

R-LADIES LANCASTER, NOVEMBER 2023

# ESTIMATING EXTREME SEA LEVELS WITH R

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# About me

- PhD in Statistics at STOR-i Centre for Doctoral Training
- PhD in partnership with EDF UK Natural Hazards & Marine Environment R&D team
- Supervisors: Jonathan Tawn and Dafni Sifnioti
- Collaborators include Jenny Samson (Environment Agency), Jo Williams (NOC), Tom Howard (Met Office) and Ivan Haigh (University of Southampton)
- Founded diverSTOR-i network - focus on safe space for coding



# Agenda

■ **01** Extreme Value Statistics

■ **02** Extremes in R

■ **03** Extreme Sea Level Estimation

■ **04** ESLestimation R package

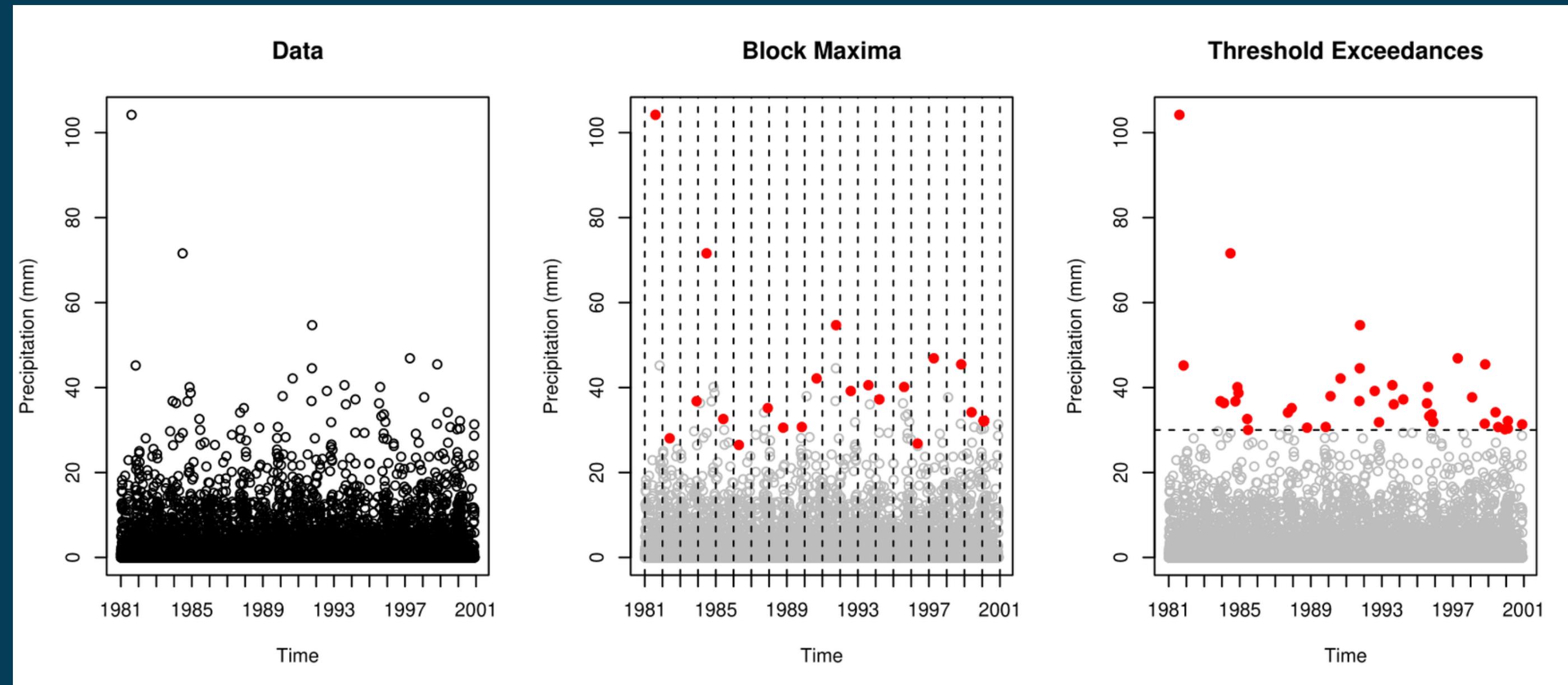
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# EXTREME VALUE STATISTICS

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# What are extremes?



# Modelling extremes

1. Block maxima -> Generalised Extreme Value distribution (GEV)
2. Threshold Exceedances -> Generalised Pareto Distribution (GPD)
  1. Scale:  $\sigma$
  2. Shape:  $\xi$
  3. Rate:  $\lambda$

## Return levels

Level we expect to be exceeded once a year with probability  $p$

- Correspond to return period  $1/p$
- E.g.  $p = 0.1$  corresponds to 10 years
- Require statistical models for extrapolation

# Applications

## Accounting for Climate Change in Extreme Sea Level Estimation

by  Eleanor D'Arcy <sup>1,\*</sup>  ,  Jonathan A. Tawn <sup>1</sup> and  Dafni E. Sifnioti <sup>2</sup> 

Modelling Extremes of Spatial Aggregates of Precipitation  
using Conditional Methods

Jordan Richards<sup>A,B</sup>, Jonathan A. Tawn<sup>A</sup>, and Simon Brown<sup>C</sup>

## A temperature dependent extreme value analysis of UK surface ozone, 1980–2019

Lily Gouldsborough   , Ryan Hossaini <sup>a,b</sup>, Emma Eastoe <sup>c</sup>, Paul J. Young <sup>a,b</sup>

## Inference for extreme spatial temperature events in a changing climate with application to Ireland

Dáire Healy<sup>1</sup>, Jonathan Tawn<sup>2</sup>, Peter Thorne<sup>3</sup>, and Andrew Parnell<sup>1</sup>

## On spatial conditional extremes for ocean storm severity

R. Shooter<sup>1</sup>  | E. Ross<sup>2</sup> | J. Tawn<sup>3</sup> | P. Jonathan<sup>3,4</sup> 

## A marginal modelling approach for predicting wildfire extremes across the contiguous United States

Eleanor D'Arcy<sup>1</sup> · Callum J. R. Murphy-Barltrop<sup>1</sup>  · Rob Shooter<sup>2</sup> · Emma S. Simpson<sup>3</sup>

## INFERENCE FOR EXTREME EARTHQUAKE MAGNITUDES ACCOUNTING FOR A TIME-VARYING MEASUREMENT PROCESS

BY ZAK VARTY<sup>1</sup>, JONATHAN A. TAWN<sup>1,\*</sup> PETER M. ATKINSON<sup>1,†</sup> AND STIJN BIERMAN<sup>2,‡</sup>

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# EXTREMES IN R

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# Packages

- evd
- extRemes
- fExtremes
- ismev
- mev
- POT

For installing R packages:

```
> install.packages('evd')  
> library(evd)
```

Check out the reference manual on CRAN

Belzile, L. R., Dutang, C., Northrop, P. J., & Opitz, T. (2023). A modeler's guide to extreme value software. *Extremes*, 1-44.

CRAN task view: <https://cran.r-project.org/web/views/ExtremeValue.html>

```
> ctv::install.views("ExtremeValue", coreOnly = TRUE)
```

# Data

Sea level observations at Heysham (m) recorded twice daily from 1964-2016.

Data is publicly available on BODC:

[https://www.bodc.ac.uk/data/hosted\\_data\\_systems/sea\\_level/uk\\_tide\\_gauge\\_network/](https://www.bodc.ac.uk/data/hosted_data_systems/sea_level/uk_tide_gauge_network/)

```
> load('Heysham.RData')  
> head(Heysham)
```

	datetime	sealevel
1	1964-01-01 12:00:00	9.86981
2	1964-01-02 01:00:00	9.80881
3	1964-01-02 13:00:00	9.99181
4	1964-01-03 02:00:00	9.29081
5	1964-01-03 14:00:00	9.56481
6	1964-01-04 02:00:00	8.83381

```
> dim(Heysham)
```

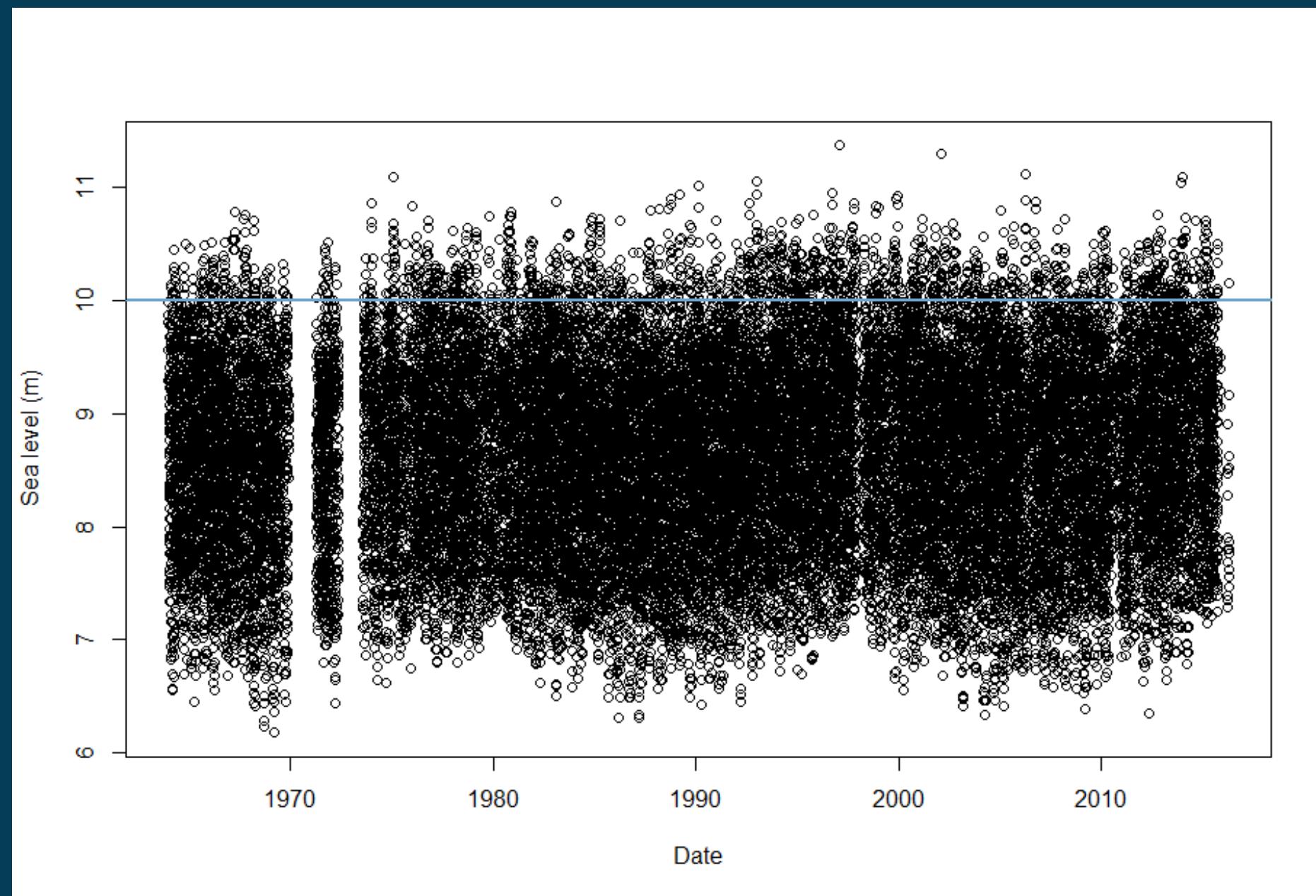
```
[1] 30827      2
```

```
> summary(Heysham)
```

	datetime	sealevel
Min.	:1964-01-01 12:00:00	Min. : 6.174
1st Qu.	:1979-05-08 15:00:00	1st Qu.: 8.009
Median	:1991-04-03 14:00:00	Median : 8.680
Mean	:1990-10-31 12:40:52	Mean : 8.650
3rd Qu.	:2003-03-23 20:30:00	3rd Qu.: 9.280
Max.	:2016-04-16 19:00:00	Max. :11.374

