Extreme Sea Level Estimation: Accounting for Seasonality, Dependence and Climate Change

Eleanor D'Arcy

Jonathan Tawn   Amélie Joly   Dafni Sifnioti

STOR-i | Lancaster University   EDF

Outline

1. Modelling motivation and considerations
   - Previous methods
   - Accounting for seasonality
   - Accounting for tidal dependence
   - Accounting for temporal dependence
   - Accounting for climate change
2. Results
   - Return level estimation
   - Uncertainty quantification
3. Implementation

Consequences of coastal flooding

- Loss of life
- Damage to property and infrastructure
- Coastal erosion
- Displacement of people
- Loss of habitats and ecosystems

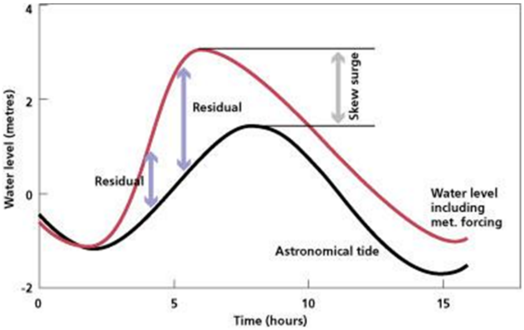
Components of Sea Levels

Tides

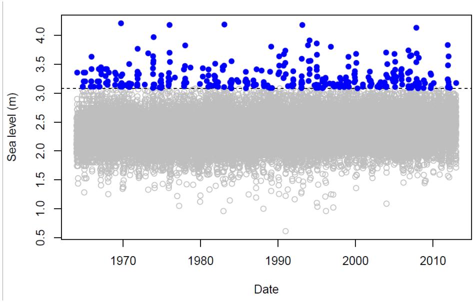
- Predictable rise and fall of the sea surface driven astronomically
- Can be well predicted far in advance

Storm surge

- Short term sea level changes caused by the weather
- Not accurately predictable



$$\text{Sea level } (Z) = \text{Skew Surge } (Y) + \text{Peak Tide } (X)$$



- Define extremes as exceedances of a high threshold
- Model using the **Generalised Pareto Distribution (GPD)**
  - Scale: σ
  - Shape: ξ
  - Rate: λ
- Estimate **return levels**: The level we expect to be exceeded once a year with probability p
  - Or, every $1/p$ years (return period)

Previous Methodology: SSJPM

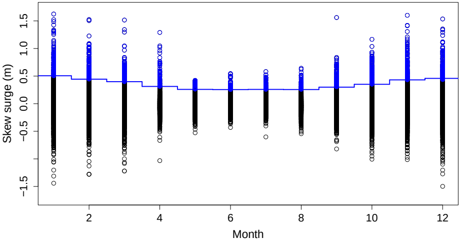
Batstone, C. et. al. (2013) *A UK best-practice approach for extreme sea-level analysis along complex topographic coastlines.* Ocean Engineering, 71, pp.28-39.

1. Skew surges are IID with distribution function
   - Model extremes using a peaks over threshold approach: GPD(σ, ξ, λ)
   - Model non-exceedances empirically
2. Tides are stationary
3. Assuming skew surge and peak tide are independent, for tidal cycle i,

$$\mathbb{P}(Z_i \leq z) = \mathbb{P}(X_i + Y_i \leq z) = \mathbb{P}(Y_i \leq z - X_i) = F_Y(z - X_i)$$

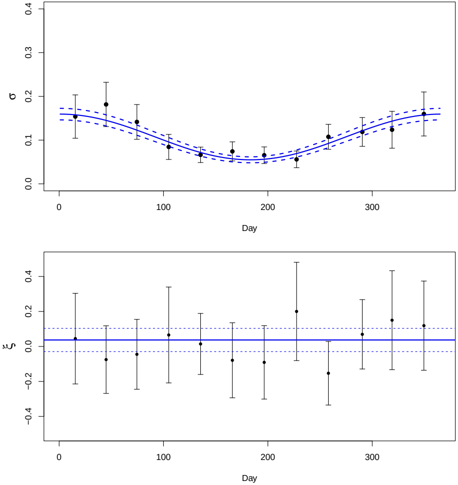
1. Methodology: Accounting for Non-stationarity

