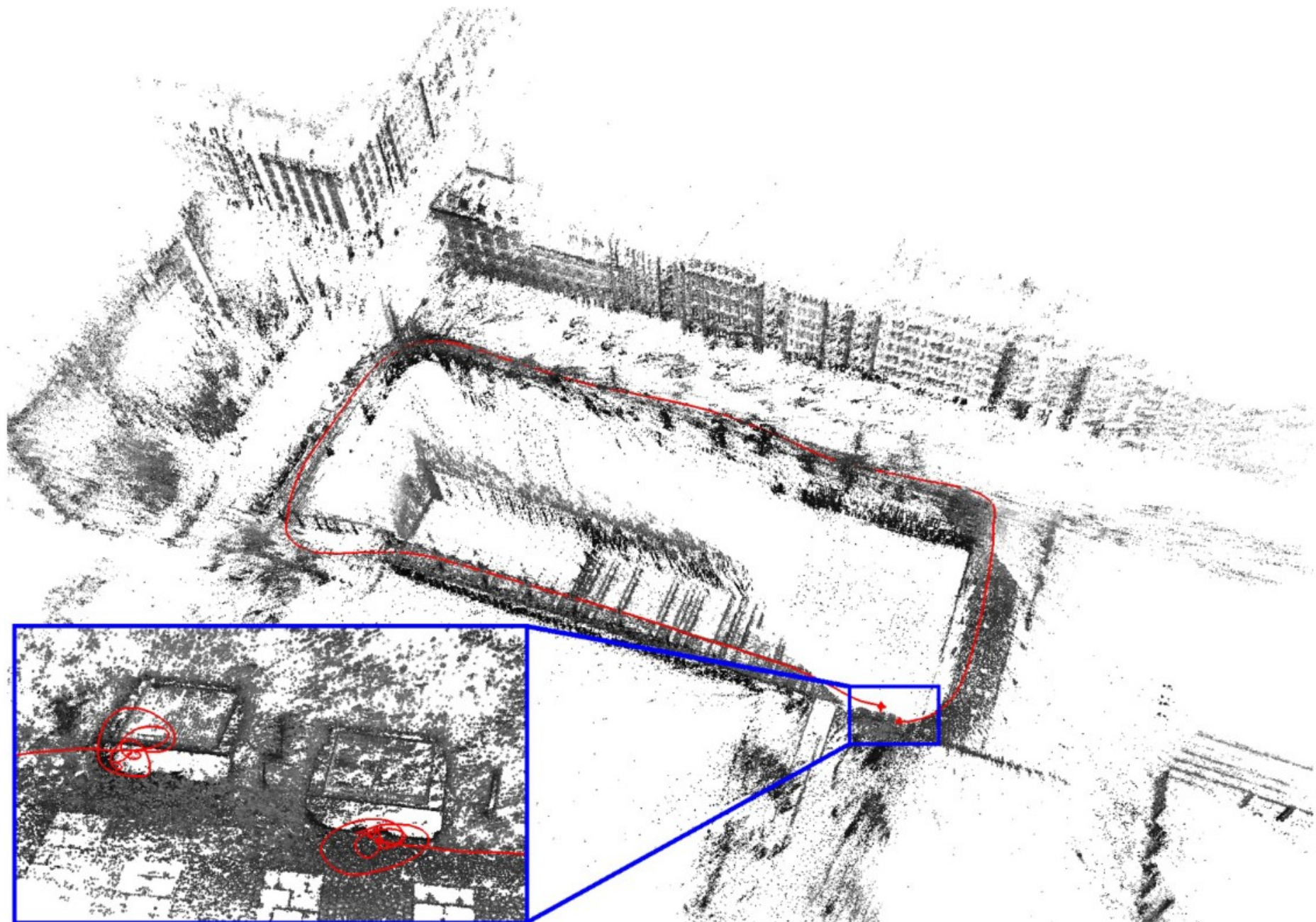


Simultaneous Localization and Mapping

Feature based approach



SLAM vs Odometry

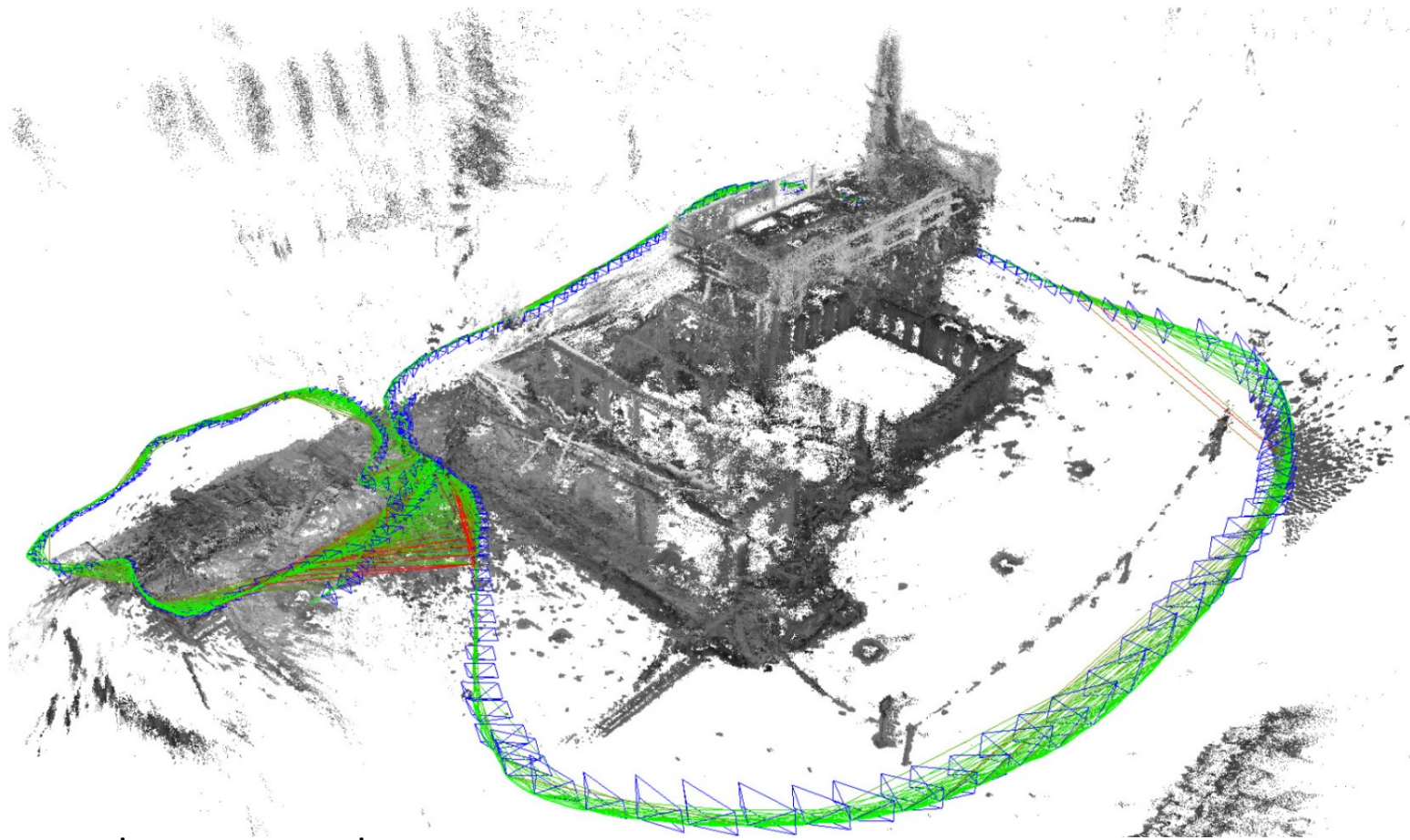
- Past: Odometry had no loop closure
- Now: closing gap
- Odometry focuses on localization
- SLAM focuses on both mapping and localization

Measurements, Sensors

- Camera (RGB, RGB-D, TOF, infra, wide-narrow, stereo...)
- Inertial Measurement Unit - IMU
 - accelerometer, gyroscope, magnetometer
- GPS
- Lidar
- Ultrasonic sensor
- Augmented environment (MoCap, AprilTag...)
- Microphone, Rotary sensor, WiFi, LiFi, Bluetooth...

