期望性质

$$\sum_x |x|p(x)\text{存在或}\int |x|p(x)\mathrm{d}x\text{存在} \to \mathrm{E}[X] \triangleq \sum_x xp(x)\text{或}\,\mathrm{E}[X] \triangleq \int xp(x) \tag{1}$$

$$\mathrm{E}[X+Y] = \mathrm{E}[X] + \mathrm{E}[Y] \tag{2}$$

$$\mathrm{E}[cX] = c\,\mathrm{E}[X] \tag{3}$$

$$\mathrm{E}[X-c] = \mathrm{E}[X] - c \tag{4}$$

$$X, Y\text{相互独立} \to \mathrm{E}[XY] = \mathrm{E}[X]\,\mathrm{E}[Y] \tag{5}$$

$$\begin{aligned}
\mathrm{E}[X+Y] &= \sum_{x=1}^{n}\sum_{j=1}^{m}(x_i+y_j)p_{ij} \\
&= \sum_{i=1}^{n}(x_i\sum_{j=1}^{m}p_{ij} + \sum_{j=1}^{m}y_jp_{ij}) \\
&= \sum_{i=1}^{n}(x_ip_i + \sum_{j=1}^{m}y_jp_{ij}) \\
&= \sum_{i=1}^{n}(x_ip_i) + (\sum_{i=1}^{n}\sum_{j=1}^{m}y_jp_{ij}) \\
&= \mathrm{E}[X] + (\sum_{j=1}^{m}y_jp_j) \\
&= \mathrm{E}[X] + \mathrm{E}[Y]
\end{aligned}$$

$$\begin{aligned}
\mathrm{E}[X+Y] &= \int\int(x+y)p(x,y)\mathrm{d}x\mathrm{d}y \\
&= \int([y\int p(x,y)\mathrm{d}x] + \int xp(x,y)\mathrm{d}x)\mathrm{d}y \\
&= \int([yp(y)] + \int xp(x,y)\mathrm{d}x)\mathrm{d}y \\
&= \mathrm{E}[Y] + \int\int xp(x,y)\mathrm{d}x\mathrm{d}y \\
&= \mathrm{E}[Y] + \int\int xp(x,y)\mathrm{d}y\mathrm{d}x \\
&= \mathrm{E}[Y] + \mathrm{E}[X]
\end{aligned}$$

方差性质

$$\mathrm{Var}[X] \triangleq \mathrm{E}[(X-\mathrm{E}[X])^2] = \mathrm{E}[X^2] - (\mathrm{E}[X])^2 \tag{6}$$

$$\mathrm{Var}[X+Y] = \mathrm{Var}[X] + \mathrm{Var}[Y] + 2\mathrm{Cov}(X,Y) \tag{7}$$

$$\mathrm{Var}[cX] = c^2\mathrm{Var}[X] \tag{8}$$

$$\mathrm{Var}[X-c] = \mathrm{Var}[X] \tag{9}$$

$$\begin{aligned}
&\mathrm{Var}[X+Y] = \mathrm{E}[(X+Y)^2] - (\mathrm{E}[X+Y])^2 \\
&= \mathrm{E}[X^2+2XY+Y^2] - (\mathrm{E}[X]+\mathrm{E}[Y])^2 \\
&= \mathrm{E}[X^2+2XY+Y^2] - \mathrm{E}[X]^2 - 2\,\mathrm{E}[X]\,\mathrm{E}[Y] - \mathrm{E}[Y]^2 \\
&= \mathrm{E}[X^2] - \mathrm{E}[X]^2 + \mathrm{E}[Y^2] - \mathrm{E}[Y]^2 + 2(\mathrm{E}[XY] - \mathrm{E}[X]\,\mathrm{E}[Y]) \\
&= \mathrm{Var}[X] + \mathrm{Var}[Y] + 2\mathrm{Cov}[X,Y]
\end{aligned}$$