A simple algorithm for the transport of gamma rays in a medium

F. Arqueros and G. D. Montesinos
Departamento de Física Atómica, Molecular y Nuclear, Facultad de Ciencias Físicas, Universidad Complutense, E-28040 Madrid, Spain

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A simple Monte Carlo algorithm for the simulation of the passage of γ rays of about 1 MeV in a medium is presented. In this energy range the only relevant processes are Compton scattering and photoelectric absorption. The algorithm allows the visualization of the photon tracks as well as the calculation of many quantities of interest. Several problems for a layer and a cylinder are proposed. In particular, the energy transferred to electrons in a water cylinder as a function of depth and the line shape of a NaI scintillator is calculated. © *2003 American Association of Physics Teachers.*
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I. INTRODUCTION

Understanding the effects of the passage of ionizing radiation through matter is important in several fields, particularly those dealing with radiation protection and measurement. Therefore, the phenomenology of radiation-matter interactions are included in many undergraduate physics programs. The interaction of an ionizing particle with a single atom follows well established laws. In particular, a γ ray can either be absorbed due to the photoelectric effect, scattered by an atom, or converted into an electron–positron pair in the field of an atom. However, the effects of a γ-ray beam passing through a medium are usually the result of many individual interactions because a photon can interact many times before it is absorbed or escapes from the medium. In addition, most of the above processes release kinetic energy to electrons, which also interact with the medium.

The most reliable way to predict the effects of a γ-ray beam is by a Monte Carlo simulation. Because fast computers are available, many codes have been developed that allow a detailed simulation of the passage of radiation through matter. For instance, the EGS4 code is successfully used in particle physics as well as in medical physics.[1]

Studying the structure of one of these simulation codes would be very helpful for obtaining a deeper knowledge of the various processes and how they are related to each other. Unfortunately, the professional programs are extremely complex for an undergraduate student even at the user level. However, in many cases it is possible to use simple algorithms that can be understood and programmed by undergraduate students. For example, Williamson and Duncan[2] have developed a simple Monte Carlo calculation for the transport of nonrelativistic electrons which allows the student to visualize the electron tracks as well as do interesting calculations. In this paper we propose a simple algorithm for the simulation of the passage of γ rays in the energy range around 1 MeV through a medium. This energy range is very interesting because it corresponds to that of γ rays emitted by most radioactive sources. Some applications of the code in radiation dosimetry and nuclear instrumentation will be discussed.

II. THE PROBLEM

The main physical processes associated with the transport of γ rays through matter are coherent scattering, incoherent scattering, the photoelectric effect, and pair creation.[3–5] In the energy range of interest the probability of coherent scattering (or Rayleigh scattering, that is, the elastic scattering of a photon with the entire atom) is very low. In addition, the energy threshold for a γ ray to be converted into an electron–positron pair is twice the rest mass of the electron (1.02 MeV). In the energy range around 1 MeV incoherent scattering (or inelastic scattering) with an atom is dominant. In this process the photon transfers energy to an atomic electron which is ejected. At smaller energies the photon can also undergo a photoelectric absorption by an atom, which emits an electron. The subsequent atomic de-excitation takes place by emission of x-ray photons and/or Auger electrons. In both processes, incoherent scattering and the photoelectric effect, the energy is transferred to atomic electrons which are stopped in a very short distance in comparison to the mean free path of the γ rays.[6]

Incoherent scattering can be approximately described by the Compton effect,[3,4] that is, the elastic scattering of a photon with a free electron (Fig. 1). From conservation of energy and momentum, the following relations can be easily deduced between the energy of the incident photon, E_0, the energy of the scattered photon, E, and the deviation angle of the photon θ,

$$\frac{E}{mc^2}=\frac{\gamma}{1+\gamma(1-\cos\theta)}, \tag{1}$$

where the parameter γ is the incident energy in electron mass units, $\gamma=E_0/mc^2$. The energy T transferred to the electron ranges from 0 (at $\theta=0$) to a maximum value of

$$T_{\max}=E_0\frac{2\gamma}{1+2\gamma} \tag{2}$$

for backscattered γ rays at $\theta=180$ degrees.

The probability distribution of θ is given by the well-known Klein–Nishina cross section[3,4]

$$\frac{d\sigma_{\mathrm{KN}}}{d\Omega}=\frac{r_e^2}{2}\frac{1}{[1+\gamma(1-\cos\theta)]^2}\times\left[1+\cos^2\theta+\frac{\gamma^2(1-\cos\theta)^2}{1+\gamma(1-\cos\theta)}\right], \tag{3}$$

which integrated over the solid angle Ω gives a total cross section

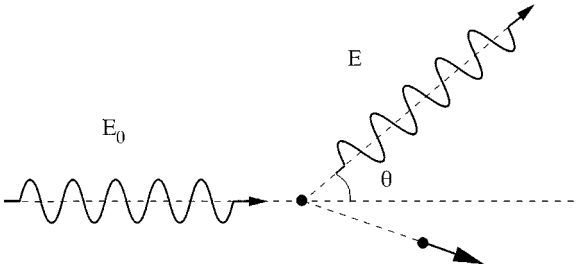

Fig. 1. Compton effect. A γ ray of energy E_0 is scattered by a free electron reducing its energy to E. The electron is emitted with energy E_0-E.

