Fan-Beam Computed Tomography

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Course Instructor:

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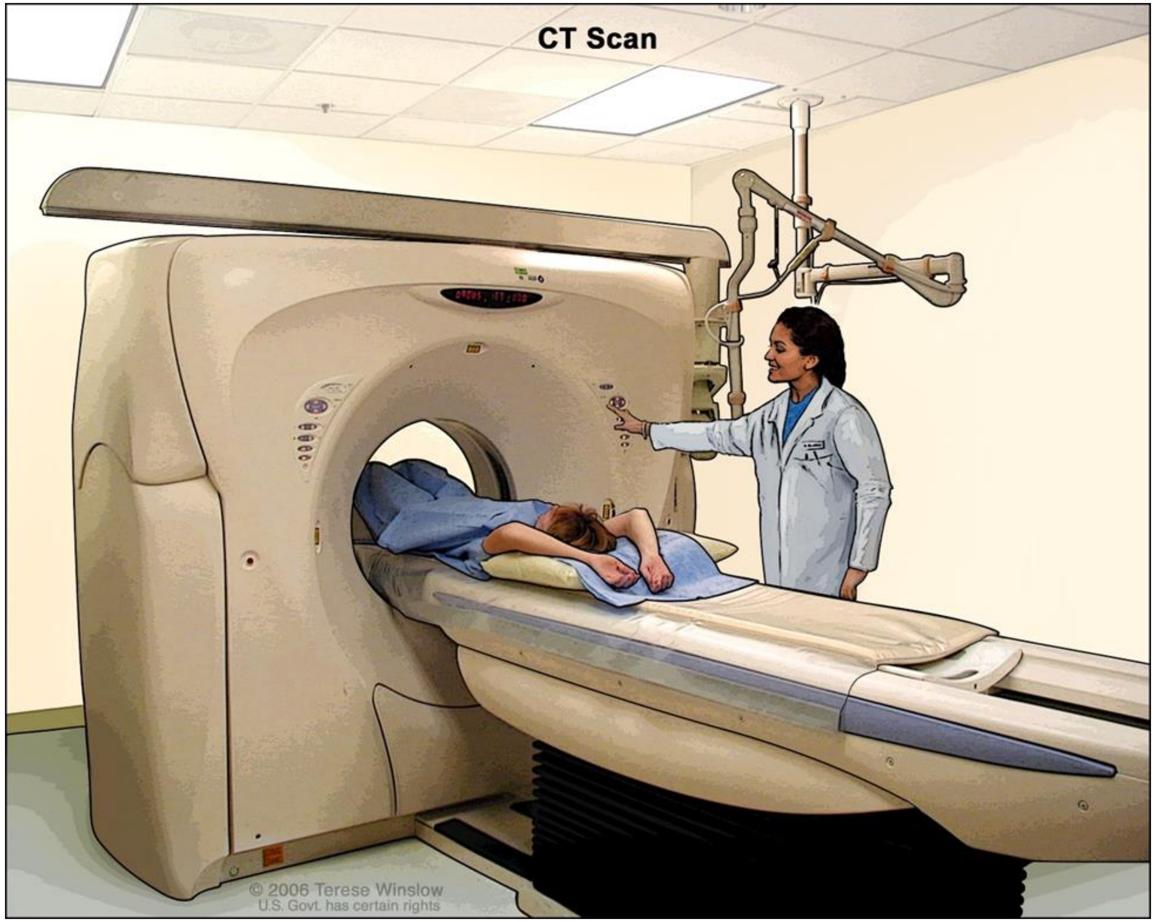
Introduction to Fan-Beam Computed Tomography

- Medical imaging scan
- Non-invasive technique
- X-ray ionization
- Third generation CT scan
  - Detector array
  - X-ray tube
  - Fan shaped beam
  - Scan time reduced



Motivation for Third Generation CT Scan

- Scan field of view determines the size of the fan beam
- Fan beam determines the number of detector elements that collect the data
- Less time-consuming scans
- Incorporated detector array
- Improved image quality

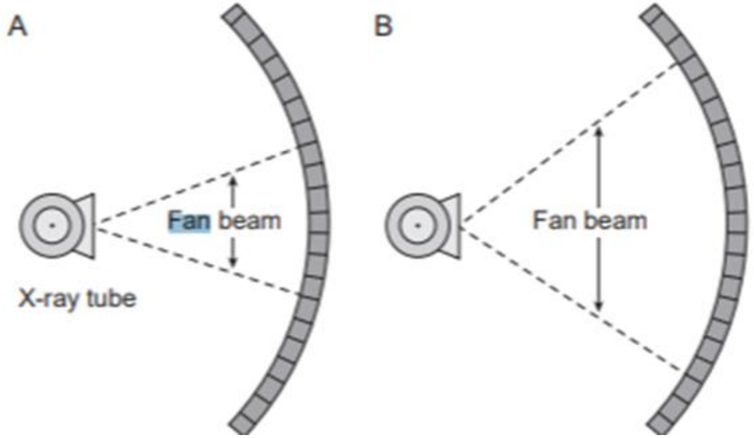


Figure 1. Amount of detector cells determined by the selection field of view.

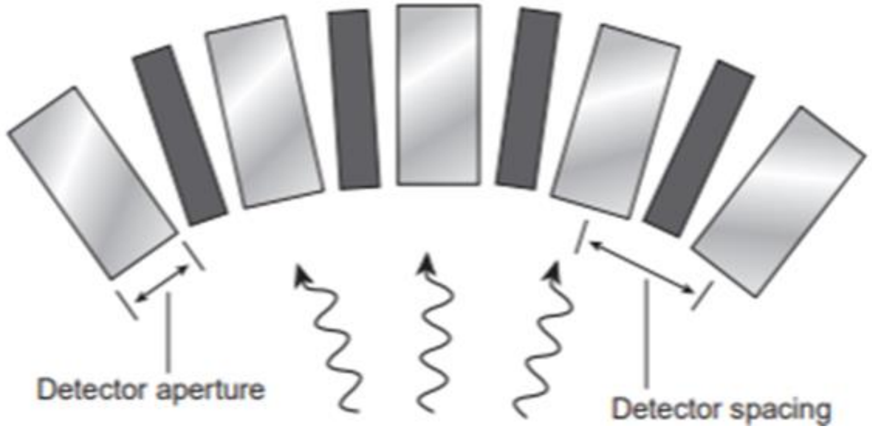


Figure 2. Amount of detector cells determined by the selection field of view.

