CPSC 221: Assignment 3

1. (a) k: 120, 130, 70, 30, 50, 20, 40, 140, 150, 60, 10, 97, 110, 96, 93

hash(k): 10, 9, 4, 8, 6, 9, 7, 8, 7, 5, 10, 9, 0, 8, 5

Resulting hash table:

0	110	
1		
2		
3		
4	70	
5	93	→ 60
6	50	
7	150	→ 40
8	96	→ 140 → 30
9	97	→ 20 → 130
10	10	→ 120

(b)

0	93
1	23
2	140
3	
4	50
5	120
6	
7	30
8	130
9	
10	10
11	
12	150
13	96
14	60
15	
16	
17	40
18	110
19	97
20	20
21	
22	

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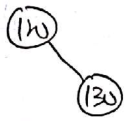
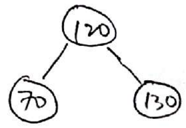
1(c)

120 →

130 →



70 →



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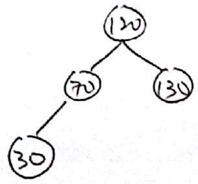
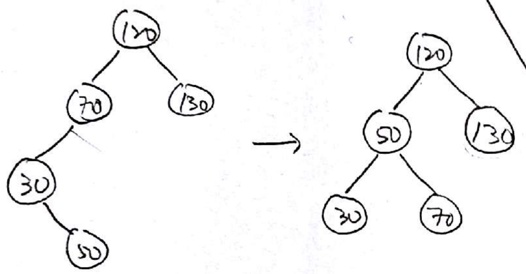
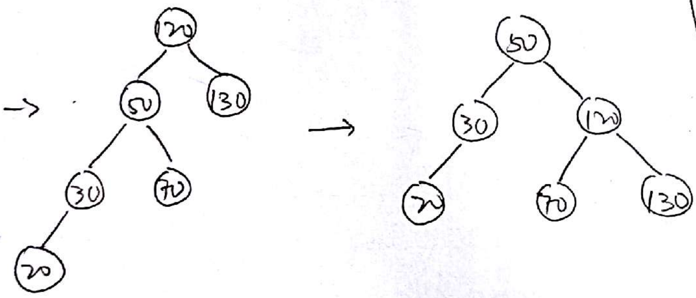


50 →



20 →



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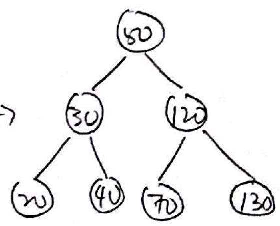
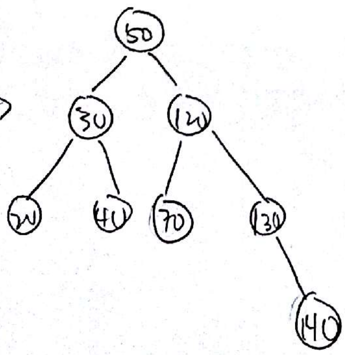


140 →



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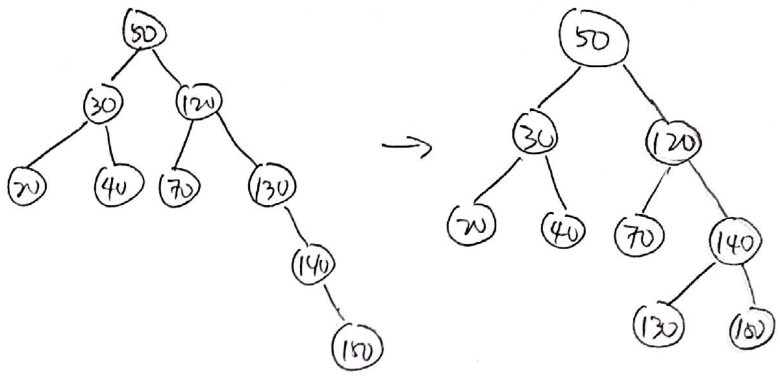

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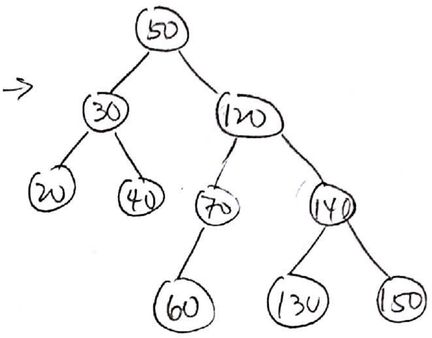