$$\sigma_{\mathrm{KN}}=2\pi r_e^2\left[\frac{1+\gamma}{\gamma^2}\left(\frac{2(1+\gamma)}{1+2\gamma}-\frac{\ln(1+2\gamma)}{\gamma}\right)+\frac{\ln(1+2\gamma)}{2\gamma}-\frac{1+3\gamma}{(1+2\gamma)^2}\right], \tag{4}$$

where $r_e=2.82\times10^{-15}$ m is the classical electron radius. If we neglect the binding energy of the atomic electrons, the cross section for incoherent scattering for an atom is given by multiplying the Klein–Nishina cross section by the atomic number Z. The effect of the binding energy can be taken into account by reducing the Z value by an amount that increases with atomic number and decreases with both increasing E_0 and θ. At about 1 MeV this correction is negligible for angles higher than a few degrees, even for heavy elements.[7] In Fig. 2 the value of $Z\sigma_{\mathrm{KN}}$ is plotted versus E_0 for lead along with the corresponding accurate incoherent cross sections[5] for comparison. Figure 3 shows the cross section per molecule for NaI [that is $(Z_{\mathrm{Na}}+Z_{\mathrm{I}})\times\sigma_{\mathrm{KN}}$]. These figures show that in the energy range where incoherent scattering dominates, the correction is negligible, and in our algorithm the Klein–Nishina formulas, Eqs. (3) and (4) have been used.

A rigorous treatment of the photoelectric effect leads to a cross section σ_{PE} that cannot be expressed by simple analytical formulas. Therefore, photoelectric cross sections are usually tabulated for E_0 and Z. See, for instance, Refs. 5 and 8–11. The implementation in our simple algorithm of these tables would make our program more complicated. Instead, we have used the following approximation for the photoelectric cross section:[12]

$$\sigma_{\mathrm{PE}}\approx\phi_0\left(b_0+\frac{b_1}{\gamma}+\frac{b_2}{\gamma^2}\right)f(\alpha Z), \tag{5}$$

where α is the fine structure constant, and ϕ_0 is given by

$$\phi_0=4\pi r_e^2(\alpha Z)^4\left(\frac{Z}{\gamma}\right). \tag{6}$$

The values of b_0, b_1, b_2, and $f(\alpha Z)$ have been tabulated[12] as a function of αZ. Fourteen values of αZ (ranging from 0.00 to 0.70) are sufficient to obtain an approximate value of σ_{PE}. Therefore, a simple algorithm for the calculation of the photoelectric cross section for any element can be carried out by implementing a very small table containing 14 sets of four parameters each [b_0, b_1, b_2, and $f(\alpha Z)$]. In Figs. 2 and 3 we have plotted the values of σ_{PE} given by Eq. (5) together with the accurate one from Storm and Israel.[8] We see that the Messel and Crawford approximation in Ref. 12 underestimates the cross section for energies beyond the K edge. As will be discussed later, for heavy elements, a lower photoelectric cross section leads to longer path lengths of the photons. However, for a particular element or compound, it is possible to find a simple law that provides an accurate value[5,8–11] of σ_{PE} in the energy range of interest of our simulations (see Sec. IV B).

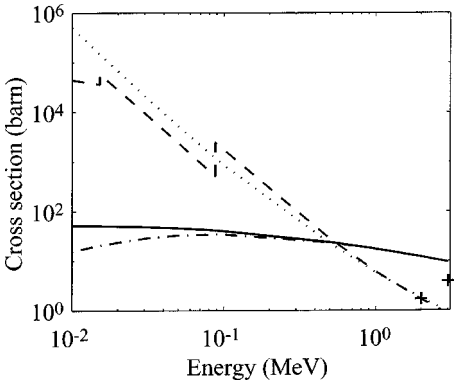

Fig. 2. Cross section per lead atom for various processes: Incoherent scattering (dashed–dotted line), Compton effect for a free electron multiplied by Z (solid line), photoelectric effect from tabulation (Ref. 8) (dashed line) and from approximation (Ref. 12) (dotted line), and pair creation (Ref. 5) (crosses).

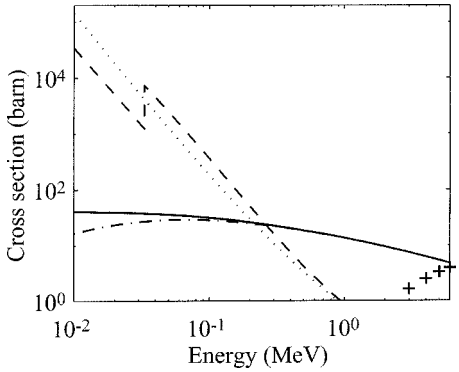

Fig. 3. Cross sections per NaI molecule (summation of the cross sections of the corresponding atoms). See the caption of Fig. 2 for labels.

