $A = \begin{bmatrix} 3 & 2 & 5 & 2 & 7 \\ 1 & 1 & 2 & 3 & 5 \\ 3 & 3 & 6 & 9 & 15 \end{bmatrix}$

Apply R_{12}

- $A \sim \begin{bmatrix} 1 & 1 & 2 & 3 & 5 \\ 3 & 2 & 5 & 2 & 7 \\ 3 & 3 & 6 & 9 & 15 \end{bmatrix}$

Apply $R_2 \to R_2 - 3R_1$ and $R_3 \to R_3 - 3R_1$

- $A \sim \begin{bmatrix} 1 & 1 & 2 & 3 & 5 \\ 0 & -1 & -1 & -7 & -8 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Apply $C_2 \to C_2 - C_1$, $C_3 \to C_3 - 2C_1$,

$C_4 \to C_4 - 3C_1$ and $C_5 \to C_5 - 5C_1$, we have

- $A \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & -7 & -8 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

- ***Example 2:*** *Reduce the matrix* $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ *to its normal form and hence find its rank.*
- **Solution:** Let us define the following terms first.
- Normal Form: *The following forms are called the normal forms of the matrix.*

$$[I_r], \qquad [I_r \quad O], \qquad \begin{bmatrix} I_r \\ O \end{bmatrix}, \qquad \begin{bmatrix} I_r & O \\ O & O \end{bmatrix}$$

where I_r is the identity matrix of order 'r' and O is null matrix of suitable order.

Rank of a Matrix: *Rank of a matrix A is the order of highest order non-vanishing (nonzero) minor of it. If the matrix is reduced to its normal form, then the rank of matrix will be equal to the order of* I_r.

- We have to reduce the following matrix to its normal form and also find the rank.

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \end{bmatrix} \text{----------} \qquad (1)$$

We have

- $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

Apply R_{12}

- $A \sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

Apply $R_2 \to R_2 - 2R_1, R_3 \to R_3 - 3R_1$ and $R_4 \to R_4 - 6R_1$

- $A \sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$

Apply $C_2 \to C_2 + C_1, C_3 \to C_3 + 2C_1,$ $C_4 \to C_4 + 4C_1$, we have

- $A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$

Apply $R_2 \to R_2 - R_3$

- $A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -6 & -3 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$
- Apply $R_3 \to R_3 - 4R_2, R_4 \to R_4 - 9R_2$
- $A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -6 & -3 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 66 & 44 \end{bmatrix}$

- $A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -6 & -3 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 66 & 44 \end{bmatrix}$
- Apply $C_3 \to C_3 + 6C_2,\ C_4 \to C_4 + 3C_2$
- $A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 66 & 44 \end{bmatrix}$
- Apply $C_3 \to \frac{1}{33} C_3$
- $A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 22 \\ 0 & 0 & 2 & 44 \end{bmatrix}$
- Apply $R_4 \to R_4 - 2R_3$

$\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$

Home Assignment

Reduce the following matrices to its normal form and hence find the rank.

1. $\begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}$ $[Ans: \rho(A) = 3]$

2. $\begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$ $[Ans: \rho(A) = 3]$

3. $\begin{bmatrix} 1 & 2 & -1 & 2 \\ -2 & -5 & 3 & 0 \\ 1 & 0 & 1 & 10 \end{bmatrix}$ $[Ans: \rho(A) = 2]$

4. $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ $[Ans: \rho(A) = 2]$

5. $\begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$ $[Ans: \rho(A) = 3]$

6. $\begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$ $[Ans: \rho(A) = 4]$

7. $\begin{bmatrix} 3 & -2 & 0 & -1 \\ 0 & 2 & 2 & 7 \\ 1 & -2 & -3 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$ $[Ans: \rho(A) = 4]$

8. $\begin{bmatrix} 3 & 4 & 1 & 1 \\ 2 & 4 & 3 & 6 \\ -1 & -2 & 6 & 4 \\ 1 & -1 & 2 & -3 \end{bmatrix}$ $[Ans: \rho(A) = 4]$

9. $\begin{bmatrix} 2 & -4 & 3 & 1 & 0 \\ 1 & -2 & 1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{bmatrix}$ $[Ans: \rho(A) = 4]$

- ***Example 3:*** *Find non singular matrices P and Q such that PAQ in a normal form. Also find the rank and inverse of the matrix if exits for* $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$.

