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Assignment – 1

1.1 A.1] $G_1(s) = \frac{10}{s+10}$

1)

→ Pole: It is the value of s at which $G_1(s) \to \infty$.

$\therefore$ pole: $s = -10$

$G_1(0) = \frac{10}{0+10} = 1.$

2) comparing with standard form, $G(s) = \frac{1}{1+\frac{s}{\omega_0}}$, $\boxed{\omega_0 = 10}$

$\therefore$ putting $s = j\omega$,

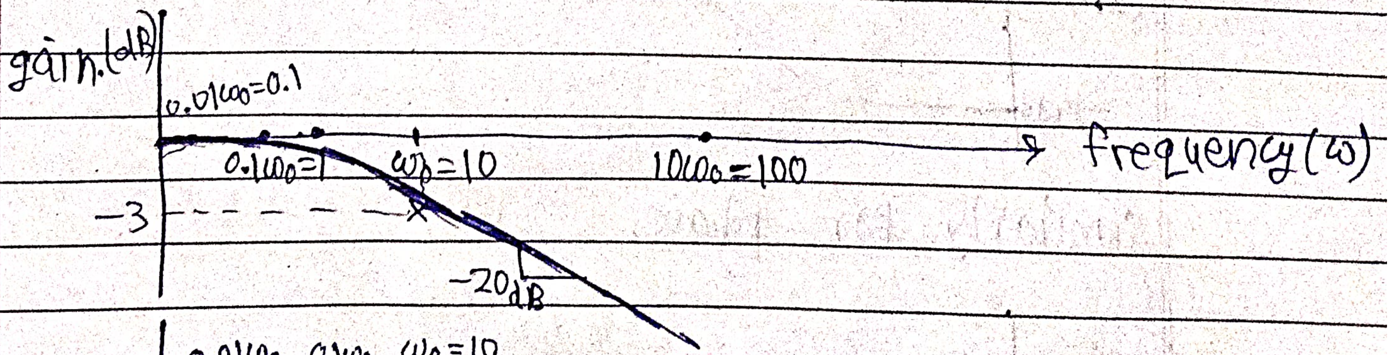
$$G(s) = \frac{1}{1+j\frac{\omega}{\omega_0}} = \frac{1 - j\frac{\omega}{\omega_0}}{1+\frac{\omega^2}{\omega_0^2}}$$

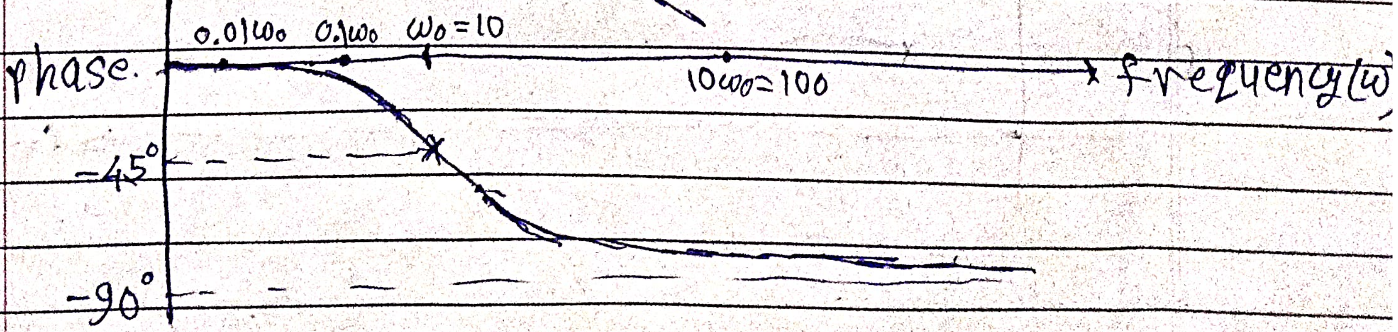
$\therefore$ Real $= \frac{1}{1+\frac{\omega^2}{\omega_0^2}}$, Imag $= \frac{-\omega/\omega_0}{1+\frac{\omega^2}{\omega_0^2}}$

$$\boxed{|G(j\omega)| = \left(1+\frac{\omega^2}{\omega_0^2}\right)^{-1/2}}, \quad \arg(G(j\omega)) = \tan^{-1}\left(-\frac{\omega}{\omega_0}\right)$$

$\therefore$ gain $\Rightarrow 20\log_{10}\left(1+\frac{\omega^2}{\omega_0^2}\right)^{-1/2}$

phase $= \tan^{-1}\left(\frac{-\omega}{\omega_0}\right)$.