III. THE SIMULATION

A. The ingredients

When a photon is in a medium, it traverses a certain distance s (in a straight line) before it interacts with an atom. As is well known, the magnitude of s follows the statistical distribution[3]

$$p(s)ds=\frac{1}{\lambda}e^{-s/\lambda}\,ds, \tag{7}$$

with a mean free path

$$\lambda=\langle s\rangle=\frac{1}{n\sigma}, \tag{8}$$

where n is the number of atoms per unit volume, and σ is the total cross section per atom, which in our simple model is given by

$$\sigma = \sigma_{PE} + Z\sigma_{KN}. \tag{9}$$

The cross section for a molecule (or compound) can be obtained by adding up the corresponding cross sections of the atoms.

The interaction of the primary γ ray with the medium gives rise either to photoelectric absorption or to Compton scattering with probabilities proportional to the corresponding cross sections [Eqs. (4) and (5), respectively]. In the first case all the energy is transferred to the medium at the position of the interaction, while in the second case the photon is scattered in a direction given by the polar and azimuthal angles θ and ϕ, respectively. The value of ϕ follows a uniform probability distribution between 0 and 2π, and θ follows the probability distribution

$$p(\theta)d\theta = \frac{d\sigma_{KN}}{d\Omega} 2\pi \sin\theta \, d\theta. \tag{10}$$

The energy of the scattered photon can be calculated by Eq. (1) and the amount of energy released to the medium is $T = E_0 - E$. Assuming that the direction of the primary photon in the lab coordinate system is given by the angles θ_1 and ϕ_1, the direction of the scattered photon (θ_2, ϕ_2) can be obtained as

$$\cos\theta_2 = \cos\theta_1 \cos\theta + \sin\theta_1 \sin\theta \cos\phi, \tag{11a}$$

$$\cos(\phi_2 - \phi_1) = \frac{\cos\theta - \cos\theta_2 \cos\theta_1}{\sin\theta_1 \sin\theta_2}, \tag{11b}$$

$$\sin(\phi_2 - \phi_1) = \frac{\sin\theta \sin\phi}{\sin\theta_2}. \tag{11c}$$

In general, these relations can be applied for any subsequent Compton collision.

B. The algorithm

An algorithm for the transport of γ rays in a medium for simple geometries like a layer with infinite parallel faces or a cylinder can be easily developed. For simplicity, we assume that the medium consists of a single material that is described by the constituent atom, molecule, or compound, and the density.

The simulation of the random processes is carried out by a Monte Carlo method.[13] The parameters that have to be randomly generated are s, θ, ϕ, and the selection of the processes (photoelectric or Compton) when an interaction occurs.

A simple method for the generation of a random variable y with a probability density $p(x)$ defined in the interval (a,b) relies on the fact that r defined as

$$r = Q(y) = \int_a^y p(x)dx \tag{12}$$

is itself a random variable with a uniformly distributed probability in the interval (0,1). Therefore y can be obtained from

$$y = Q^{-1}(r), \tag{13}$$

where r is a random number in the interval (0,1) that can be provided by a computer. This method is feasible as long as $Q(y)$ is a function that can be inverted. In our case we apply this method to generate the path traversed between two interactions [Eq. (7)]. From Eqs. (12) and (13), it is easily inferred that the random variable s is related to r by

$$s = -\frac{\ln r}{n\sigma}. \tag{14}$$

This method also can be easily applied for the generation of ϕ,

$$\phi = 2\pi r. \tag{15}$$

Unfortunately, this direct method cannot be used for the generation of θ. Although Eq. (10) can be integrated, θ cannot be obtained analytically from r. In this case we can apply the acceptance–rejection method.[13] Lower and upper limits, K_l and K_u respectively, are defined for $p(x)$. Also lower and upper limits, x_l and x_u are defined for x. Two random numbers r_x and r_y are generated with uniform probability in the intervals $x_l \leqslant r_x \leqslant x_u$ and $K_l \leqslant r_y \leqslant K_u$, respectively. In other words, random points are generated in a rectangular area. If $r_y < p(r_x)$, the trial is accepted and the value $x = r_x$ is used. Otherwise, the value is rejected. It can be demonstrated that the probability of x follows the $p(x)$ function.[13] This procedure is less efficient than the direct method in which all random numbers generated by the computer are used, but the acceptance–rejection method can be applied to any bounded function. This method can be applied for the generation of θ for Compton scattering.

Finally the selection of the process, Compton scattering or photoelectric absorption, can be implemented by splitting the (0,1) interval in two subintervals with a length proportional to the corresponding cross sections. A γ ray moves inside the medium until either it is absorbed by the photoelectric effect or leaves the medium, that is, until the position of the next randomly generated interaction occurs outside the medium.

