subject	date	keywords
topic		

K22 T3

Given $\sum_{k=1}^{\infty} \left(\frac{2}{3}\right)^{k+2}$, ⓐ Determine if converge

ⓑ If so, state sum

$r = \frac{2}{3}$

$a_1 = \frac{8}{27}$

$$\sum_{k=0}^{\infty} \overset{\frac{8}{27}}{\cancel{\left(\frac{2}{3}\right)^3}} \cdot \left(\frac{2}{3}\right)^k = \sum_{k=0}^{\infty} a_1 r^k$$

$$\left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^4 + \left(\frac{2}{3}\right)^5 + \cdots$$

$$\left(\frac{2}{3}\right)^3 \left(1 + \frac{2}{3} + \left(\frac{2}{3}\right)^2 + \cdots\right)$$

$$a_0 (1 + r + r^2 + \cdots)$$

Yes converge!

Geometric Series

$$S = \frac{a_0}{1-r} = \frac{\frac{8}{27}}{1-\frac{2}{3}} = \frac{\frac{8}{27}}{\frac{1}{3}} = \frac{24}{27}$$

$$\boxed{= \frac{8}{9}}$$

$$\sum_{K=2}^{\infty} \frac{1}{K^2-1}$$

$$\frac{1}{K^2-1} = \frac{1}{(K+1)(K-1)} \overset{\text{want}}{=} \frac{A}{K+1} + \frac{B}{K-1}$$

$$1 = A(K-1) + B(K+1)$$

$$K=1: 1 = A(0) + B(2) \rightarrow B = \tfrac{1}{2}$$

$$K=-1: 1 = A(-2) + B(0) \rightarrow A = \tfrac{-1}{2}$$

$$\sum_{K=2} \frac{1}{K^2-1} = \sum \frac{-1}{2(K+1)} + \frac{1}{2(K-1)}$$

$$S_1 = \frac{-1}{6} + \frac{1}{2}$$

$$S_2 = \frac{-1}{6} + \frac{1}{2} \quad \frac{-1}{8} + \frac{1}{4}$$

$$S_3 = \cancel{\frac{-1}{6}} + \frac{1}{2} - \frac{1}{8} + \frac{1}{4} \quad -\frac{1}{10} + \cancel{\frac{1}{6}}$$

$$S_4 = \frac{1}{2} - \cancel{\frac{1}{8}} + \frac{1}{4} - \frac{1}{10} \quad -\frac{1}{12} + \cancel{\frac{1}{8}}$$

$$S_n = \frac{1}{2} + \frac{1}{4} - \frac{1}{2n+2} - \frac{1}{2n+4} \Rightarrow \sum_{K=2}^{\infty} f = \frac{1}{2} + \frac{1}{4}$$

$$= \frac{3}{4}$$

$$\sum_{k=1}^{\infty} (-1)^k \frac{(2k-1)!}{3^k}$$

Ast? No

k	1	2	3
f	$\frac{-1}{3}$	$\frac{2}{3}$	$\frac{-120}{27}$

Ratio

$$\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \frac{\frac{(2(k+1)-1)!}{3^{k+1}}}{\frac{(2k-1)!}{3^k}}$$

$$= \frac{(2k+1)!\,3^k}{(2k-1)!\,3^{k+1}}$$

$$= \frac{(2k+1)(2k)}{3}$$

$$\lim_{k\to\infty} \text{———''———''———} = \infty > 1$$

$\boxed{\text{Ratio Test: Does Not Converge}}$

AKA Diverge

$$\sum_{k=1}^{\infty} \frac{3k^3 - 2k^2 + 4}{k^7 - k^3 + 2} \leftarrow a_k$$

Limit comparison test : $\frac{1}{k^4} \leftarrow b_k$ p-series w/ $p = 4 > 1$ converges