1.2 A.2) $G_2(s) = \frac{s-2}{s+10}$

1)

→ zero : $s=2$, pole: $s=-10$, $G_2(0) = \text{DC gain} = \frac{-1}{5} = -0.2$

2)

→ $G_2(s) = \frac{s-2}{s+10}$

$\therefore$ gain ⇒ gain of $(s-2)$ − gain of $(s+10)$

gain of $s-2$ ⇒ $s-2 = 2\left(\frac{s}{2}-1\right) = \frac{2}{\frac{1}{\frac{s}{2}-1}} = \frac{-2}{\frac{1}{1-\frac{s}{2}}} = \frac{-2}{\frac{1}{1+\left(\frac{s}{-2}\right)}}$

$\boxed{\therefore \text{gain of } s-2 = \text{gain}(-2) - \text{gain}\left(\frac{1}{1-\frac{s}{2}}\right)}$

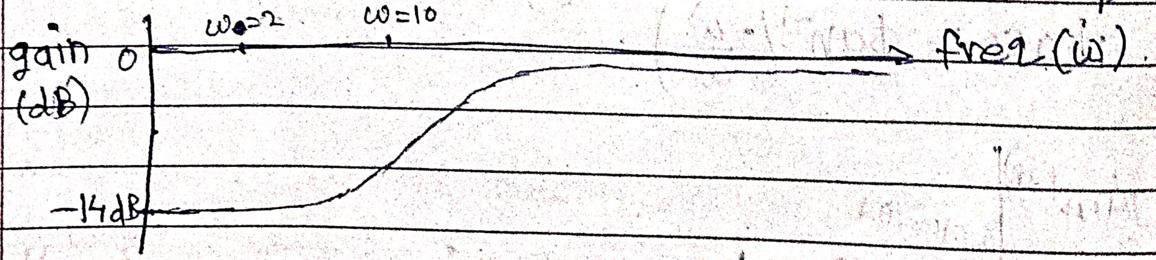
gain of $s+10$ ⇒ $s+10 = 10\left(1+\frac{s}{10}\right) \Rightarrow \frac{10}{\left(\frac{1}{1+\frac{s}{10}}\right)}$

$\boxed{\therefore \text{gain of } s+10 = \text{gain of } 10 - \text{gain}\left(\frac{1}{1+\frac{s}{10}}\right)}$

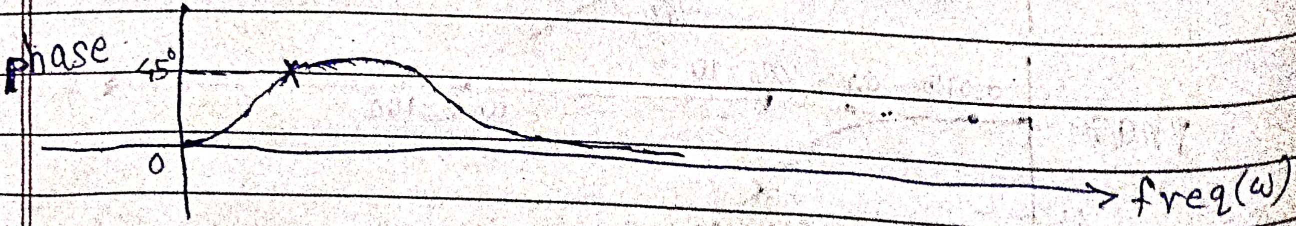
$\therefore \text{gain}\left(\frac{s-2}{s+10}\right) = \text{gain}(-2) - \text{gain}(10) + \text{gain}\left(\frac{1}{1+\frac{s}{10}}\right) - \text{gain}\left(\frac{1}{1+\frac{s}{(-2)}}\right)$

★ gain(-2) = 6 dB, gain(10) = 20 dB {constant}.

$\therefore$ Adding gain plots, for all 4, resultant plot ⇒



similarly, for phase,



1.3] A3] $G_3(s) = \dfrac{100}{s^2+10s+100}$

1]

→ For pole, $s^2+10s+100=0 \Rightarrow s = \dfrac{-10 \pm \sqrt{100-400}}{2} = \dfrac{-10 \pm \sqrt{-300}}{2}$

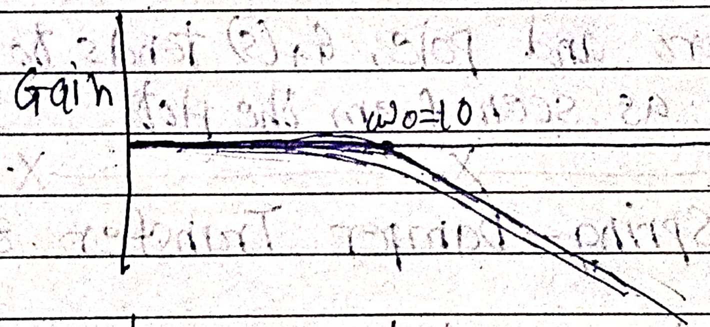
$\therefore s = \dfrac{-10 \pm 10\sqrt{3}i}{2} \Rightarrow \boxed{s = -5-5\sqrt{3}i \;\&\; s = -5+5\sqrt{3}i}$