Challenges of Fan-Beam Computed Tomography

- Ionizing radiation
  - X-ray
- Time
  - Less time consuming but still is long
- Ring artifacts
  - Due to having the same bank of detectors being used repeatedly, small misalignment of a single detector results in visible ring artifact

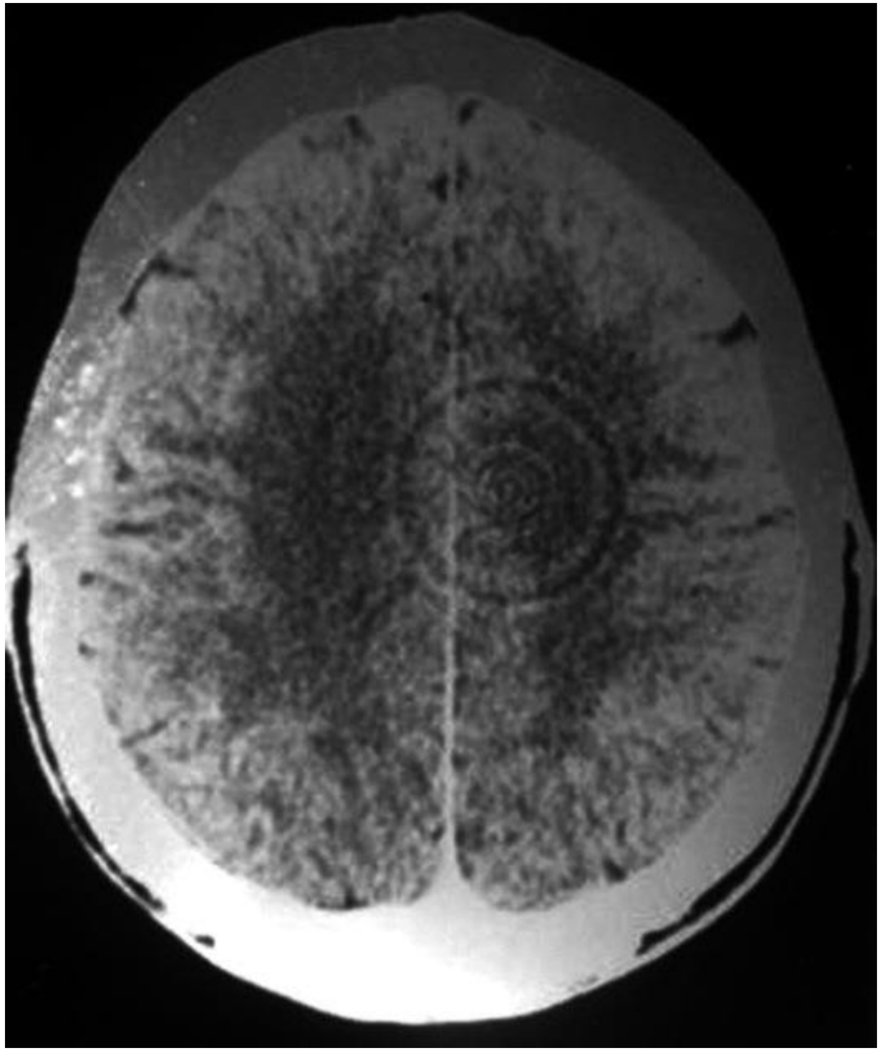
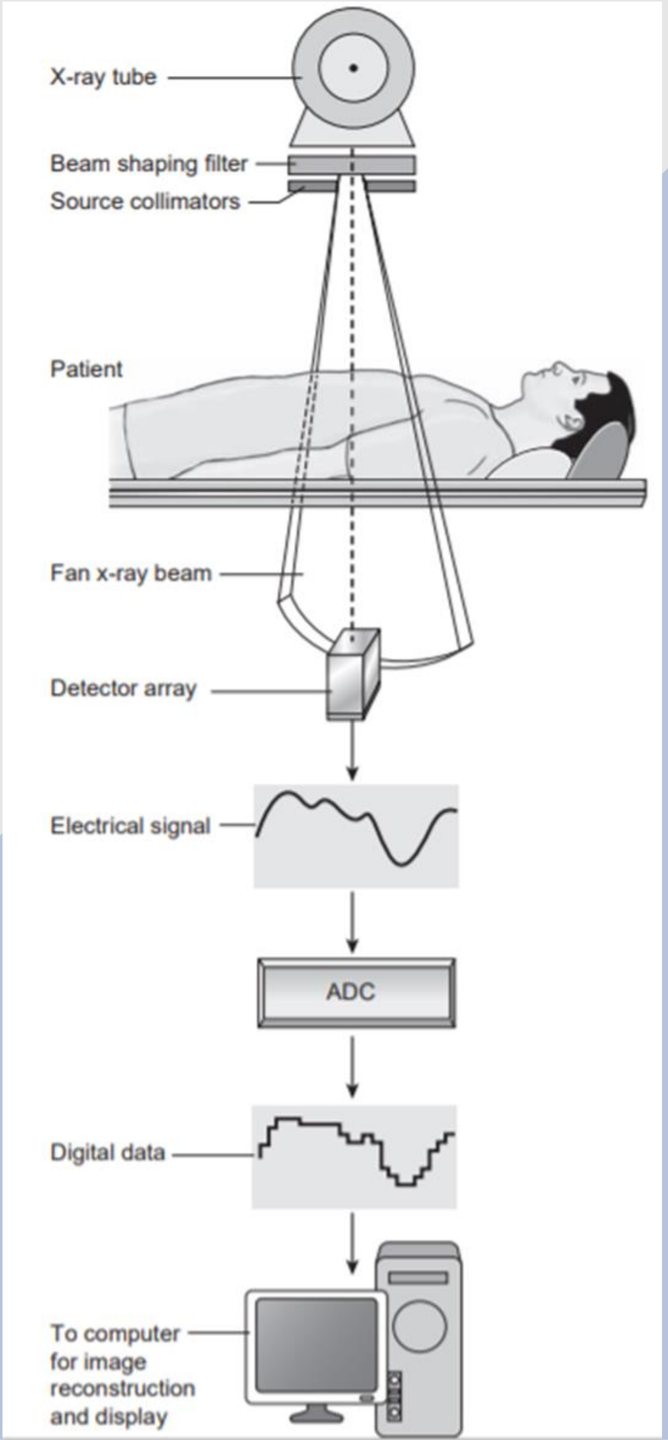
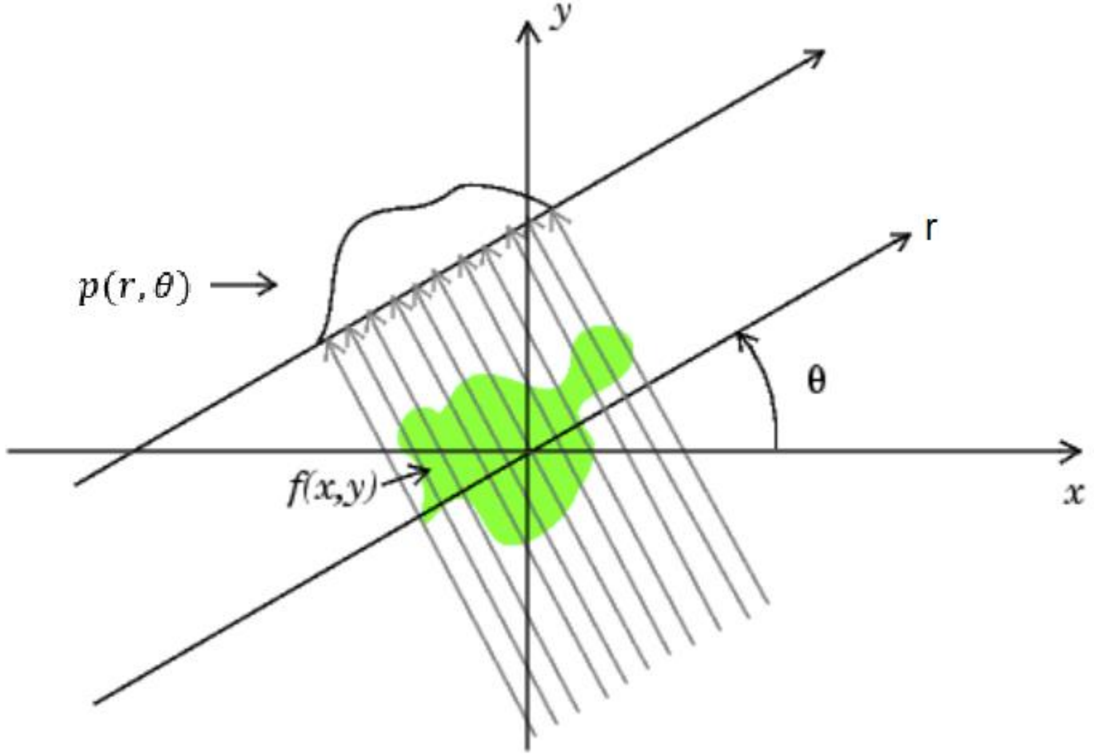