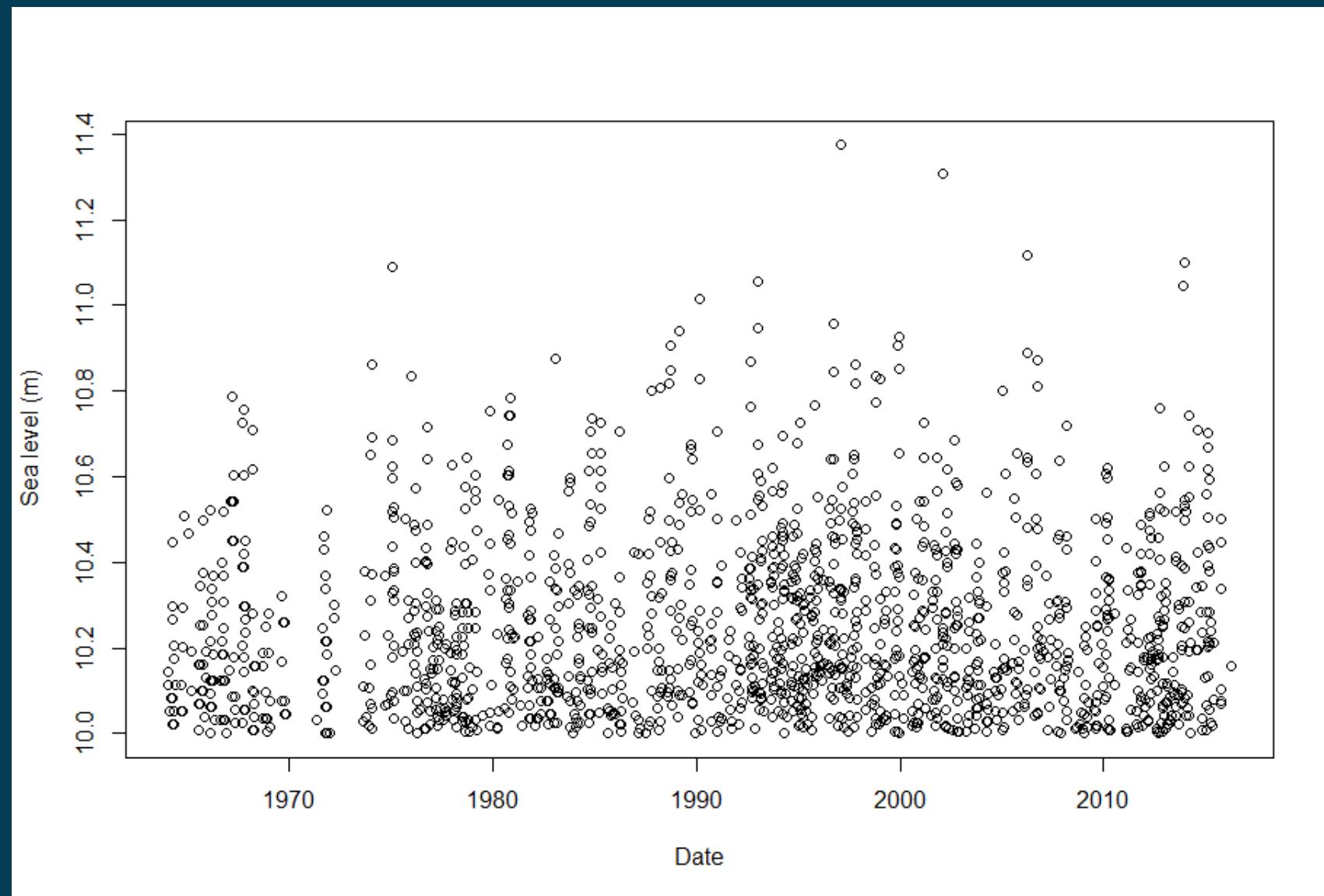
# Data visualisation

```
> plot(Heysham$datetime, Heysham$sealevel, xlab='Date', ylab='Sea level (m)')  
> abline(h=10)
```



# Extreme observations

```
> plot(Heysham$datetime[which(Heysham$sealevel>10)],  
      Heysham$sealevel[which(Heysham$sealevel>10)],  
      xlab='Date', ylab='Sea level (m)')
```



# Model fitting

```
> library(evd)  
> model <- gpd.fit(Heysham$sealevel, 10)
```

Number of exceedances

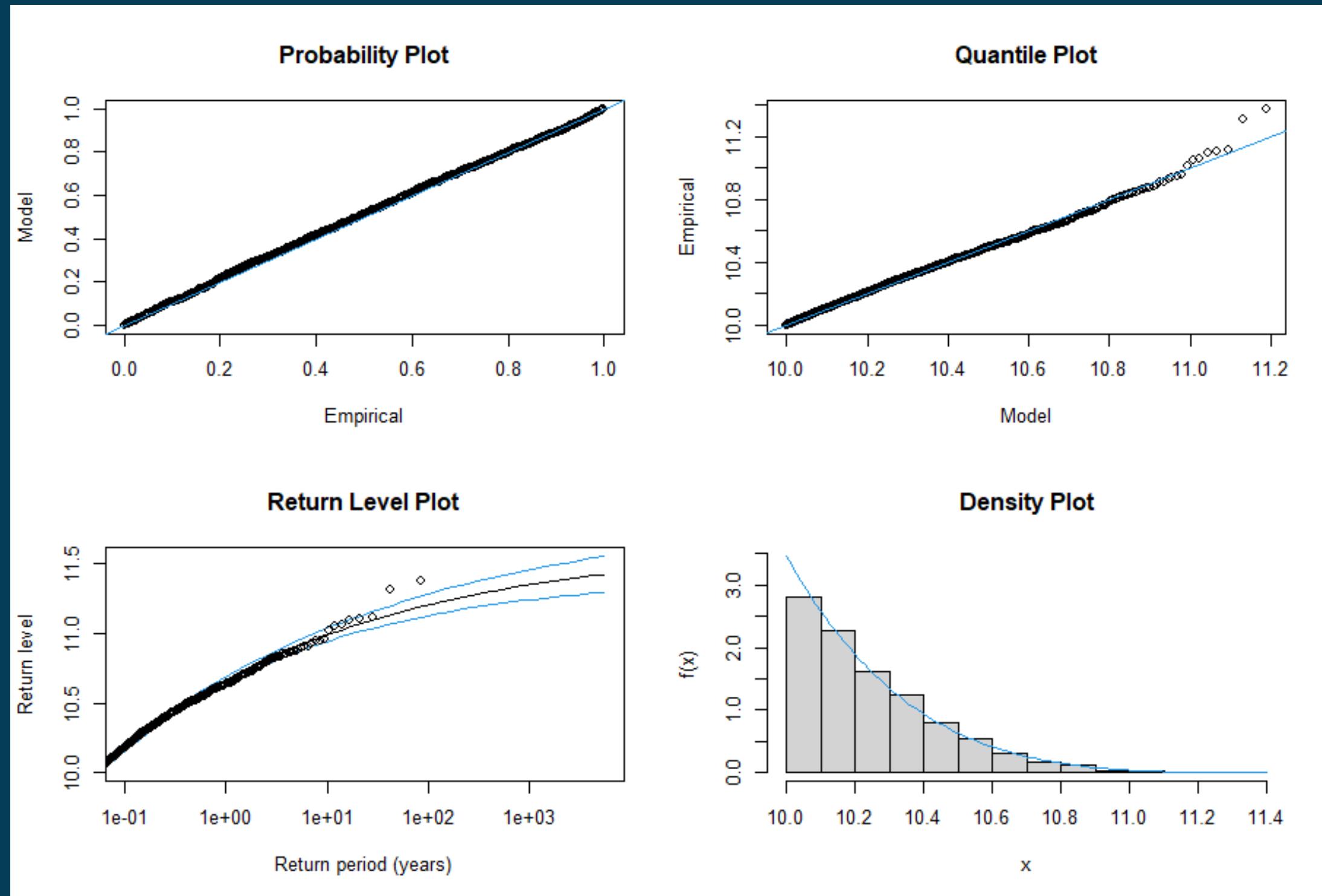
Parameter estimates

Standard error

```
$threshold  
[1] 10  
  
$nexc  
[1] 1590  
  
$conv  
[1] 0  
  
$nllh  
[1] -663.1941  
  
$mle  
[1] 0.2895677 -0.1777515  
  
$rate  
[1] 0.05157816  
  
$se  
[1] 0.008554979 0.016246434
```

# Model diagnostics

```
> gpd.diag(model)
```



# Model fitting - extRemes

```
> library(extRemes)
> model <- fevd(Heysham$sealevel, threshold = 10, type='GP' )
> model
```

```
fevd(x = Heysham$sealevel, threshold = 10, type = "GP")
[1] "Estimation Method used: MLE"

Negative Log-Likelihood value: -663.1941

Estimated parameters:
  scale      shape
0.2895837 -0.1777924

Standard Error Estimates:
  scale      shape
0.008554627 0.016238489
```

# Return levels

```
> return.level(model, do.ci=TRUE)
fevd(x = Heysham$sealevel, threshold = 10, type = "GP")
[1] "Normal Approx."


|                       | 95% lower CI | Estimate | 95% upper CI |
|-----------------------|--------------|----------|--------------|
| 2-year return level   | 10.73380     | 10.77441 | 10.81501     |
| 20-year return level  | 10.99302     | 11.06142 | 11.12983     |
| 100-year return level | 11.10902     | 11.20261 | 11.29619     |


> return.level(model, return.period = c(1000, 10000), do.ci=TRUE)
fevd(x = Heysham$sealevel, threshold = 10, type = "GP")
[1] "Normal Approx."


|                         | 95% lower CI | Estimate | 95% upper CI |
|-------------------------|--------------|----------|--------------|
| 1000-year return level  | 11.21151     | 11.34577 | 11.48004     |
| 10000-year return level | 11.25973     | 11.44084 | 11.62196     |


```

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# EXTREME SEA LEVEL ESTIMATION

