Motion of a Tennis Ball

Group 11 – Matthew Boles, Emily Dulieu, Raul Hurnall, Ross McCarthy, Benjamin Rogers, Eleanor Ward

Model Flight of Tennis Ball

We used forward Euler time stepping to model the flight of the tennis ball. The trajectory was computed until the ball touched the ground.

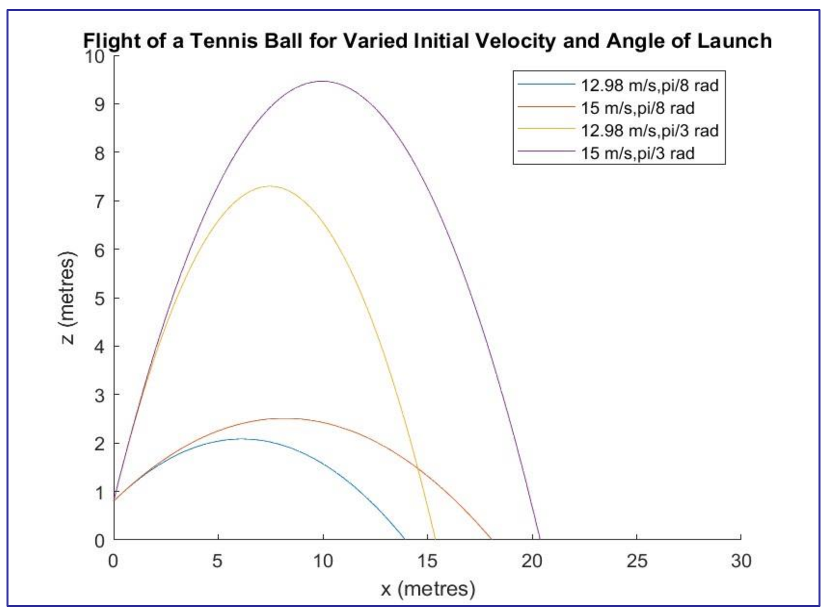

Figure 1

We altered the values of alpha (angle of projection) and the velocity to see how it affected the trajectory of the tennis ball.

A larger velocity with constant alpha resulted in the range of the tennis ball increasing, which can be seen in via the difference in the distance travelled between the purple and yellow graphs and the orange and blue graphs.

A greater alpha with constant velocity also led to a greater range, however this time, a higher maximum can be seen with greater values of alpha.

Comparing with an Analytical Solution:

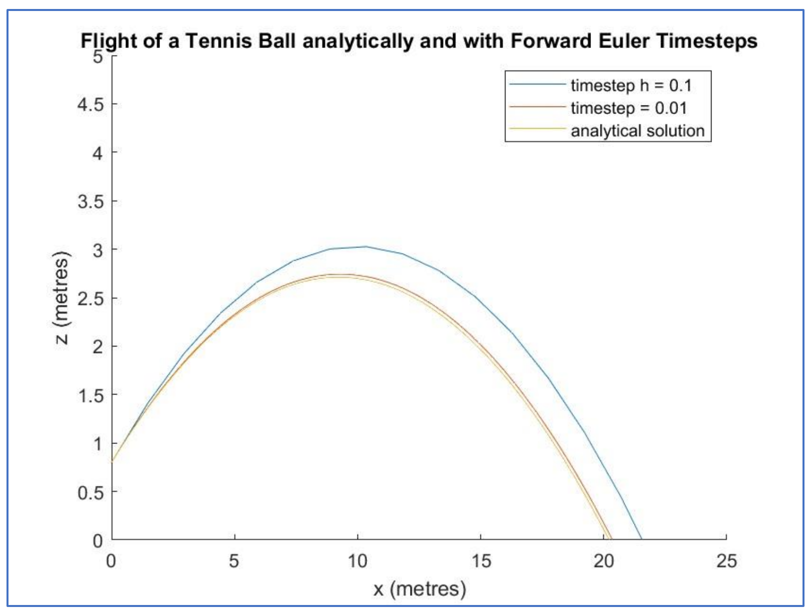

Figure 2

To test the correctness of our code, we compared the forward Euler method with varying timesteps to an analytical solution. We plotted the analytical solution along with 2 values for the timestep on the same graph and used the same velocity (16m/s) and angle of projection (pi/8). The analytical solution showed the ball travelling 20.180m whilst the timestep of 0.1 and 0.01 was 22.173 and 20.399 respectively. From this, we can see that the smaller the timestep, the closer the model of the flight of the ball is to the analytical solution. Therefore, the smaller the timestep, the more accurate the forward Euler is.

Model Flight of Tennis Ball with Drag

We investigated the effect on drag on the flight of the tennis ball up until it first hits the ground.