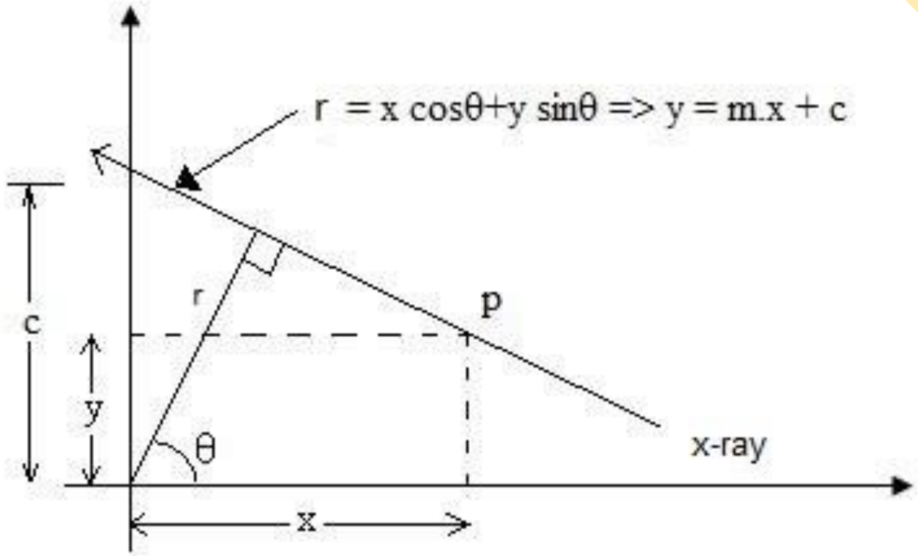
Figure 3. Strong ring artifact of cerebral CT scan.

Working Principle: Setup



Working Principle: Parallel Beam





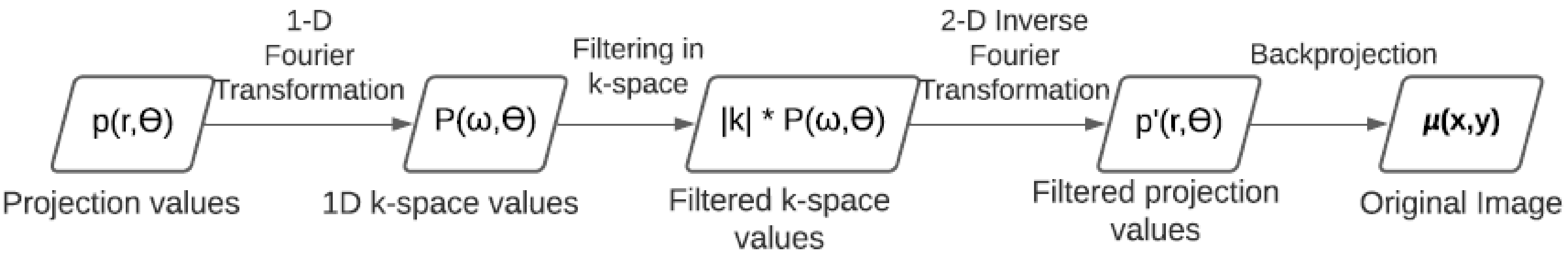
The projection data p(r,ϴ) is the line integral along the x-ray path represented by the equation:

r= x cosϴ + y cosϴ

Image acquired from: https://www.researchgate.net/figure/Equation-of-a-line-in-Cartesian-and-polar-spaces_fig3_323021825

Obtaining Original Image μ(x,y) from p(r,ϴ)

We know, for parallel beam CT reconstruction:



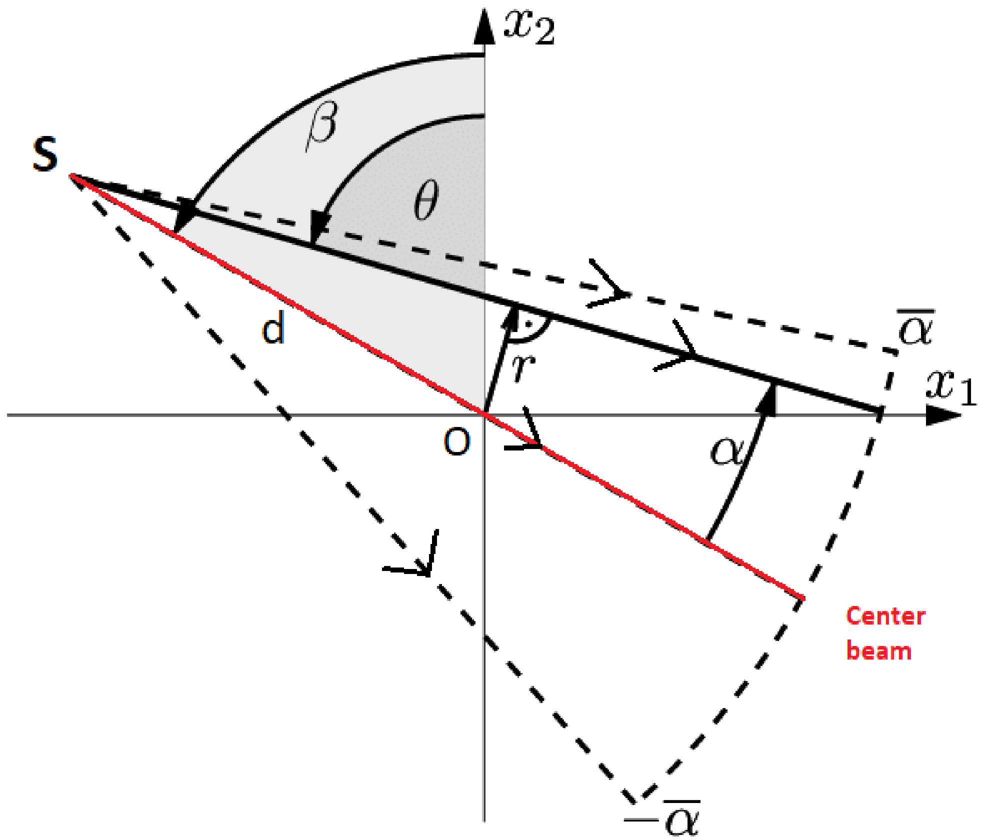
Most CT scanners work in the spatial domain via convolutions:

