Probability

Probability space is $(\Omega, \mathcal{F}, \mathbb{P})$. Probability function is *any* function $\mathbb{P} : \mathcal{F} \to \mathbb{R}$ that satisfies

1. $\forall E, 0 \leq \mathbb{P}(E) \leq 1$
2. $\mathbb{P}(\Omega) = 1$
3. $\mathbb{P}\left(\bigcup_{i \geq 1} E_i\right) = \sum_{i \geq 1} \mathbb{P}(E_i)$

Events E and F are independent if and only if

$$\mathbb{P}(E \cap F) = \mathbb{P}(E) \cdot \mathbb{P}(F)$$

Conditional probability

$$\mathbb{P}(E \mid F) = \frac{\mathbb{P}(E \cap F)}{\mathbb{P}(F)}$$

Law of Total Probability

$$\mathbb{P}(B) = \sum_{i=1}^{n} \mathbb{P}(B \cap E_i) = \sum_{i=1}^{n} \mathbb{P}(B \mid E_i)\mathbb{P}(E_i)$$

Bayes' Law

$$\mathbb{P}(E_i \mid B) = \frac{\mathbb{P}(E_i \cap B)}{\mathbb{P}(B)} = \frac{\mathbb{P}(B \mid E_i)\mathbb{P}(E_i)}{\sum_{j=1}^{n} \mathbb{P}(B \mid E_j)\mathbb{P}(E_j)}$$

Linearity of Expectations

$$\mathbb{E}\left[\sum_{i=1}^{n} X_i\right] = \sum_{i=1}^{n} E[X_i]$$

Jensen's Inequality. If f is a convex function, then

$$\mathbb{E}[f(X)] \geq f(\mathbb{E}[X])$$

Intermediate value

$$f'(\xi) = \frac{f(b) - f(a)}{b - a}$$

$$\frac{1}{b-a} \int_a^b f(x)\, dx = f(z)$$

Given $h(x) \geq 0$ and f is continuous then

$$\frac{\int_a^b f(x)h(x)\, dx}{\int_a^b h(x) dx} = f(z)$$

Power series

$$\Sigma a_k (x - x_0)^k$$

Radius of convergence

$$\lim_{k \to \infty} \left| \frac{a_k}{a_{k+1}} \right| \quad \text{or} \quad \lim_{k \to \infty} \frac{1}{\sqrt[k]{|a_k|}}$$

Taylor's theorem

$$T_n(x; x_0) = \Sigma_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!}(x - x_0)^k$$

Integral form remainder

$$R_n(x; x_0) = \int_{x_0}^{x} \frac{f^{(n+1)}(t)}{n!}(x - t)^n\, dt$$

$$R_n(x; x_0) = (x - x_0)^n \varepsilon(x), \quad \lim_{x \to x_0} \varepsilon(x) = 0$$

Cauchy form remainder

$$R_n(x; x_0) = \frac{f^{(n+1)}(\xi_x)}{n!}(x - \xi_x)^n (x - x_0)$$

Lagrange form remainder

$$R_n(x; x_0) = \frac{f^{(n+1)}(\xi_x)}{(n+1)!}(x - x_0)^{n+1}$$