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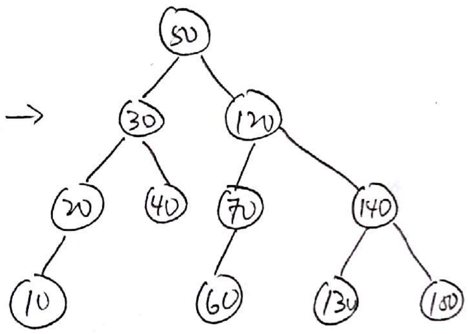

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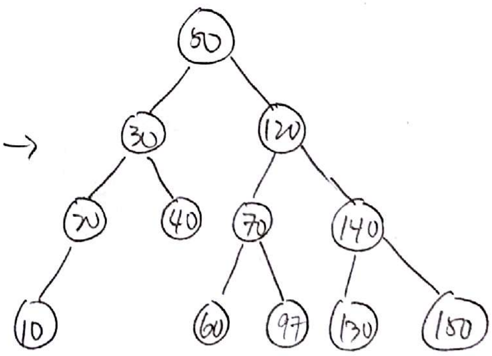

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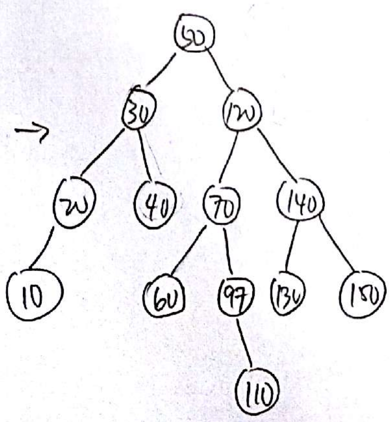

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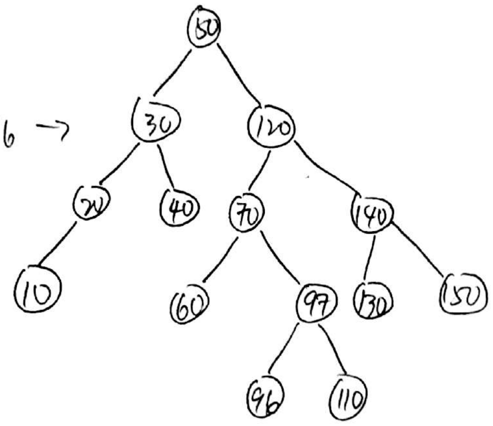
96 →



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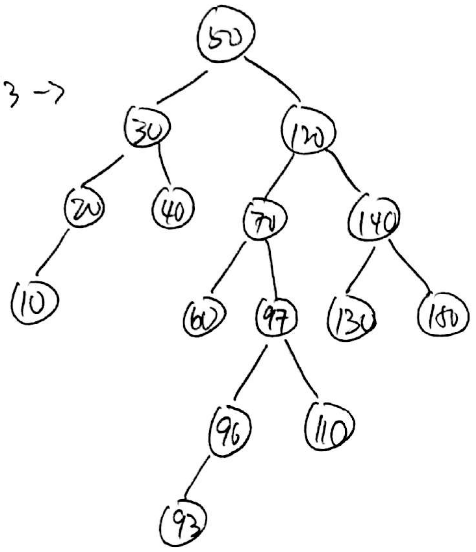

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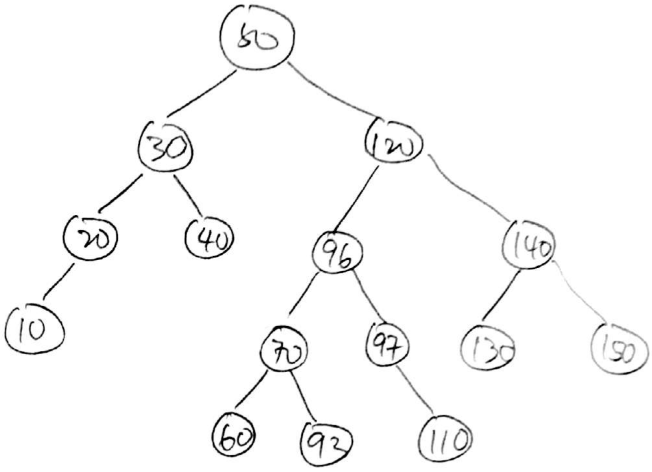