Maps

- Metric vs Topological
- Implicit representations
 - Occupancy grids
 - Depth fields
 - Light fields, radiance fields
- Explicit representations
 - Point clouds
 - Keyframes
 - Meshes
- Graphs
 - Pose graph, factor graph, covisibility graph, scene graph...
- Grids
 - Voxel grid, Multi resolution, Hierarchical, Octree, k-d tree



Basic formulation

- Probabilistic model
 - \mathbf{Y} noisy measurements
 - \mathbf{X} unknown model parameters (map, trajectory)
- Maximum Likelihood approach
 - $\mathbf{X}_{\text{opt}} = \text{argmax } P(\mathbf{Y}|\mathbf{X})$
 - Only the (approx.) best solution
- Maximum A Posteriori approach
 - $P(\mathbf{X}|\mathbf{Y}) = P(\mathbf{Y}|\mathbf{X}) * P(\mathbf{X}) * c$
 - Complete distribution over the parameters
 - Prior often unfeasible

EKF vs Keyframe based SLAM

- EKF: Extended Kalman Filter
 - Strict probabilistic approach
 - Hard to detect/incorporate loop closures
 - Marginalization is difficult
 - Better suited to factor graphs
- Keyframe based
 - Sparser approach
 - Easier long-term association for loop closure and bundle adjustment
 - More robust

V/VI-SLAM

- Visual vs Visual+Inertial Odometry
- Most common
- IMU measurements
 - More information, higher accuracy
 - More complexity
- Differences
 - Pose graph optimization
 - Initialization, calibration
 - Local/global Bundle adjustment

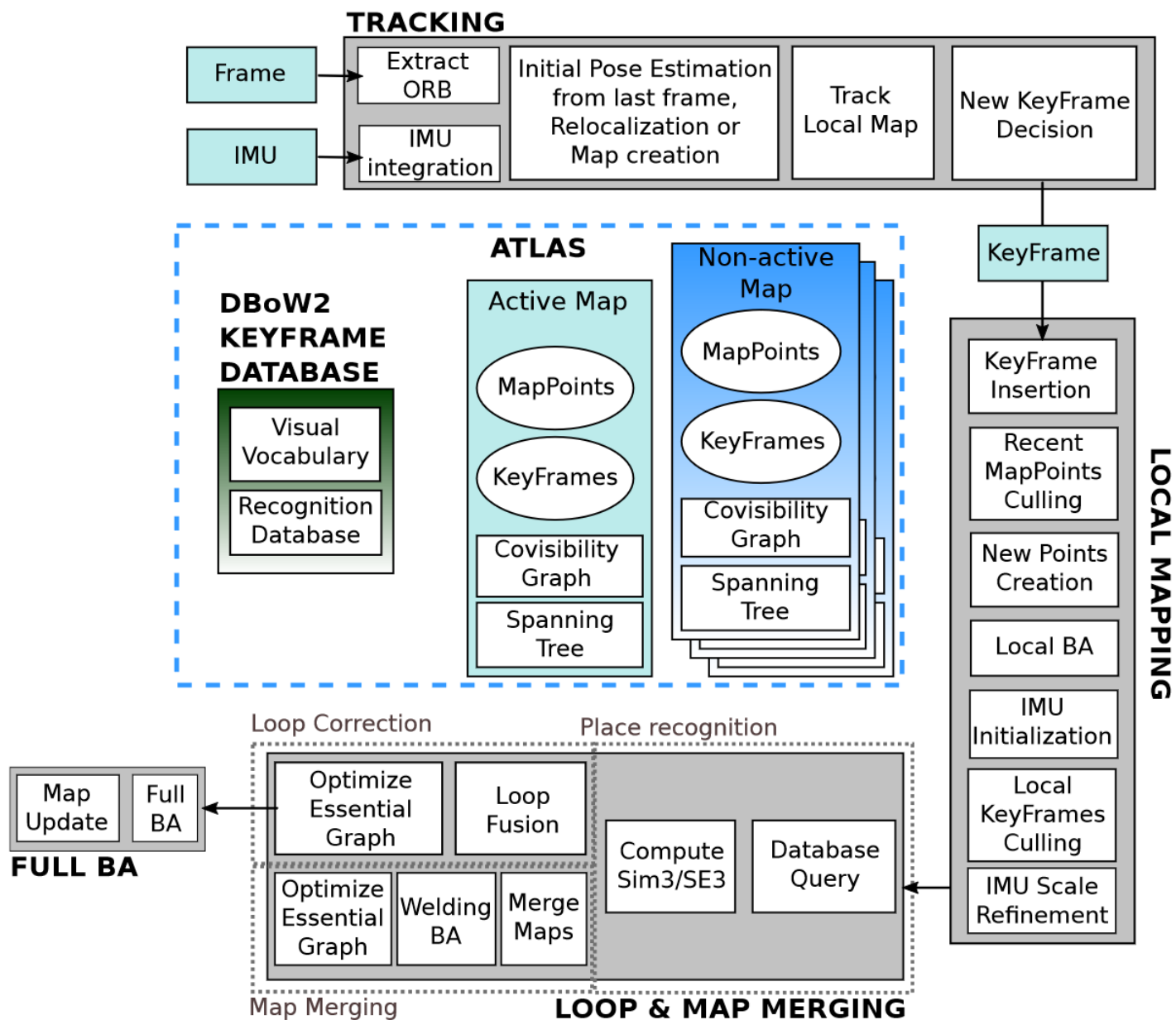
V/VI-SLAM Approaches

- Direct vs Indirect
 - Indirect: raw measurements preprocessed $\rightarrow \mathbf{Y}$
(e.g. features, optical flow, line detection)
typically geometric error
 - Direct: light (radiant energy or radiance) as \mathbf{Y}
typically photometric error (geometric for depth measurements)
- Dense vs Sparse
 - Dense: all pixels are used during the estimation
keeps geometrical prior: notion of neighborhood, leads to dense Hessians
 - Sparse: “special” pixels are selected (corners, line segments)
keypoint positions conditionally independent given the camera parameters
 - (Semi-dense: not all pixels, but larger patches)

Sparse + Indirect

- Most common
- Map
 - Keyframes, keyframe descriptors
 - Feature points (2D and 3D) with descriptors
 - Pose graph
- Localization
 - Feature point extraction
 - Feature/frame descriptor generation
 - Image retrieval
 - Feature matching
 - Ransac+PnP

Basic steps

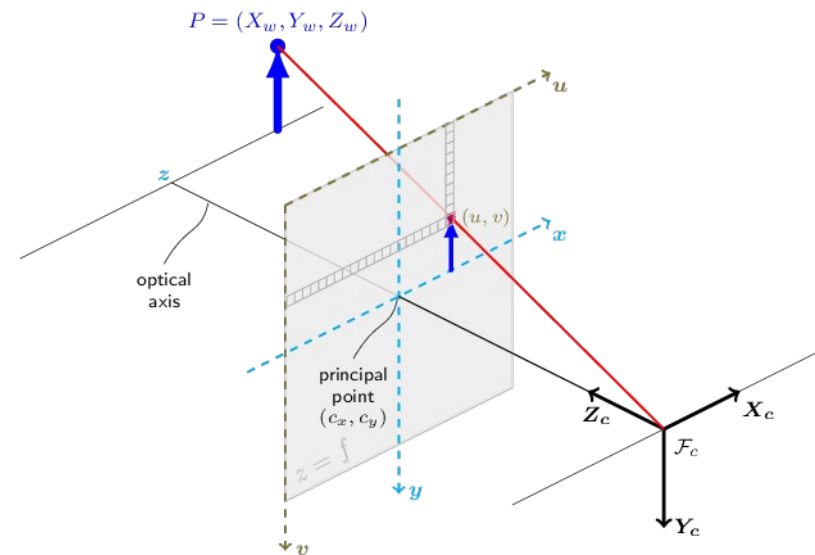
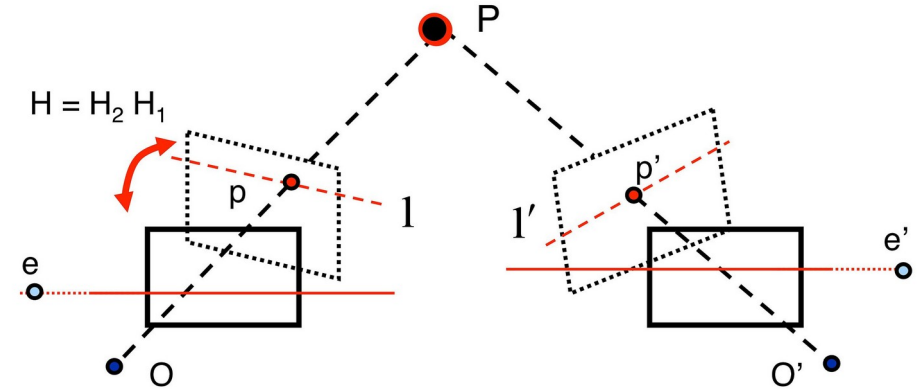


