Low-gain based controller development for a 2-DOF helicopter model

--- Report for "RW Gillette awards"

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This project develops a low-gain based controller for a two degree-of-freedom (2- DOF) helicopter model. The pitch and yaw angles of a 2-DOF helicopter are regulated to their corresponding reference angles by providing appropriate input voltages to the main and tail motors of the 2-DOF helicopter using the developed low-gain based controller. From the existing controllers/algorithms, the input voltages of both motors are easy to get saturated, which definitely impacts the output performance. This project aims to remove the impact of the input saturation by designing a controller based on a low-gain methodology. At the end, the designed controller will be simulated on the mathematical model of 2-DOF helicopter and also be applied on the 2-DOF helicopter. The output performance will be analyzed and compared with other existing controllers.

This apparatus that was used as the model is called Quanser AERO. Also, Simulink and MATLAB were used to program it. The theory for the control was with implementing low gain to an LQR controller and adding an integrator. Big hurdles were overcome with learning the material as well as why certain things were not working as expected. With this, my mentor guided me through many of these things and therefore increased my knowledge of how-to problem solve through control systems. With this the controller does work well for going to constant positions but still needs to be modified for oscillating positions.

Figure 1: Quanser AERO

The linearized state-space model of the Quanser AERO is represented by:

$$\dot{x} = Ax + Bu \quad and \quad y = Cx + Du,$$

where $x^T = [\theta(t), \quad \psi(t), \quad \dot{\theta}(t), \quad \dot{\psi}(t)]$, $y^T = [\theta(t), \quad \psi(t)]$, $u^T = [V_p(t), \quad V_y(t)]$, and

$$\mathrm{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -K_{sp}/J_p & 0 & -D_p/J_p & 0 \\ 0 & 0 & 0 & -D_y/J_y \end{bmatrix}, \mathrm{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ K_{pp}/J_p & K_{py}/J_p \\ K_{yp}/J_y & K_{yy}/J_y \end{bmatrix}, \mathrm{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \text{and } \mathrm{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

In the above, θ(t) and $\psi(t)$ are the pitch and yaw position respectively, $\dot{\theta}(t)$ and $\dot{\psi}(t)$ are the pitch and yaw velocity respectively, $V_p(t)$ and $V_y(t)$ are the voltage inputs for the yaw and pitch motors respectively, J_y $(0.0220\, kg \cdot m^2)$ and J_p $(0.0219\, kg \cdot m^2)$ are the total moment of inertia about an axis, D_y $(0.0220\, kg \cdot m^2/s)$ and D_p $(0.0071\, kg \cdot m^2/s)$ are the damping about the yaw and pitch axis respectively, K_{yy} $(0.0022\, kg \cdot m^2/s^2V)$ and K_{pp} $(0.0375\, kg \cdot m^2/s^2\mathrm{V})$ are torque thrust gains from the yaw and pitch rotor respectively, K_{py} $(0.0021\, kg \cdot m^2/s^2\mathrm{V})$ and K_{yp} $(-0.0027\, kg \cdot m^2/s^2\mathrm{V})$ are the cross-torque thrust gain acting on pitch/yaw from the yaw/pitch rotor respectively, and K_{sp} $(0.0375\, kg \cdot m^2/s^2\mathrm{V})$ is the stiffness about the pitch axis. This state-space model would then be controlled by a Linear Quadratic Regulator (LQR) controller. An LQR controller is how you must minimize the cost function with adjusting u(t) in:

$$J = \int_0^{\infty} x^T(t) \cdot Q \cdot x(t) + u(t)^T \cdot R \cdot u(t)\, dt$$

The Q and R are diagonal matrices such that whichever matrix, and/or elements, are bigger would mean that it would reach the setpoint faster for which state or input it corresponds to. And that is simplified to the algebraic Riccati equation (ARE) that solves for S with:

$$A^T S + SA - SBR^{-1}B^T S + Q = 0$$

Then a gain matrix $\mathrm{K} = R^{-1}B^T\mathrm{S}$ is used to set the input vector u(t) with $u(t) = -Kx(t)$.

The principle of low gain is with how the following properties hold:

1. There exists a constant M such that the feedback gain $|F_\varepsilon| \leq M$ for any $\varepsilon \in (0, \varepsilon^*]$;
2. $A + BF_\varepsilon$ *is Hurwitz stable for any* $\varepsilon \in (0, \varepsilon^*]$;
3. For any $\mathrm{x}(0) \in R^n$, the closed − loop system with $\mathrm{u} = F_\varepsilon \mathrm{x}$ satisfies $\lim_{\varepsilon \to 0} |\mathrm{y}|_{L_2} = 0$

And these conditions are satisfied with how the gains were not extremely high to have its norm be bounded by a constant, A+BF$_\varepsilon$ is stable with its eigenvalues negative to be in the left-hand plane, and how the positions start from zero. With this the input voltages would not have its magnitude be above 24 V or less than -24 V. Finally, an integrator is as shown in Figure 2 where the inputs to this are the difference of the pitch and yaw setpoints and their current positions, to be the error, that is added with the voltage also set by the LQR controller to then be inputted to the machine or model. Therefore, the integrator rids of steady state error for the controller by having the increasing error increase the voltage input with it integrating the error over time.