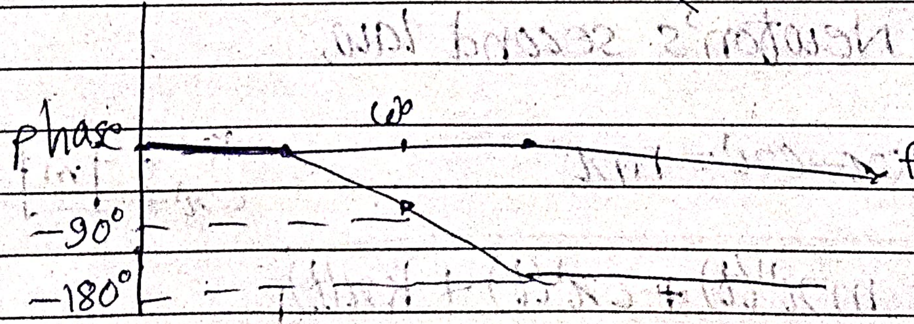
2.] BODE plots ⇒

Comparing to standard form, $G(s) = \dfrac{\omega_0^2}{s^2+2z\omega_0 s+\omega_0^2}$

$\therefore \omega_0 = 10,\; 2z\omega_0 = 10 \Rightarrow \boxed{z = \frac{1}{2} = 0.5}$

When $z=0.5$, for $\omega=\omega_0$, gain $=0$.





1.4] A.4] $G_4(s) = \dfrac{0.1s+1}{0.01s+1}$

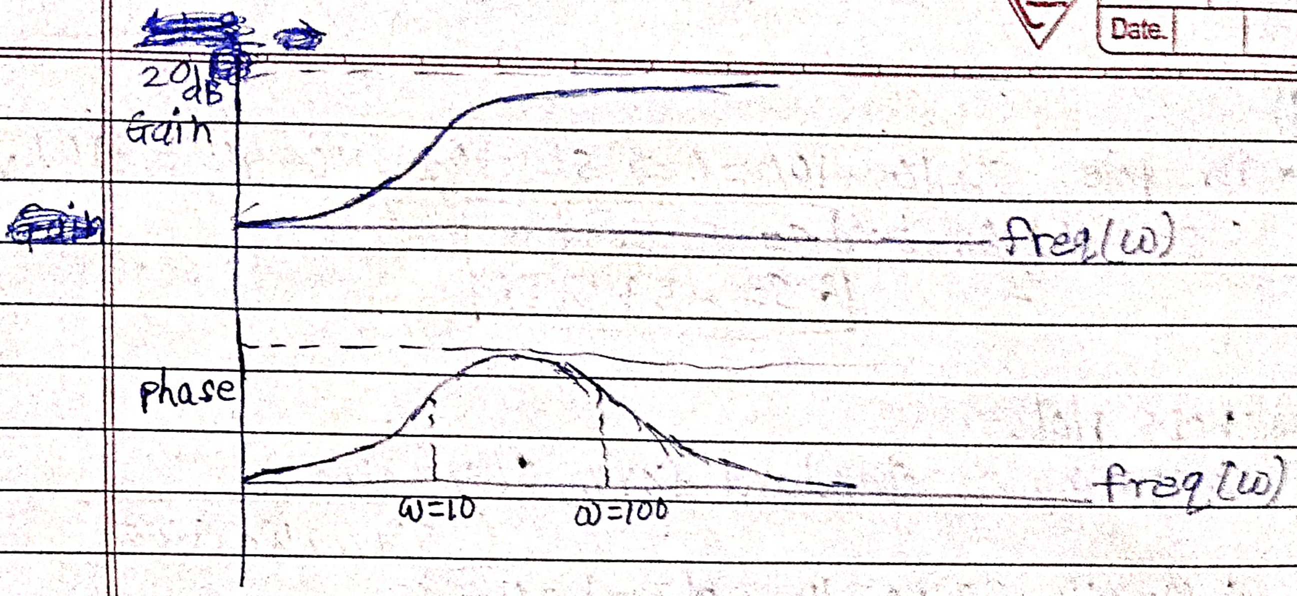
1]

→ zero at $s = \dfrac{-1}{0.1} \Rightarrow \boxed{s=-10}$

pole at $s = \dfrac{-1}{0.01} = -100 \Rightarrow \boxed{s=-100}$

2]

Gain $\Rightarrow$ gain$\left(1+\frac{s}{10}\right)$ $-$ gain$\left(1+\frac{s}{100}\right)$



4) $\rightarrow$ zero is at $\omega=10$,
pole at $\omega=100$.
$\therefore$ Between zero and pole, $G_4(s)$ tends to add positive phase as seen from the plot.

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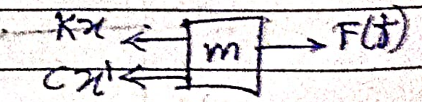
2 Part B: Mass-Spring-Damper Transfer ~~Equation~~ Function

B1)
1) Using Newton's second law,

$\therefore F(t) - kx - cx' = mx''$



$\therefore \boxed{F(t) = m x''(t) + c x'(t) + k x(t)}$

2) Applying Laplace's transform,
$\boxed{F(s) = (ms^2 + cs + k)X(s).}$

3) Transfer function:-
$\therefore \boxed{G(s) = \frac{X(s)}{F(s)} = \frac{1}{ms^2+cs+k}}$

B2) Numerical Example and Bode Plots.