- **Solution:** We have to find non-singular matrices P and Q such that PAQ is in normal form, where

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} \qquad \text{---------} \qquad (1)$$

Now consider

$$\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Apply $R_1 \rightarrow R_1 - R_2$, we have

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix}1 & 0 & 0\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix} = \begin{bmatrix}1 & -1 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} A \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

Apply $R_2 \to R_2 - 2R_1$, we have

$$\begin{bmatrix}1 & 0 & 0\\ 0 & -3 & 4\\ 0 & -1 & 1\end{bmatrix} = \begin{bmatrix}1 & -1 & 0\\ -2 & 3 & 0\\ 0 & 0 & 1\end{bmatrix} A \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

Apply $C_2 \to C_2 + C_3$, we have

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 4\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}1 & -1 & 0\\ -2 & 3 & 0\\ 0 & 0 & 1\end{bmatrix} A \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 1 & 1\end{bmatrix}$$

Apply $C_3 \to C_3 - 4C_2$, we have

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}1 & -1 & 0\\ -2 & 3 & 0\\ 0 & 0 & 1\end{bmatrix} A \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & -4\\ 0 & 1 & -3\end{bmatrix}$$

$$\Rightarrow \qquad I \quad = \quad P \quad A \quad Q$$

Here $P = \begin{bmatrix}1 & -1 & 0\\ -2 & 3 & 0\\ 0 & 0 & 1\end{bmatrix}$ and $Q = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & -4\\ 0 & 1 & -3\end{bmatrix}$ we have

Here $P = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -4 \\ 0 & 1 & -3 \end{bmatrix}$

Now we have to find the A^{-1}.

Since P and Q are nonsingular matrices, so P^{-1} and Q^{-1} exist.

Consider

$$PAQ = I$$

Operating P^{-1} and Q^{-1} on both sides, we have

$$P^{-1}(PAQ)Q^{-1} = P^{-1}(I)Q^{-1}$$

$$\Rightarrow (P^{-1}P)A(QQ^{-1}) = P^{-1}Q^{-1}$$

$$\Rightarrow (I)A(I) = P^{-1}Q^{-1} [\because AA^{-1} = A^{-1}A = I, \qquad IA = A]$$

$$\Rightarrow A = P^{-1}Q^{-1}$$

Taking inverse on both the sides

$$A^{-1} = (P^{-1}Q^{-1})^{-1}$$

$$= (Q^{-1})^{-1}(P^{-1})^{-1} [\because (AB)^{-1} = B^{-1}A^{-1}]$$

$$\therefore \boxed{A^{-1} = QP} [\because (A^{-1})^{-1} = A]$$

Here $P = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -4 \\ 0 & 1 & -3 \end{bmatrix}$

Putting the values of P and Q , we have

$$A^{-1} = QP = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -4 \\ 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+0 & -1+0+0 & 0+0+0 \\ 0-2+0 & 0+3+0 & 0+0-4 \\ 0-2+0 & 0+3+0 & 0+0-3 \end{bmatrix}$$

$$\therefore \boxed{A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}}$$

Again Consider $AA^{-1} = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

- ***Example 4:*** *Find non singular matrices P and Q such that PAQ in a normal form. Also find the rank and inverse of the matrix if exits for* $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$

- **Solution:** We have to find non-singular matrices P and Q such that PAQ is in normal form, where

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix} \qquad \text{- - - - - - - - -} \qquad (1)$$

Now consider

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Apply $R_2 \to R_2 - 2R_1, R_3 \to R_3 - 3R_1$, we have

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -2 & -5 \\ \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \end{bmatrix}$$

$$\begin{bmatrix}1 & 2 & 3 & 4\\0 & -3 & -2 & -5\\0 & -6 & -4 & -22\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\-2 & 1 & 0\\-3 & 0 & 1\end{bmatrix}A\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$$

Apply $C_2 \to C_2 - 2C_1$, $C_3 \to C_3 - 3C_1$, $C_4 \to C_4 - 4C_1$, we have

$$\begin{bmatrix}1 & 0 & 0 & 0\\0 & -3 & -2 & -5\\0 & -6 & -4 & -22\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\-2 & 1 & 0\\-3 & 0 & 1\end{bmatrix}A\begin{bmatrix}1 & -2 & -3 & -4\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$$

