

subject: Foundations of Computing II

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keywords: Recursion, Induction, Strong Induction

topic: Exam 1 Debrief

8 $a_n = 4a_{n-1} + 12a_{n-2}$, $a_0 = 2$, $a_1 = 4$

$\rightsquigarrow a_{n+2} - 4a_{n+1} + 12a_n = 0$

$\rightarrow r^2 - 4r + 12 = 0 = (r-6)(r+2) \Rightarrow r = 6, -2$

$\Rightarrow a_n = \alpha 6^n + \beta(-2)^n$

$a_0 = 2 = \alpha + \beta$

$a_1 = 4 = 6\alpha - 2\beta$

$\Rightarrow \alpha = \beta = 1$

$\Rightarrow a_n = 6^n + (-2)^n$

7 $f(n) = 5f(n-1) + 1$, $f(0) = 1$

$f(n) = 5f(n-1) + 1$

$= 5(5f(n-2) + 1) + 1 = 5^2 f(n-2) + 5 + 1$

$= 5(5^2 f(n-3) + 1) + 5 + 1 = 5^3 f(n-3) + 5^2 + 5 + 1$

$\rightarrow f(n) = \frac{5^{n+1} - 1}{4}$

$f(3) = 156$

6 $2a_n - x a_{n+1} = 0$, $a_2 = 2$, $a_1 = 6$

$2(2) - x(6) = 0 \Rightarrow x = \frac{2}{3} \rightarrow a_0 = 18$

$a_n = \frac{1}{3} a_{n-1} \rightarrow a_n = \left(\frac{1}{3}\right)^n a_0 = 18 \cdot \left(\frac{1}{3}\right)^n$

5 $a_1 = 1$, $a_2 = 3$, $a_n = a_{n-2} + 2a_{n-1}$. Prove a_n odd

Proof (BWOSI)

Base cases: $a_1 = 1$ (odd) ✓ $a_2 = 3$ (odd) ✓

Suppose our claim is strong for all $i \in \{1, \dots, k\}$ for $k \in \mathbb{N}$

$a_{k+1} = 2a_k + a_{k-1} = 2(2m+1) + (n+1) = 2(2m+n+1) + 1 \in \mathcal{O}$ ■

4 Prove $1 + 4n \leq 2^n$ for $n \geq 5$

Proof (BWOI)

Base case $(n=5)$ $1 + 4(5) = 21 < 32 = 2^5$

Suppose our claim is true for some $k \in \mathbb{N}$

Thus $1 + 4(k+1) = 1 + 4k + 4 < 2^k + 4 = 2^k + 2^2 \overset{k>2}{<} 2^{k+1}$ ■

3 $S(n) = 2 + 4 + 6 + \dots + 2n$. Prove $S(n) = n^2 + n$

Proof (BWOI):

Base case $(n=1)$ $2 = 1^2 + 1$ ✓

Suppose our claim is true for some $k \in \mathbb{N}$

Thus $S(k+1) = S(k) + 2(k+1) = k^2 + k + 2k + 1 = (k+1)^2 + (k+1)$ ■

2 $f(1) = \frac{1}{2}$, $f(n) = f(n-1) + \frac{1}{n(n+1)}$. Prove $f(n) = \frac{n}{n+1}$

Proof (BWOI):

Base case $(n=1)$ $\frac{1}{1+1} = \frac{1}{2} = f(1)$ ✓

Suppose our claim is true for some $k \in \mathbb{N}$

Thus $f(k+1) = f(k) + \frac{1}{(k+1)(k+2)} = \frac{k}{k+1} + \frac{1}{(k+1)(k+2)} = \frac{k^2 + 2k + 1}{(k+1)(k+2)} = \frac{k+1}{k+2}$ ■

1 $a_n = 2n + 3$, $n \geq 1$

$a_n = a_{n-1} + 2$, $a_1 = 5$

$b_n = 3^n + 1$, $n \geq 2$

$b_n - b_{n-1} = 3^n - 3^{n-1} = 3 \cdot 3^{n-1} - 3^{n-1} = 2 \cdot 3^{n-1} \rightarrow b_n = b_{n-1} + 2 \cdot 3^{n-1}$, $b_0 = 2$

$c_n = 1 + 2^3 + 3^3 + \dots + n^3$, $n \geq 1$

$c_n - c_{n-1} = n^3 \rightarrow c_n = c_{n-1} + n^3$, $c_1 = 1$

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