Topology

Pair (M, d) is metric space where M is a set and $d : M \times M \to [0, \infty)$ is a metric. It satisfies

1. $d(x, z) \leq d(x, y) + d(y, z)$
2. $d(x, y) = d(y, x)$
3. $d(x, y) = 0 \Leftrightarrow x = y$

Probability

Probability space is $(\Omega, \mathcal{F}, \mathbb{P})$. Probability function is *any* function $\mathbb{P} : \mathcal{F} \to \mathbb{R}$ that satisfies

1. $\forall E, 0 \leq \mathbb{P}(E) \leq 1$
2. $\mathbb{P}(\Omega) = 1$
3. $\mathbb{P}\left(\bigcup_{i \geq 1} E_i\right) = \sum_{i \geq 1} \mathbb{P}(E_i)$

Events E and F are independent if and only if

$$\mathbb{P}(E \cap F) = \mathbb{P}(E) \cdot \mathbb{P}(F)$$

Conditional probability

$$\mathbb{P}(E \mid F) = \frac{\mathbb{P}(E \cap F)}{\mathbb{P}(F)}$$

Law of Total Probability

$$\mathbb{P}(B) = \sum_{i=1}^{n} \mathbb{P}(B \cap E_i) = \sum_{i=1}^{n} \mathbb{P}(B \mid E_i)\,\mathbb{P}(E_i)$$

Bayes' Law

$$\mathbb{P}(E_i \mid B) = \frac{\mathbb{P}(E_i \cap B)}{\mathbb{P}(B)} = \frac{\mathbb{P}(B \mid E_i)\,\mathbb{P}(E_i)}{\sum_{j=1}^{n} \mathbb{P}(B \mid E_j)\,\mathbb{P}(E_j)}$$

Linearity of Expectations

$$\mathbb{E}\left[\sum_{i=1}^{n} X_i\right] = \sum_{i=1}^{n} E[X_i]$$

Jensen's Inequality. If f is a convex function, then

$$\mathbb{E}[f(X)] \geq f(\mathbb{E}[X])$$

Distributions

Distribution	PMF/PDF	EV	Variance
Bern(p)	p	p	pq
Geom(p)	$q^{n-1}p$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Bin(n, p)	$\binom{n}{k}p^k q^{n-k}$	np	npq
Pois(μ)	$\frac{e^{-\mu}\mu^k}{k!}$	μ	μ
U$[a, b]$	$\frac{1}{b-a}$	$\frac{b+a}{2}$	$\frac{b^2-a^2}{12}$
Exp(λ)	$\lambda e^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$

Intermediate value

$$f'(\xi) = \frac{f(b) - f(a)}{b - a}$$

$$\frac{1}{b - a}\int_a^b f(x)\,dx = f(z)$$

Given $h(x) \geq 0$ and f is continuous then

$$\frac{\int_a^b f(x)h(x)\,dx}{\int_a^b h(x)\,dx} = f(z)$$

Power series

$$\Sigma a_k (x - x_0)^k$$

Radius of convergence

$$\lim_{k \to \infty}\left|\frac{a_k}{a_{k+1}}\right| \quad \text{or} \quad \lim_{k \to \infty}\frac{1}{\sqrt[k]{|a_k|}}$$

Taylor's theorem

$$T_n(x; x_0) = \Sigma_{k=0}^{n}\frac{f^{(k)}(x_0)}{k!}(x - x_0)^k$$

Integral form remainder

$$R_n(x; x_0) = \int_{x_0}^{x}\frac{f^{(n+1)}(t)}{n!}(x - t)^n\,dt$$

$$R_n(x; x_0) = (x - x_0)^n \varepsilon(x), \quad \lim_{x \to x_0}\varepsilon(x) = 0$$

Cauchy form remainder

$$R_n(x; x_0) = \frac{f^{(n+1)}(\xi_x)}{n!}(x - \xi_x)^n(x - x_0)$$

Lagrange form remainder

$$R_n(x; x_0) = \frac{f^{(n+1)}(\xi_x)}{(n+1)!}(x - x_0)^{n+1}$$