$m = 1kg$, $k = 16 N/m$, $c = 4 \frac{Ns}{m}$

1) $\boxed{G(s) = \frac{1}{s^2+4s+16}}$

2)

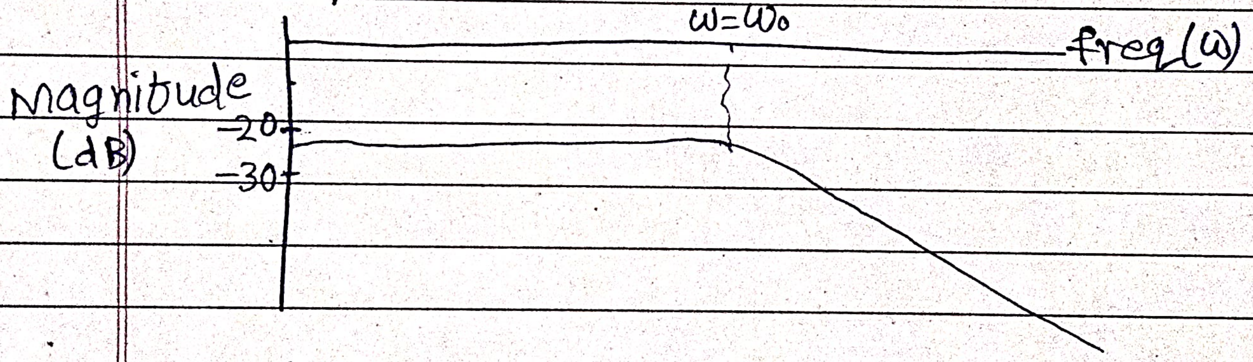
→ For pole, $s^2+4s+16=0$.

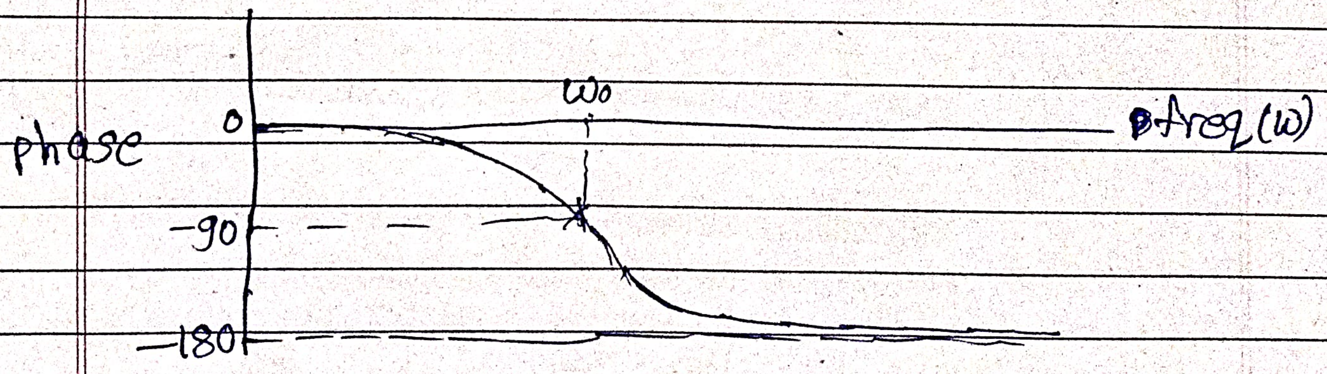
$\therefore s = \frac{-4 \pm \sqrt{4^2-4\times16}}{2} = \frac{-4\pm\sqrt{16-4\times16}}{2} = \frac{-4\pm\sqrt{16\times(-3)}}{2}$