C. The program

The above algorithm has been implemented in MATLAB[14] by the authors. The conversion to other languages such as Basic, Fortran, and C is straightforward. The flow diagram of the algorithm is shown in Fig. 4. The input parameter are the number of chemical elements, number of atoms of every species per molecule or compound, atomic number, atomic weight of the atoms, density, and the energy of the γ rays. The program next computes the various cross sections for the first interaction. Next, the path traversed before the first collision takes place is randomly generated using Eq. (14). If the collision occurs inside the medium, the kind of process is chosen. If it turns out to be Compton scattering, the direction and energy of the photon is updated, and the final position is computed. If the photon is still inside the medium, the next path is generated and the procedure is repeated. After every interaction the energy transferred to the medium (T for a Compton scattering or E_0 for a photoelectric absorption) and the corresponding coordinates are stored. A trial ends either when the photon escapes from the medium or a photoelectric absorption takes place.

We recommend that readers write their own code following the above discussion. However, the program developed by the authors can be freely downloaded.[15]

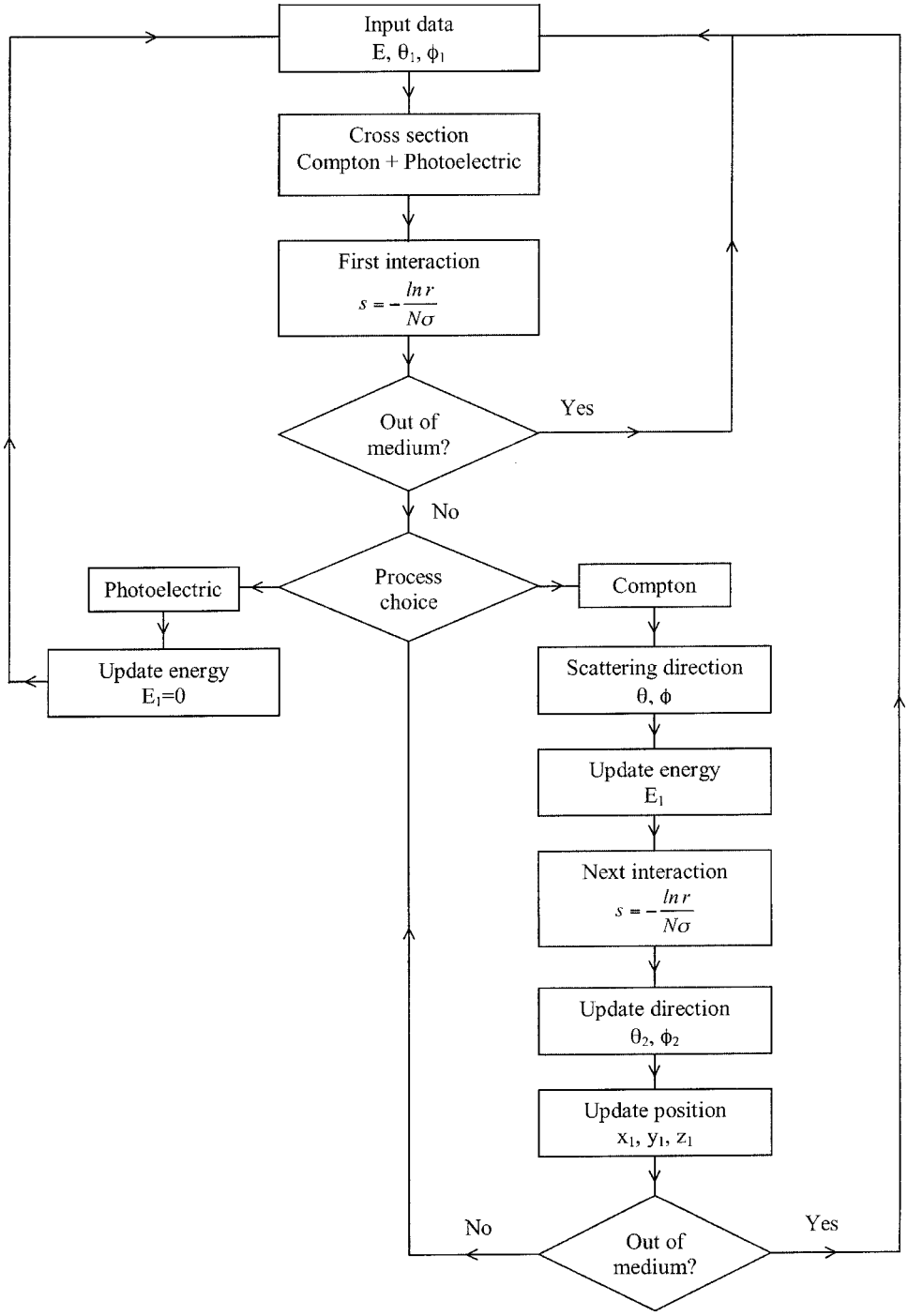

Fig. 4. Flow diagram of the algorithm for the simulation of the passage of γ rays through matter.