Skew Surge Modelling



- Monthly threshold u_j, $j = 1, \ldots, 12$
- For $Y \leq u_j$, we use the monthly empirical distribution $\tilde{F}_Y^{(j)}$
- For $Y > u_j$, we use a non-stationary GPD

Skew Surge Modelling

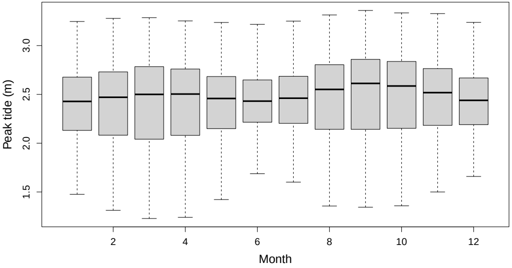


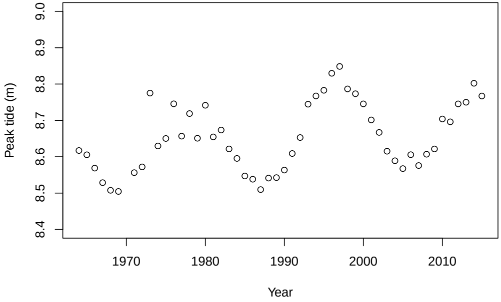
For $Y > u_j$, $Y \sim GPD(\sigma_d, \xi, \lambda_d)$ where

- $\sigma_d = \alpha_\sigma + \beta_\sigma \sin\left(\frac{2\pi}{365}(d - \phi_\sigma)\right)$ for $\alpha_\sigma, \beta_\sigma, \phi_\sigma \in \mathbb{R}$ and $d = 1, \dots, 365$,
- $\xi \in \mathbb{R}$ is kept fixed,
- $g(\lambda_d) = \alpha_\lambda + \beta_\lambda \sin\left(\frac{2\pi}{365}(d - \phi_\lambda)\right)$ for $\alpha_\sigma, \beta_\sigma, \phi_\sigma \in \mathbb{R}$, $d = 1, \dots, 365$ and $g(\cdot)$ the logit link

JPM

$$\mathbb{P}(Z_i \leq z) = \mathbb{P}(X_i + Y_i \leq z) = \mathbb{P}(Y_i \leq z - X_i) = F_Y(z - X_i)$$



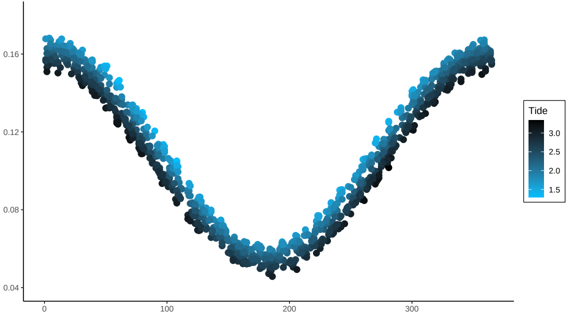


2. Methodology: Accounting for SS-PT Dependence

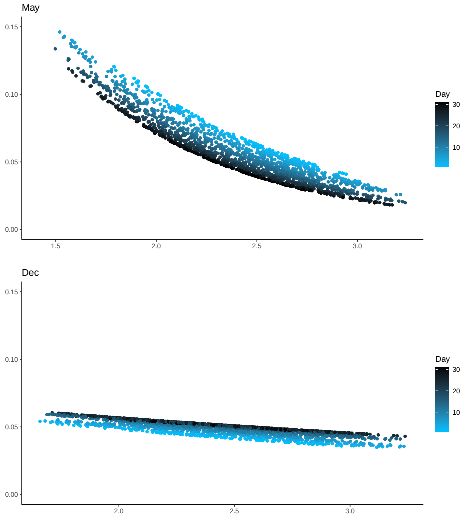
Skew Surge-Peak Tide Dependence

- Can be assumed as independent at most sites
- Extreme skew surges tend to occur on lower peak tides
- Add a tidal covariate to the scale parameter

$$\sigma_{d,x} = \alpha_\sigma + \beta_\sigma \sin\left(\frac{2\pi}{365}(d-\phi_\sigma)\right) + \gamma_\sigma x$$