We did this using forward Euler equations:

$$\mathcal{X}(n+1) = \mathcal{X}(n) + h\mathcal{U}n$$
$$\mathcal{Z}(n+1) = \mathcal{Z}(n) + h\mathcal{W}n$$
$$\mathcal{U}(n+1) = \mathcal{U}(n) + h\mathcal{U}n*drag$$
$$\mathcal{W}(n+1) = \mathcal{W}(n) + h\mathcal{W}n*(-g-drag)$$

Where $drag = (-0.5C D\rho|v|v) / m$

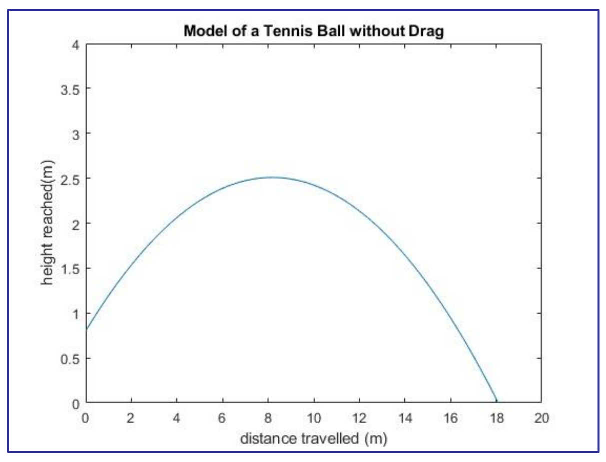

Figure 3

In Figure 2 and Figure 3 we used the same velocity (15m/s) and angle of projection (pi/8) so that we could compare the flight of the tennis ball with drag and without drag. We can see that with drag, the range of the tennis ball is shorter, not quite reaching the 18m that it does without drag.

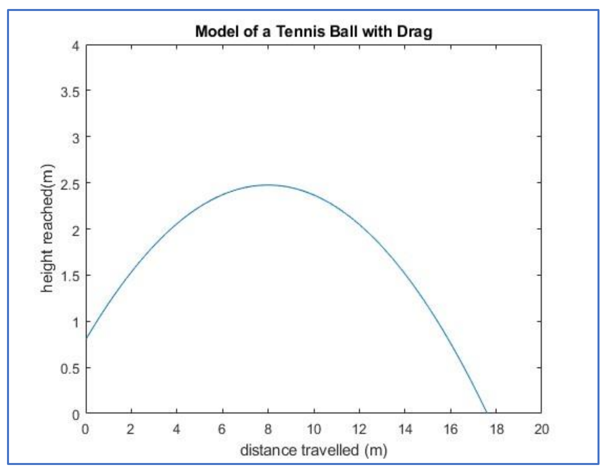

Figure 4

From figure 4 we see that as the coefficient of drag increases, the range of the flight of the ball decreases, hence they are inversely proportional. We scaled the drag up by multiplying it by 10 to portray this, while keeping the velocity and angle of projection constant. From this, it is much clearer to see the effect of the drag on the ball as it now travels just past 14m, a much shorter distance.

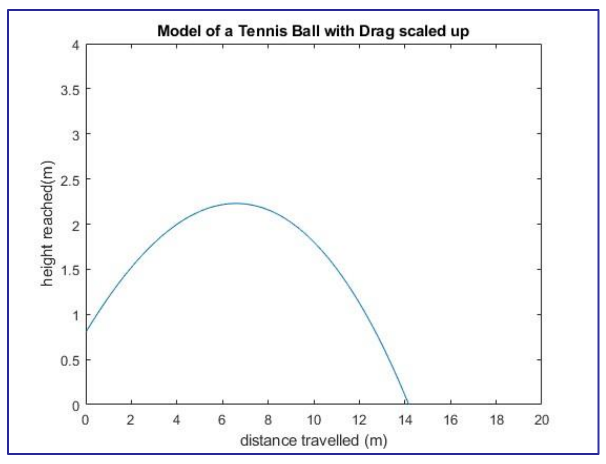

Figure 5

To check the correctness of our numerical solution, we could set the drag to zero on the one with drag. The we compare this graph to the graph without drag to see if they are identical. If so, then our numerical solution is correct, and we have succeeded.