IV. APPLICATION TO SEVERAL PRACTICAL CASES

The simple model can be used to visualize the γ-ray tracks in the medium. Many interesting features of the passage of γ rays through matter can be observed in this way. For example, in Fig. 5 (upper panel) the graphical output of our program for an Al layer 30.2 cm (5 λ) thick corresponding to a 150 photons beam of 1 MeV γ rays is shown. In this case, the γ rays often suffer several Compton collisions before they are absorbed. It is also clearly seen that backscattering is the consequence of a number of collisions with high scattering angles. Students can obtain a first impression of these features, which strongly depend on the medium and the γ-ray energy.

The program optionally shows the positions where the energy has been deposited (either by photoelectric absorption or Compton scattering). Figure 5 (lower panel) shows the corresponding result for a 1000 photon beam. This plot is very useful for the visualization of the spatial distribution of the energy left in the medium, which is of much interest for radiation dosimetry.

The program also generates histograms corresponding to the angle and energy distributions of the photons escaping from the medium. In addition, the spectrum of the energy transferred to the medium as well as its spatial distribution is calculated.

The predictions of the simple algorithm have been compared with those of the EGS4 code. For light elements the

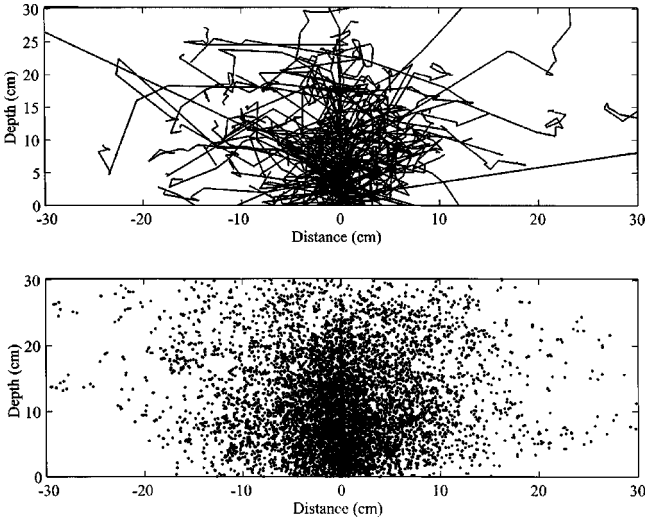

Fig. 5. A beam of 1 MeV γ-rays impinging on an Al layer of 30.2 cm (5 λ) width. The upper panel shows the tracks of the γ rays. The lower panel shows the positions where a γ ray has deposited energy either by photoelectric absorption or by Compton scattering.

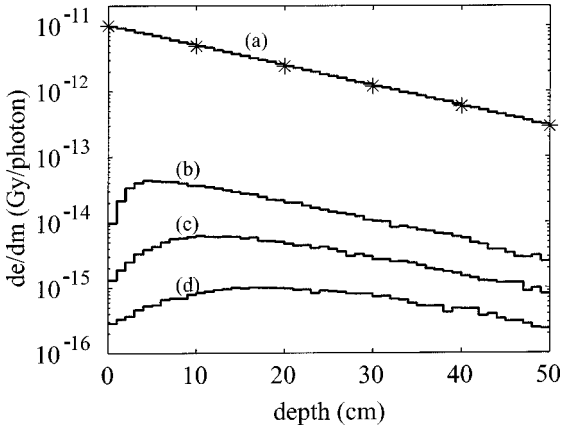

Fig. 6. Kerma in a water cylinder of 10 cm radius and 50 cm length versus radial distance for a 1 MeV γ-ray beam impinging perpendicularly to the cylinder axis. The average energy transferred per unit mass, per photon for radial distances of (a) 0.0 cm, (b) 0.8 cm, (c) 3.6 cm, and (d) 9.6 cm are shown. The predictions of Eq. (21) (stars) are also shown.

agreement is very good. However, for heavy elements (like Pb), the simple algorithm overestimates the path length of photons because Eq. (5) underestimates the photoelectric cross section in the energy range of the simulation (see Figs. 2 and 3). For light elements, incoherent scattering is dominant and, therefore, the inaccuracy of Eq. (5) does not give rise to any obervable effects.

A. Energy deposition

One of the most relevant quantities related to the effect of the radiation on a medium is the amount of energy that is absorbed and the corresponding spatial distribution. Consider a γ-ray beam traversing a medium along the z axis. The beam fluence[16] Φ, the number of γ rays per unit area, decreases with path length z as

$$\Phi(z)=\Phi_0 e^{-\mu z}, \tag{16}$$

where μ is the linear attenuation coefficient (that is, the collision probability of a photon per unit path length) given by $\mu=\sigma n$ [see Eq. (7)].

The quantity that most directly connects the description of the radiation beam with its effects is the *kerma*, which is defined as the kinetic energy transferred from photons to electrons per unit mass in a volume element.[16] In our case the energy transferred to electrons arises either from incoherent scattering or photoelectric absorption. The unit of kerma is 1 joule per kg. (In radiation dosimetry this unit is called the *gray* and is represented by G.)