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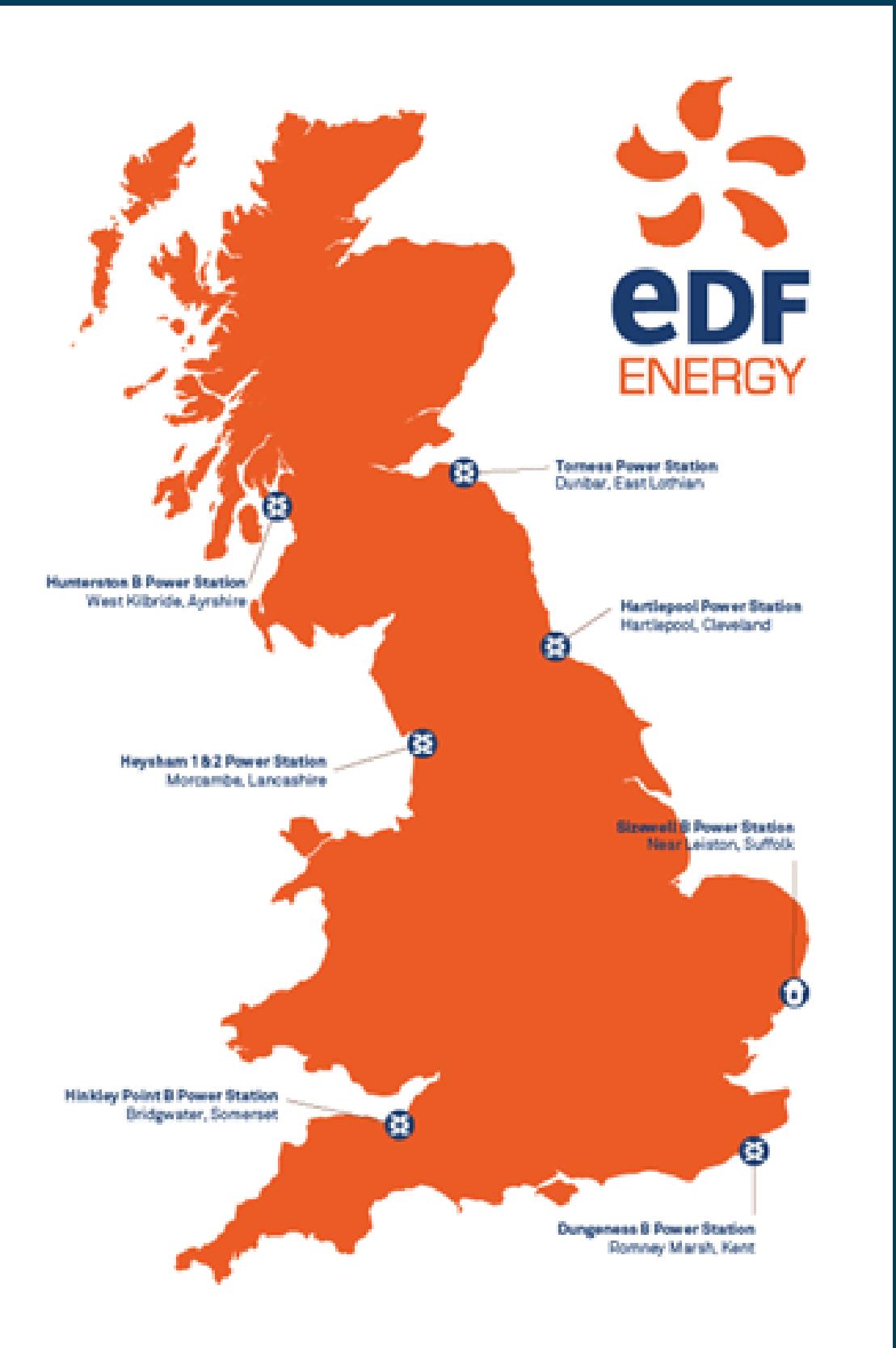
Eleanor D'Arcy  
Lancaster University

# Motivation

Consequences of coastal flooding:

- Loss of life
- Damage to property and infrastructure
- Coastal erosion
- Displacement of people
- Loss of habitats and ecosystems

The Office for Nuclear Regulation are interested in  
10,000 year return level estimates



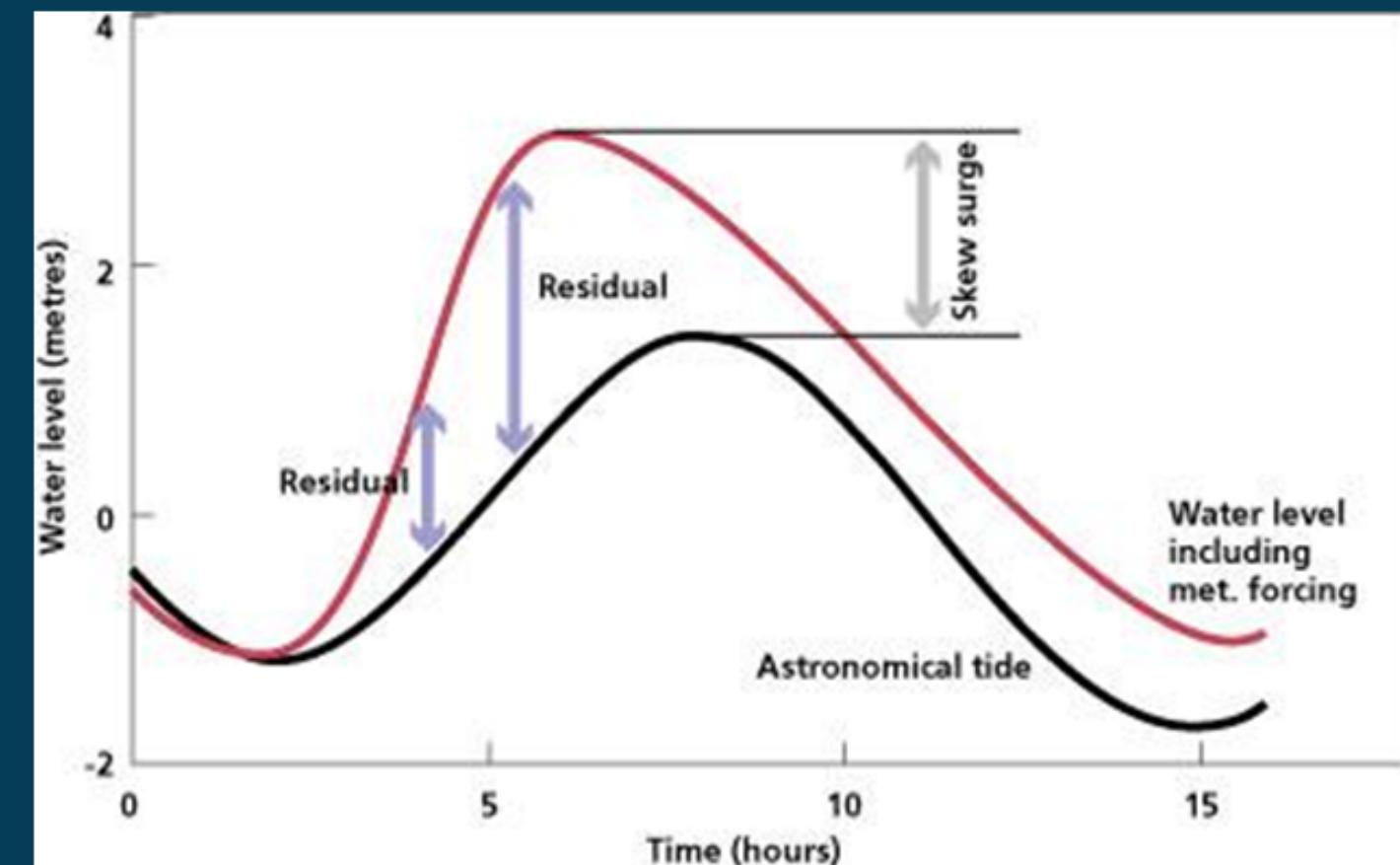
# Sea level components

## Tides

- Predictable rise and fall of the sea surface
- Can be well predicted far in advance

## Storm surge

- Short term sea level changes caused by the weather
- Not accurately predictable



**Sea Level = Mean Sea Level + Skew surge + Peak Tide**

# Current methodology

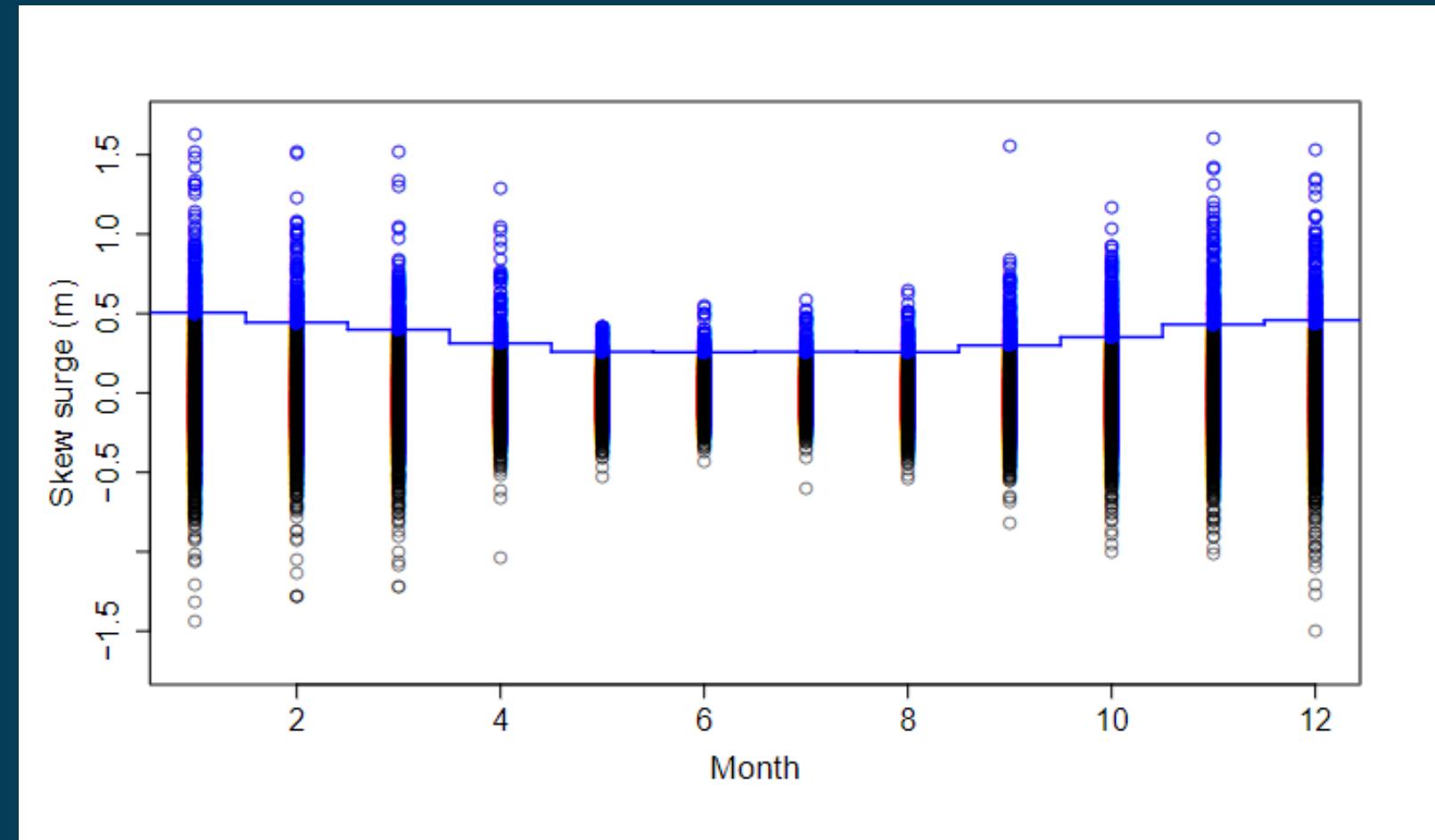
- Skew surges are independent and identically distributed
  - Model extremes using a GPD
- Tides are stationary
- Skew surge and peak tide are independent of each other

Batstone, C. et. al. (2013) *A UK best-practice approach for extreme sea-level analysis along complex topographic coastlines*. Ocean Engineering, 71, pp.28-39.