Skew Surge-Peak Tide Dependence



- Relationship is non-stationary
- Add a tidal covariate to the rate parameter,

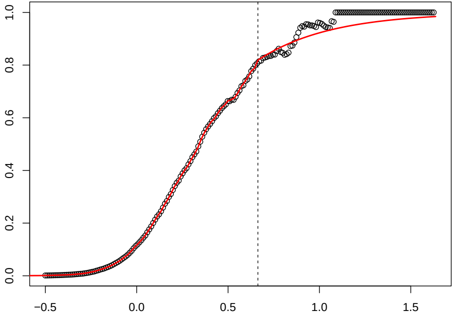
$$g(\lambda_{d,x}) = g(\lambda_d) + \tilde{x}\left[\alpha_\lambda + \beta_\lambda \sin\left(\frac{2\pi}{365}(d - \phi_\lambda)\right)\right]$$

for $\alpha_\lambda, \beta_\lambda, \phi_\lambda \in \mathbb{R}$, $d = 1, \ldots, 365$, g the logit link and $\tilde{x}$ standardised tide

3. Methodology: Accounting for Temporal Dependence

Skew Surge Temporal Dependence

Extremal index: $\theta(y, r) = \lim_{y \to y^F} \mathbb{P}(\max\{Y_2, \dots, Y_r\} < y | Y_1 > y) \in [0, 1]$



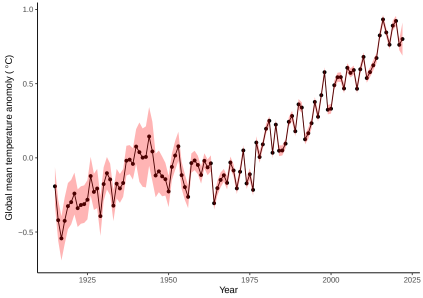
Model θ in terms of skew surge level y

$$\hat{\theta}(y, r) = \begin{cases} \tilde{\theta}(y, r) & \text{if } y \leq v \\ 1 - [1 - \tilde{\theta}(v, r)] \exp\left(-\frac{y-v}{\psi}\right) & \text{if } y > v, \end{cases}$$

where $\tilde{\theta}$ is the runs estimate and r run length

4. Methodology: Accounting for Climate Change

Climate Change



- Magnitude of extreme events is not changing
- Occurrence of extreme events is increasing

$$g(\lambda_{d,x,k}) = g(\lambda_{d,x}) + \sum_{s=1}^{4} \delta_s GMT_k \mathbb{1}_{d \in \mathscr{S}_s}$$