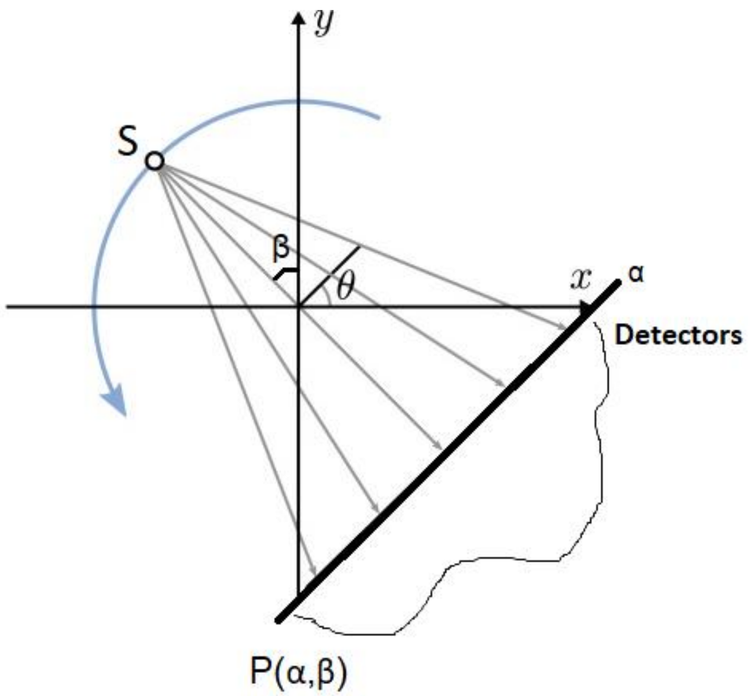
convolution in space domain = multiplication in frequency domain

$$\mu(x,y) = \underbrace{\int_0^{\pi} \underbrace{\int_{-\infty}^{\infty} s(xcos\theta + ysin\theta - r, \theta) p(r,\theta) dr}_{\text{Convolution}}\, d\theta}_{\text{Backprojection}}$$

F: Fourier Transform; IF: Inverse Fourier Transform

Parameters for Fan beam CT



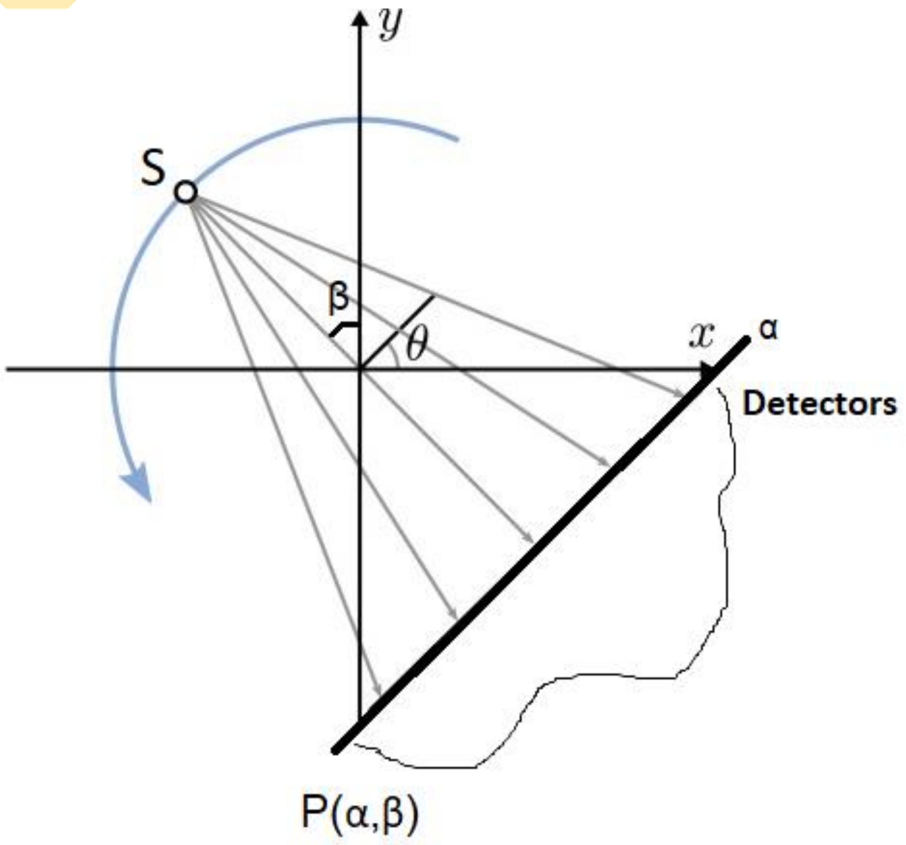


<u>Defining parameters:</u>

- **β = angle between vertical axis and center beam**
- **α = angle between center beam and any other beam produced by the source at angle β**
- **d = distance of the source from C(x,y)**

Image acquired from: https://www.sciencedirect.com/science/article/abs/pii/S0263822313006090

Projection Data for Fan-Beam: p(α,β)



r= x cosϴ + y cosϴ ??

Data collected by detectors at each angle β represents a projection function p(α,β).

The range of values for α is determined by **fan range** and **spacing between detectors.**

In projection function p(α,β):
X- axis = values of α
Y- axis = Intensity(I_0)

Image acquired from: https://www.ncbi.nlm.nih.gov/books/NBK546157/figure/ch8.fig16a/

Representing X-ray Path in Terms of ϴ, d and β

Geometrically:

$\Theta = \beta + \alpha$

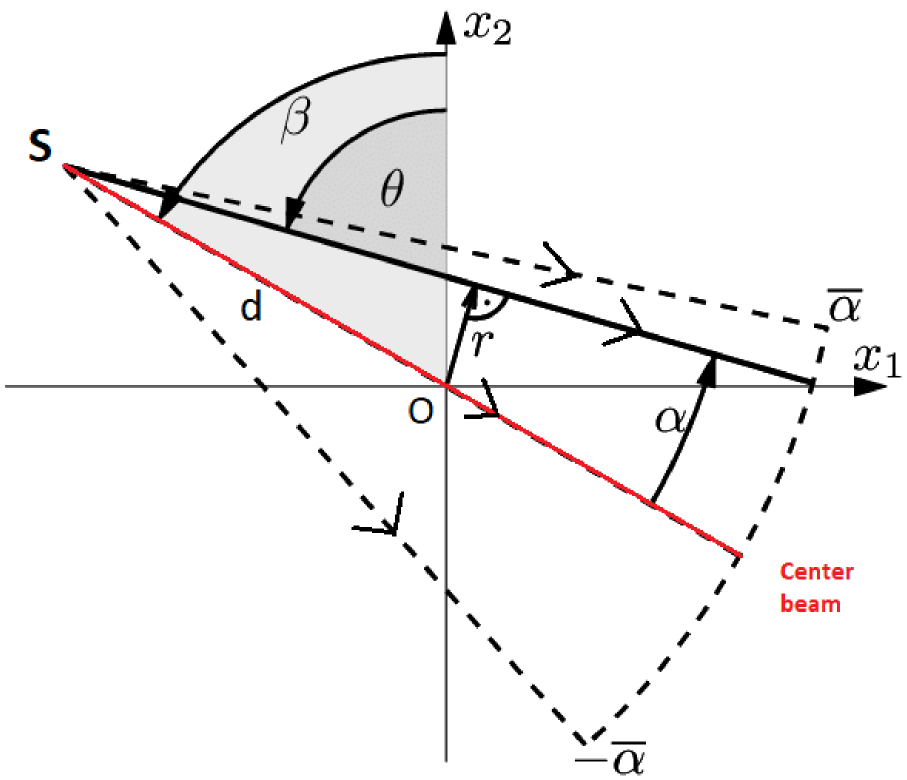
$r = d \sin\alpha$

Equation of the line:

$r = x \cos\Theta + y \cos\Theta$

Replacing r and ϴ with ϴ, d and β we get:

$d \sin\alpha = x \cos(\beta + \alpha) + y \cos(\beta + \alpha)$



$d \sin\alpha = x \cos(\beta + \alpha) + y \cos(\beta + \alpha)$

This is the equation of the x-ray path along which signal is attenuated.