**Aim: Correct these and estimate extreme sea levels**

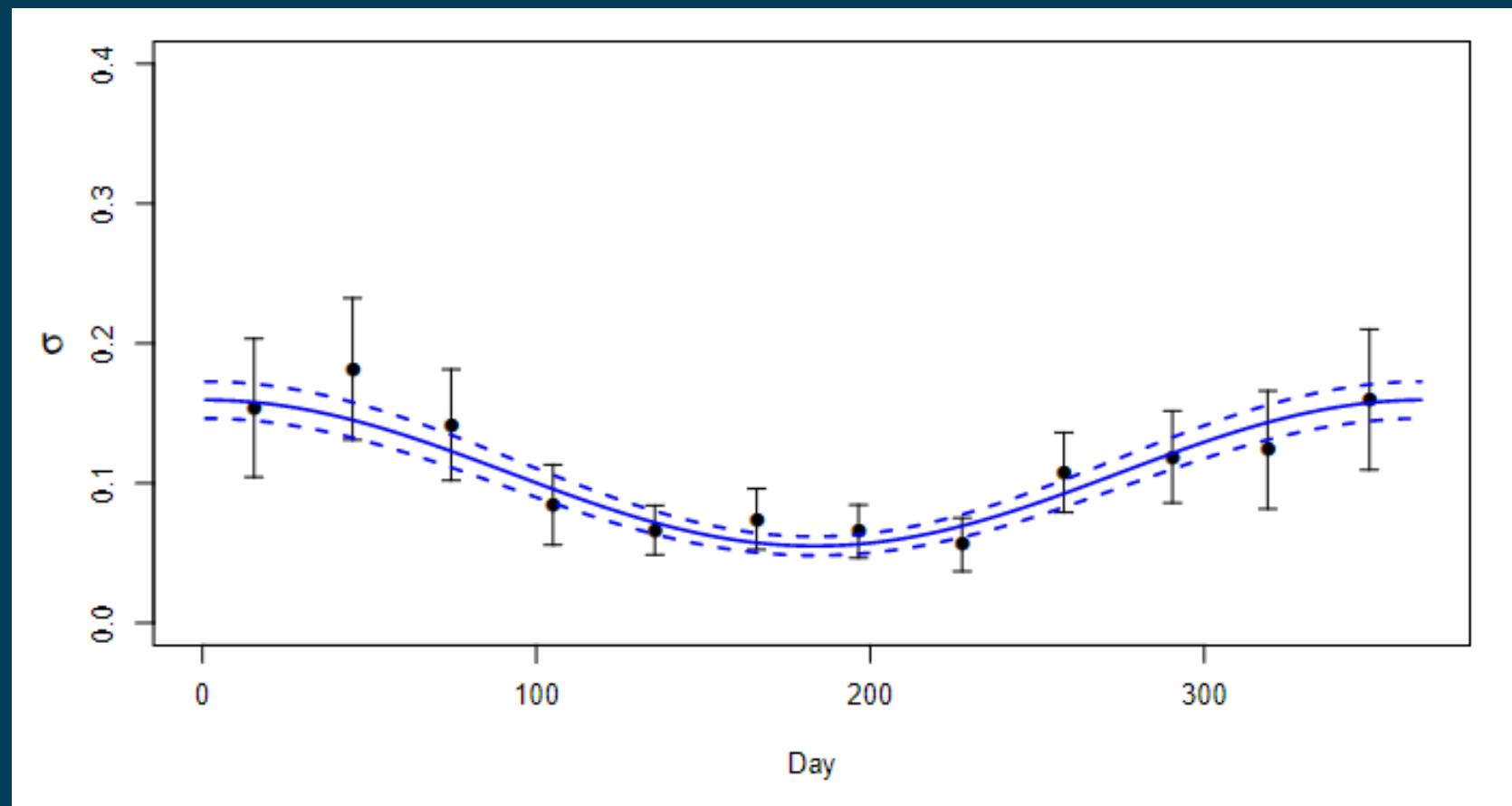
# 1: Accounting for Seasonality

# Skew surge seasonality



- Monthly threshold
- For non-extremes, we use the monthly empirical distribution
- For extremes, we use the non-stationary GPD

# Skew surge modelling



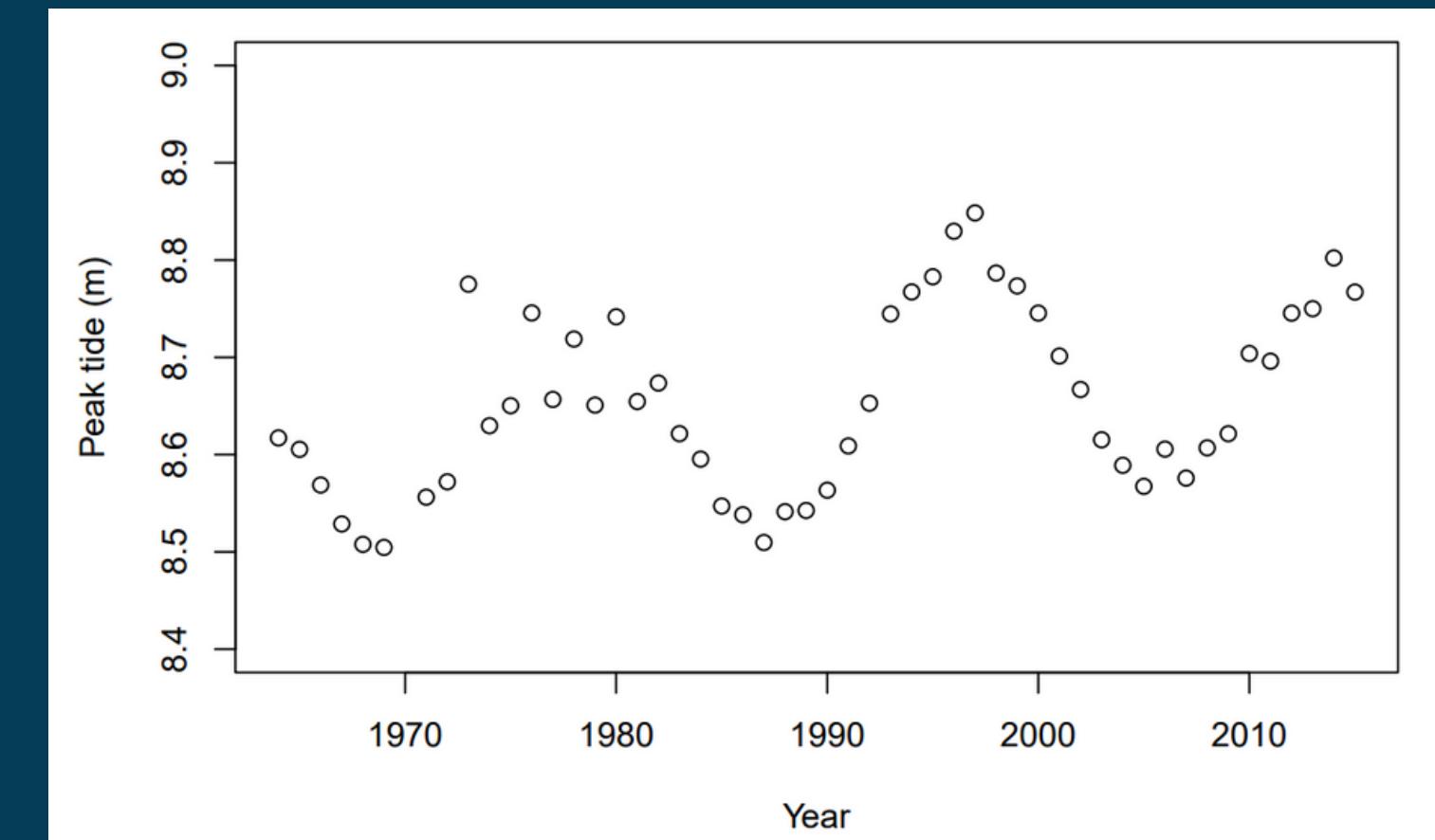
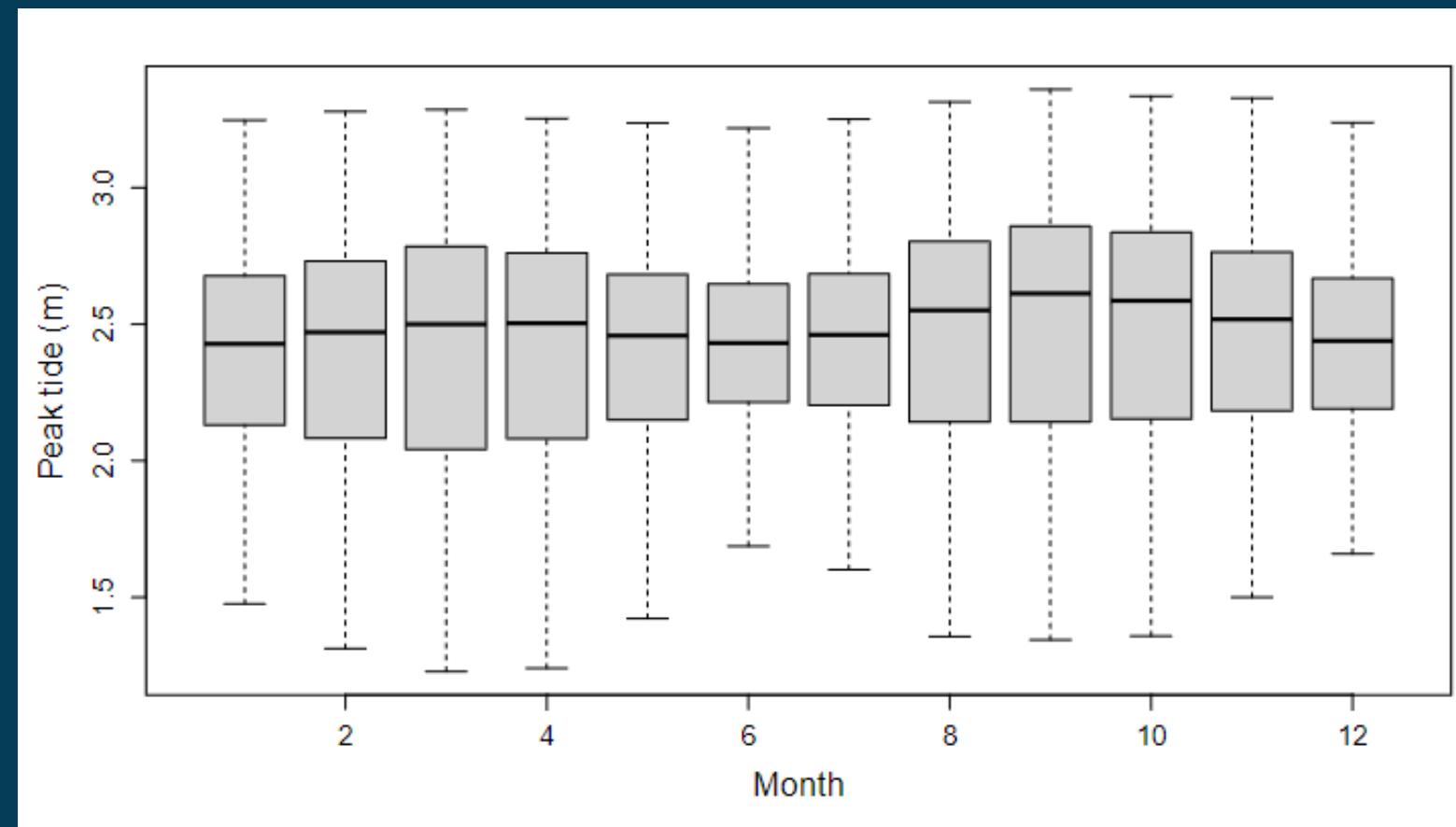
$$Y \sim GPD(\sigma_d, \xi, \lambda_d)$$

daily covariate

$$\sigma_d = \alpha_\sigma + \beta_\sigma \sin\left(\frac{2\pi}{365}(d - \phi_\sigma)\right)$$