Tracking – Taking new measurements

- Undistort image
- Color, exposure balancing...
- Rectify (stereo): epipolar lines parallel
- Pinhole camera model

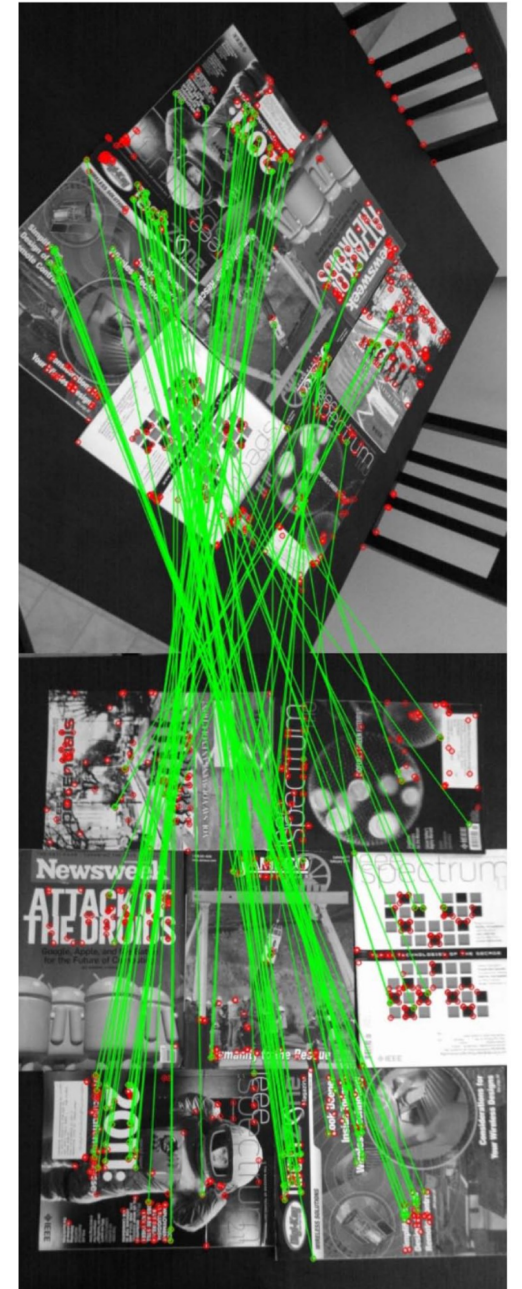
$$s \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

$$(k_1, k_2, p_1, p_2[, k_3[, k_4, k_5, k_6[, s_1, s_2, s_3, s_4[, \tau_x, \tau_y]]]])$$



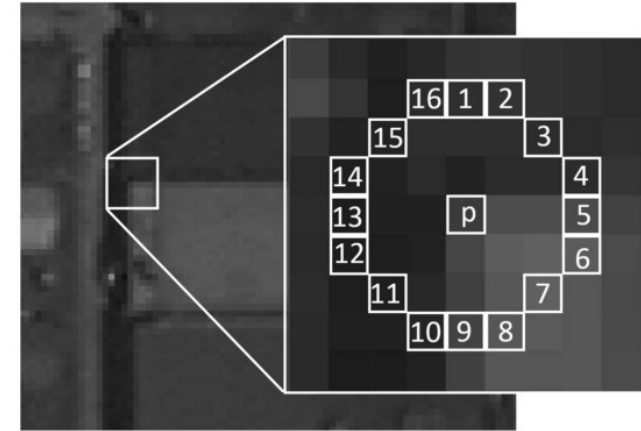
Tracking- Feature extraction

- High gradient points: edges, corners
- Optionally multi scale and oriented
- Shi-Tomasi, Harris corner detection: fast, inaccurate
- FAST: fast, single scale, not oriented
- SIFT: slowest, multi scale, oriented, patented
- SURF: slow, multi scale, oriented (inaccurate), patented
- ORB: fast, multi scale, oriented, free



ORB: Oriented Fast and Rotated BRIEF

- FAST threshold for circular ring around center -> involves edges too
- Harris corner filtering, top N points
- Scale pyramid for multi scale
- Orientation from center of mass
- BRIEF (Binary robust independent elementary feature)
 - Binary intensity tests in the patch
- Rotate tests according to feature orientation
- Use greedy algorithm to find best test pairs



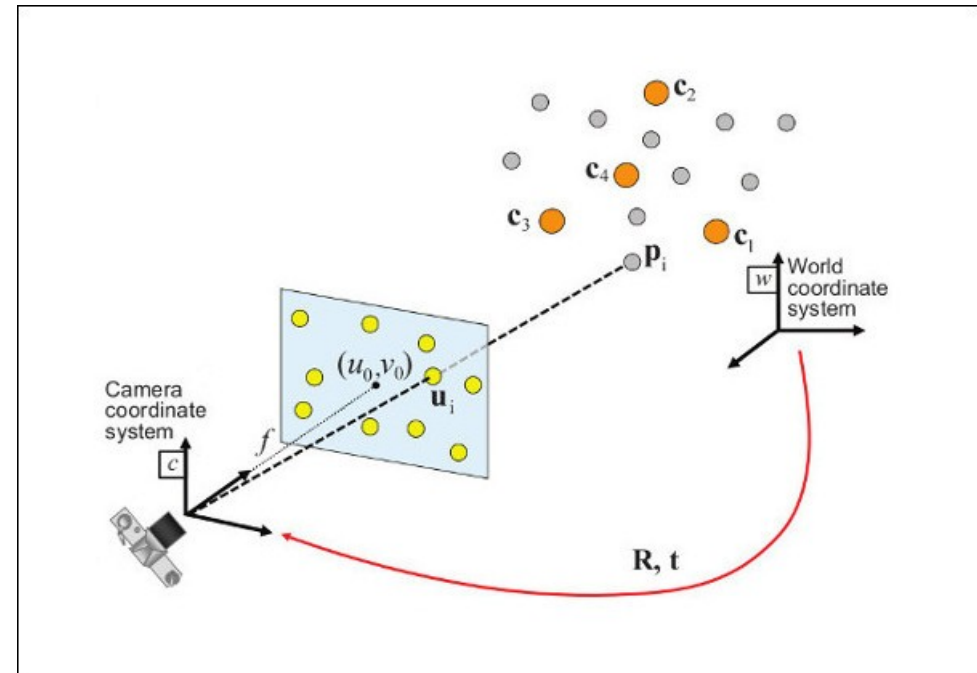
$$m_{pq} = \sum_{x,y} x^p y^q I(x, y)$$

$$C = \left(\frac{m_{10}}{m_{00}}, \frac{m_{01}}{m_{00}} \right)$$

$$\theta = \text{atan2}(m_{01}, m_{10})$$

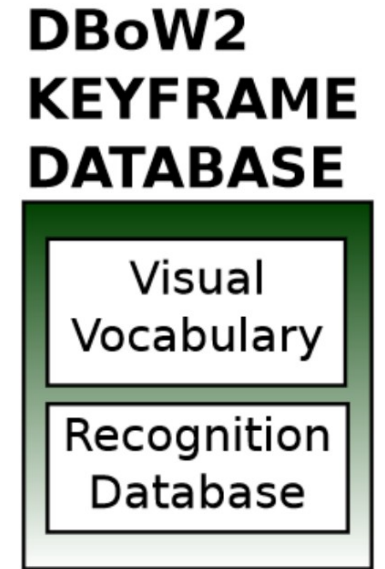
Tracking- Pose estimation

- Current keyframe with feature points
- Feature matching based on descriptors (cosine similarity)
- RANSAC + PnP
- Inliers, outliers
- If fails
 - relocalization
- If succeeds
 - Keyframe optimization