The kerma K due to the primary collisions of a γ-ray beam can be obtained from

$$K=\Phi\left(\frac{\mu}{\rho}\right)\overline{E}_{\mathrm{tr}}, \tag{17}$$

where the mass attenuation coefficient μ/ρ is defined as the linear attenuation coefficient divided by the medium density ρ. $\overline{E}_{\mathrm{tr}}$ is the mean energy transferred per collision. In photoelectric absorption almost all the energy is transferred to the medium, while in a Compton scattering, T depends on θ according to Eq. (1). Hence, the value of $\overline{E}_{\mathrm{tr}}$ is given by

$$\overline{E}_{\mathrm{tr}}=\frac{\sigma_{\mathrm{PE}}}{\sigma}E_0+Z\frac{\sigma_{\mathrm{tr,KN}}}{\sigma}E_0, \tag{18}$$

where

$$\sigma_{\mathrm{tr,KN}}=\int_{T=0}^{T=T_{\max}}\frac{T}{E_0}\frac{d\sigma_{\mathrm{KN}}}{d\Omega}d\Omega. \tag{19}$$

The evaluation of kerma for water is of particular interest in radiotherapy because water is the main component of many biological tissues. As is well known,[3,5] the attenuation coefficient for a molecule (or compound) is given by

$$\frac{\mu}{\rho}=\sum_i\frac{\mu_i}{\rho_i}w_i, \tag{20}$$

where w_i is the weight fraction of the atoms in the molecule (or compound). Extensive data on attenuation coefficients for both atomic elements and compounds of interest in medical physics can be found in Refs. 5, 9, 10, and 17.

Because of the exponential attenuation of Φ with depth, the kerma due to primary collisions follows the relation

$$K=\Phi_0\frac{\mu}{\rho}\overline{E}_{\mathrm{tr}}e^{-\mu z}. \tag{21}$$

In the following we will calculate the kerma produced by 1 MeV γ rays in a beam of 0.5 cm^2 area in water. For this case the mass attenuation coefficient equals 0.0707 $\mathrm{cm}^2\,\mathrm{g}^{-1}$ [Eqs. (9) and (20)]. and $\overline{E}_{\mathrm{tr}}$ equals 0.440 MeV [Eqs. (18) and (19)]. These values can also be found in Ref. 5. From these results, a kerma value at the entrance of the medium ($z=0$) of 0.0622 MeV/g (that is 1.03×10^{-11} G) per incident photon is inferred. In Fig. 6 the exponential attenuation of the kerma along the beam path, Eq. (21), is shown. Unfortunately, this simple calculation does not account for the energy transferred to the medium by Compton scattered γ rays (see Fig. 5). However, our algorithm can easily obtain the energy that is released as kinetic energy to the atomic electrons in every collision, including those carried out by these secondary photons outside the beam path.

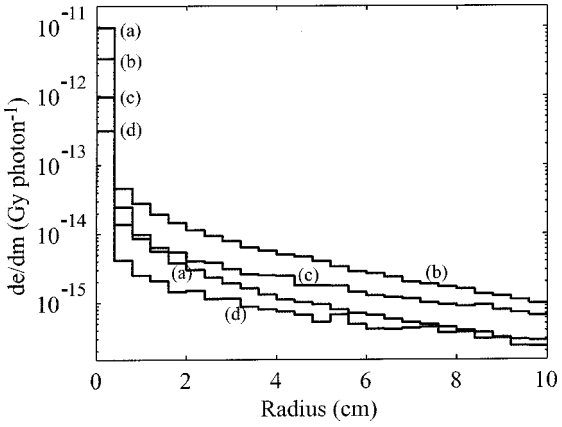

Fig. 7. Kerma in water (same case as Fig. 6) versus radial distance for depths of (a) 0.5 cm, (b) 15.5 cm, (c) 33.5 cm, and (d) 49.5 cm.

Let us consider a γ-ray beam along the axis of a water cylinder of radius 10 cm and length 50 cm. Because kerma is the energy per unit mass, we need to divide the medium in volume elements (cells) and then compute the amount of energy transferred in each cell. The smaller the cell, the higher the spatial resolution for the calculation of the kerma, but also higher are the statistical fluctuations of the results. In our case, each cell, is symmetrical about the cylinder axis and is defined by the radial distance to the axis r and the depth z. The size of the cells is $\Delta r = 0.4$ cm and $\Delta z = 1.0$ cm. Therefore, the volume of the cell is $2\pi r \Delta r \Delta z$. In Fig. 6 we have plotted the energy transferred (on the average) per unit mass, per photon, as a function of depth for several radial distances (0.0, 0.8, 3.6, and 9.6 cm). As expected, Eq. (21) is in agreement with the $r=0$ curve. Figure 7 shows the energy transferred as a function of r for several depths (0.5, 15.5, 33.5, and 49.5 cm), showing that a non-negligible amount of energy is released outside the beam path. The effect of secondary γ rays in radiation dosimetry is crucial, but unfortunately the contributing kerma cannot be predicted by simple formulas. Monte Carlo simulation is a very valuable tool for the corresponding calculation.