- Spatial pooling

Results

Accounting for Non-stationarity

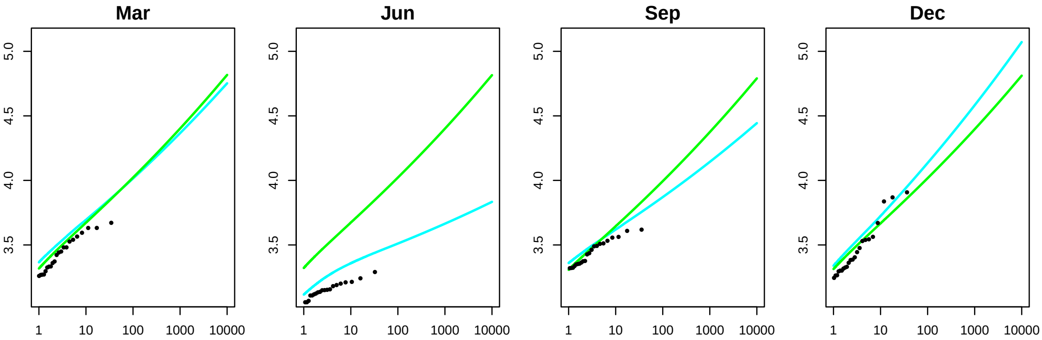


Figure: **Empiricals**, Current, Accounting for seasonality

Accounting for Non-stationarity

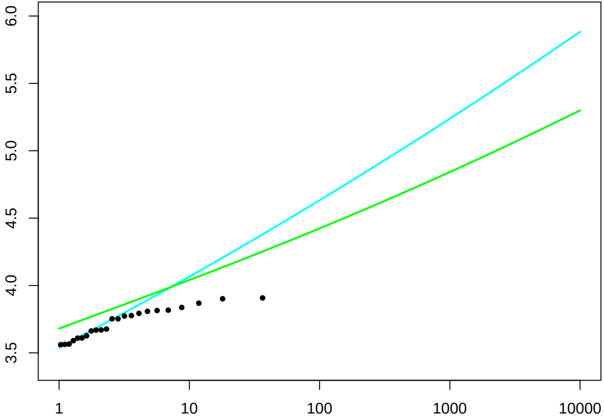


Figure: **Empiricals**, Current, Accounting for seasonality

Accounting for Dependence

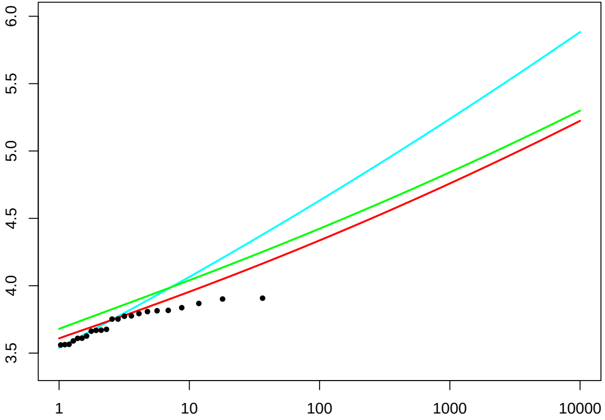


Figure: **Empiricals**, Current, Accounting for seasonality, Accounting for SS-PT dependence

Accounting for Dependence

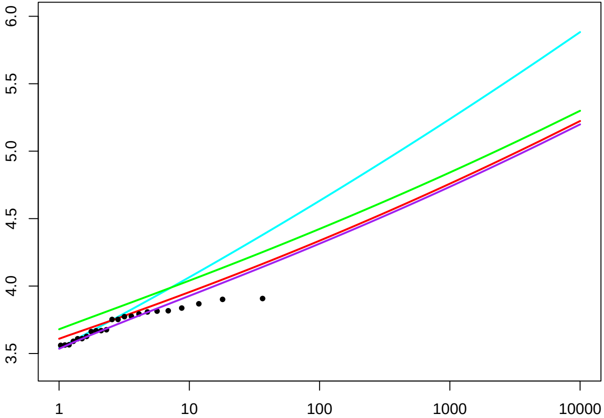


Figure: **Empiricals**, Current, Accounting for seasonality, Accounting for SS-PT dependence, Accounting for SS temporal dependence

Results at Other Sites

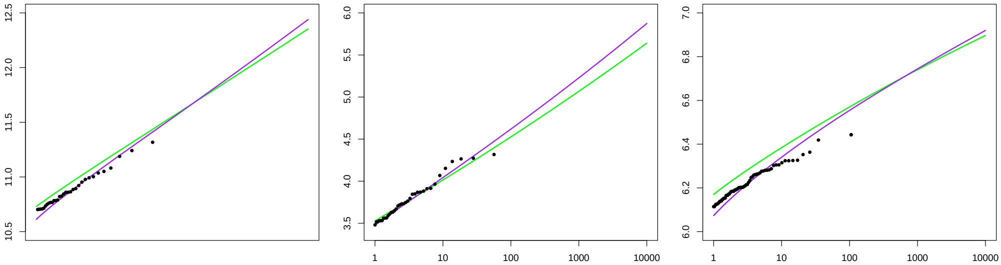
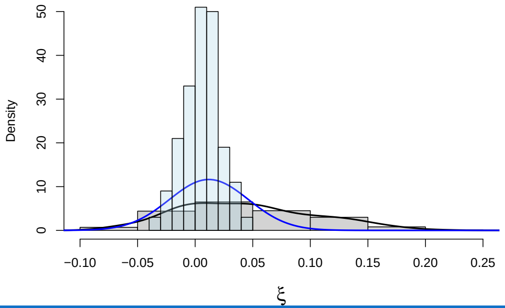
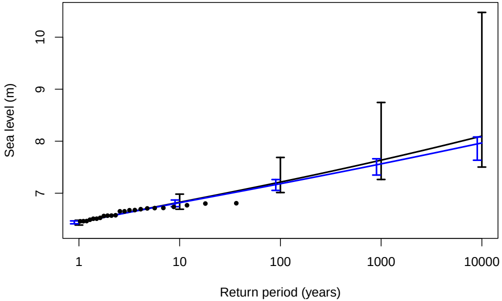