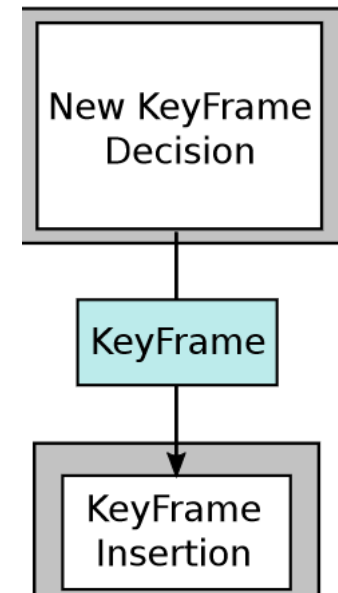
Relocalization

- Input: map, measurements
- Output: **T** pose estimation
- Keyframe based approach:
 - Image retrieval with BoW
 - Pose estimation with feature matching
- DBoW2: Bags of Binary Words for Fast Place Recognition in Image Sequence
 - General visual vocabulary
 - Inverted index
 - Updated recognition database
 - Multiple solutions



Tracking- Keyframe selection

- Based on heuristics
 - Time, distance, failed tracking, unbalanced map, feature density...
- Previous keyframe fixed
- New keyframe added to map



Local Mapping – Keyframe insertion

- Update Pose graph
- Calculate BoW descriptor
- Find covisible keyframes
- Match features points
- Discard duplicate feature points
- Triangulate depth
- Project 3D keypoints

Local Mapping – Local Bundle Adjustment

- New keyframe optimized (camera pose Sim(3) or SE(3), 3D features)
- Based on correspondences
- Moving window of keyframes
- Projection (j-th image to i-th)
- Energy term
- Loss function
- Reprojection error (geometric)
- Covariance as weight associated with feature scale
- Usually first-order approximations (Levenberg–Marquardt alg.)

$$\pi_i(\mathbf{T}_{iw}, \mathbf{X}_{w,j}) = \begin{bmatrix} f_{i,u} \frac{x_{i,j}}{z_{i,j}} + c_{i,u} \\ f_{i,v} \frac{y_{i,j}}{z_{i,j}} + c_{i,v} \end{bmatrix}$$

$$\begin{bmatrix} x_{i,j} & y_{i,j} & z_{i,j} \end{bmatrix}^T = \mathbf{R}_{iw} \mathbf{X}_{w,j} + \mathbf{t}_{iw}$$

$$\mathbf{e}_{i,j} = \mathbf{x}_{i,j} - \pi_i(\mathbf{T}_{iw}, \mathbf{X}_{w,j})$$

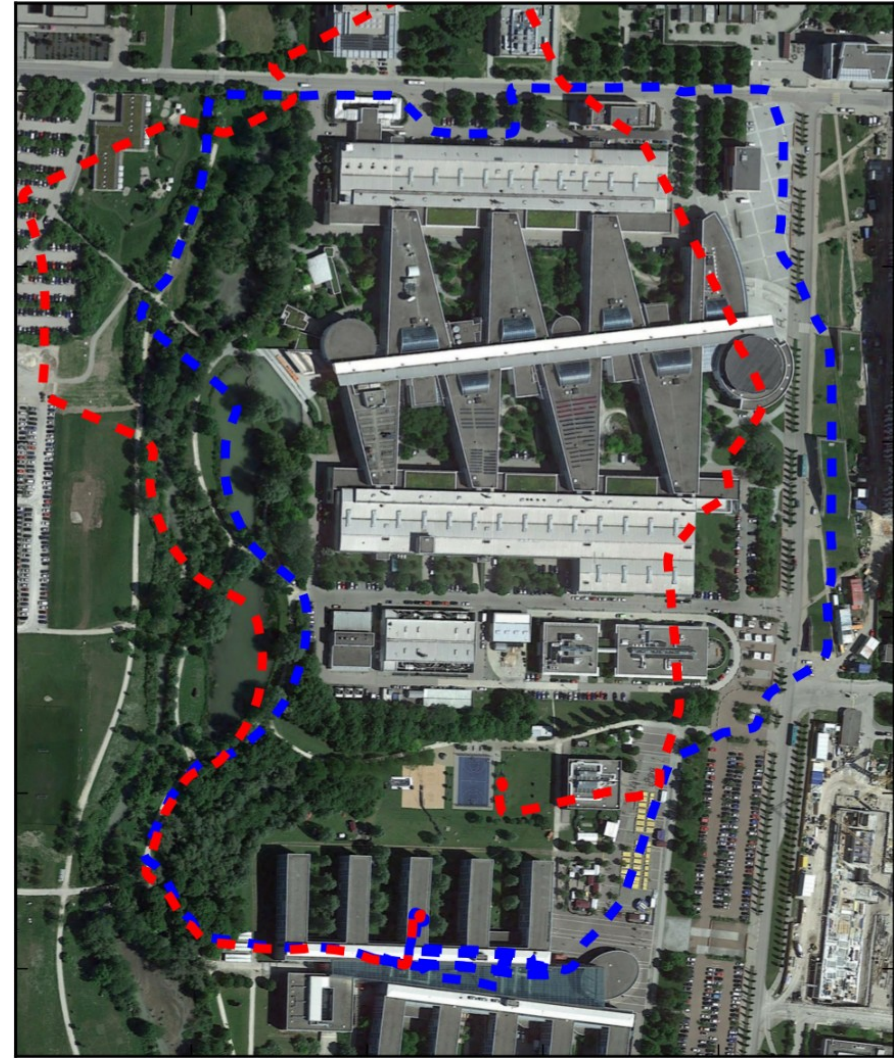
$$C = \sum_{i,j} \rho_h(\mathbf{e}_{i,j}^T \boldsymbol{\Omega}_{i,j}^{-1} \mathbf{e}_{i,j})$$

Local Mapping – Keyframe culling

- Discard duplicated keyframes, feature points
- Balance map density
- Keep current environment densely mapped to help tracking
- Sparsify later

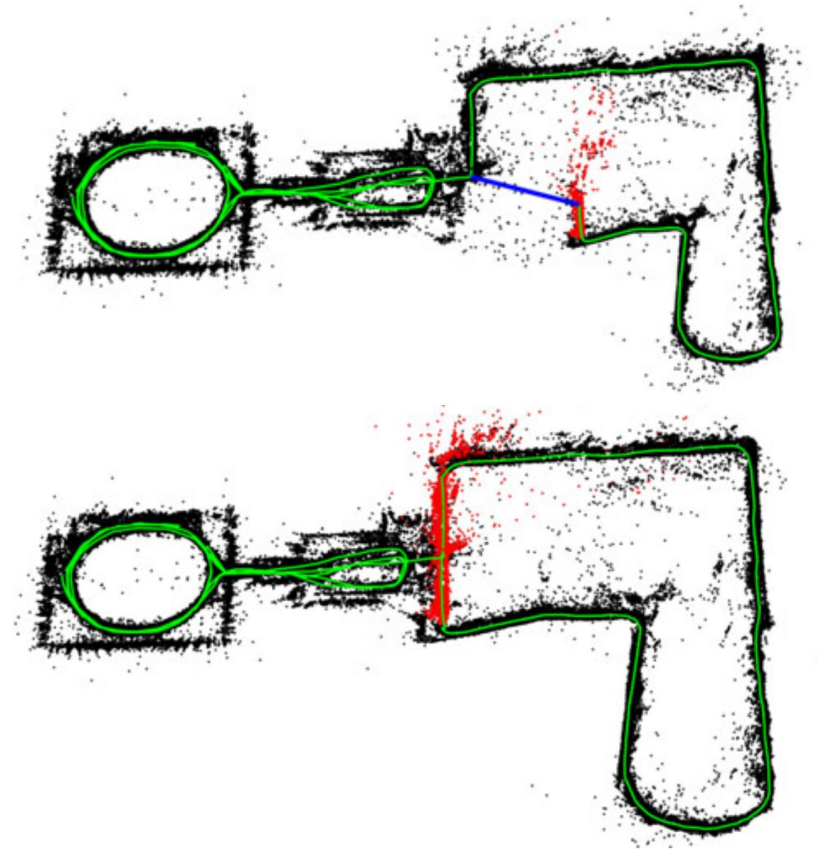
Drift

- Odometry contains inaccuracies
- Even with IMU with perfect calibration
 - dead reckoning
- GPS helps but not accurate enough
- EKF solutions suffer from drift as well
- Limitation: maximum mid-term data association
- Global consistency not enforced