Image acquired from: https://www.sciencedirect.com/science/article/abs/pii/S0263822313006090

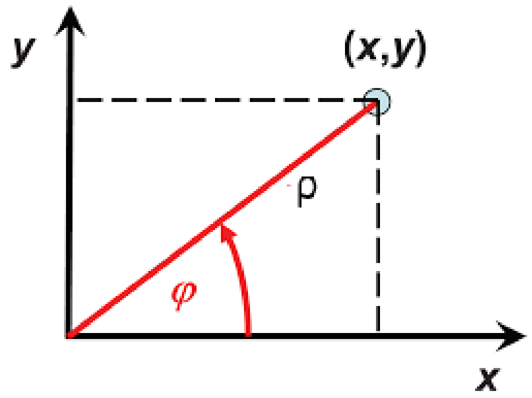
Obtaining Original Image μ(x,y) from p(α,β)

Replacing *B(Rf)* and with convolution formula we get:

$$\mu(x,y) = \underbrace{\int_0^{\pi} \underbrace{\int_{-\infty}^{\infty} s(x\cos\theta + y\sin\theta - r,\theta)\,p(r,\theta)\,dr}_{\text{Convolution}}\, d\theta}_{\text{Backprojection}} \qquad \text{.......................(i)}$$

For fan beam projections we have,

- p(α,β) instead of p(r,ϴ)
- ϴ = β + α
- r = d sinα
- μ(ρ,ϕ) instead of μ(x,y) in polar coordinates
- x = ρcosϕ and y = ρsinϕ



F: Fourier Transform; IF: Inverse Fourier Transform
Image acquired from: https://seos-project.eu/laser-rs/laser-rs-c03-s01-p01.html

Obtaining Original Image $\mu(\rho,\phi)$ from p(α,β)

Replacing all required variables for fan beam geometry we get:

$$\mu(\rho,\varphi) = \frac{1}{2}\int_0^{2\pi}\int_{-\propto}^{\propto} s(\rho cos\varphi\cos(\alpha+\beta) + \rho sin\varphi\sin(\alpha+\beta) - dsin\alpha)p(\propto,\beta)\, dcos\alpha\, d\alpha\, d\beta$$

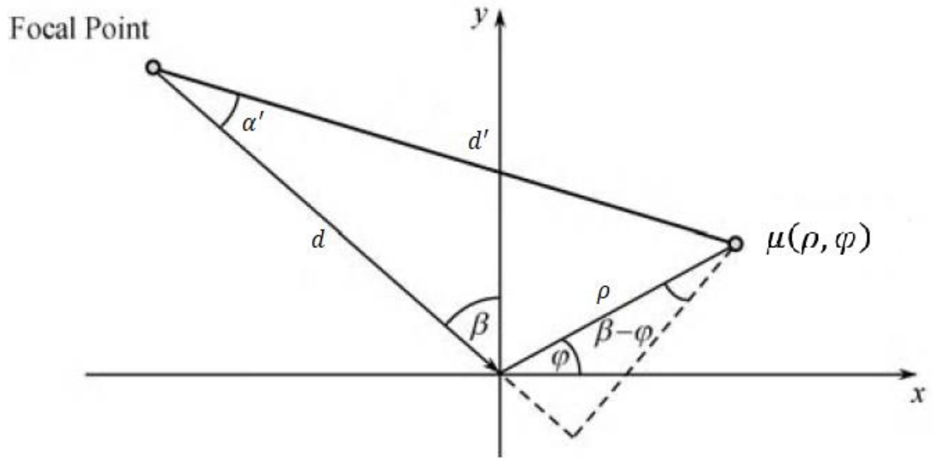
With further simplification using trigonomeric identities:

$$\mu(\rho,\varphi) = \frac{1}{2}\int_0^{2\pi}\int_{-\propto}^{\propto} s(\rho\cos(\beta+\alpha-\varphi) - dsin\alpha)p(\propto,\beta)\, dcos\alpha\, d\alpha\, d\beta$$

To convert the above notation into convolution form we define two terms:

α' : It *is angle between the central beam and x-ray beam passing through the point μ(ρ,ϕ)* and

d' : It is the distance between Source and μ(ρ,ϕ).



Geometrically, $\boxed{\rho cos(\beta+\alpha-\varphi) - dsin\alpha = d'sin(\alpha'-\alpha)}$

Image acquired from: Zeng, G. L. (2010). *Medical image reconstruction: A conceptual tutorial*. Higher Education Press. Pages: 58

Obtaining Original Image $\mu(\rho,\phi)$ from $p(\alpha,\beta)$

Now, we have,

$$\mu(\rho,\varphi) = \frac{1}{2}\int_0^{2\pi}\int_{-\propto}^{\propto} s(d'\sin(\alpha'-\alpha))\, p(\propto,\beta)\, dcos\alpha\, d\alpha\, d\beta$$

Using a special property of the ramp filter: $h(D'\sin\gamma) = \left(\frac{\gamma}{D'\sin\gamma}\right)^2 h(\gamma)$ for ds(d'sin(α'-α)) we get

$$\mu(\rho,\varphi) = \underbrace{\int_0^{2\pi}\frac{1}{(d')^2}\underbrace{\int_{-\pi/2}^{\pi/2} s\_fan(\alpha'-\alpha)\, p(\propto,\beta)\, cos\alpha\, d\alpha}_{\text{Convolution}}\, d\beta}_{\text{Backprojection}}$$

where,

$$s\_fan(\alpha) = \frac{d}{2}\left(\frac{\alpha}{sin\alpha}\right)^2 s(\alpha)$$

This is the fan beam convolution backprojection algorithm.

Creating Fan-Beam Projections on MATLAB

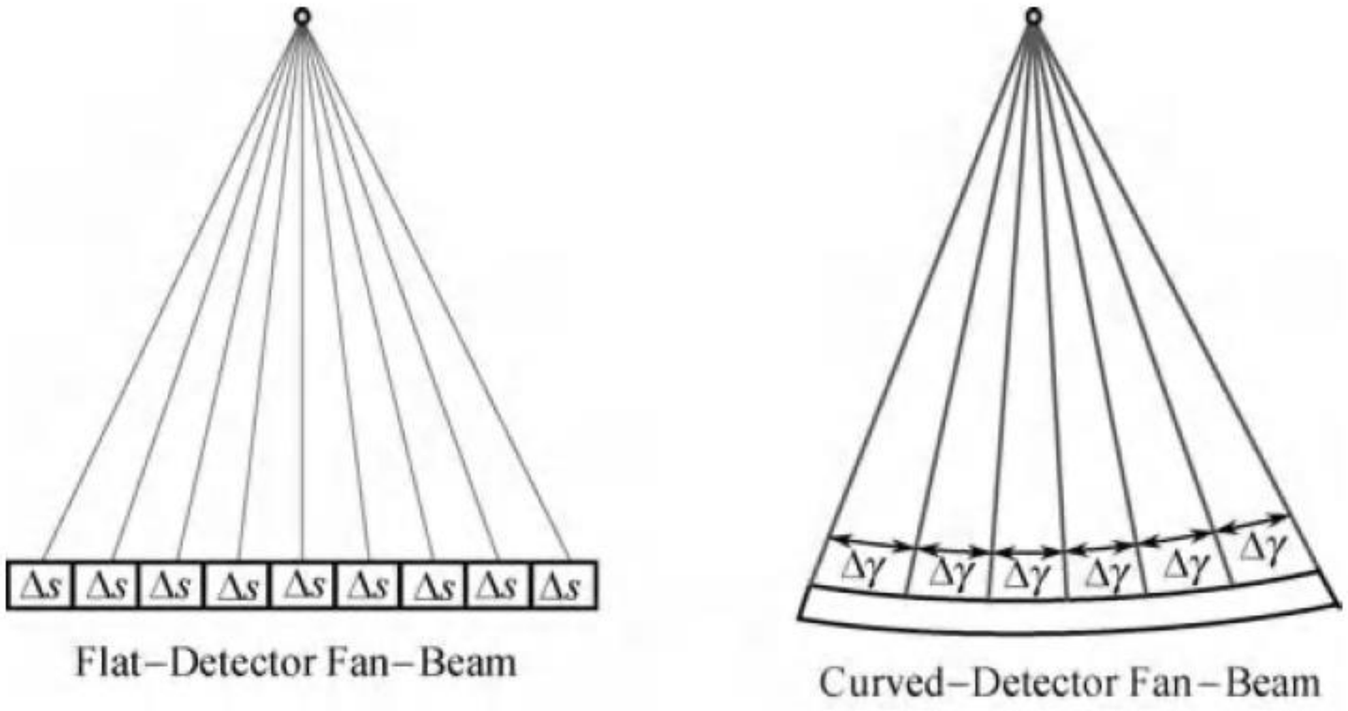


Fig. 3.6. Flat and curved detector fan-beam geometries.