The quantity that is of more interest in radiotherapy and radiobiology is the absorbed dose, which is the energy per unit mass actually retained in the medium.[5] At this energy range the differences between the kerma and the absorbed dose are due to the track length of the ejected electrons, because the energy is actually deposited in the medium at a certain distance from the point where the photon interacts.

For 1 MeV photons the energy of the most energetic electrons is 0.8 MeV [Eq. (2)]. According to Ref. 5, the corresponding range in water is about 0.33 cm. This range is smaller than our cell size and thus Figs. 6 and 7 nearly give the absorbed dose. At higher energies, however, the electron track is non-negligible and the spatial distribution of dose does not equal that of kerma.

In radiation therapy it is extremely important to determine the absorbed dose at any point of the irradiated volume. For an accurate simulation of the dose it would also be necessary to simulate the tracks of the ejected electrons.

B. Response of a NaI scintillator

Another interesting application of our simple algorithm is the simulation of the response of a NaI scintillator to γ rays. Spectrometers based on a NaI scintillator are a very popular device in nuclear physics laboratories, and because of their low cost, they can be found in many undergraduate laboratories. The device consists of a NaI crystal attached to a photomultipler that detects the light pulse generated by the scintillator when a γ ray is absorbed. The energy absorbed in the scintillator is converted into visible light by luminescence, giving rise to a photomultiplier pulse height distribution that can be analyzed by a multichannel analyzer. In the ideal case all the γ-ray energy is deposited, and the light pulse has an amplitude proportional to the γ-ray energy. In practice, the amplitude spectrum is broadened by several instrumental effects. (See Ref. 4 for a detailed discussion on the various broadening sources.)

In a real detector, photons very often leave the scintillator after suffering one (or more) Compton collisions thus depositing an energy smaller than E_0. This effect gives rise to the well-known γ-ray line shape of inorganic scintillators,[4] which consist of the so-called photopeak (total absorption) together with a continuous spectrum in the energy interval from 0 to the Compton edge given by Eq. (2).

Our program can be used for an approximate calculation of the line shape of a NaI spectrometer. As mentioned, for heavy elements, our simple algorithm overestimates the path length of a photon and consequently overestimates the number of γ rays escaping from the scintillator, in particular through the side surface. To achieve a more accurate simulation, we have slightly modified the algorithm for this particular case. Instead of using the general formula of Messel and Crawford [Eq. (5)], we have computed the photo-cross-section of NaI from the accurate data of Fig. 3 (Ref. 5). From Fig. 3 the cross section can be approximately obtained as

$$\sigma_{\mathrm{PE}}(\mathrm{cm}^2) = 2.12\times 10^{-3} E^{-2.89} \quad (E < 0.033 \text{ MeV}), \tag{22a}$$

$$\sigma_{\mathrm{PE}}(\mathrm{cm}^2) = 2.35\times 10^{-3} E^{-2.78} \quad (E > 0.033 \text{ MeV}). \tag{22b}$$

Using the simple algorithm with Eq. (22) the spectrum of the energy absorbed by the NaI scintillator in our student lab has been calculated, and the result has been compared with the experimental line shape. The NaI scintillator is cylindrical with a 1.5″ diameter and 2.0″ height. The γ-rays are emitted by an isotropic source of 0.2 cm radius located at a distance of of 3.7 cm to entrance face of the scintillator. The energy of the γ-rays is 1.275 MeV, which corresponds to an isolated peak of the nucleus ^{22}Na with which the simulation will be compared. In principle, this energy is over the threshold for pair creation. However, as can be seen in Fig. 3, the corresponding cross section is negligible in comparison to Compton scattering.

Figure 8(a) shows the simulated spectrum consisting of a pronounced photopeak which contains 7.15% of the input γ rays together with a continuum spectrum due to photons escaping after one Compton collision (14.7% efficiency) or more than one (8.13%). For comparison with the experimental data, the theoretical line shape in Fig. 8 has to be folded into the experimental broadening function. The experimental shape of the photopeak was found to be Gaussian with a full width at half-maximum weakly dependent on the energy (90 KeV at around 1.2 MeV) has been used for that purpose. If we fold the spectrum of Fig. 8(a) with this Gaussian, we obtain the result shown in Fig. 8(b) which can be compared with the experimental spectrum. The comparison has been