Loop Closing – Loop detection

- BoW
- Cosine similarity
- Multiple candidates, local window
- Inlier based verification
- Estimate relative \mathbf{T} (Sim(3) or SE(3)) transformation
 - E.g. Ransac + Horn algorithm (3D to 3D)
- Refine relative pose with feature matching-based optimization
- Merge keyframes
- Update factor graph or pose graph optimization
- Full BA

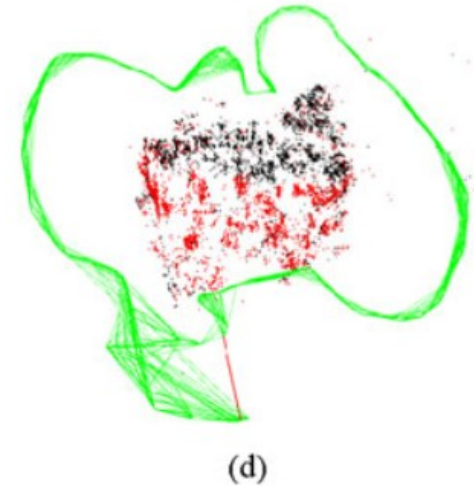
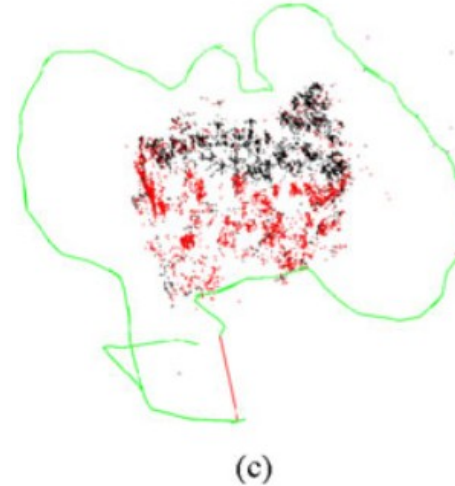
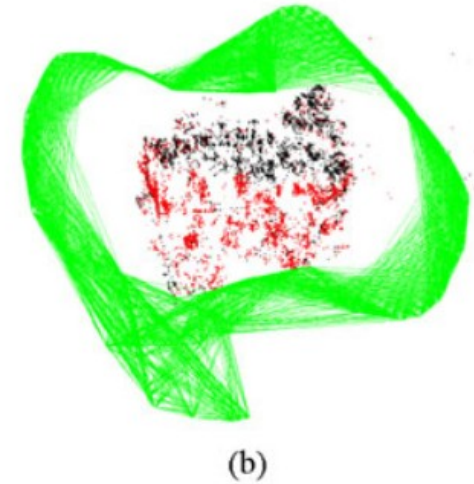
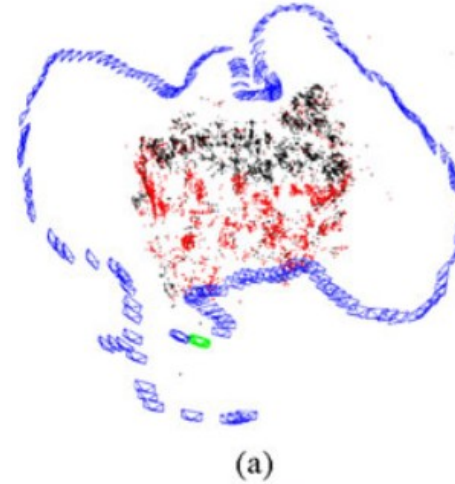


Loop Closing – Pose graph optimization

- Optimize only the keyframe poses
- Optionally expressed by a factor graph (preferred if uncertainty is modelled)

$$\mathbf{e}_{i,j} = \log_{\text{Sim}(3)} (\mathbf{S}_{ij} \mathbf{S}_{jw} \mathbf{S}_{iw}^{-1})$$

$$C = \sum_{i,j} (\mathbf{e}_{i,j}^T \mathbf{\Lambda}_{i,j} \mathbf{e}_{i,j})$$



Full Bundle adjustment

- Formulation similar to Local BA
- Computationally demanding
- All keyframe poses and keypoint positions are optimized

$$\{\mathbf{X}^i, \mathbf{R}_l, \mathbf{t}_l | i \in \mathcal{P}_L, l \in \mathcal{K}_L\} = \operatorname{argmin}_{\mathbf{X}^i, \mathbf{R}_l, \mathbf{t}_l} \sum_{k \in \mathcal{K}_L \cup \mathcal{K}_F} \sum_{j \in \mathcal{X}_k} \rho(E_{kj})$$

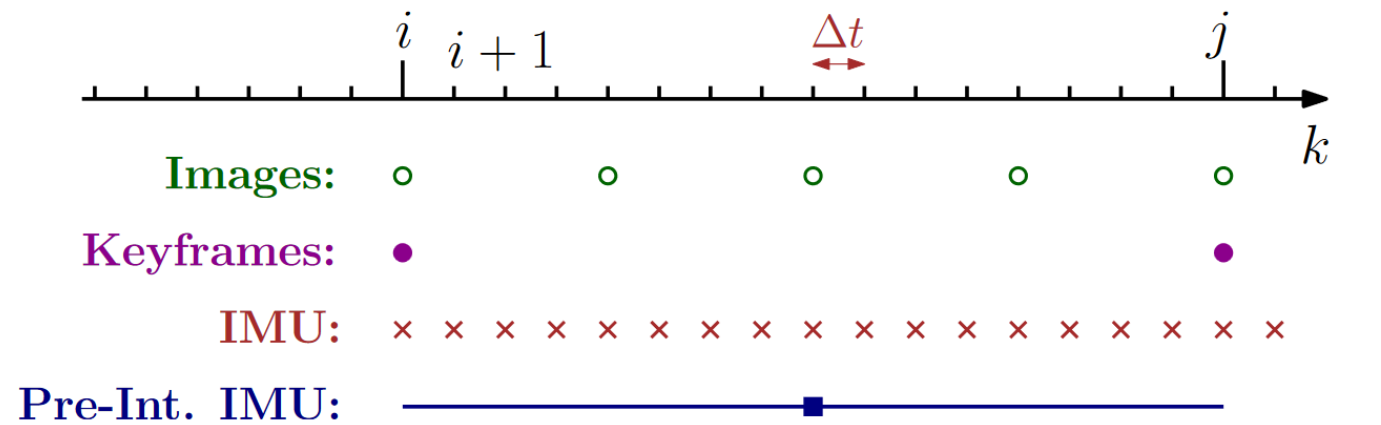
$$E_{kj} = \left\| \mathbf{x}_{(\cdot)}^j - \pi_{(\cdot)}(\mathbf{R}_k \mathbf{X}^j + \mathbf{t}_k) \right\|_{\Sigma}^2$$

Mono vs Stereo

- Depth uncertainty
- Scale drift
- $SE(3)$ – $Sim(3)$
- Mono
 - scale calibration
 - scale optimization during pose graph optimization
- Stereo
 - image rectification
 - \mathbf{Y} contains inverse depth
 - Stereo keypoint: (u_L, v_L, u_R)
 - RGB-D can replace
 - Meaningful only for close observations (for translation at least)