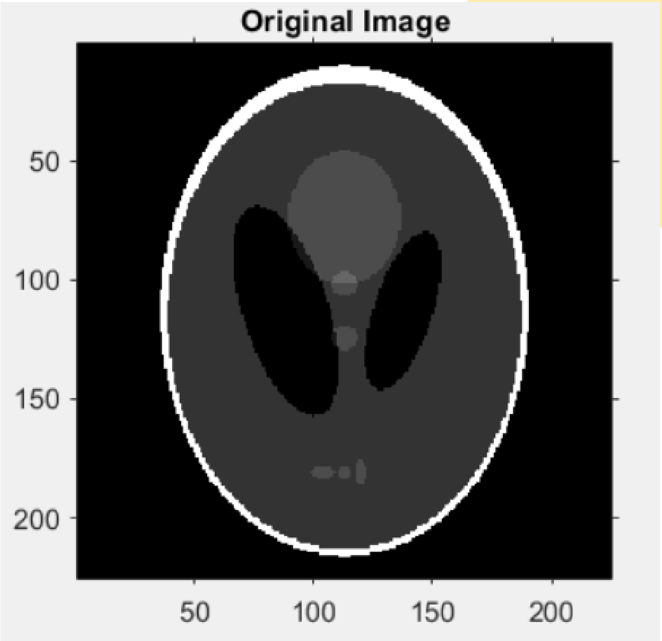
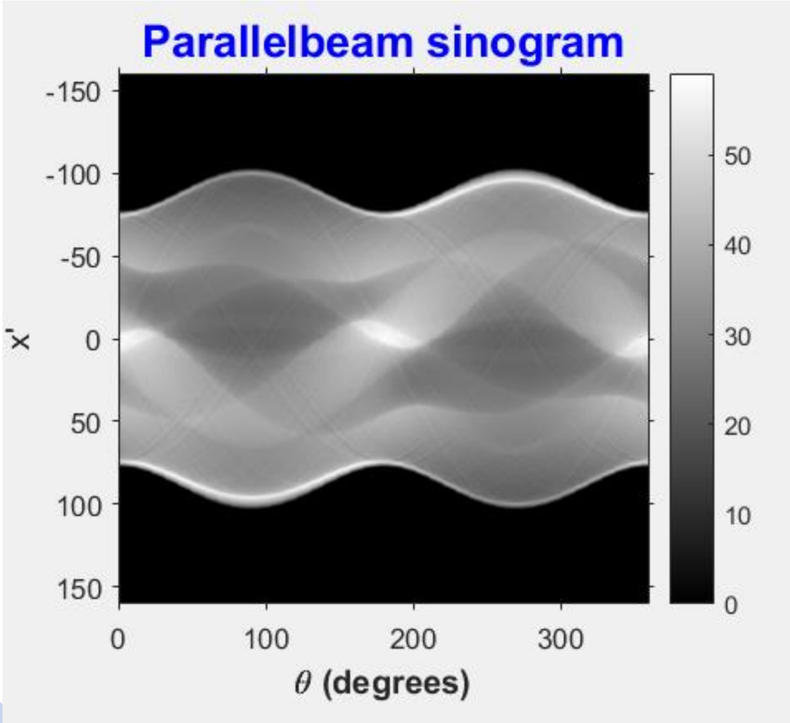
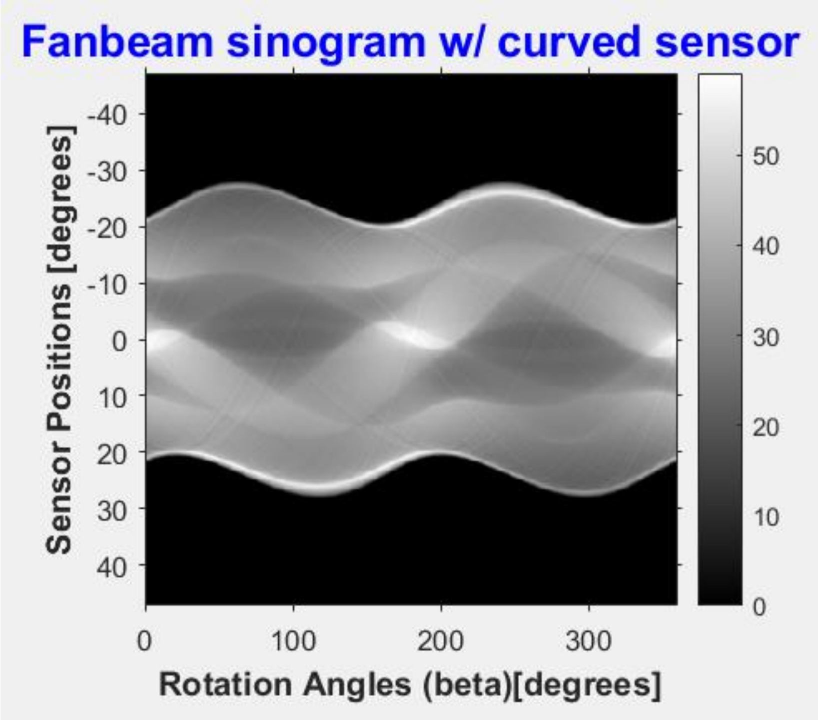
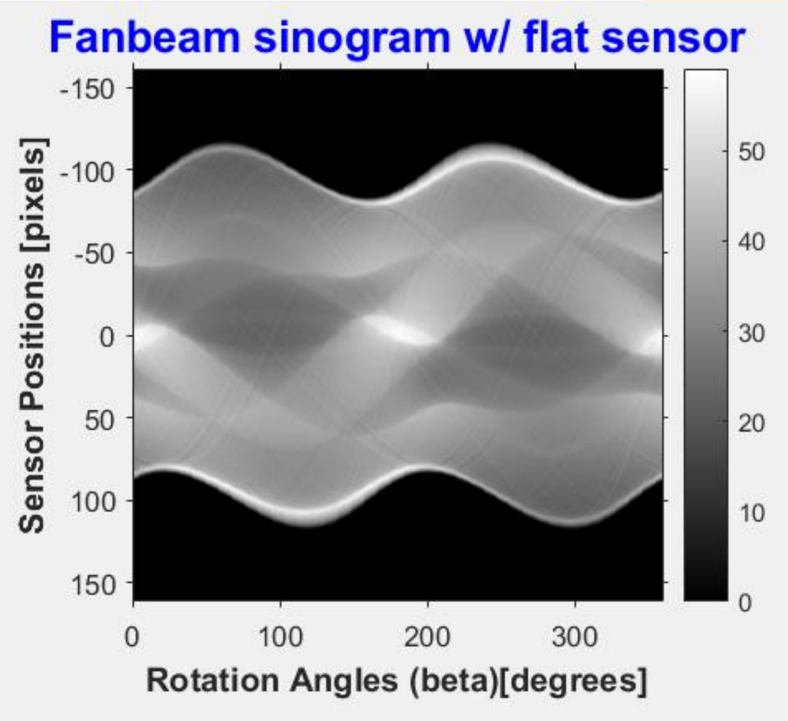