Higher Order Time Stepping Scheme

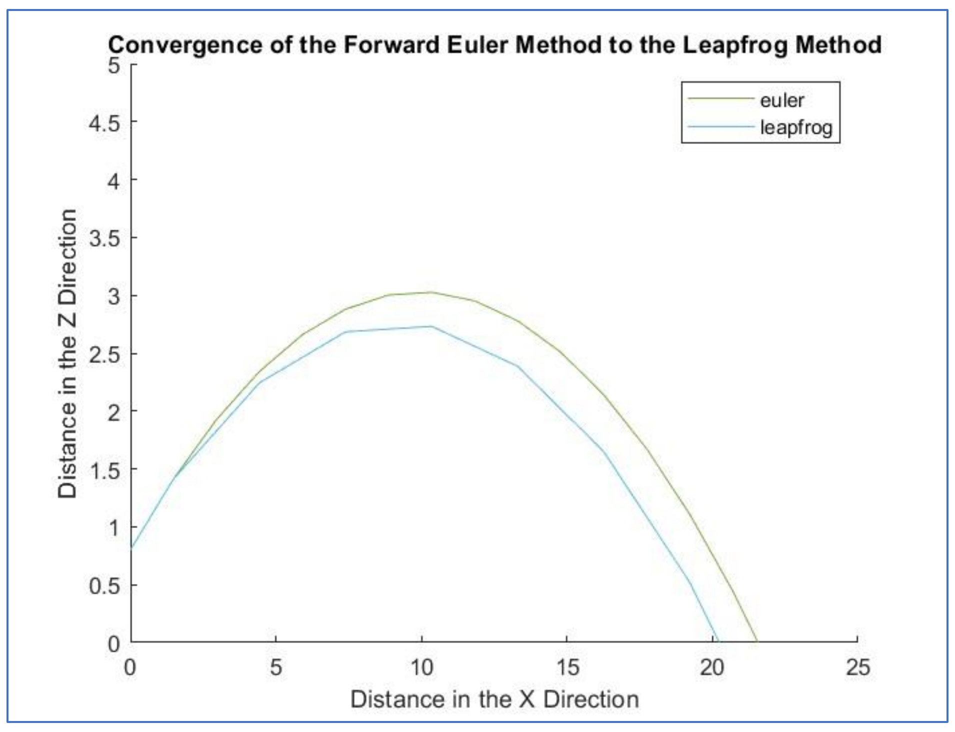

Figure 6

When choosing the higher order time-stepping scheme, we chose the leapfrog method to compare against Forward Euler. Leapfrog is a Second Order time-stepping Method which uses the second previous value x(n-1) compared to Forward Euler which uses x(n). Due to this we had to use Forward Euler to generate the second value of x, z, v and u. With v and u being the respective velocities in the vertical and horizontal direction. The purpose of using the Leapfrog method was to gain a more accurate numerical approximation which was successful as seen in the graph to the left as Forward Euler overshoots quite significantly.