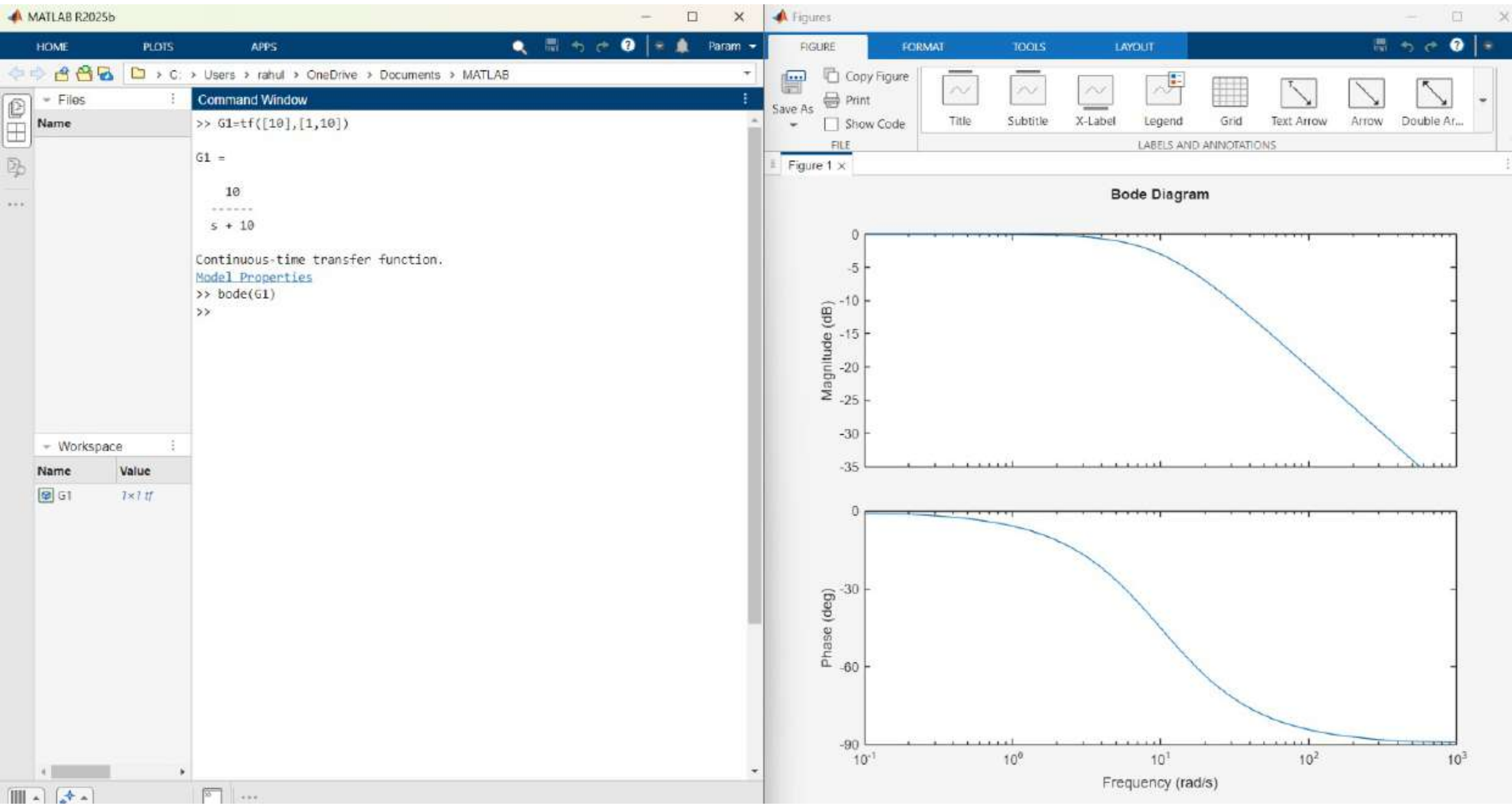
$= \frac{-4\pm4\sqrt{3}i}{2} = -2\pm2\sqrt{3}i$.

$\therefore$ poles are $s=-2+2\sqrt{3}i$, $s=-2-2\sqrt{3}i$.

3) Bode plots:-






MATLAB R2025b
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MATLAB
Files
Name
Command Window
>> G1=tf([10],[1,10])
G1 =
10
------
s + 10
Continuous-time transfer function.