$$\lim_{k\to\infty} \frac{a_k}{b_k} = \lim_{k\to\infty} \frac{3k^3 - 2k^2 + 4}{k^7 - k^3 + 2} \cdot \frac{k^4}{1} = 3 < \infty$$

$$\boxed{\text{LCT} \Rightarrow \text{Converges!}}$$

$$\sum_{k=2}^{\infty} \frac{1}{x \ln x}$$

Integral test

$$\int_2^{\infty} \frac{1}{x \ln x}\, dx$$

$$u = \ln x$$

$$du = \frac{1}{x}\, dx$$

$$dx = x\, du$$

$$\int_{\ln 2}^{\infty} \frac{\cancel{x}\, du}{\cancel{x}\, u} = \int_{\ln 2}^{\infty} \frac{du}{u} = \ln|u| \Big|_{\ln 2}^{\infty}$$

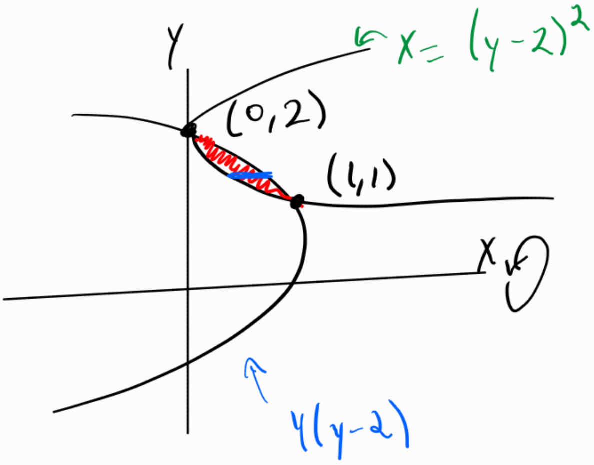
$$= \ln(\infty) - \ln|\ln 2| = \infty$$

$$\boxed{\text{Diverges}}$$

KS22 T1

Method of cylindrical shells, volume of solid given by

$x = y(2-y)$, $x = (y-2)^2$ about x-axis



$$2\pi \int_1^2 y\left(y(2-y) - (y-2)^2 \right) dy$$

$$2\pi \int_1^2 y\left(2y - y^2 - (y^2 - 4y + 4)\right) dy$$

$$2\pi \int_1^2 (2y^2 - y^3 - y^3 + 4y^2 - 4y)\, dy$$

$$2\pi \int_1^2 (-2y^3 + 6y^2 - 4y)\, dy$$

$$2\pi \left[\frac{-2}{4} y^4 + \frac{6}{3} y^3 - \frac{4}{2} y^2 \right]_1^2$$

$$= \text{something} \ ☺$$

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$$

$$\Delta x = \frac{b-a}{n}$$

$$x_k^* \in [a+(k-1)\Delta x,\ a+k\Delta x]$$

$$\int_0^1 \sqrt{1+4x^2}\,dx$$

$$\int \sqrt{1+\tan^2(\theta)}\;\frac{1}{2}\sec^2(\theta)\,d\theta$$

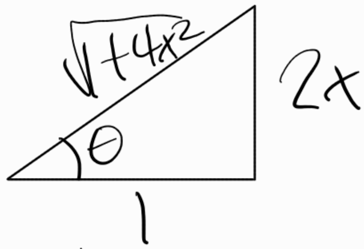
$$=\frac{1}{2}\int \underline{\sqrt{1+\tan^2\theta}}\,\sec^2(\theta)\,d\theta$$

$$=\frac{1}{2}\int \sqrt{\sec^2\theta}\,\sec^2(\theta)\,d\theta$$

$$=\frac{1}{2}\int \sec^3\theta\,d\theta$$

$$a^2+b^2=c^2$$

$$c=\sqrt{a^2+b^2}$$



$$x=\frac{1}{2}\tan(\theta)$$

$$dx=\frac{1}{2}\sec^2(\theta)\,d\theta$$

$$1+\tan^2(\theta)=\sec^2(\theta)$$