Final AVL tree

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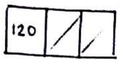
1d) B+ tree M = 5 L = 3

120 →



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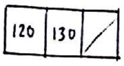

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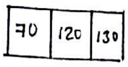

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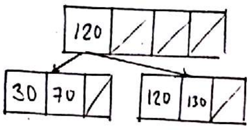

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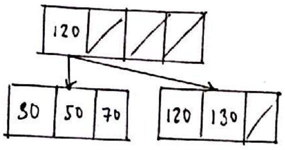

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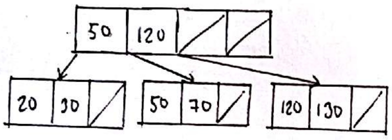

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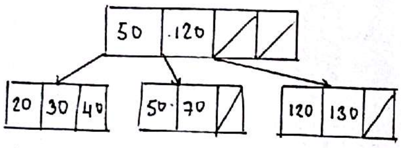

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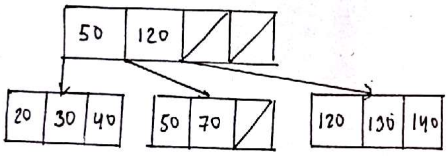

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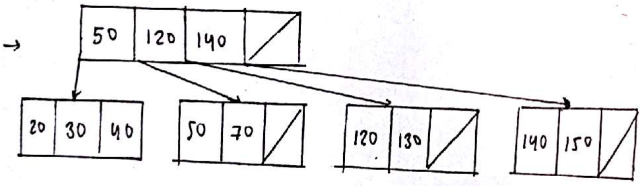

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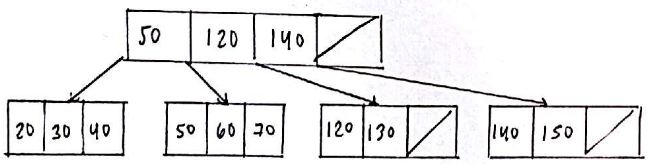

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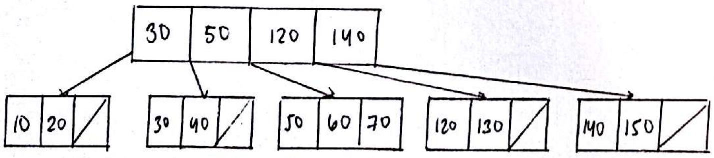

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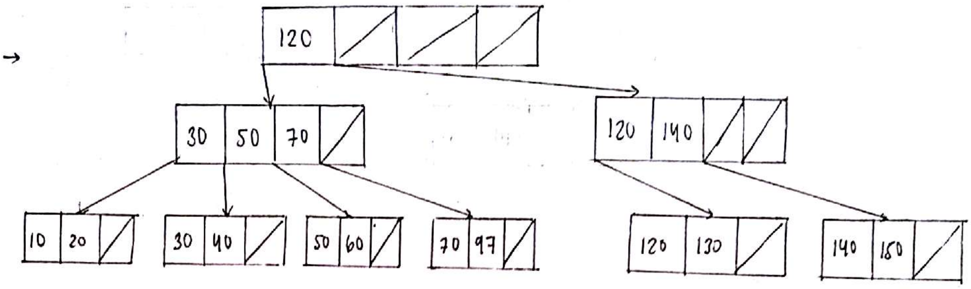

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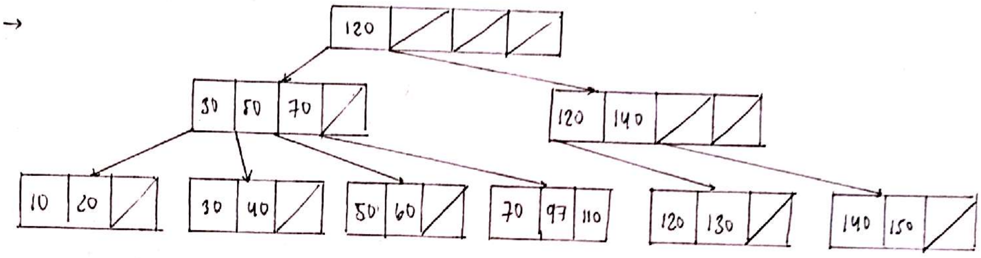

