C$61C Spring 2018 Discussion 0 – Number Representation

1 Unsigned Integers

If we have an n-digit unsigned numeral $d_{n-1}d_{n-2}\dots d_0$ in radix (or base) r, then the value of that numeral is $\sum_{i=0}^{n-1} r^i d_i$, which is just fancy notation to say that instead of a 10's or 100's place we have an r's or r^2's place. For the three radices binary, decimal, and hex, we just let r be 2, 10, and 16, respectively.

Recall also that we often have cause to write down unreasonably large numbers, and our preferred tool for doing that is the IEC prefixing system:

Ki (Kibi) = 2^{10}	Mi (Mebi) = 2^{20}	Gi (Gibi) = 2^{30}	Ti (Tebi) = 2^{40}
Pi (Pebi) = 2^{50}	Ei (Exbi) = 2^{60}	Zi (Zebi) = 2^{70}	Yi (Yobi) = 2^{80}

1.1 We don't have calculators during exams, so let's try this by hand

1. Convert the following numbers from their initial radix into the other two common radices:
   (a) 0b10010011
   (b) 0xD3AD
   (c) 63
   (d) 0b00100100
   (e) 0xB33F
   (f) 0
   (g) 39
   (h) 0x7EC4
   (i) 437
2. Write the following numbers using IEC prefixes:
   - 2^{16}
   - 2^{27}
   - 2^{43}
   - 2^{36}
   - 2^{34}
   - 2^{61}
   - 2^{47}
   - 2^{58}
3. Write the following numbers as powers of 2:
   - 2 Ki
   - 512 Ki
   - 16 Mi
   - 256 Pi
   - 64 Gi
   - 128 Ei

2 $igned Integers

Unsigned binary numbers work for natural numbers, but many calculations use negative numbers as well. To deal with this, a number of different schemes have been used to represent signed numbers, but we will focus on two's complement, as it is the standard solution for representing signed integers.

2.1 Two's complement

- Most significant bit has a negative value, all others are positive. $o the value of an n-digit two's complement number can be written as $\sum_{i=0}^{n-2} 2^i d_i - 2^{n-1} d_n$.
- Otherwise exactly the same as unsigned integers.
- A neat trick for flipping the sign of a two's complement number: flip all the bits and add 1.
- Addition is exactly the same as with an unsigned number.
- Only one 0, and it's located at 0b0.

2.2 Exercises

For questions 1 – 3, assume an 8 bit integer and answer each one for the case of an unsigned number, biased number with a bias of -127, and two's complement number, indicating if it cannot be answered with a specific representation.

1. What is the largest integer? The largest integer's representation + 1?
   (a) [Unsigned]
   (b) [Biased]
   (c) [Two's Complement]
2. How do you represent the numbers 0, 1, and -1?
   (a) [Unsigned]
   (b) [Biased]
   (c) [Two's Complement]
3. How do you represent 17, -17?
   (a) [Unsigned]
   (b) [Biased]
   (c) [Two's Complement]
4. What is the largest integer that can be represented by *any* encoding scheme that only uses 8 bits?

5. Prove that the two's complement inversion trick is valid (i.e. that x and $\overline{x} + 1$ sum to 0).

6. Explain where each of the three radices shines and why it is preferred over other bases in a given context.

3 Counting

Bitstrings can be used to represent more than just numbers. In fact, we use bitstrings to represent *everything* inside a computer. And, because we don't want to be wasteful with bits it is important that to remember that n bits can be used to represent 2^n distinct things. For each of the following questions, answer with the minimum number of bits possible.

3.1 Exercises

1. How many bits do we need to represent a variable that can only take on the values $0, \pi$ or e?

2. If we need to address 3 TiB of memory and we want to address every byte of memory, how long does an address need to be?

3. If the only value a variable can take on is e, how many bits are needed to represent it.