Model Properties
>> bode(G1)
>>
Workspace
Name
Value
G1
1×1 tf
Figures
FIGURE
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TOOLS
LAYOUT
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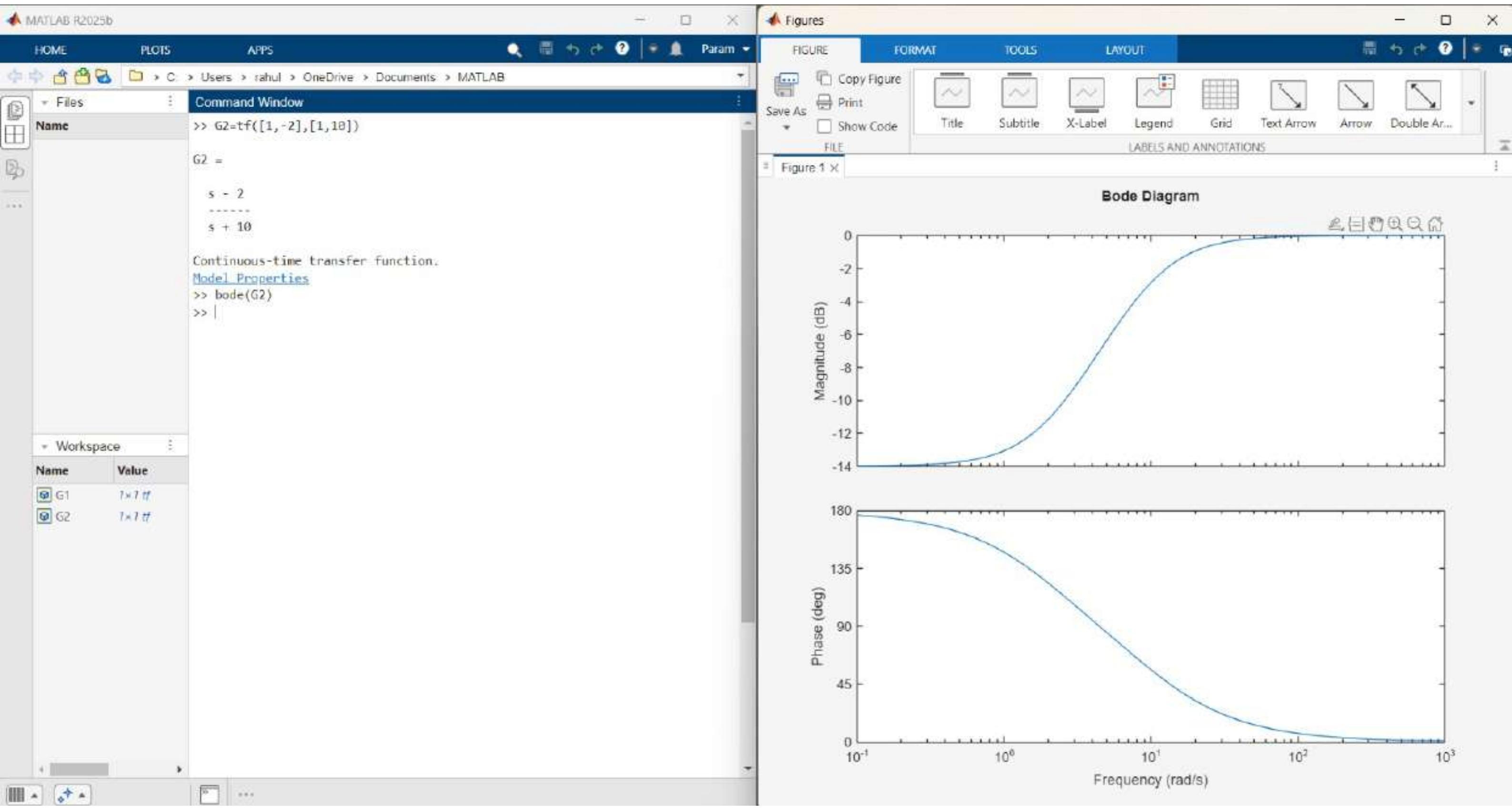
LABELS AND ANNOTATIONS
Figure 1
Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/s)