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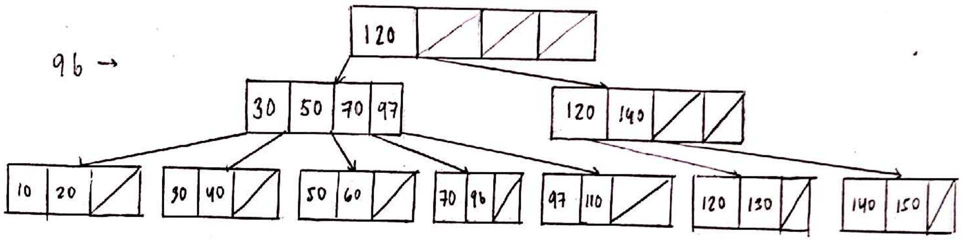

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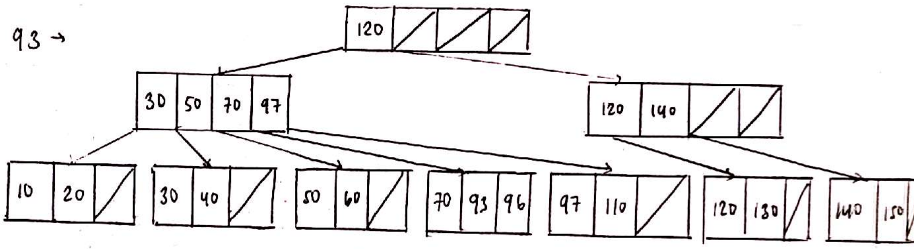

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2. • First consider the case where each computer is directly connected to 1 or more of the computers (call them nodes). The possible number of connections for each node are 1, 2, 3, 4, 5. But there are six nodes, so by pigeonhole principle, there must be at least two nodes that share the same number of connections.

• Now consider the case that one computer is connected to 0 other computers. This reduces the problem to the previous case, but with 5 computers, and then the same argument applies.

• In the case where two or more computers have 0 connections, the condition is obviously satisfied.

Thus, we have shown that at least two computers have the same number of connections

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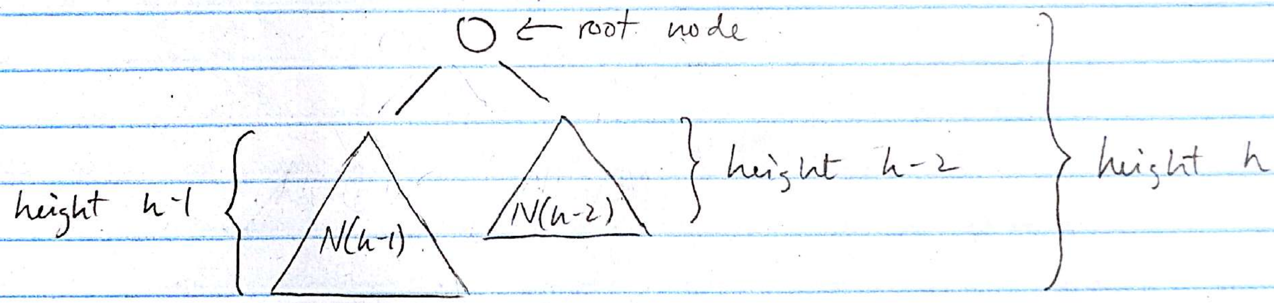
3. Consider:

$N(0) = 1$
$N(1) = 2$
$N(2) = 4$
$N(3) = 7$

The resulting recurrence relation is:

$$N(h) = N(h-1) + N(h-2) + 1$$
$$N(0) = 1, \quad N(1) = 2$$

This recurrence relation is correct because we can easily build the AVL tree of height h with the smallest number of nodes by starting with a root node and attaching AVL trees (with the smallest number of nodes) of height h-1 and h-2 which satisfies the requirement that the difference between the heights of the children are less than or equal to 1. The tree can be drawn as shown



Now to prove by induction that $N(h) = F(h+3) - 1$

Base Case:

$N(0) = 1 = F(3) - 1 = 2 - 1$
$N(1) = 2 = F(4) - 1 = 3 - 1$

Induction Step: Now assume $\begin{cases} N(h) = F(h+3) - 1 & \text{is true} \\ N(h-1) = F(h+2) - 1 \end{cases}$

Now we show that $N(h) = F(h+3) - 1$ holds for $h+1$

$$N(h+1) = N(h) + N(h-1) + 1 = F(h+3) - 1 + F(h+2) - 1 + 1$$
$$= F(h+3) + F(h+2) - 1$$
$$N(h+1) = F(h+4) + 1 = F((h+1)+3) + 1$$

QED