Visual-Inertial SLAM

- Measurements:
 - Acceleration (a_B)
 - Angular velocity (ω_B)
- Needed:
 - Velocity (v_B)
 - Position (p_B)
 - Rotation (R_B)
- Extra parameters
 - Biases (b_a, b_g)
 - Gravity (g_w)
- Euler integration (or Runge-Kutta)
- IMU preintegration



$$\mathbf{R}_{WB}^{k+1} = \mathbf{R}_{WB}^k \text{Exp} \left((\boldsymbol{\omega}_B^k - \mathbf{b}_g^k) \Delta t \right)$$

$${}_W \mathbf{v}_B^{k+1} = {}_W \mathbf{v}_B^k + \mathbf{g}_W \Delta t + \mathbf{R}_{WB}^k (\mathbf{a}_B^k - \mathbf{b}_a^k) \Delta t$$

$${}_W \mathbf{p}_B^{k+1} = {}_W \mathbf{p}_B^k + {}_W \mathbf{v}_B^k \Delta t + \frac{1}{2} \mathbf{g}_W \Delta t^2 + \frac{1}{2} \mathbf{R}_{WB}^k (\mathbf{a}_B^k - \mathbf{b}_a^k) \Delta t^2$$

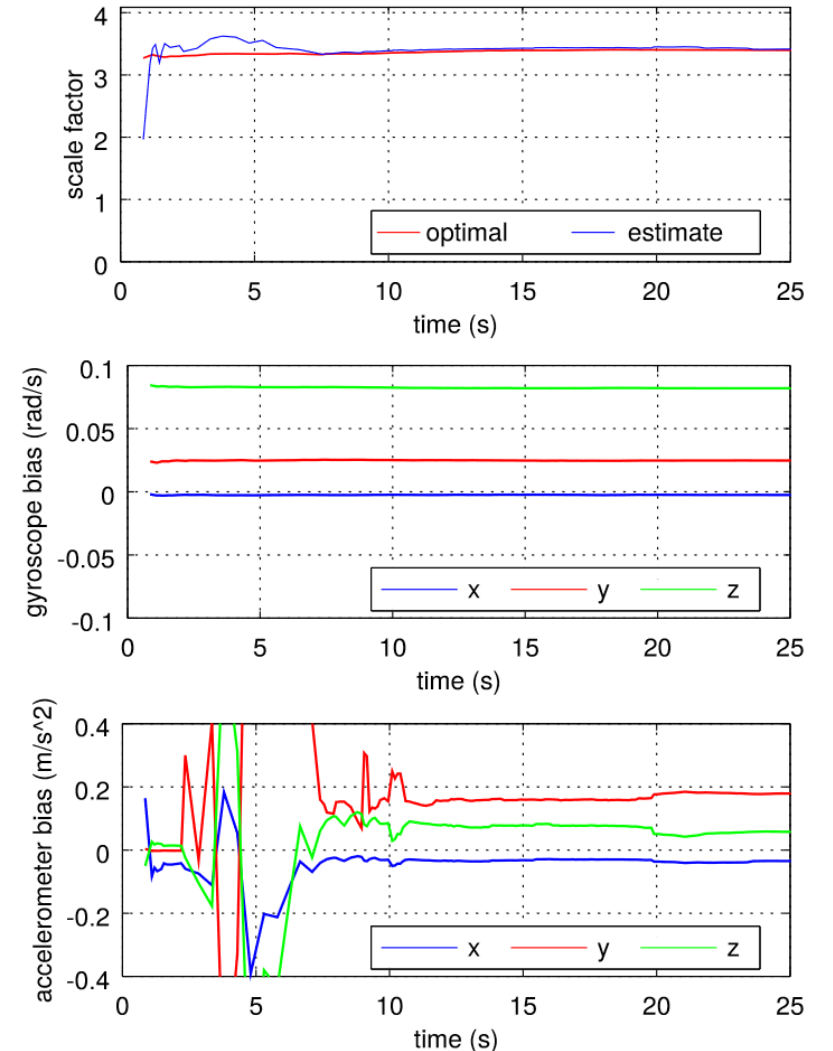
$$\mathbf{R}_{WB}^{i+1} = \mathbf{R}_{WB}^i \Delta \mathbf{R}_{i,i+1} \text{Exp} \left((\mathbf{J}_{\Delta R}^g \mathbf{b}_g^i) \right)$$

$$\begin{aligned} {}_W \mathbf{v}_B^{i+1} &= {}_W \mathbf{v}_B^i + \mathbf{g}_W \Delta t_{i,i+1} \\ &+ \mathbf{R}_{WB}^i \left(\Delta \mathbf{v}_{i,i+1} + \mathbf{J}_{\Delta v}^g \mathbf{b}_g^i + \mathbf{J}_{\Delta v}^a \mathbf{b}_a^i \right) \end{aligned}$$

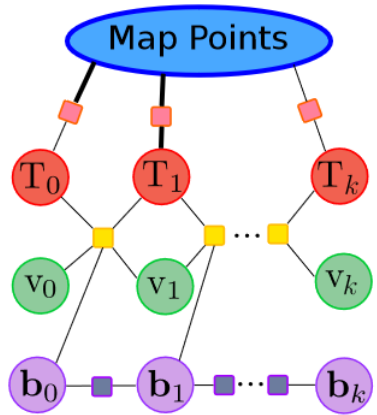
$$\begin{aligned} {}_W \mathbf{p}_B^{i+1} &= {}_W \mathbf{p}_B^i + {}_W \mathbf{v}_B^i \Delta t_{i,i+1} + \frac{1}{2} \mathbf{g}_W \Delta t_{i,i+1}^2 \\ &+ \mathbf{R}_{WB}^i \left(\Delta \mathbf{p}_{i,i+1} + \mathbf{J}_{\Delta p}^g \mathbf{b}_g^i + \mathbf{J}_{\Delta p}^a \mathbf{b}_a^i \right) \end{aligned}$$

Visual-Inertial SLAM- Calibration

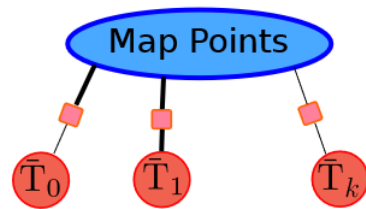
- IMU initialization
- Gyroscope bias estimation
- Scale and gravity estimation
- Accelerometer Bias Estimation
- Scale and Gravity Direction Refinement