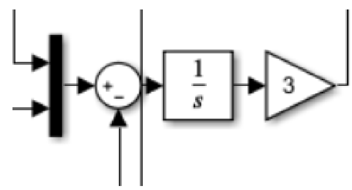

Figure 2: Integrator

The company's LQR controller was quite accurate on the linearized model as shown by Figure 3. However, the motors do oversaturate as shown by Figure 4. In the following, the left figure is for pitch and the right for yaw.

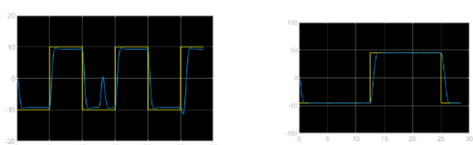

Figure 4: Company LQR Simulation Positions

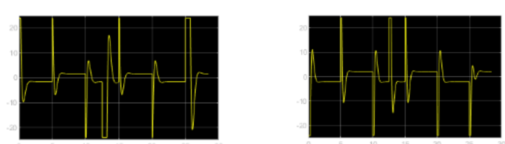

Figure 4: Company LQR Simulation

Therefore, with the principles of low gain on an LQR controller and using a constant setpoint the simulator yielded the results in Figure 5 and the input voltages do not saturate as shown by Figure 6.

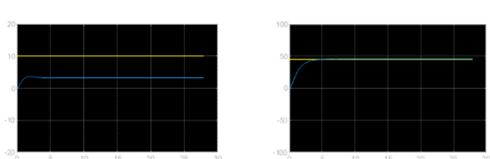

Figure 6: Low-Gain Setpoint Simulation Positions

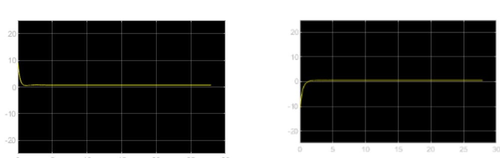

Figure 6: Low-Gain Setpoint Simulation Voltages

As seen here the pitch has steady state error, this is as the pitch and yaw do not have the same system type. And after an integrator was added here is what the simulator yielded the results in Figure 7. And here are the voltage inputs as shown by Figure 8.

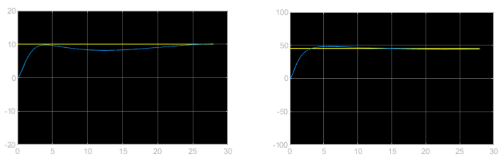

Figure 8: Low-Gain with Integrator Simulation Position

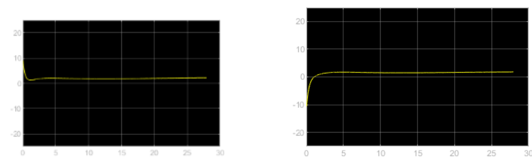

Figure 8: Low-Gain with Integrator Simulation Voltages

Here are the results on the machine as shown by Figure 9. Motor inputs are as shown by Figure 10

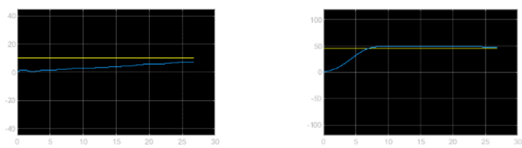

Figure 10: Low-Gain with Integrator Actual Machine Position

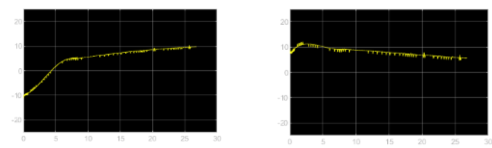

Figure 10: Low-Gain with Integrator Actual Machine Voltage

The results on the machine are different as compared to the simulation as the simulation uses a linearized model while the machine assumes a non-linearized model to be different. For the square wave input, the controller would need to be sped up, which will be investigated in the future.

This project has provided me experience with how work that applies control theory is like. These results would be disseminated through a technical report. And this effort would be sustained through future work with possible additional funding. This project has an impact for electrical engineering with being able to know how to apply the principles of low gain to complex control systems. And other disciplines like mathematics, mechanical engineering, and even computer science (with robot control) could be able to use this research in their work.

Overall, my mentor provided me with great insight to problems and taught me how become a better problem solver with control engineering as well as a lot about the field of control theory. And despite COVID-19, the Zoom meeting and email communications were able to convey many of these things to be able to help the project go smoothly.

References:

1. Quanser AERO - 2DOF Lab Guide (https://www.quanser.com/products/quanser-aero/)
2. Research Paper: A set-theoretic model reference adaptive control architecture with dead-zone effect <- used this paper to get value of constants for the state space model
3. Introduction to Linear Quadratic Regulator (LQR) Control (https://www.youtube.com/watch?v=wEevt2a4SKI)
4. Research paper section: CHAPTER 2 H2 and H1 low-gain feedback – Continuous-time systems
5. Dr. Zhang's notes on low-gain & integrators