Image acquired from: Zeng, G. L. (2010). *Medical image reconstruction: A conceptual tutorial*. Higher Education Press. Pages: 54

Parallel vs Fan-Beam Sinogram (d = 220)



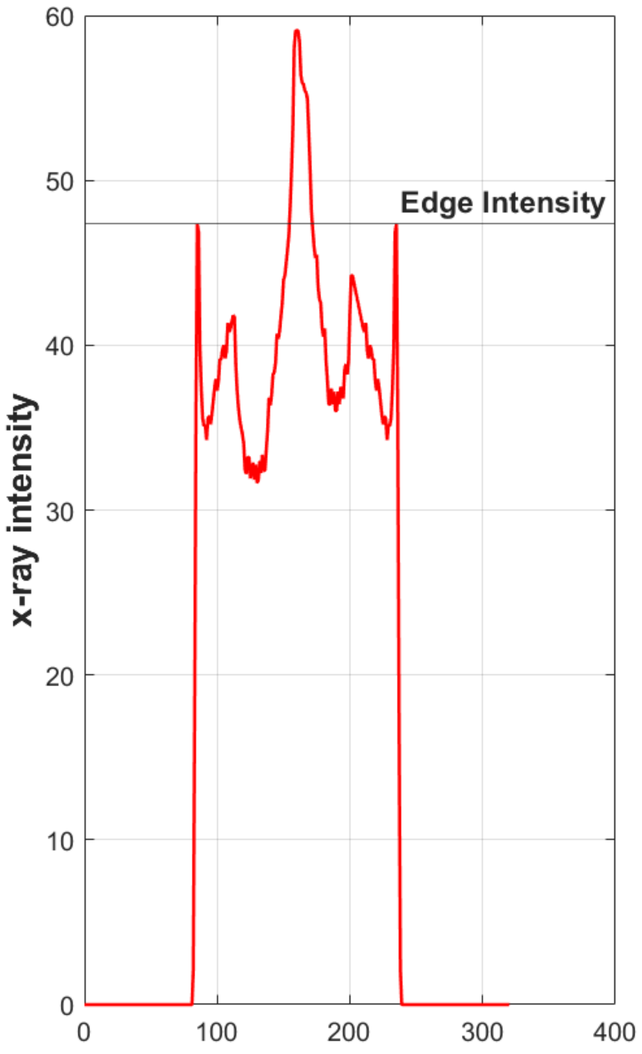




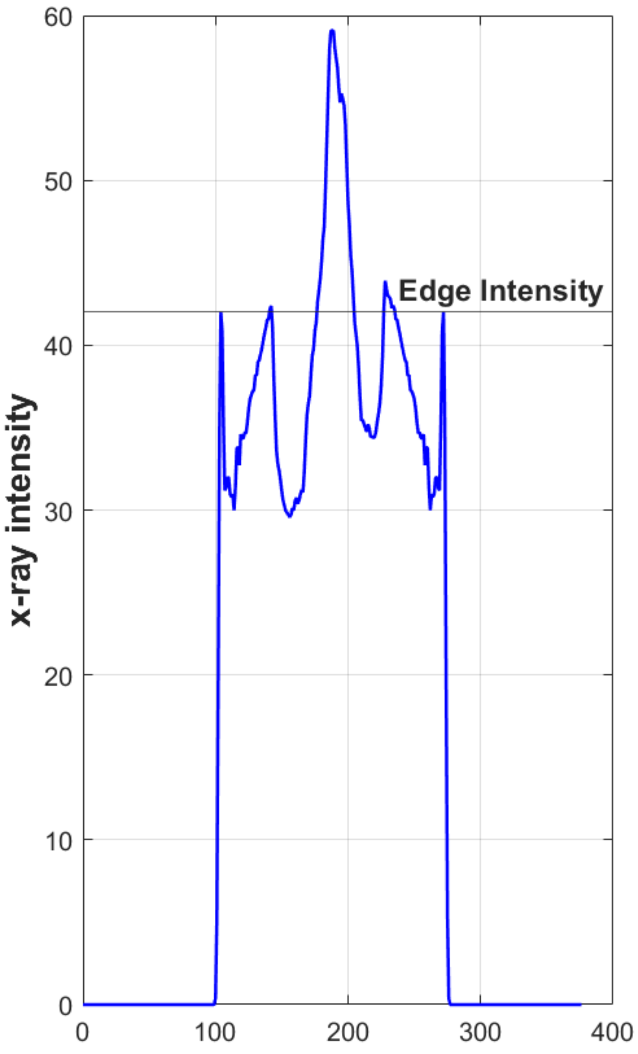
Observation: The central part of all sinograms are similar but the edges seem to have shifted.
Cause: As we move farther away from the central beam, projection values for parallel and fan beam rays start varying more.