MATLAB R2025b
HOME
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Command Window
>> G2=tf([1,-2],[1,10])
G2 =
s - 2
------
s + 10
Continuous-time transfer function.
Model Properties
>> bode(G2)
Workspace
G1
G2
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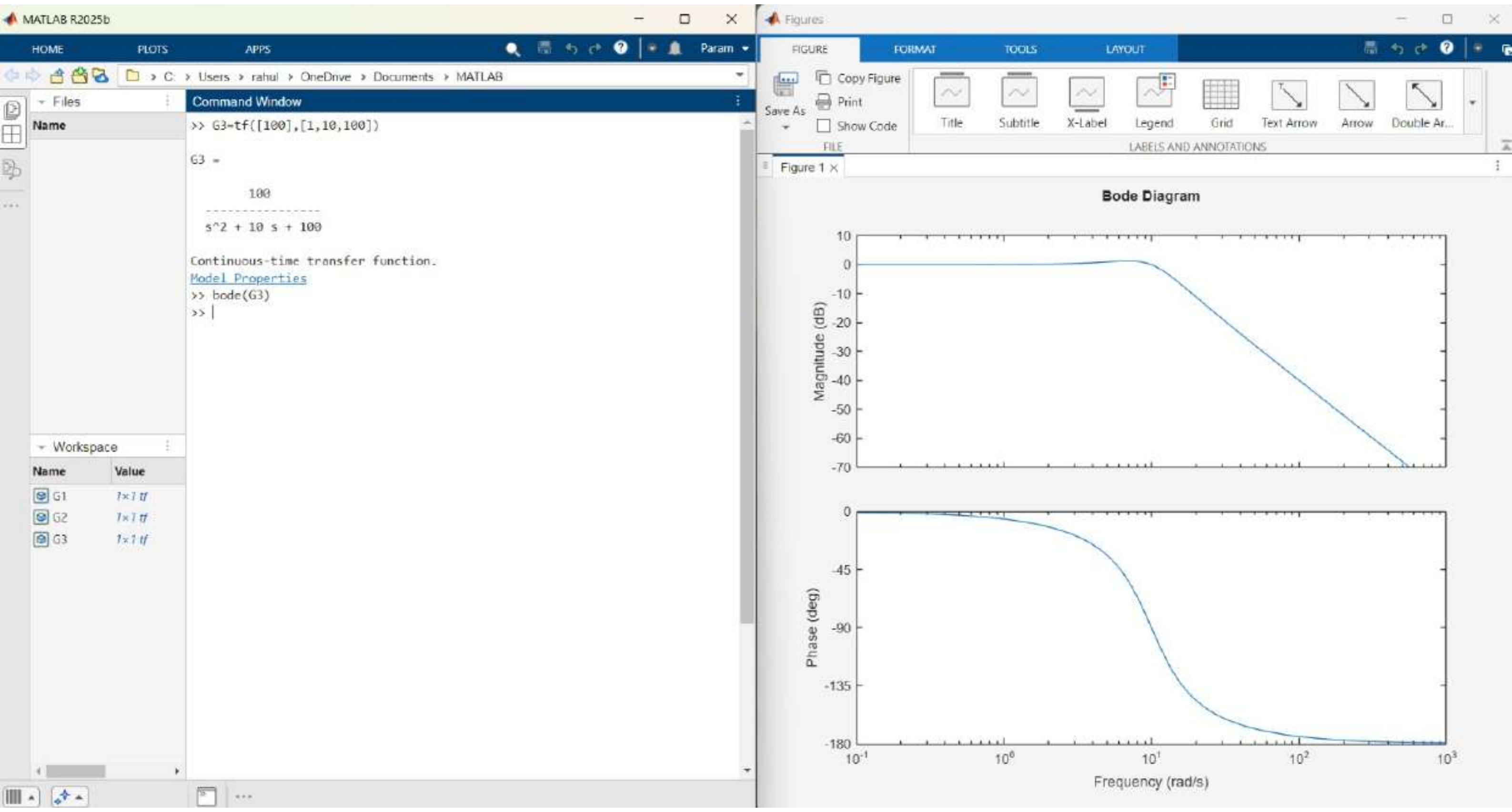
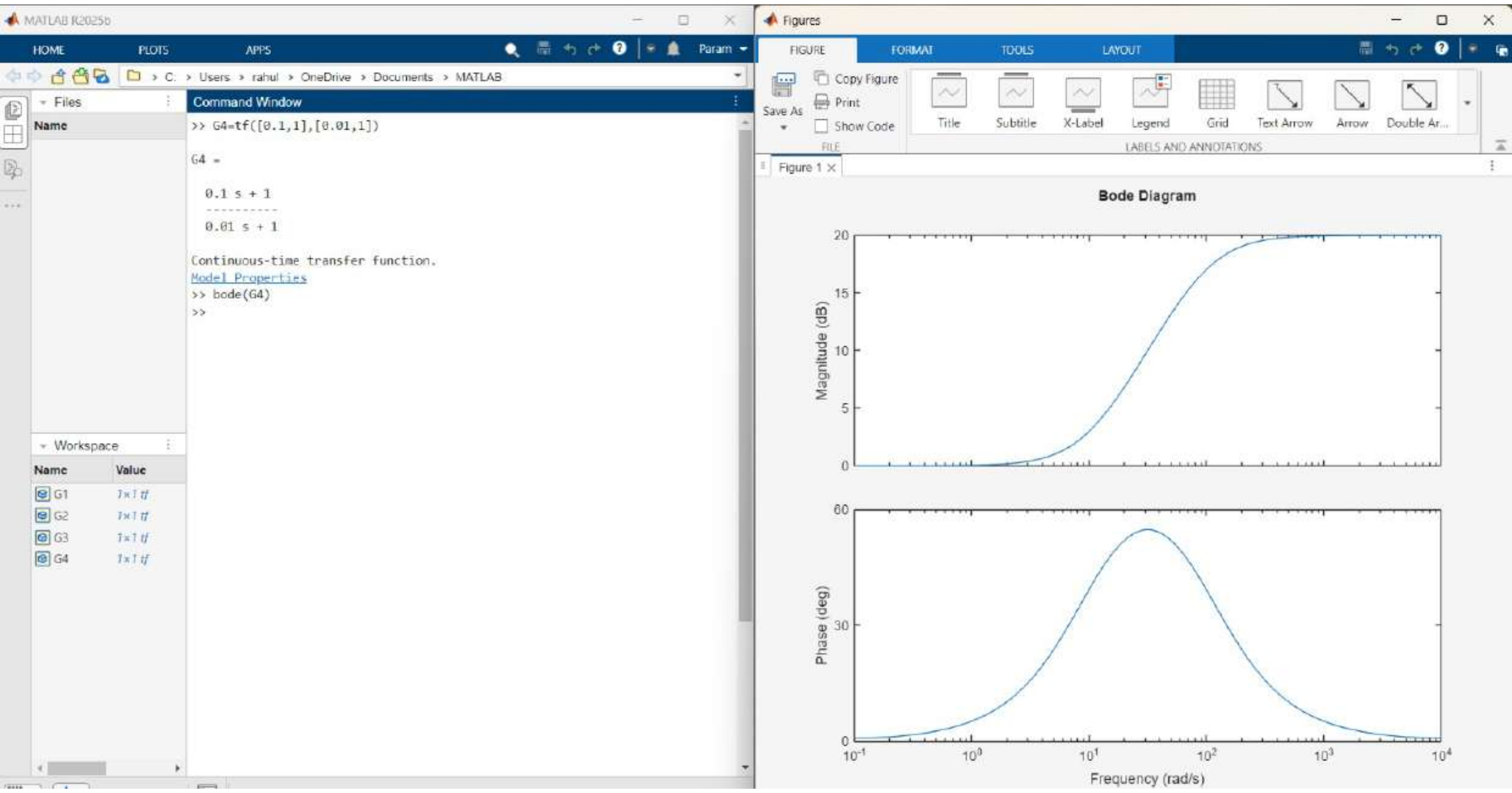
LAYOUT
Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/s)