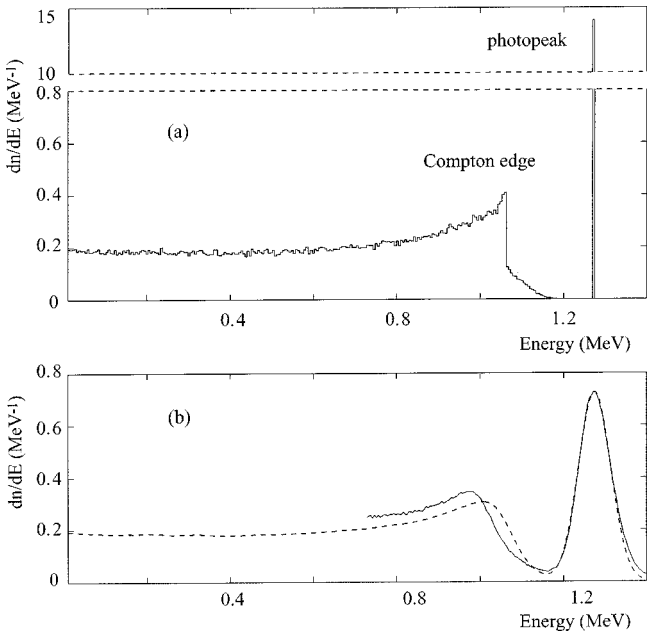

Fig. 8. Line shape of a NaI scintillator (cylinder of 1.5″ diameter and 2″ height) for γ rays of 1.275 MeV energy. (a) Simulation results for the spectrum of the energy absorbed by the scintillator. (b) Experimental line shape (solid) and theoretical line shape (dashed) obtained by folding the energy spectrum of plot (a) with a Gaussian of 90 KeV width.

restricted to energies higher than 0.7 MeV. At lower energies the spectrum is dominated by the profile of the very intense annihilation peak at 0.511 MeV. The experimental spectrum shown in Fig. 8(b) has been obtained by subtracting the laboratory background spectrum and has been normalized to the intensity of the simulated photopeak. The spectrum shows reasonable agreement between simulation (dashed line) and experiment (solid line) for the Compton continuum. The discrepancy is due to secondary experimental effects not taken into account in the simulation.

The efficiency of the light collection depends on the position of radiation interaction. On the other hand, the radioactive source is attached to a cylindrical holder with a hole through which the photons escape. Many γ rays hit the aluminum hole walls, thus degrading the energy spectrum of the beam. In addition, γ rays also interact with the front and side faces of the scintillator as well as with the photomultiplier. All these effects tend to smooth the sharp cutoff of the continuum spectrum at the Compton edge energy.

V. CONCLUSIONS

We have discussed a simple algorithm for the simulation of the passage of γ rays with energy around 1 MeV in a medium. In this energy range the only relevant processes are incoherent scattering and photoelectric absorption. The Klein–Nishina formula accounts accurately for the cross section of the former, while the latter can be approximately described by an analytical formula. The energy at which the cross section for pair creation, σ_{pair}, is significant in comparison to Compton scattering is much higher than 1 MeV; in particular for light elements (Al, for example) $\sigma_{\text{pair}}=\sigma_{\text{KN}}$ at about 15 MeV. Therefore, the algorithm is also applicable to energies well above 1 MeV.

The algorithm is simple enough to be understood and converted to a computer program by an undergraduate student with a basic knowledge of programming. The program can be used as a tool to assist students in becoming familiar with the phenomenology of the interaction of gamma rays in matter. The dependence on the material properties and energy can be easily tested.

We have used the simple algorithm for the calculation of the energy deposited in a cylinder of water versus depth and radial distance. We also simulated the line shape of a NaI scintillator and compared the shape with a spectrum measured in the laboratory. Finally two problems dealing with the effect of multiple Compton scattering are proposed in the Appendix.

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APPENDIX

We include here some suggested problems that can be solved by using the simple algorithm discussed in this paper.

Problem 1. (a) A pencil beam of 1 MeV γ rays impinges perpendicularly on a layer of Al 0.2 λ thick (the mean free path for 1 MeV γ rays in Al is 6.02 cm). Plot the probability distributions per unit solid angle of a photon to be transmitted and backscattered, respectively, with an angle θ between the beam direction and that of the escaping photon (do not take into account those γ rays that cleanly traverse the layer without interacting). Compare the results of the simple algorithm with the predictions of the Klein–Nishina formula Eq. (3) for single Compton scattering.

(b) Plot the probability of an interacting photon for escaping forward or backward as a function of the output energy. Calculate the energy bounds of a photon after a single Compton scattering using Eq. (1) for both forward and backscattered γ rays. Are the escaping photons bounded to these limits? Why?

(c) Repeat the calculations for 1.0 λ and 5.0 λ thicknesses.

Problem 2. Repeat the calculations of Problem 1 for layers of Pb (mean free path of 1.25 cm). Check that the energy of the escaping photons are basically bounded to the limits imposed by Eq. (1). Why?

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Tantalus' Cup. The mythological Tantalus angered the gods, and was condemned to hang from the limbs of a tree over a pool of water. When he tried to drink, the water level receded from his lips. Tantalus' cup is used as a paradox in introductory physics. The unsuspecting user pours water into the cup, and the water level rises evenly in the cup and the inside end of the syphon tube. All is well until the water level just starts to exceed the highest point in the curving tube. At this point syphon action commences, and the cup empties down to the level of the bottom of the tube. This large example at Harvard University is 27 cm high. (Photograph and notes by Thomas B. Greenslade, Jr., Kenyon College)