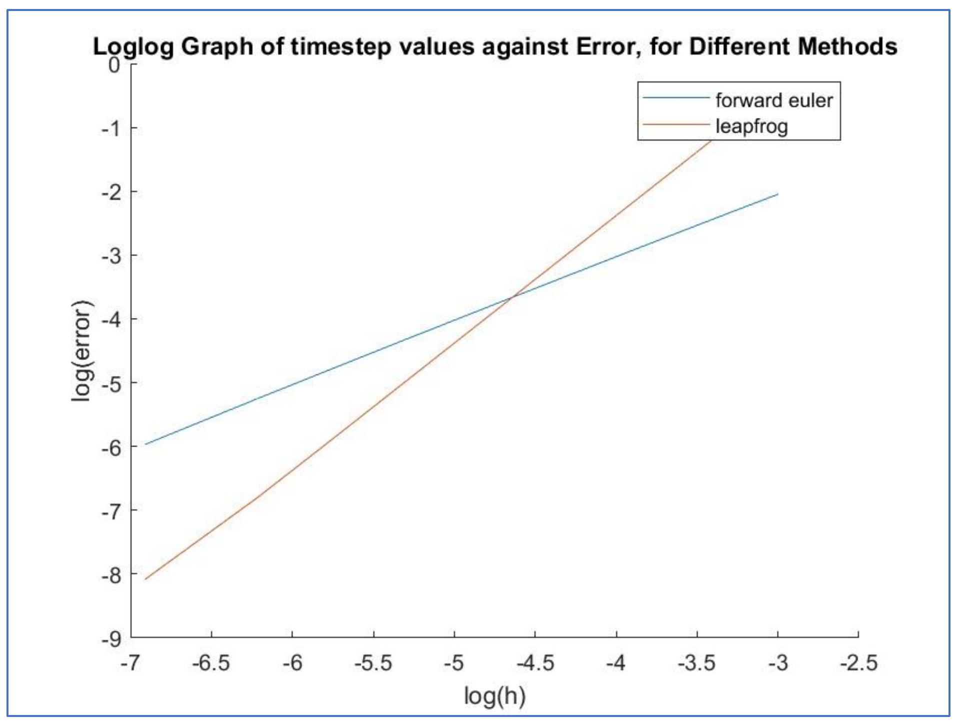

Figure 7

To test the error in our code, we found a value at a certain time, in our case t = 3, for a very small timestep that would act as our "perfect solution". We then calculated the x and y values for the Leapfrog and Forward Euler methods at that time for a range of timesteps and found the error for each one. We plotted this on a loglog graph as the order of the error in Leapfrog is higher and can be seen more clearly. The Leapfrog has a gradient of 2 as opposed to Forward Euler with a gradient of one, showing Leapfrog has an error of order h^2. This means if you double the timestep, the error increases by a factor of 4. In both methods, the error tends to 0 as h tends to 0, but leapfrog will tend to 0 faster, so for small h values is more accurate than Forward Euler. For extreme values of h, the method becomes unstable and the error isn't reasonable to consider.

Magnus Effect

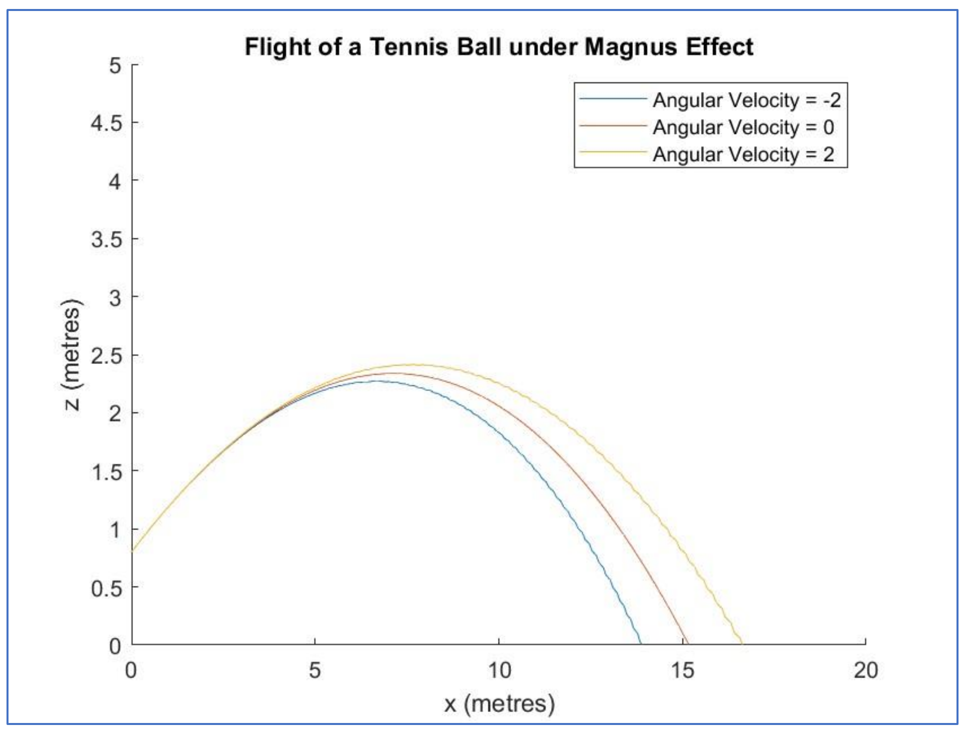

Figure 8

Lift Force, F = p * v * w * A
Where:
p = air density (1.2kgm^-3)
v = velocity
w = angular velocity
A = cross-sectional area (29 * 10^-4 m^2)

The Magnus Effect:

The Magnus effect is a phenomenon observed in fluid dynamics, particularly in the context of aerodynamics. It explains the deviation in the trajectory of a rotating object when it moves through a fluid.

When a rotating object, such as a spinning tennis ball, moves through the air, it creates a pressure difference on opposite sides of the object. This pressure difference is a result of the Magnus force, which is perpendicular to both the direction of the object's velocity and the axis of rotation. On observing a tennis player hit forehand groundstrokes (to produce topspin) and backhand slices (to produce backspin), we can intuitively say that topspin makes the ball arc down faster (from the downwards force caused by the effect), and backspin makes the ball 'glide' through the air (from the upwards forced caused by the effect).

The Magnus effect is more visible on the tennis ball when the angular velocity has been increased. This results in a greater falling force, causing a faster drop of the ball. If the air density is lowered, the magnus effect is reduced, resulting in the tennis ball having a higher swerve within its flight. It also makes the ball dip downwards if it has a topspin. The greater the cross-sectional area of the tennis ball, the more air particles the ball will collide with, causing a larger force impeding the tennis ball's flight.

As seen in figure 8, we have compared 3 different angular velocities (–2, 0, 2). From our research, we can see that a negative angular velocity results in a larger dip within the ball's flight.

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