ALGORITHM DESIGN

Exercise Sheet 1

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General Terms: You can work on the exercises in teams of two people, and hand them in accordingly. However, make sure that each one of you has fully understood every solution you present. Do not copy any work from other students, the internet or other sources, and do not share your work with others outside your team. If at any point, any part of the exercises you hand in is apparent to be a copy of other work, this will result in the following consequences: All of your exercises, previous as well as upcoming ones, will be treated as if you did not hand them in at all, and you will have to participate in the written exam to make up for this. Please note that there will be no exception made, even if you are the original author of work someone else copied, or if your exercise partner is the one responsible. Therefore, please make sure to only choose a partner that you trust, and do not hand out your exercise solutions to others.

Late Policy: If you hand in your exercises after the due date, each day that you are late will result in a discount to your score, i.e. you will only receive 90% of the points if you are one day late, 80% on the second day after the due date, and 70% on the third. If you are late by more than three days, the assignment will get zero points in total.

Formal Requirements: Please typeset your solutions (11 pt at least), and state the names of both team members at the beginning of each sheet you hand in. Make sure to name the exact exercise each part of the solution refers to, otherwise it will not be graded. Please start a new page for each main exercise in the assignment (i.e. exercise 1, 2, etc., but not for each subquestion in them). Make sure your solution takes no more than one page for each main exercise (plus at most one extra page for the code, if an implementation is required). Everything after the first page will not be taken into account. So, for example, when the assignment has five exercises, please hand in five pages, one for each exercise. All solutions have to be sent via email to

birbas@diag.uniroma1.it

The subject of the email should be "Algorithm Design HW1-[Full-Name]", and the submitted file should be named "AlgorithmDesignHW1-[Full name].pdf".

Office Hours: There will be special hours for questions about the exercises announced via the piazza site. Presumably, this will take place in form of a zoom meeting .

Exercise 1

A group of n kids (denoted by K) wants to play football. Each kid has a level denoted by $\ell_k \in \{1, \ldots, L\}$, and they want to be partitioned into 3 (not necessarily of the same size) groups, with the same aggregated level.

a. Provide an algorithm that finds whether such partition exists, and if so, returns it.

b. What is the complexity of the algorithm as a function of n and L?

c. Prove the correctness of the algorithm.

Exercise 2

There is a network N of n people, in which every person i is associated with a subset of people $F_i \subseteq N$. For every $i \in N$, it holds that $i \in F_i$, and if person j is in F_i, then it must be that i is in F_j as well. Your goal is to advertise a product to every person in N. To do so, you need to choose a set of advertisers among N that will promote it, each will advertise it among his associates.

Prove that the problem of finding whether there exists a set of size at most k of advertisers that can promote your product, is NP-Complete.

Exercise 3

You play the following game: At each time $t = 1, \ldots, n$, you see a prize of value of x_t, and you need to decide whether to take it and go home, or continue to the rest of the prizes. All x_t are distributed independently from the uniform distribution over $[0, 1]$. A **dynamic** threshold-strategy is described by $\tau_1, \ldots, \tau_n$, where you select the first x_i that exceeds the corresponding τ_i. A **static** threshold-strategy is described by a single parameter τ where you select the first x_i that exceeds τ.

a. Calculate the best dynamic threshold-strategies for $n = 2, 3$, along with their expected performances.

b. Calculate the best static threshold-strategies for $n = 1, 2$, along with their expected performances.

c. What is the expectation of the best dynamic threshold as a function of n (write a recursive formula)?

Exercise 4

You have k houses located on a line in locations $L_1 < L_2 < \ldots < L_k$. You need to build hospitals on the line (i.e., to choose locations $H_1, \ldots, H_m$), so that all houses will be at most d far from a hospital (i.e., for all i, $|L_i - H_j| \leq d$ for some j).

Your goal is to design an algorithm that finds the minimal amount m of hospitals that are needed to be built.

a. Provide an algorithm that finds the minimal value m of hospitals that are needed, as well as a valid values of $H_1, \ldots, H_m$.

b. What is the complexity of the algorithm as a function of k?

c. Prove the correctness of the algorithm.

Exercise 5

Consider a graph $G = (V, E)$, with integer capacities on the edges, such that for all $e \in E \setminus \{e^*\}$, it holds that c_e is an even number, and c_{e^*} is odd. Suppose that there is a maximum flow in this graph with odd flow.

a. Prove/Disprove: It must be that in every maximum flow, there is a flow in e^* (i.e., $f_{e^*} > 0$).

b. Prove/Disprove: It must be that in every maximum flow, there is a **full** flow in e^* (i.e., $f_{e^*} = c_{e^*}$).