parameters

# Peak tide seasonality



Note: Tides are deterministic so don't require modelling

# 2: Skew surge-peak tide dependence

# SS-PT dependence

- Can be assumed as independent at most sites
- Extreme skew surges tend to occur on lower peak tides
- For extreme skew surge,

$$Y \sim GPD(\sigma_{d,t}, \xi, \lambda_{d,t})$$

- Relationship changes throughout the year

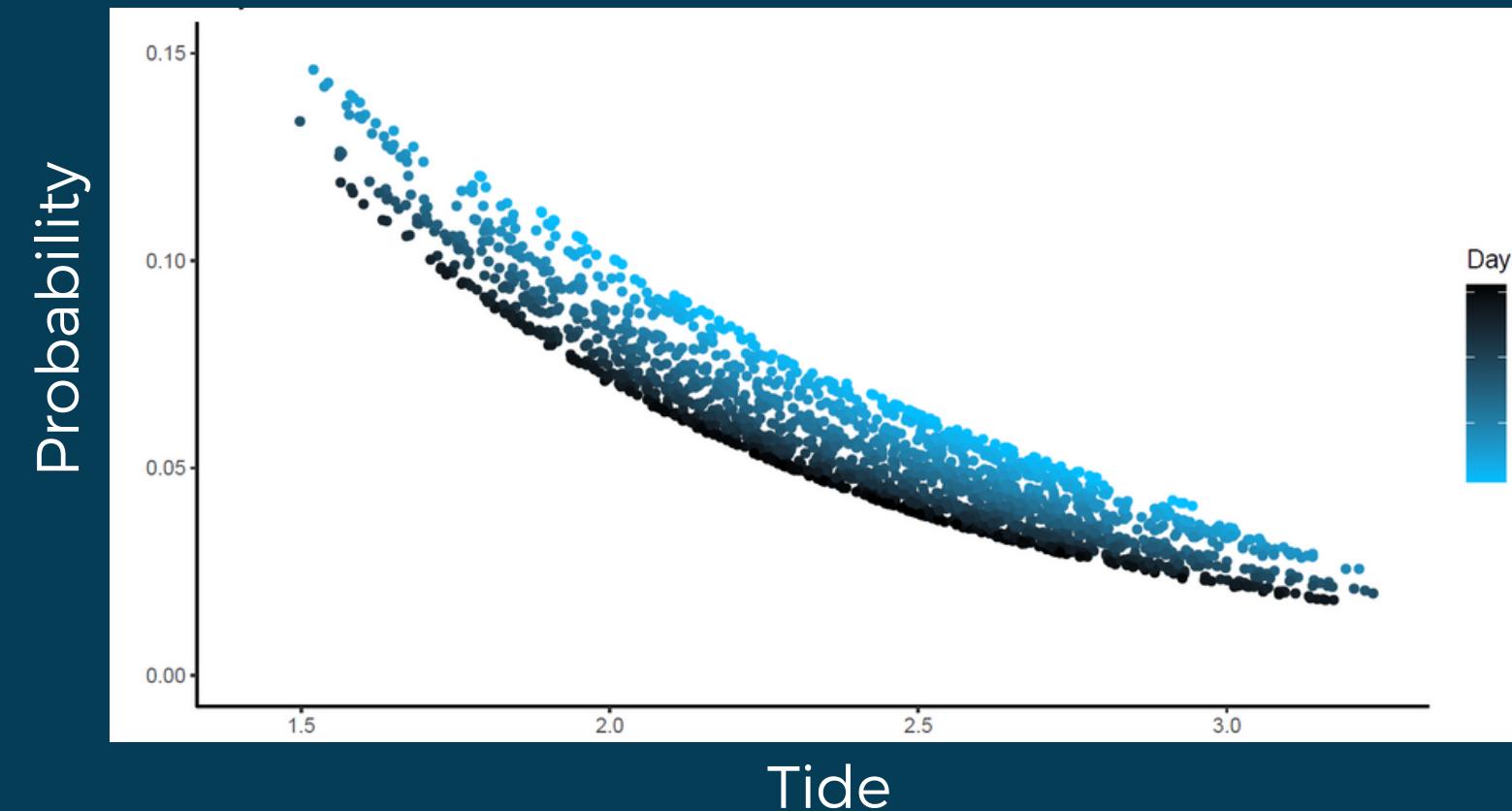
# SS-PT dependence



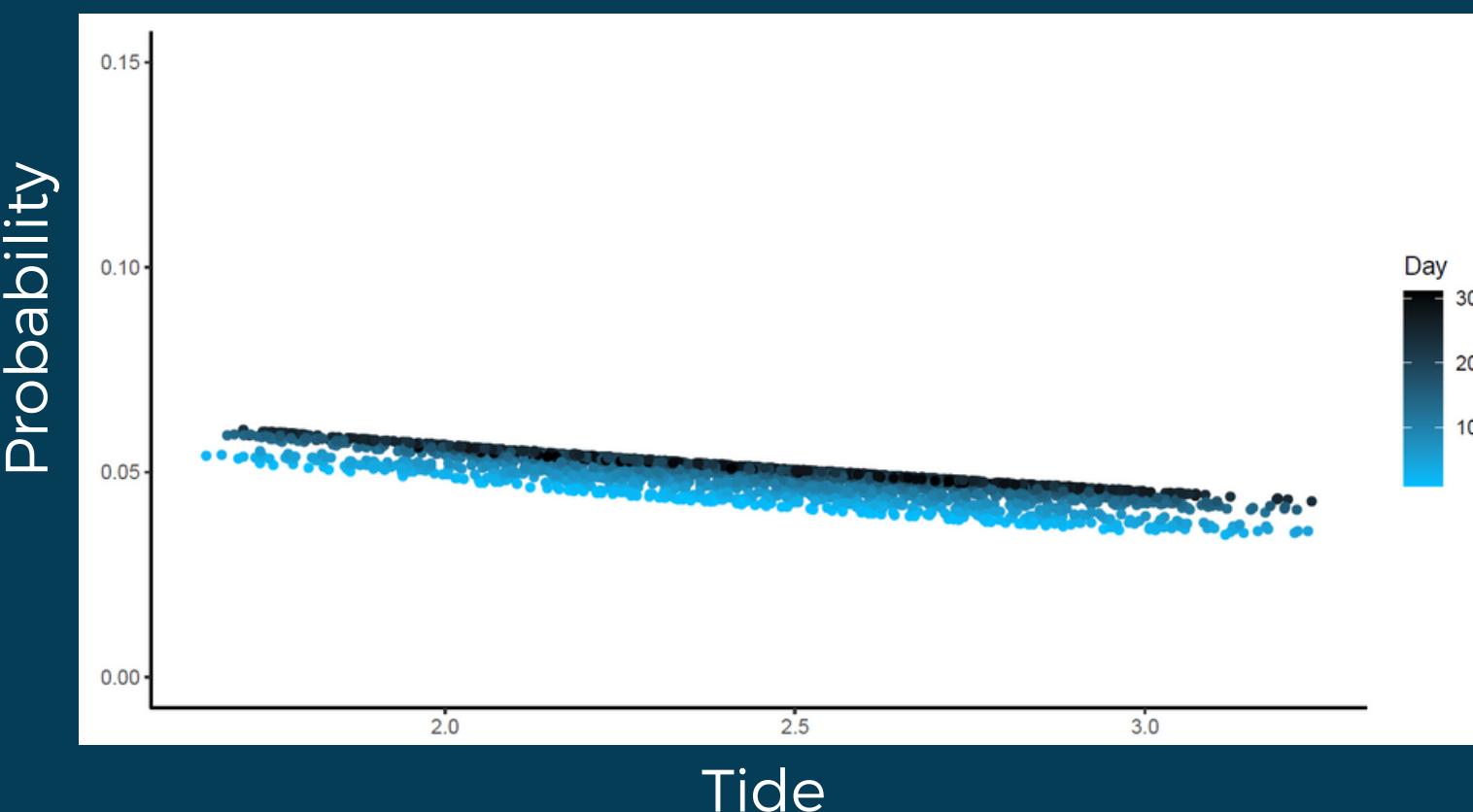
link function

$$g(\lambda_{d,x}) = g(\lambda_d) + \tilde{x} \left[ \alpha_\lambda + \beta_\lambda \sin \left( \frac{2\pi}{365} (d - \phi_\lambda) \right) \right]$$