MATLAB R2025b
>> G3=tf([100],[1,10,100])
G3 =
100
----
s^2 + 10 s + 100
Continuous-time transfer function.
Model Properties
>> bode(G3)
Workspace
G1 1x1 tf
G2 1x1 tf
G3 1x1 tf
Figures
Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/s)


MATLAB R2025b
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APPS
C: > Users > rahul > OneDrive > Documents > MATLAB
Files
Name
Command Window
>> G4=tf([0.1,1],[0.01,1])
G4 =
0.1 s + 1

0.01 s + 1
Continuous-time transfer function.
Model Properties
>> bode(G4)
>>
Workspace
Name
Value
G1 1x1 tf
G2 1x1 tf
G3 1x1 tf
G4 1x1 tf
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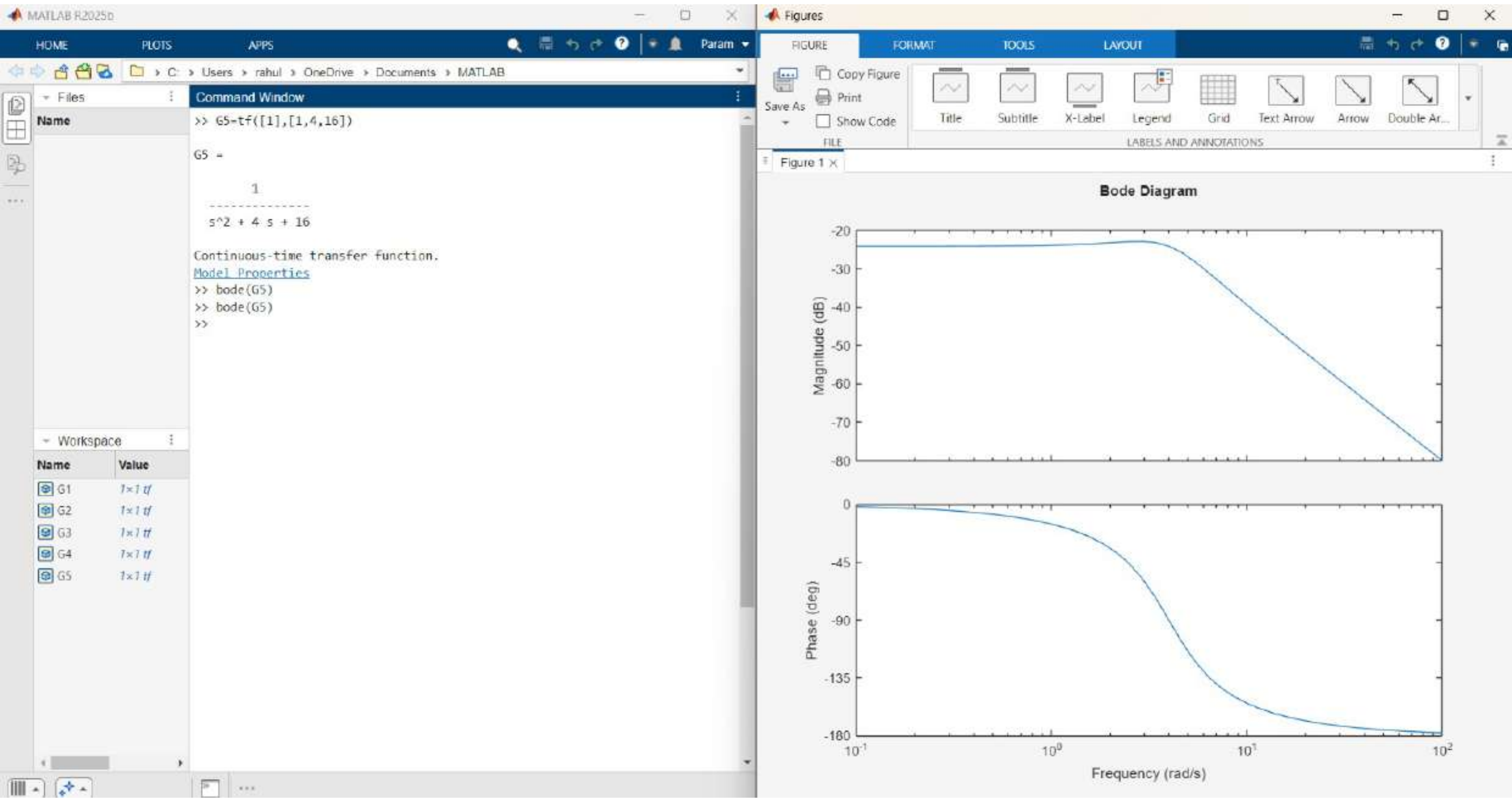
MATLAB R2025b
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Files
Name
Command Window
>> G5=tf([1],[1,4,16])
G5 =
1
----
s^2 + 4 s + 16
Continuous-time transfer function.
Model Properties
>> bode(G5)
>> bode(G5)
>>
Workspace
Name
Value
G1 1×1 tf
G2 1×1 tf
G3 1×1 tf
G4 1×1 tf
G5 1×1 tf
Figures
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LABELS AND ANNOTATIONS
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Frequency (rad/s)