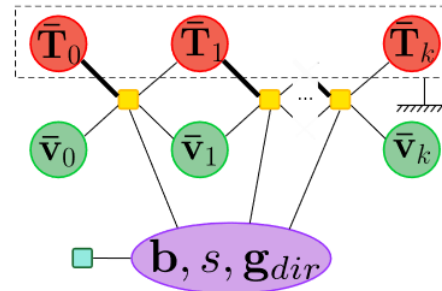
Visual-Inertial SLAM- ORB-SLAM 3



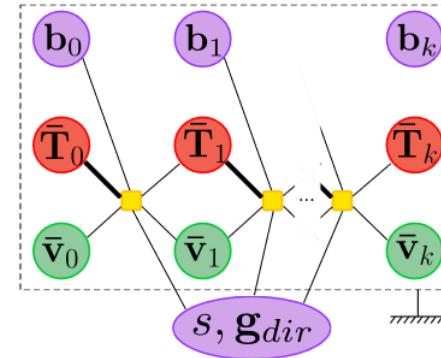
(a) Visual-Inertial



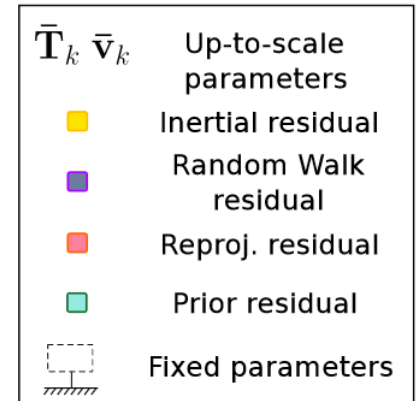
(b) Visual-Only



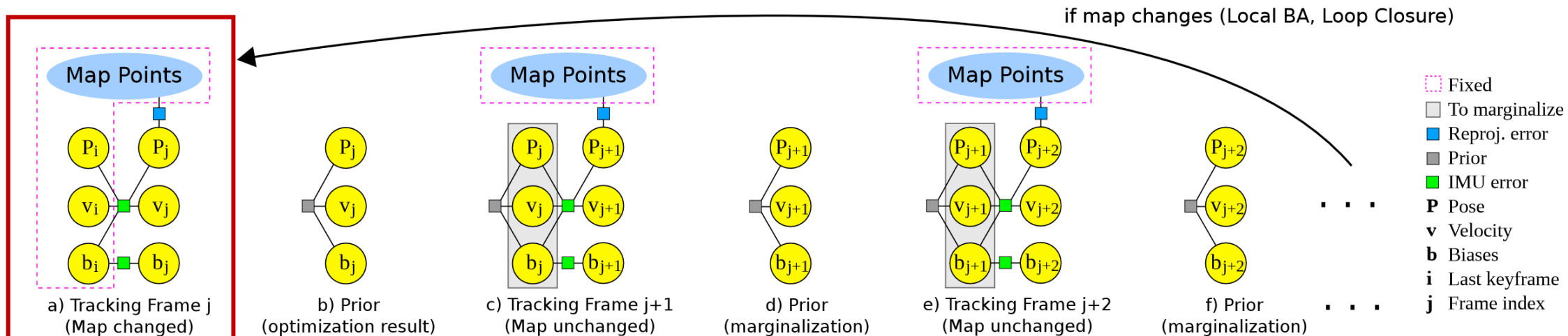
(c) Inertial-Only



(d) Scale and Gravity



Visual-Inertial SLAM- Tracking



$$\theta = \left\{ \mathbf{R}_{WB}^j, {}_W\mathbf{p}_B^j, {}_W\mathbf{v}_B^j, \mathbf{b}_g^j, \mathbf{b}_a^j \right\}$$

$$\theta^* = \underset{\theta}{\operatorname{argmin}} \left(\sum_k \mathbf{E}_{\text{proj}}(k, j) + \mathbf{E}_{\text{IMU}}(i, j) \right)$$

$$\mathbf{E}_{\text{proj}}(k, j) = \rho \left(\left(\mathbf{x}^k - \pi(\mathbf{X}_C^k) \right)^T \Sigma_k \left(\mathbf{x}^k - \pi(\mathbf{X}_C^k) \right) \right)$$

$$\mathbf{X}_C^k = \mathbf{R}_{CB} \mathbf{R}_{BW}^j \left(\mathbf{X}_W^k - {}_W\mathbf{p}_B^j \right) + {}_C\mathbf{p}_B$$

$$\mathbf{E}_{\text{IMU}}(i, j) = \rho \left(\left[\mathbf{e}_R^T \mathbf{e}_v^T \mathbf{e}_p^T \right] \Sigma_I \left[\mathbf{e}_R^T \mathbf{e}_v^T \mathbf{e}_p^T \right]^T \right) + \rho \left(\mathbf{e}_b^T \Sigma_R \mathbf{e}_b \right)$$

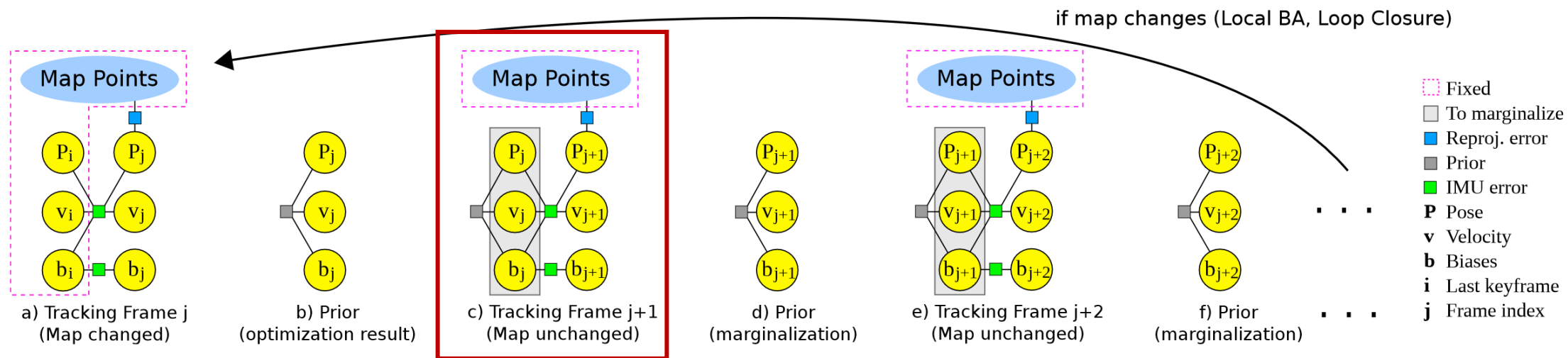
$$\mathbf{e}_R = \operatorname{Log} \left(\left(\Delta \mathbf{R}_{ij} \operatorname{Exp} \left(\mathbf{J}_{\Delta R}^g \mathbf{b}_g^j \right) \right)^T \mathbf{R}_{BW}^i \mathbf{R}_{WB}^j \right)$$

$$\mathbf{e}_v = \mathbf{R}_{BW}^i \left({}_W\mathbf{v}_B^j - {}_W\mathbf{v}_B^i - \mathbf{g}_W \Delta t_{ij} \right) - \left(\Delta \mathbf{v}_{ij} + \mathbf{J}_{\Delta v}^g \mathbf{b}_g^j + \mathbf{J}_{\Delta v}^a \mathbf{b}_a^j \right)$$

$$\mathbf{e}_p = \mathbf{R}_{BW}^i \left({}_W\mathbf{p}_B^j - {}_W\mathbf{p}_B^i - {}_W\mathbf{v}_B^i \Delta t_{ij} - \frac{1}{2} \mathbf{g}_W \Delta t_{ij}^2 \right) - \left(\Delta \mathbf{p}_{ij} + \mathbf{J}_{\Delta p}^g \mathbf{b}_g^j + \mathbf{J}_{\Delta p}^a \mathbf{b}_a^j \right)$$

$$\mathbf{e}_b = \mathbf{b}^j - \mathbf{b}^i$$

Visual-Inertial SLAM- Tracking



$$\theta = \left\{ \mathbf{R}_{WB}^j, \mathbf{p}_W^j, \mathbf{v}_W^j, \mathbf{b}_g^j, \mathbf{b}_a^j, \mathbf{R}_{WB}^{j+1}, \mathbf{p}_W^{j+1}, \mathbf{v}_W^{j+1}, \mathbf{b}_g^{j+1}, \mathbf{b}_a^{j+1} \right\}$$

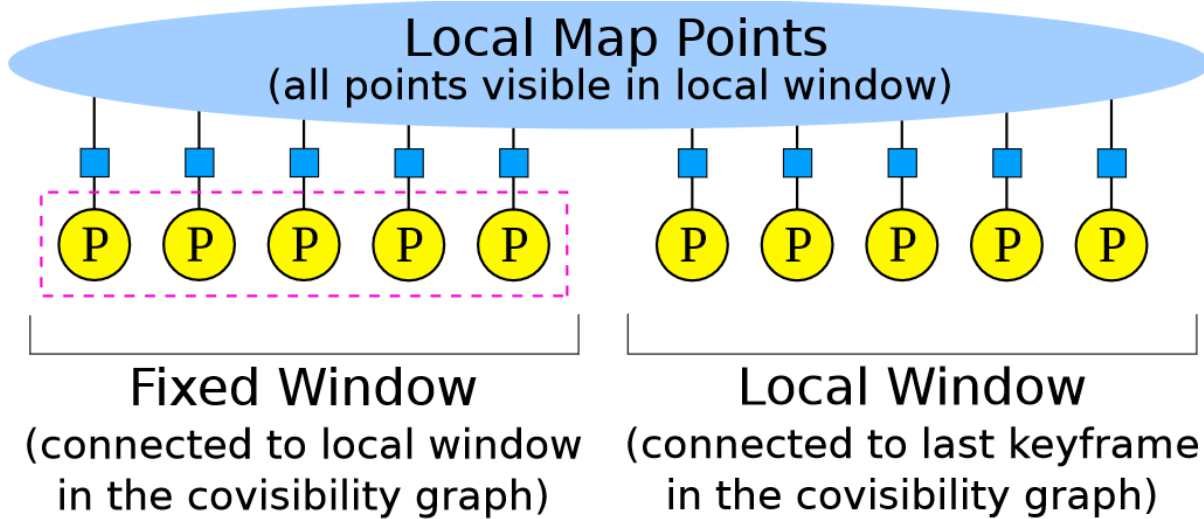
$$\theta^* = \underset{\theta}{\operatorname{argmin}} \left(\sum_k \mathbf{E}_{\text{proj}}(k, j+1) + \mathbf{E}_{\text{IMU}}(j, j+1) + \mathbf{E}_{\text{prior}}(j) \right)$$

