

subject Linear Algebra

date

keywords

Determinants

Cofactor Matrices

Invertible Matrix Theorem

topic 5.2 (Down with) Determinants

Test 3

Final

Monday

5/1 11:30am (8:25 class)

5/3 8am (9:30 class)

A function

$$\det: M_{n \times n} \rightarrow \mathbb{R}$$

 $\in M_{2 \times 2}$ 

i.e. for  $A \in M_{n \times n}$ ,  $\det A \in \mathbb{R} \rightarrow A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ ;  $\det A = ad - bc$

$$\text{Ex } A = \begin{bmatrix} 7 & 2 \\ 4 & 3 \end{bmatrix}; \det A = 7(3) - (2)(4) = -29$$

A geometric interpretation

If you hit a unit square w/  $A$ ,  
area will scale by  $-29$  (negative = rotation)

$$\text{For } A \in M_{n \times n}, [a_{ij}] = A$$

$\uparrow \quad \uparrow$   
 row column

$$\det A = a_{11} \det A_{11} - a_{12} \det A_{12} + \dots + (-1)^{n+1} a_{1n} \det A_{1n}$$

$\uparrow$   
 $ij$  cofactor  
 matrix

$A$  w/o  $i$ -th row,  $j$ -th column

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & -1 & 5 \\ 0 & 4 & 0 \end{bmatrix}$$

$$\det A = 7 \det \begin{bmatrix} -1 & 5 \\ 4 & 0 \end{bmatrix} - 2 \det \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix} + 4 \det \begin{bmatrix} 0 & -1 \\ 0 & 4 \end{bmatrix}$$

Cofactor expansion across 1st row

$$\begin{bmatrix} 7 & 2 & 4 \\ 0 & -1 & 5 \\ 0 & 4 & 0 \end{bmatrix} \quad \begin{bmatrix} 7 & 2 & 4 \\ 0 & -1 & 5 \\ 0 & 4 & 0 \end{bmatrix} \quad \begin{bmatrix} 7 & 2 & 4 \\ 0 & -1 & 5 \\ 0 & 4 & 0 \end{bmatrix}$$

(1,1) cofactor    (1,2) cofactor    (1,3) cofactor

$$= 7(0 - 20) - 2(0 - 0) + 4(0) = -140$$

$$\det A = 7 \det \begin{bmatrix} -1 & 5 \\ 4 & 0 \end{bmatrix} - 0 \det \begin{bmatrix} 2 & 4 \\ 4 & 0 \end{bmatrix} + 0 \det \begin{bmatrix} 2 & 4 \\ -1 & 5 \end{bmatrix}$$

Zero's, baby!

$$= 7(-20) = -140$$

Cofactor expansion down 1st column

↑  
 You can choose  
 any row or column

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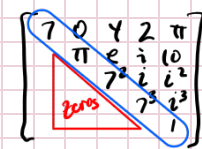
$$A = \begin{bmatrix} 7 & 0 & 7 & 2 & \pi \\ 0 & \pi & e & i & 10 \\ 0 & 0 & 7^2 & i & i^2 \\ 0 & 0 & 0 & 7^3 & i^3 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \det A = 7 \det \begin{bmatrix} \pi & e & i & 10 \\ 0 & 7^2 & i & i^2 \\ 0 & 0 & 7^3 & i^3 \\ 0 & 0 & 0 & 1 \end{bmatrix} + 0 \quad \leftarrow \text{down first row}$$

$$= 7 \cdot (\pi \det \begin{bmatrix} 7^2 & i & i^2 \\ 0 & 7^3 & i^3 \\ 0 & 0 & 1 \end{bmatrix}) \quad \leftarrow$$

$$= 7\pi (7^2 \det \begin{bmatrix} 7^3 & i^3 \\ 0 & 1 \end{bmatrix})$$

$$= 7(\pi)(7^2)(7^3)(1) = 7^6 \pi$$

Upper Triangular



all the fun stuff is in the "upper triangle"

Main diagonal

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix} \quad \leftarrow \text{signs for general } (i,j) \text{ cofactor expansion}$$

## Invertable Matrices

Fun fact: consider  $A \in M_{n \times n}$  upper triangular. If there's a zero on the main diagonal, then

- $\det A = 0$
- $A$  not invertible

thus non zero determinants detect invertibility!

## Facts

- $\det A = 0 \iff A$  not invertible
  - $\det(AB) = (\det A)(\det B)$
  - $\det(A^{-1}) = \frac{1}{\det(A)}$
  - $\det(I_n) = 1$
- $\det(A+B) \neq (\det A) + \det B$