Apply $R_2 \to (-1)R_2$, we have

$$\begin{bmatrix}1 & 0 & 0 & 0\\0 & 3 & 2 & 5\\0 & -6 & -4 & -22\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\2 & -1 & 0\\-3 & 0 & 1\end{bmatrix}A\begin{bmatrix}1 & -2 & -3 & -4\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$$

Apply $C_2 \to C_2 - C_3$, we have

$$\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 2 & 5\\0 & -2 & -4 & -22\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\2 & -1 & 0\\-3 & 0 & 1\end{bmatrix}A\begin{bmatrix}1 & 1 & -3 & -4\\0 & 1 & 0 & 0\\0 & -1 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}1&0&0&0\\0&1&2&5\\0&-2&-4&-22\end{bmatrix}=\begin{bmatrix}1&0&0\\2&-1&0\\-3&0&1\end{bmatrix}A\begin{bmatrix}1&1&-3&-4\\0&1&0&0\\0&-1&1&0\\0&0&0&1\end{bmatrix}$$

Apply $R_3 \to R_3 + 2R_2$, we have

$$\begin{bmatrix}1&0&0&0\\0&1&2&5\\0&0&0&-12\end{bmatrix}=\begin{bmatrix}1&0&0\\2&-1&0\\1&-2&1\end{bmatrix}A\begin{bmatrix}1&1&-3&-4\\0&1&0&0\\0&-1&1&0\\0&0&0&1\end{bmatrix}$$

Apply $C_3 \to C_3 - 2C_2$, $C_4 \to C_4 - 5C_2$, we have

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&-12\end{bmatrix}=\begin{bmatrix}1&0&0\\2&-1&0\\1&-2&1\end{bmatrix}A\begin{bmatrix}1&1&-5&-9\\0&1&-2&-5\\0&-1&3&5\\0&0&0&1\end{bmatrix}$$

Apply C_{34}, we have

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&-12&0\end{bmatrix}=\begin{bmatrix}1&0&0\\2&-1&0\\1&-2&1\end{bmatrix}A\begin{bmatrix}1&1&-9&-5\\0&1&-5&-2\\0&-1&5&3\\0&0&1&0\end{bmatrix}$$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&-12&0\end{bmatrix}=\begin{bmatrix}1&0&0\\2&-1&0\\1&-2&1\end{bmatrix}A\begin{bmatrix}1&1&-9&-5\\0&1&-5&-2\\0&-1&5&3\\0&0&1&0\end{bmatrix}$$

Apply $R_3(-1/12)$, we have

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\end{bmatrix}=\begin{bmatrix}1&0&0\\2&-1&0\\-1/12&1/6&-1/12\end{bmatrix}A\begin{bmatrix}1&1&-9&-5\\0&1&-5&-2\\0&-1&5&3\\0&0&1&0\end{bmatrix}$$

$$\Rightarrow \qquad I \qquad = \qquad P \qquad A \qquad Q$$

Here $P=\begin{bmatrix}1&0&0\\2&-1&0\\-1/12&1/6&-1/12\end{bmatrix}$ and $Q=\begin{bmatrix}1&1&-9&-5\\0&1&-5&-2\\0&-1&5&3\\0&0&1&0\end{bmatrix}$

Rank of the given matrix is 3.

Since the matrix is not a square matrix, so A^{-1} does not exist.

Home Assignment

Find non-singular matrices P and Q such that PAQ is in normal form. Hence find the rank of A and also find A^{-1}.

1. $\begin{bmatrix} 2 & -2 & 3 \\ 3 & -1 & 2 \\ 1 & 2 & -1 \end{bmatrix} \left\{ Ans: \rho(A) = 3,\ A^{-1} = \frac{1}{5}\begin{bmatrix} -3 & 4 & -1 \\ 5 & -5 & 5 \\ 7 & -6 & 4 \end{bmatrix} \right\}$

2. $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix} \left\{ Ans: \rho(A) = 3,\quad A^{-1} = \frac{1}{2}\begin{bmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 2 & -3 & 1 \end{bmatrix} \right\}$

3. $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} \left\{ Ans: \rho(A) = 3,\ A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} \right\}$

4. $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \left\{ Ans: \rho(A) = 3,\ A^{-1} = \frac{1}{2}\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \right\}$

5. $\begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix} \{ Ans: \rho(A) = 3,\quad A^{-1} does\ not\ exist \}$

Questions?

Thanks