$$\mathbf{E}_{\text{prior}}(j) = \rho \left(\begin{bmatrix} \mathbf{e}_R^T & \mathbf{e}_v^T & \mathbf{e}_p^T & \mathbf{e}_b^T \end{bmatrix} \Sigma_p \begin{bmatrix} \mathbf{e}_R^T & \mathbf{e}_v^T & \mathbf{e}_p^T & \mathbf{e}_b^T \end{bmatrix}^T \right)$$

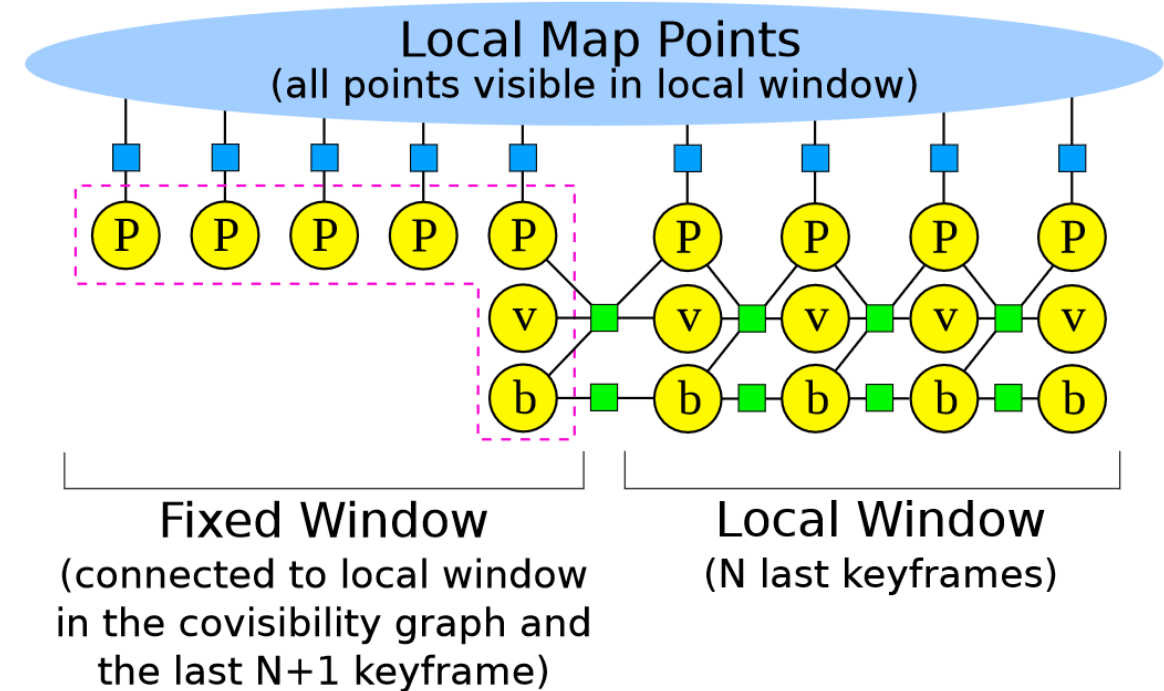
$$\mathbf{e}_R = \operatorname{Log} \left(\bar{\mathbf{R}}_{BW}^j \mathbf{R}_{WB}^j \right) \quad \mathbf{e}_v = {}_W \bar{\mathbf{v}}_B^j - {}_W \mathbf{v}_B^j$$

$$\mathbf{e}_p = {}_W \bar{\mathbf{p}}_B^j - {}_W \mathbf{p}_B^j \quad \mathbf{e}_b = \bar{\mathbf{b}}^j - \mathbf{b}^j$$

Visual-Inertial SLAM – Local BA



ORB-SLAM's Local BA



Visual-Inertial ORB-SLAM's Local BA

	SLAM or VO	Pixels used	Data association	Estimation	Relocali- zation	Loop closing	Multi Maps	Mono	Stereo	Mono IMU	Stereo IMU	Fisheye	Accuracy	Robustness	Open source
Mono-SLAM [13], [14]	SLAM	Shi Tomasi	Correlation	EKF	-	-	-	✓	-	-	-	-	Fair	Fair	[15] ¹
PTAM [16]–[18]	SLAM	FAST	Pyramid SSD	BA	Thumbnail	-	-	✓	-	-	-	-	Very Good	Fair	[19]
LSD-SLAM [20], [21]	SLAM	Edgelets	Direct	PG	-	FABMAP PG	-	✓	✓	-	-	-	Good	Fair	[22]
SVO [23], [24]	VO	FAST+ Hi.grad.	Direct	Local BA	-	-	-	✓	✓	-	-	✓	Very Good	Very Good	[25] ²
ORB-SLAM2 [2], [3]	SLAM	ORB	Descriptor	Local BA	DBoW2	DBoW2 PG+BA	-	✓	✓	-	-	-	Exc.	Very Good	[26]
DSO [27]–[29]	VO	High grad.	Direct	Local BA	-	-	-	✓	✓	-	-	✓	Fair	Very Good	[30]
DSM [31]	SLAM	High grad.	Direct	Local BA	-	-	-	✓	-	-	-	-	Very Good	Very Good	[32]
MSCKF [33]–[36]	VO	Shi Tomasi	Cross correlation	EKF	-	-	-	✓	-	✓	✓	-	Fair	Very Good	[37] ³
OKVIS [38], [39]	VO	BRISK	Descriptor	Local BA	-	-	-	-	-	✓	✓	✓	Good	Very Good	[40]
ROVIO [41], [42]	VO	Shi Tomasi	Direct	EKF	-	-	-	-	-	✓	✓	✓	Good	Very Good	[43]
ORB-SLAM-VI [4]	SLAM	ORB	Descriptor	Local BA	DBoW2	DBoW2 PG+BA	-	✓	-	✓	-	-	Very Good	Very Good	-
VINS-Fusion [7], [44]	VO	Shi Tomasi	KLT	Local BA	DBoW2	DBoW2 PG	✓	-	✓	✓	✓	✓	Good	Exc.	[45]
VI-DSO [46]	VO	High grad.	Direct	Local BA	-	-	-	-	-	✓	-	-	Very Good	Exc.	-
BASALT [47]	VO	FAST	KLT (LSSD)	Local BA	-	ORB BA	-	-	-	-	✓	✓	Very Good	Exc.	[48]
Kimera [8]	VO	Shi Tomasi	KLT	Local BA	-	DBoW2 PG	-	-	-	-	✓	-	Good	Exc.	[49]
ORB-SLAM3 (ours)	SLAM	ORB	Descriptor	Local BA	DBoW2	DBoW2 PG+BA	✓	✓	✓	✓	✓	✓	Exc.	Exc.	[5]

Out of scope

- Dense and direct approaches
- Deep learning-based solutions
- Multi map SLAM
- Image retrieval
- Map initialization

Sources

1. Engel, J., Koltun, V., & Cremers, D. (2016). Direct Sparse Odometry. *ArXiv*. /abs/1607.02565
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3. Tardos, J. D. (2016). Visual-Inertial Monocular SLAM with Map Reuse. *ArXiv*. <https://doi.org/10.1109/LRA.2017.2653359>
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