Parallel vs. Fan-Beam Projection Values

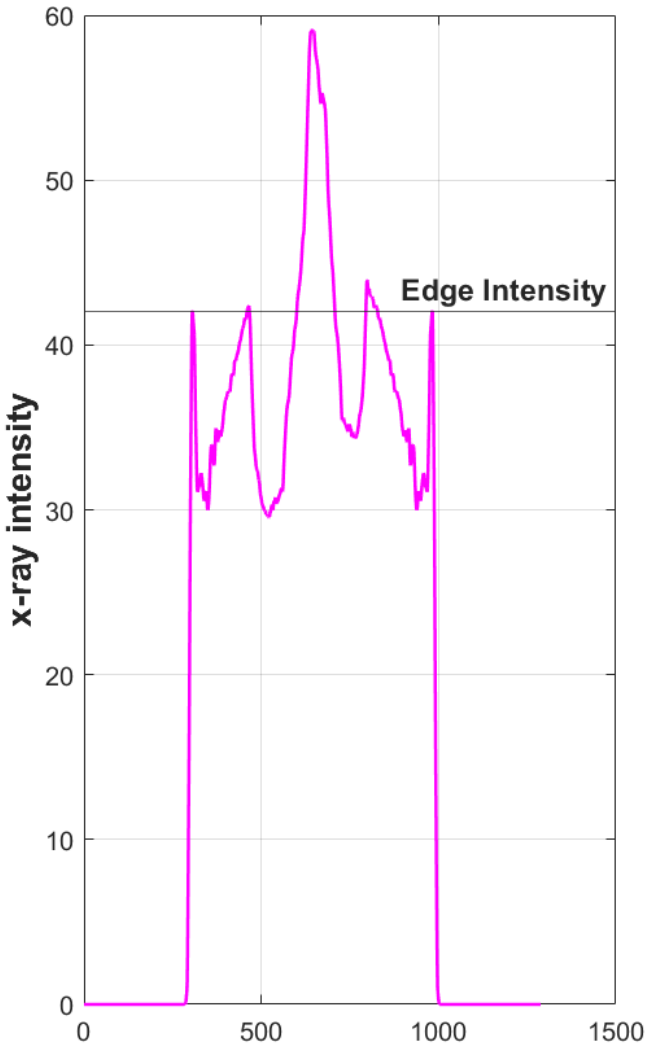
Parallel beam



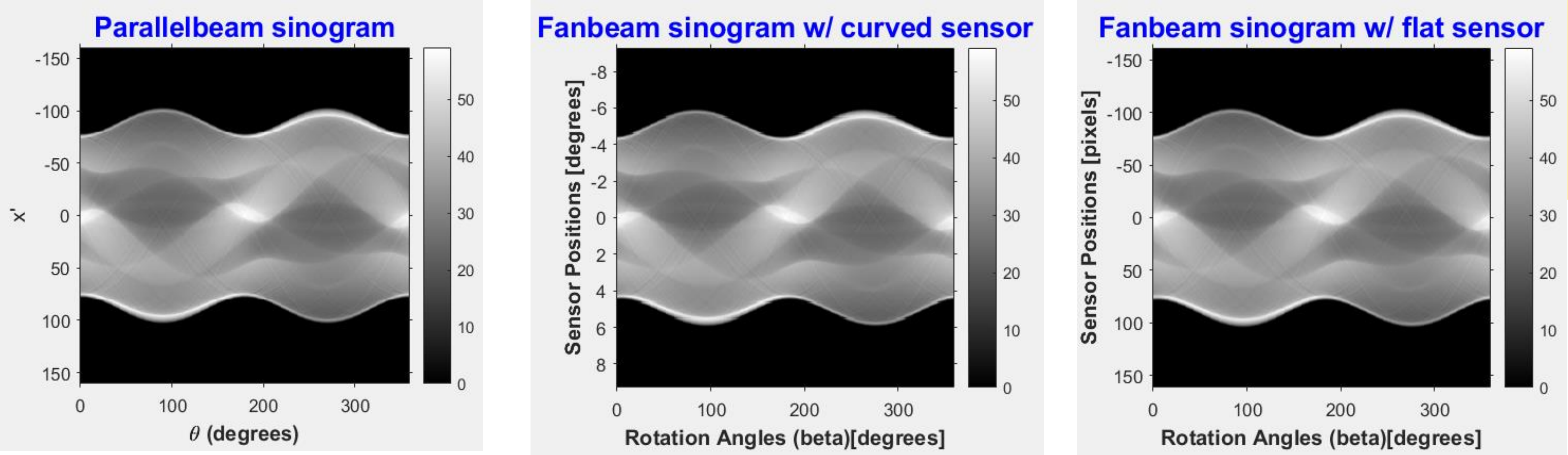
Fanbeam with curved sensor



Fanbeam with flat sensor

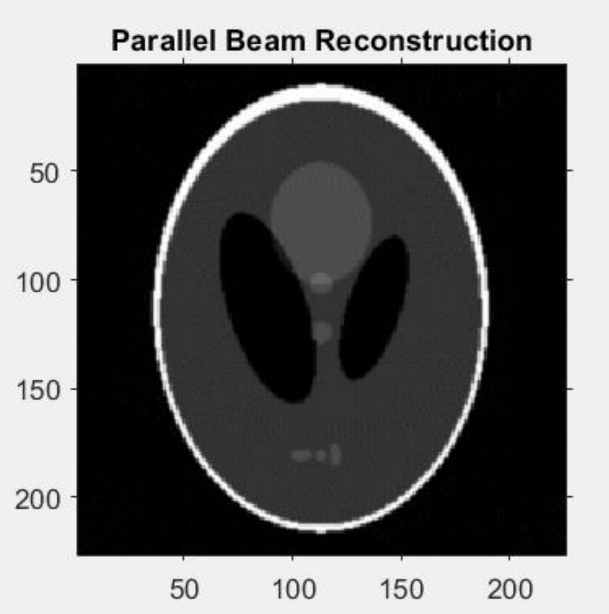


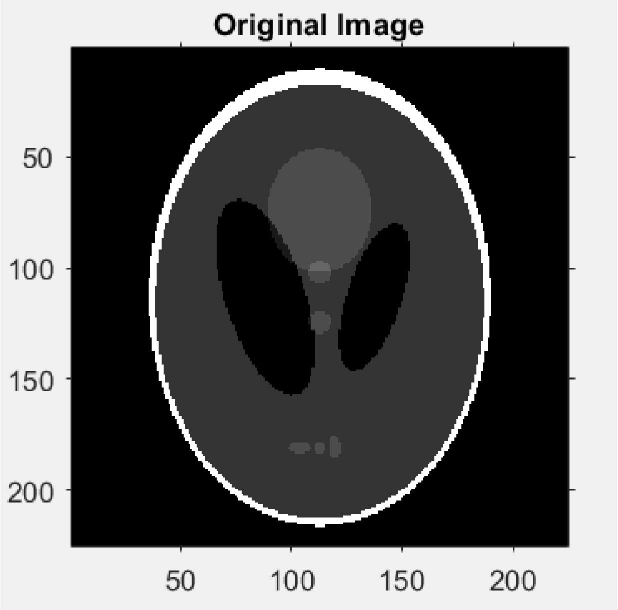
Parallel vs Fan-Beam Sinogram (d = 1000)

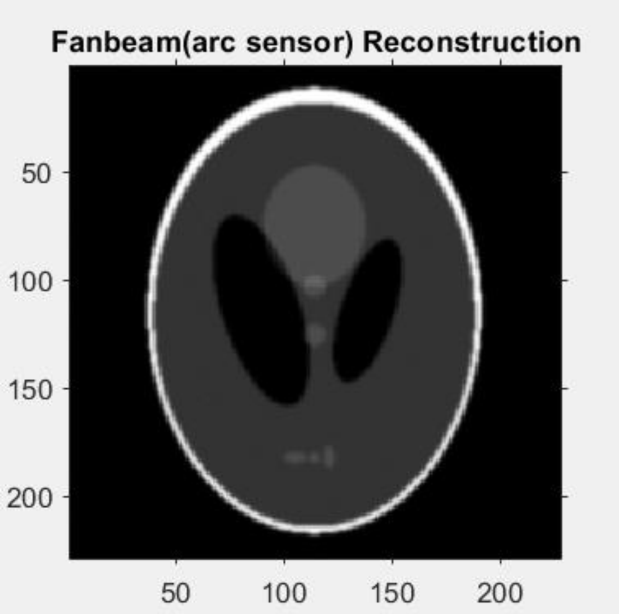


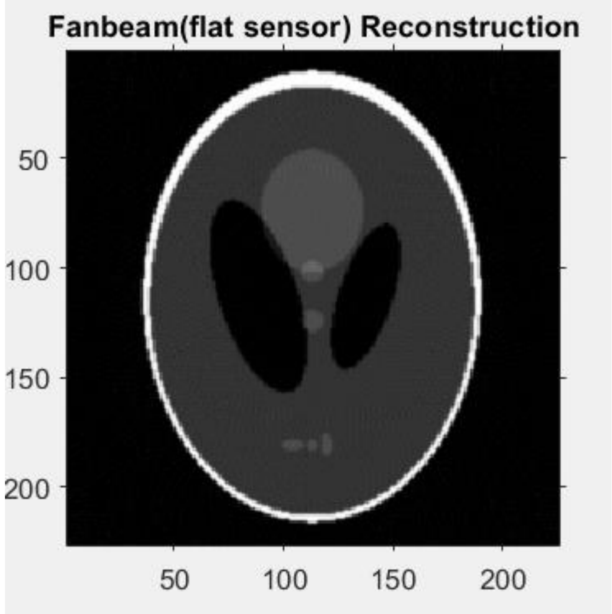
Observation: All sinograms look similar.
Cause: Greatly increasing distance of the source from the center makes fan beam behave like parallel beams

Image Reconstruction









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