# Modeling & Simulation

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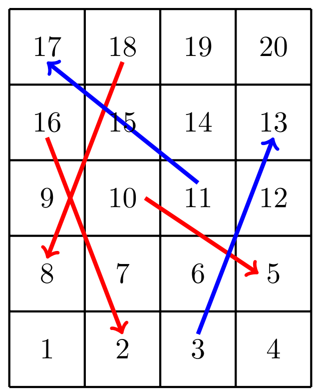
## A. Simulating a Snakes-and-Ladders game.

Consider the following snakes-and-ladders game. Let $N$ be the number of tosses to reach the finish using a fair dice. Write a R program to calculate the expectation of $N$. Also, draw a plot in which the $x$-axis shows the number of rolls, and the $y$-axis shows the percentage of games that were completed in that number of rolls.




## B. Single-Server Queuing Model.

Write a R program to simulate a single-server queue, with the following assumptions.

- Assume inter-arrival times are independent and identically distributed (IID) random variables.
- Assume service times are IID, and are independent of inter-arrival times.
- Queue discipline is FIFO.
- Start empty and idle at time 0.
- First customer arrives after an inter-arrival time, not at time 0.
- Stopping rule: When $n^{\text{th}}$ customer has completed delay in queue, stop simulation.
- Update system, state variables, clock, event list, statistical counters, all after execution of each event.

You are expected to create a function with three arguments

1. Inter-arrival rate,
2. Service rate, and

3. Stopping rule.

Quantities to be estimated are

1. Expected average delay in queue (excluding service time) of the 20 customers completing their delays,
2. Expected average number of customers in queue (excluding any in service), and
3. Expected utilization (proportion of time busy) of the server.

Note:

(a) The system begins at time 0 with no customers in the system and an idle server.

(b) The times between arrivals are mutually independent and identically distributed exponential random variates with arrival rate $\lambda = 1$ (you can assume initially).

(c) The service times are mutually independent and identically distributed exponential random variates with service rate $\mu = .5$ (you can assume initially).

(d) The server does not take any breaks.

(e) A customer departs the system once the service is complete.

(f) When $20^{\text{th}}$ customer has completed delay in queue, stop simulation.

## C. Simulation of a harbour port.

A harbour port has three berths 1, 2 and 3. At any given time Berth1 can accommodate two small ships, or one medium ship. Berth2 and Berth3 can each handle one large ship, two medium ships or four small ships.

The interarrival time of ships is 26 hours, exponentially distributed, and small, medium, and large ships are in the proportions 5:3:2 respectively. Queuing for berths is on a first come first served basis, except that no medium or small ship may go to a berth for which a large ship is waiting, and medium ships have a higher priority than small ships.

Unloading times for ships are exponentially distributed with mean times as follows: small ships, 15 hours; medium ships, 30 hours; and large ships, 45 hours. The loading times are as follows:

- Small ships: $24 \pm 6$ hours uniformly distributed.
- Medium ships: $36 \pm 10$ hours uniformly distributed.
- Large ships: $56 \pm 12$ hours uniformly distributed.

The tide must be high for large ships to enter or leave Berths 2 and 3. Low tide lasts 3 hours, high tide, 10 hours. Write a R program to simulate the harbour port and

1. run the simulation for 500 days,
2. determine the distribution of transit times of each type of ship, and
3. determine the utilization of the three berths.