May



December

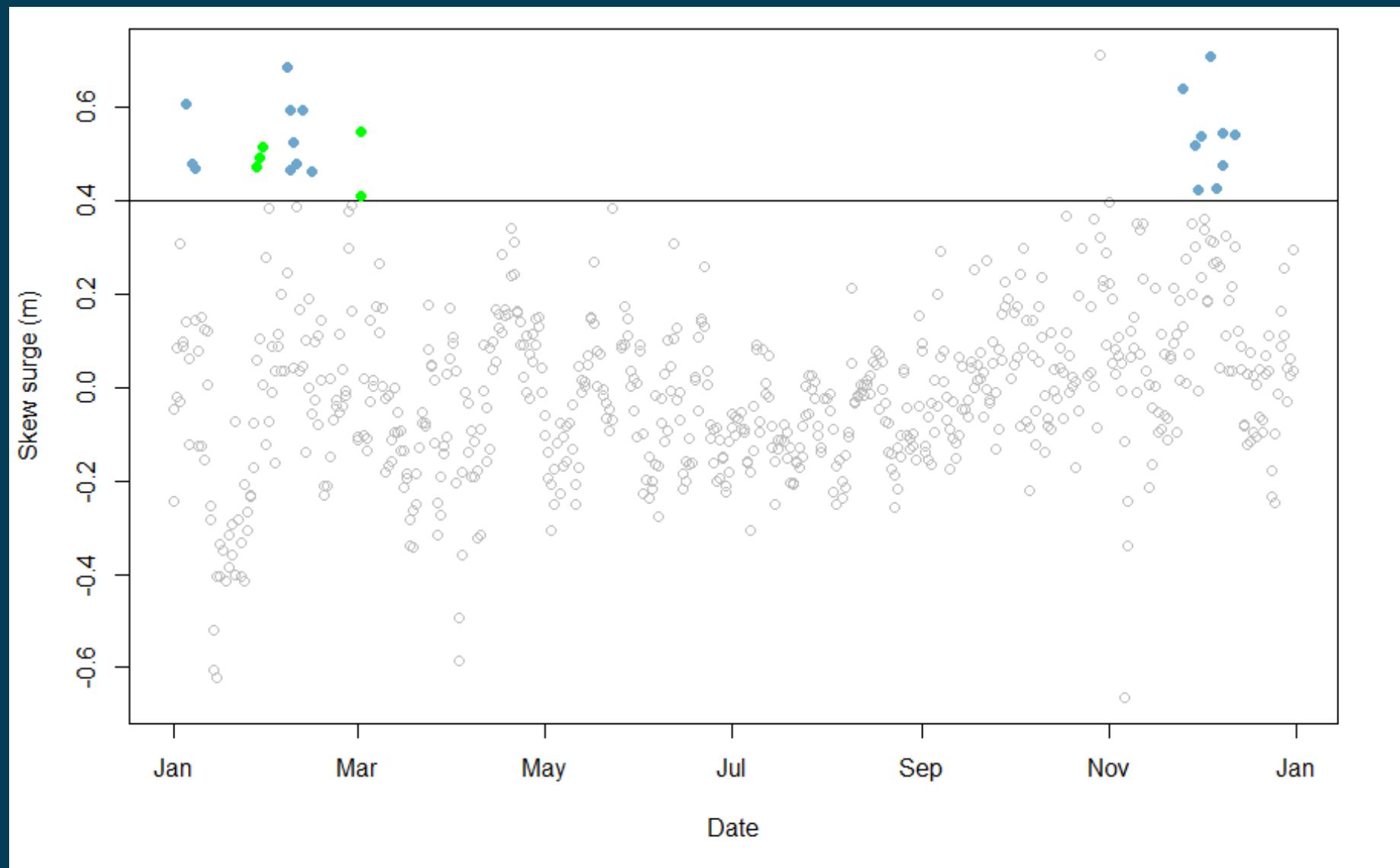


# 3: Skew surge temporal dependence

# Clusters of extremes

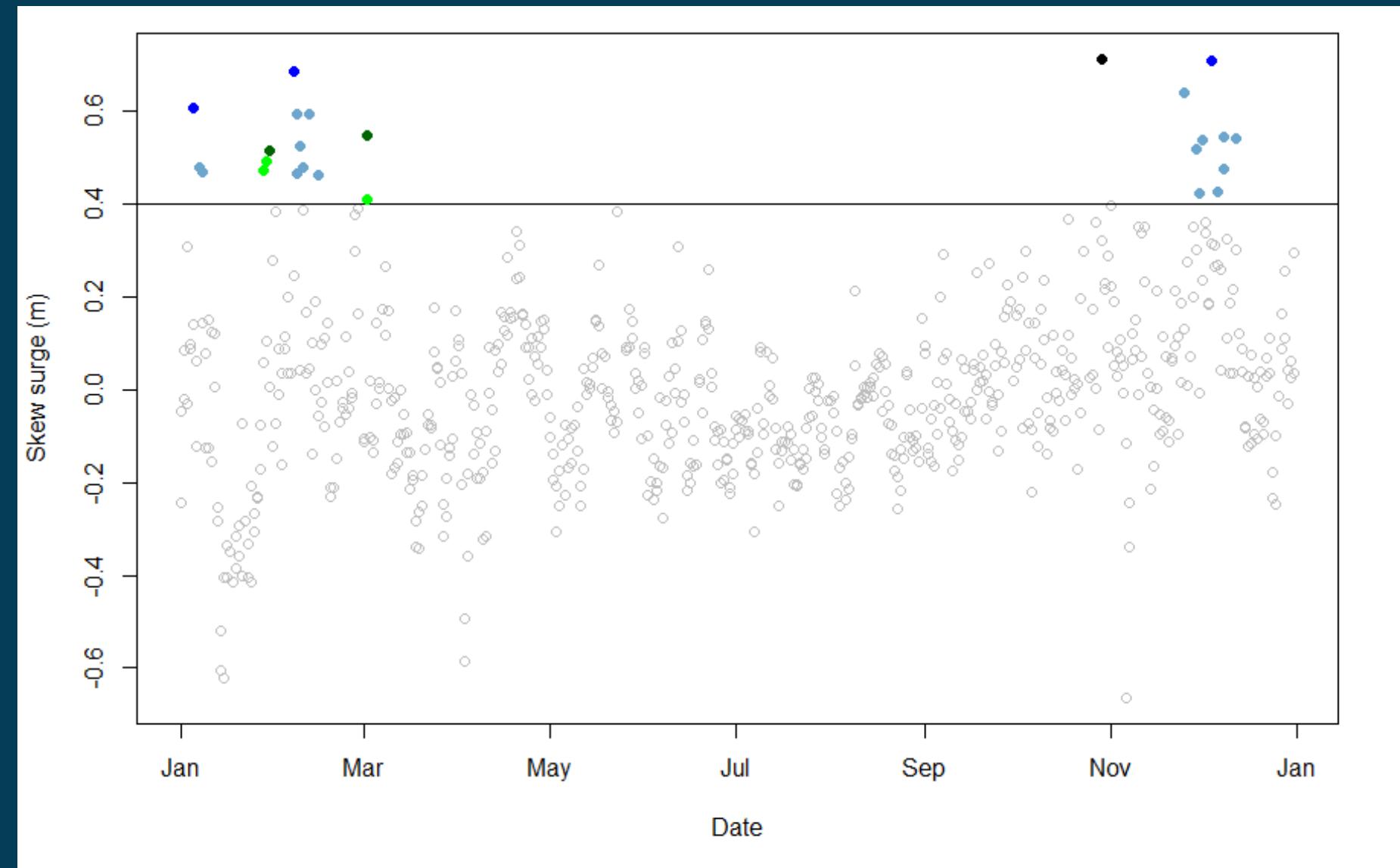
Clusters of extreme skew surge occur when a storm lasts multiple days.

