

subject

Foundations of Computing II

date

4 Mar 24

keywords

Second Order NH

topic

Nonhomogeneous Recurrence Relations, Cont.

General Steps

$$c_0 a_n + c_1 a_{n-1} + c_2 a_{n-2} = f(n)$$

associated characteristic eqn $c_0 r^2 + c_1 r + c_2 = 0 \rightarrow$ Find roots r_1, r_2

$$a_n^{(h)} = \alpha r_1^n + \beta r_2^n \quad (r_1 \neq r_2)$$

$$= \alpha r^n + \beta n r^n \quad (r_1 = r_2)$$

Specific $f(n)$ Suppose $f(n) = K r^n$. Then...

a) $a_n^{(p)} = A r^n$ for $r \neq r_1, r \neq r_2$

b) $a_n^{(p)} = A n r^n$ for $r = r_1, r \neq r_2$

c) $a_n^{(p)} = A n^2 r^n$ for $r = r_1 = r_2$

Example

$$a_n - 10a_{n-1} + 21a_{n-2} = n, \quad a_0 = 5, a_1 = 10$$

$$\hookrightarrow r^2 - 10r + 21 = 0 \rightarrow (r-3)(r-7) \rightarrow r=3, 7$$

$$\hookrightarrow a_n^{(h)} = \alpha 7^n + \beta 3^n$$

$$a_n^{(p)} = A_1 n + A_0$$

$$\rightarrow a_{n-1}^{(p)} = A_1 (n-1) + A_0$$

$$\rightarrow a_{n-2}^{(p)} = A_1 (n-2) + A_0$$

Replace $n \rightarrow (n-1), (n-2)$

Def'n of recurrence

$$A_1 n + A_0 - 10(A_1 (n-1) + A_0) + 21(A_1 (n-2) + A_0) = n$$

Match coefficients

$$\rightarrow 12A_1 n + (12A_0 - 32A_1) = n$$

$$\rightarrow A_1 = \frac{1}{12}, A_0 = \frac{2}{9}$$

$$\rightarrow a_n = a_n^{(h)} + a_n^{(p)} = \alpha 7^n + \beta 3^n + \frac{1}{12}n + \frac{2}{9}$$

$$\rightarrow n=0 \rightarrow \alpha + \beta + \frac{2}{9} = 5$$

$$n=1 \rightarrow 7\alpha + 3\beta + \frac{1}{12} + \frac{2}{9} = 10$$

$$\leadsto \alpha = \frac{-167}{144}, \beta = \frac{95}{16}$$

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$$a_n - 5a_{n+1} + 6a_{n+2} = 6(7)^n \quad \text{w/ } a_0 = 20, a_1 = 100$$

$$\leadsto r^2 - 5r + 6 = 0 \rightarrow (r-2)(r-3) \rightarrow r=2, 3$$

$$7 \neq 2 \neq 3 \text{ thus } a_n^{(p)} = \gamma \cdot 7^n$$

$$\rightarrow a_n = a_n^{(h)} + a_n^{(p)} = \alpha \cdot 3^n + \beta \cdot 2^n + \gamma \cdot 7^n$$

$$\rightarrow a_2 = 6 \cdot 7^2 + 5 \cdot 100 - 6 \cdot 20 = 674$$

$$\rightarrow n=0: \alpha + \beta + \gamma = 20$$

$$n=1: 3\alpha + 2\beta + 7\gamma = 100$$

$$n=2: 9\alpha + 4\beta + 49\gamma = 674$$

linear algebra
my beloved

$$\rightarrow \alpha = -13.5, \beta = 18.8, \gamma = 14.7$$

$$\rightarrow a_n = 14.7(7)^n - 13.5(3)^n + 18.8(2)^n$$