Figure: **Empiricals**,Current, Proposed
Heysham (left), Lowestoft (centre), Newlyn (right)

Uncertainty Quantification

- Stationary bootstrap procedure
- Accounting for uncertainty in
  - Threshold selection
  - Empirical distributions
  - Model parameters
- Add a prior to the GPD shape parameter
  - $\xi \sim \text{Normal}(\mu, \sigma^2)$
  - Penalised likelihood





Implementation

Code

- GitHub: `eleanordarcy/ESL_estimation`
  - `ESL_estimation.R` file for return level estimation
  - Specify data and annual exceedance probability p
- *R* package in development: `ESLestimation`
- Data
  - Skew surge and peak tide
  - Covariates: Month, day, peak tide, GMT
  - Stationary across time, i.e., remove climate change effects
  - Script to convert BODC UKNTGN data into required format

Model fitting

Step 1: Fit GPD model

$Y > u_j \sim \text{GPD}(\sigma_{d,x}, \xi)$ where

$$\sigma_{d,x} = \alpha_\sigma + \beta_\sigma \sin\left(\frac{2\pi}{365}(d - \phi_\sigma)\right) + \gamma_\sigma x$$

```
> GPD.fit(data, q=0.95)
```

Model fitting

Step 2: Fit rate parameter model

$Y \sim \text{GPD}(\sigma_{d,x}, \xi, \lambda_{d,x})$ where

$$g(\lambda_{d,x}) = \alpha_\lambda^1 + \beta_\lambda^1 \sin\left(\frac{2\pi}{365}(d - \phi_\lambda^1)\right) + \tilde{x}\left[\alpha_\lambda^2 + \beta_\lambda^2 \sin\left(\frac{2\pi}{365}(d - \phi_\lambda^2)\right)\right]$$

```
> rateparam.fit(data, q=0.95)
```

Model fitting

Step 3: Fit extremal index model

$$\hat{\theta}(y,r) = \begin{cases} \tilde{\theta}(y,r) & \text{if } y \leq v \\ 1 - [1 - \tilde{\theta}(v,r)] \exp\left(-\frac{y-v}{\psi}\right) & \text{if } y > v, \end{cases}$$

where $\tilde{\theta}$ is the runs estimate and r run length.

```
> extremalindex.fit(data, run.length=10, q=0.99)
```

Return level estimation

```
> returnlevel.est(p, data, gpd_par, rate_par,
  extremalindex_par)

> CI.est(p, data, gpd_par, rate_par,
  extremalindex_par, n.boot=200, block.length=10)
```

Current Work

- Motivated by the Thames Barrier: Coastal and fluvial flooding,
- Joint work with Ivan Haigh,
- Developed an alternative approach for skew surge temporal dependence,
- Marginal modelling of extreme river flow: Temporal dependence and seasonality,
- Future work: Joint modelling skew surge and river flow.

Concluding Remarks

- D'Arcy, E., Tawn, J.A., Joly, A. and Sifnioti, D.E., 2023. Accounting for seasonality in extreme sea level estimation. Annals of Applied Statistics, 17(4), p.3500.
- D'Arcy, E., Tawn, J.A. and Sifnioti, D.E., 2022. Accounting for climate change in extreme sea level estimation. Water, 14(19), p.2956.

Thank you for listening!

e.darcy@lancaster.ac.uk
@eleanor_darcy_
eleanordarcy