Aim: Identify individual storm events



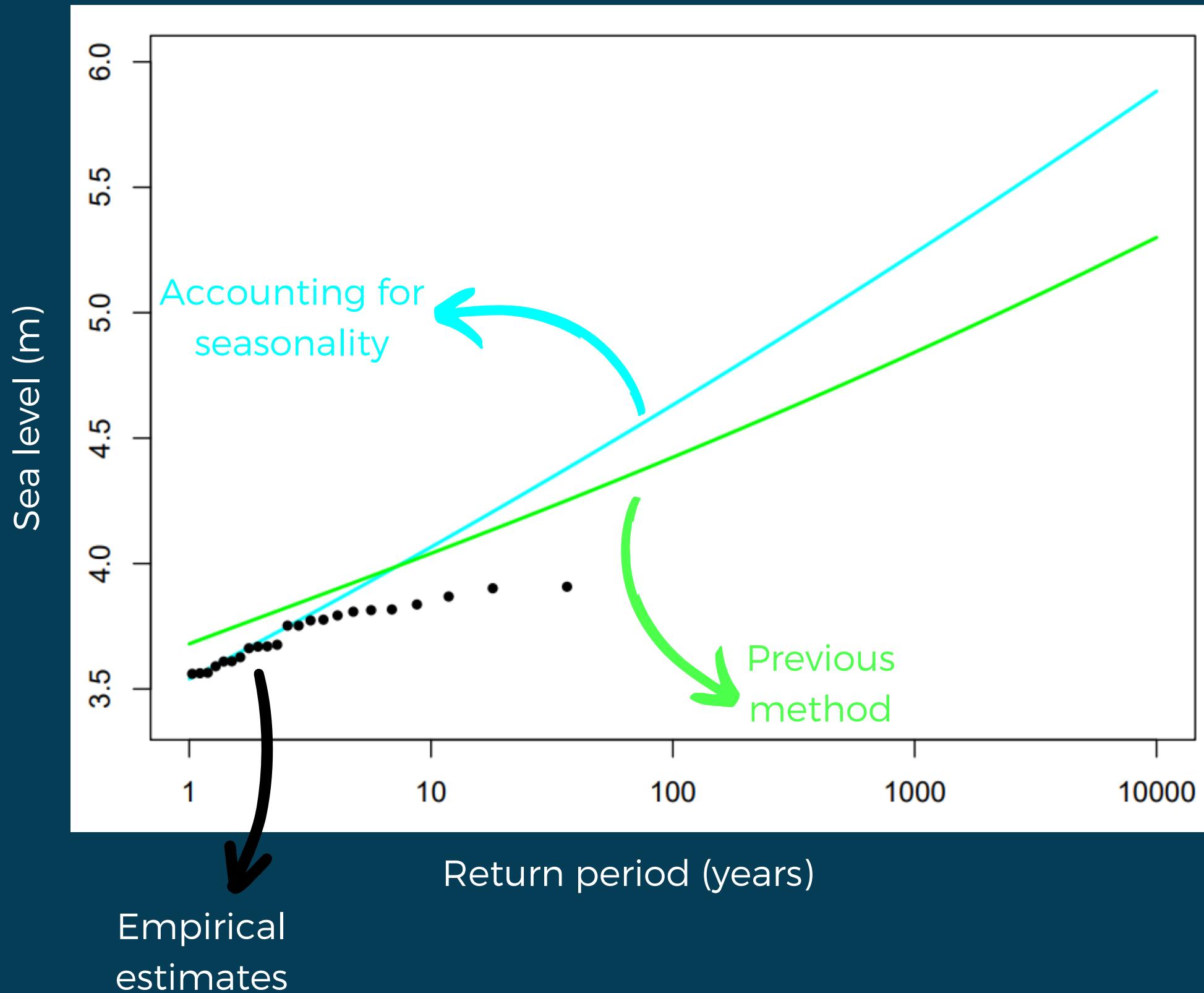
# Clusters of extremes

Decluster the data by modelling cluster maxima

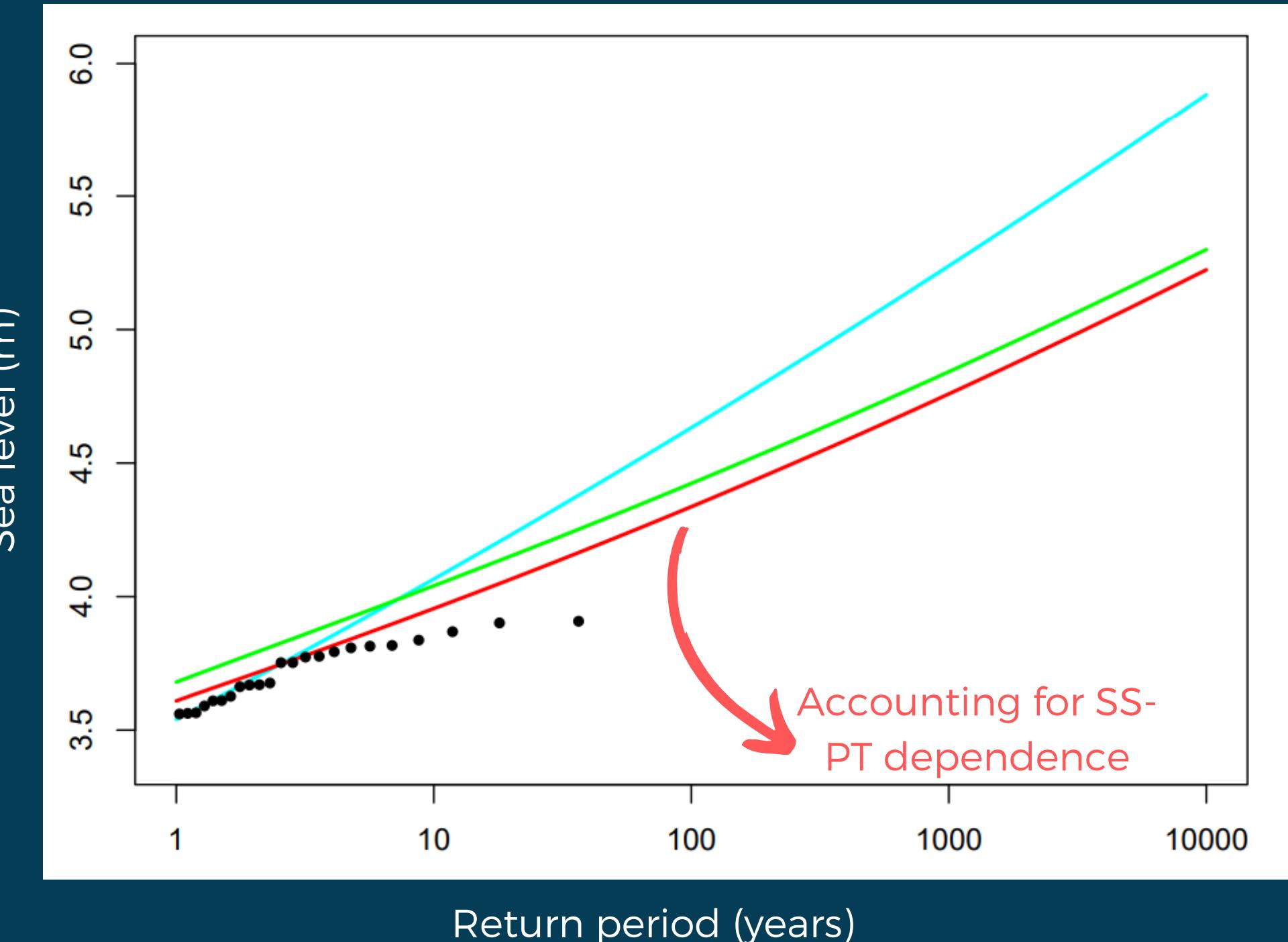


# Results

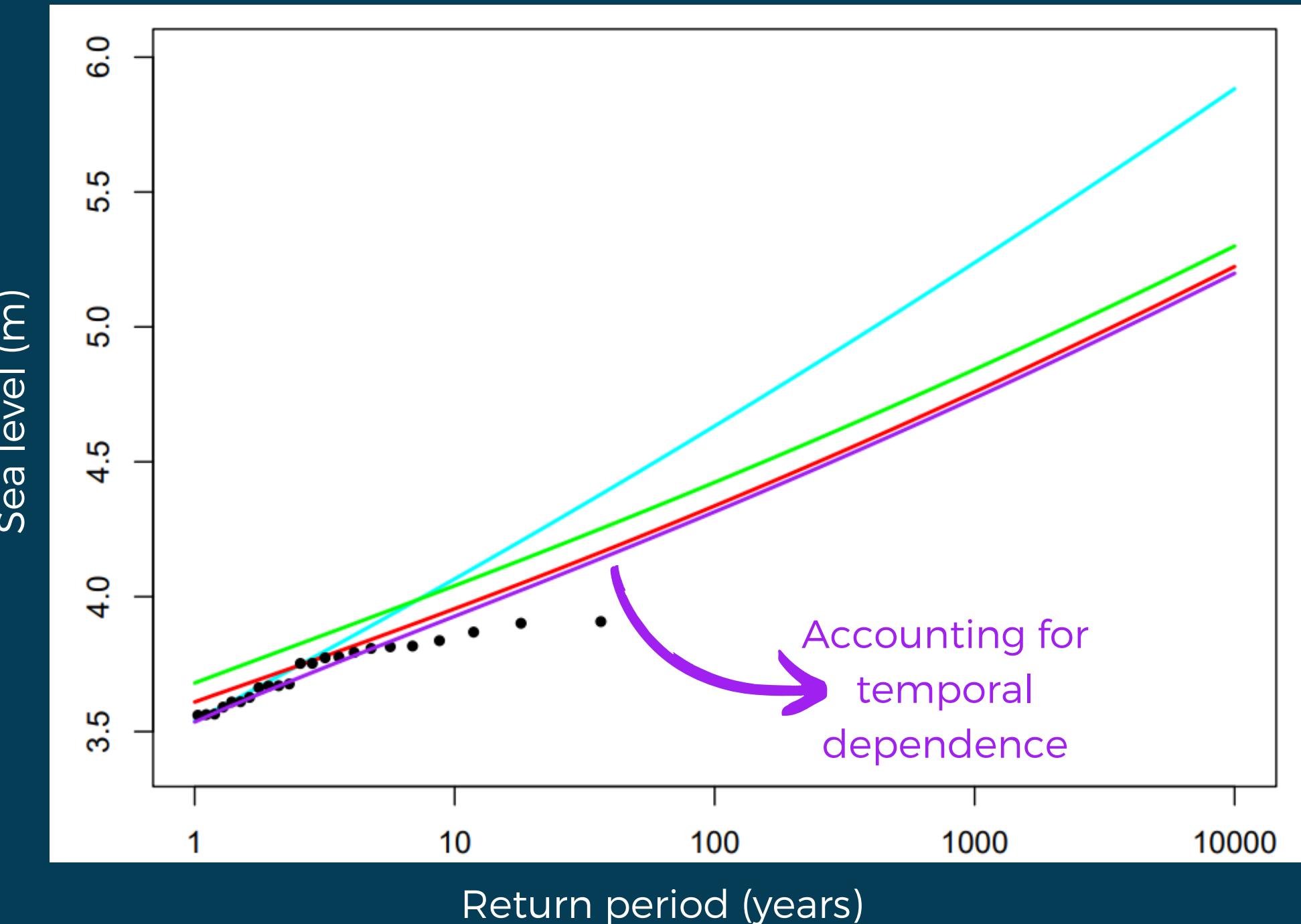
# Annual return levels



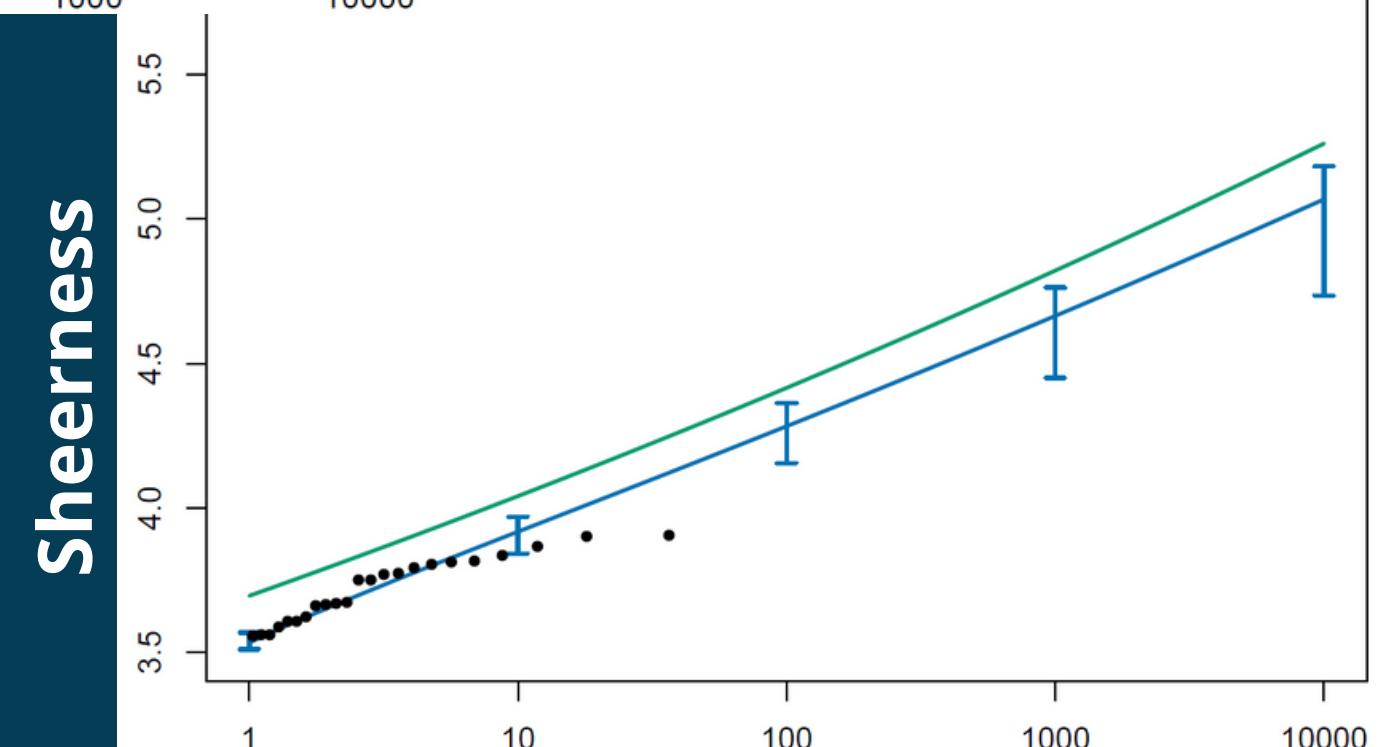
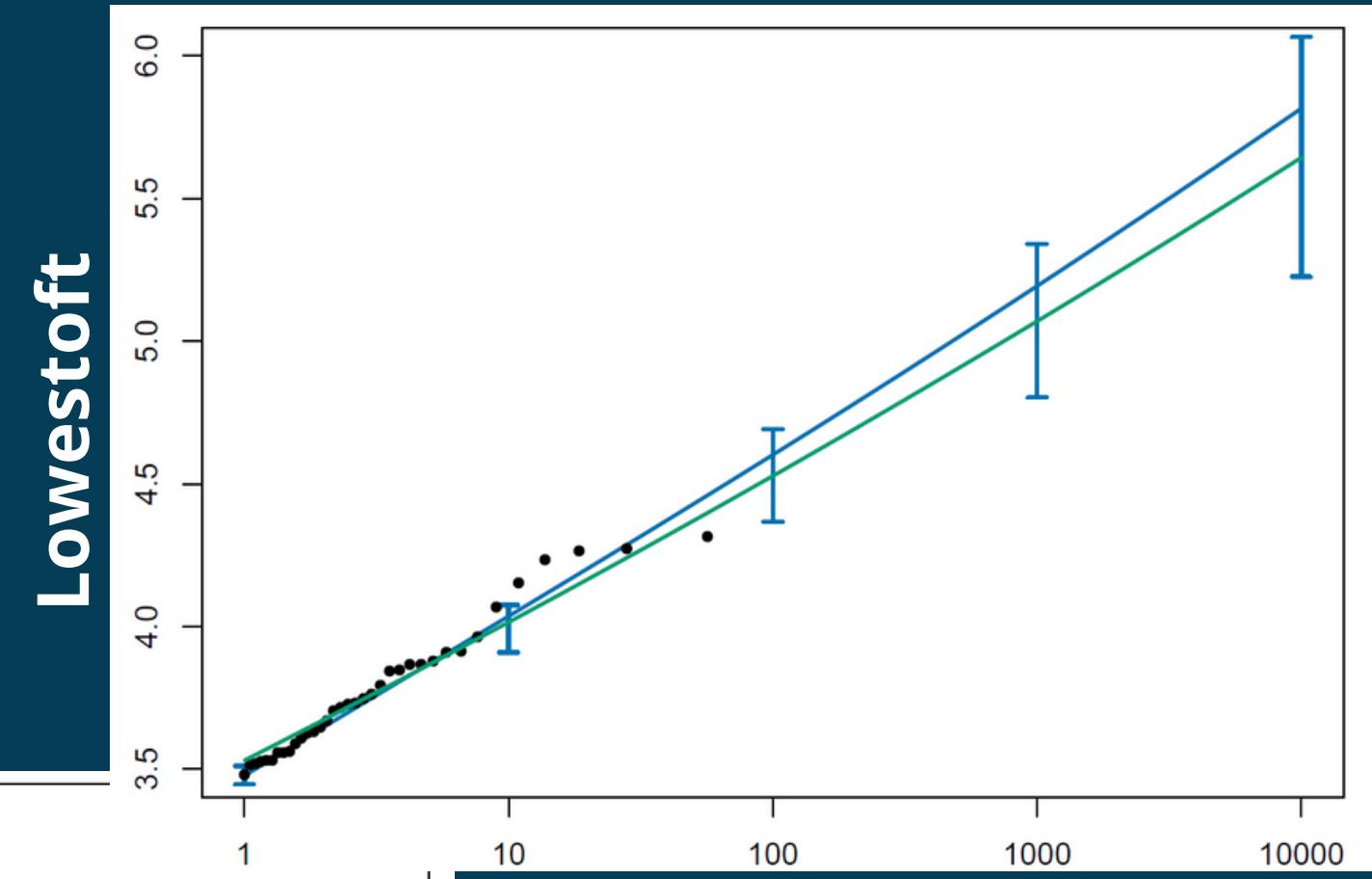
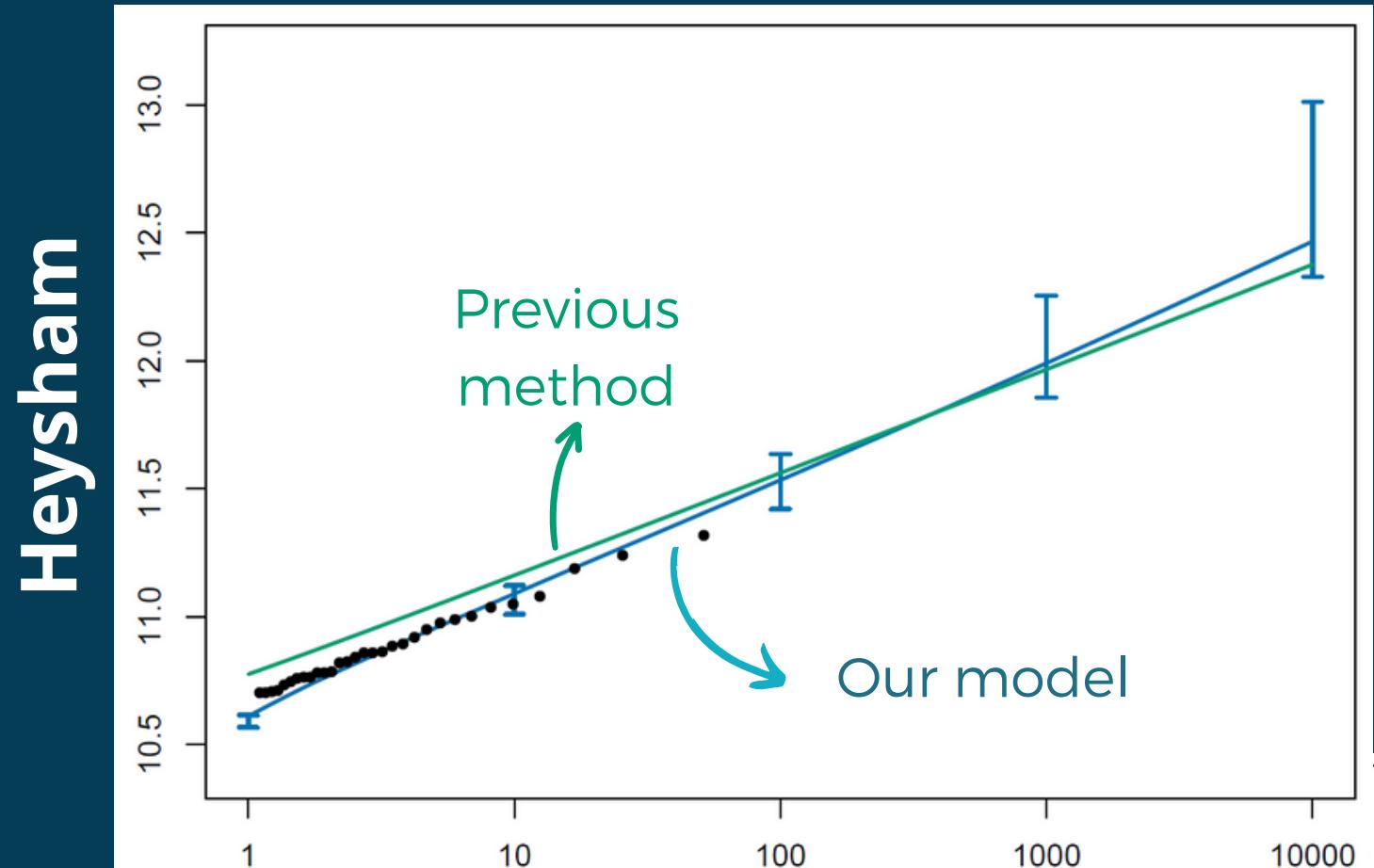
# Annual return levels



# Annual return levels



# Results at other sites



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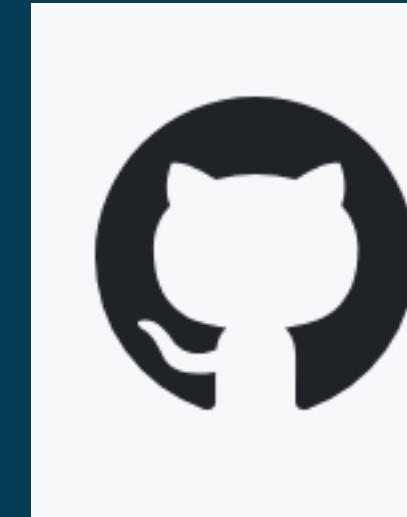
# EXTREME SEA LEVEL ESTIMATION WITH R

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# My Code

- *ESL\_estimation.R* for return level estimation from our model,
- Exploratory analysis,
- Diagnostics,
- Return level estimation from different models,
- Data formulation
- Illustrative example at Lowestoft



eleanordarcy /  
**ESL\_estimation**

R package in development  
with support from Dan Grose

# Data

- Preprocessing to remove trends in mean sea level

```
> load('data_HEY.RData')  
> head(data)
```

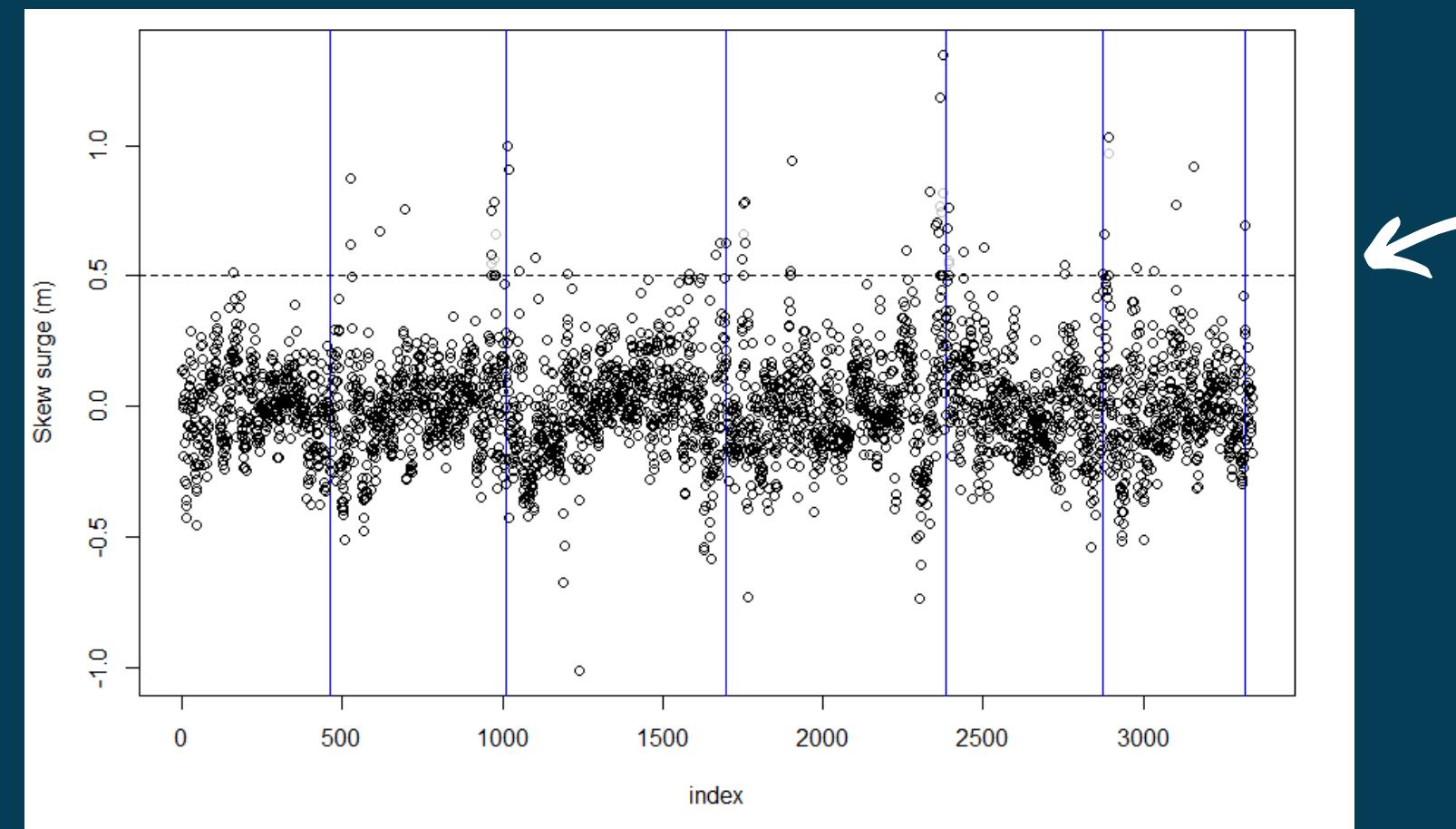
		date	day	month	year	skews	maxTide
1	1964-01-01	1		1	1964	-0.01219025	9.882
2	1964-01-02	2		1	1964	0.07980975	9.729
3	1964-01-02	2		1	1964	0.15980975	9.832
4	1964-01-03	3		1	1964	-0.04319025	9.334
5	1964-01-03	3		1	1964	0.01280975	9.552
6	1964-01-04	4		1	1964	-0.09819025	8.932

# Declustering in R

```
> library(extRemes)  
> extremalindex(data$skews, threshold = 0.5)
```

extremal.index	number.of.clusters	run.length
0.4264282	278.0000000	13.0000000

```
> EI <- decluster(data$skews, threshold = 0.5)  
> plot(EI)
```



2010-2016 data

# Model fitting

1.  $GPD(\sigma, \xi)$

$$\sigma_{d,x} = \alpha_\sigma + \beta_\sigma \sin\left(\frac{2\pi}{365}(d - \phi_\sigma)\right) + \gamma_\sigma x$$

```
> ModS4(data = data_ex, init.par = c(0.2, 0, 100, 0, 0.01))
```

2.  $GPD(\lambda)$

$$g(\lambda_{d,x}) = \alpha_\lambda^1 + \beta_\lambda^1 \sin\left(\frac{2\pi}{365}(d - \phi_\lambda^1)\right) + \tilde{x} \left[ \alpha_\lambda^2 + \beta_\lambda^2 \sin\left(\frac{2\pi}{365}(d - \phi_\lambda^2)\right) \right]$$

```
> ModR1(data = data, init.par = c(0.05, 0, 100, 0, 0, 100))
```

Shape

# Return level estimation

```
> RL_est(p, interval=c(quantile(data$sealevel, 0.5), max(sea$level)+3))
```



Annual exceedance probability

- $p=0.0001$  for 10,000 year return level
- Can be a vector

```
> CI_est(p, interval=c(quantile(data$sealevel, 0.5), max(sea$level)+3),  
n.boot=200, block.length=10, prob=0.95)
```

# Impact

- Implementation at EDF



- Working with the Environment Agency for improving existing best ESL estimates in the Coastal Flood Boundary Report



- Thames Estuary 2100 plan



# References

- Coles, S. (2001). *An introduction to statistical modeling of extreme values*. London: Springer.
- D'Arcy, E., Tawn, J. A., Joly, A., & Sifnioti, D. E. (2023). Accounting for seasonality in extreme sea-level estimation. *The Annals of Applied Statistics*, 17(4), 3500-3525.
- D'Arcy, E., Tawn, J. A., & Sifnioti, D. E. (2022). Accounting for climate change in extreme sea level estimation. *Water*, 14(19), 2956.
- GitHub repository: [github.com/eleanordarcy/ESL\\_estimation/](https://github.com/eleanordarcy/ESL_estimation/)

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THANK YOU  
ANY QUESTIONS?

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