CS 61A Orders of Growth & Linked Lists
Spring 2018 Discussion 6: March 7, 2018

1 Warmup

What is the order of growth for the following functions? Answer in terms of Θ (for example, $\Theta(n)$).

1.1
```
def fib_iter(n):
    prev, curr, i = 0, 1, 0
    while i < n:
        prev, curr = curr, prev + curr
        i += 1
    return prev
```

1.2
```
def fib_recursive(n):
   if n == 0 or n == 1:
       return n
   else:
       return fib_recursive(n - 1) + fib_recursive(n - 2)
```

1.3 Write a function that takes in a a linked list and returns the sum of all its elements. You may assume all elements in `lnk` are integers.

```
def sum_nums(lnk):
    """
    >>> a = Link(1, Link(6, Link(7)))
    >>> sum_nums(a)
    14
    """
```

2 Orders of Growth

When we talk about the efficiency of a function, we are often interested in the following: as the size of the input grows, how does the runtime of the function change? And what do we mean by "runtime"?

- `square(1)` requires one primitive operation: `*` (multiplication). `square(100)` also requires one. No matter what input `n` we pass into `square`, it always takes one operation.

input	function call	return value	number of operations
1	square(1)	$1 \cdot 1$	1
2	square(2)	$2 \cdot 2$	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$
100	square(100)	$100 \cdot 100$	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	square(n)	$n \cdot n$	1

- `factorial(1)` requires one multiplication, but `factorial(100)` requires 100 multiplications. As we increase the input size of n, the runtime (number of operations) increases linearly proportional to the input.

input	function call	return value	number of operations
1	factorial(1)	$1 \cdot 1$	1
2	factorial(2)	$2 \cdot 1 \cdot 1$	2
$\vdots$	$\vdots$	$\vdots$	$\vdots$
100	factorial(100)	$100 \cdot 99 \cdots 1 \cdot 1$	100
$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	factorial(n)	$n \cdot (n-1) \cdots 1 \cdot 1$	n

For expressing complexity, we use what is called big Θ (Theta) notation. For example, if we say the running time of a function `foo` is in $\Theta(n^2)$, we mean that the running time of the process will grow proportionally with the square of the size of the input as it increases to infinity.

- **Ignore lower order terms:** If a function requires $n^3 + 3n^2 + 5n + 10$ operations with a given input n, then the runtime of this function is $\Theta(n^3)$. As n gets larger, the lower order terms (10, $5n$, and $3n^2$) all become insignificant compared to n^3.

- **Ignore constants:** If a function requires $5n$ operations with a given input n, then the runtime of this function is $\Theta(n)$. We are only concerned with how the runtime grows asymptotically with the input, and since 5n is still asymptotically linear; the constant factor does not make a difference in runtime analysis.

Kinds of Growth

Here are some common orders of growth, ranked from no growth to fastest growth:

- $\Theta(1)$ — constant time takes the same amount of time regardless of input size

- $\Theta(\log n)$ — logarithmic time
- $\Theta(n)$ — linear time
- $\Theta(n \log n)$ — linearithmic time
- $\Theta(n^2)$, $\Theta(n^3)$, etc. — polynomial time
- $\Theta(2^n)$, $\Theta(3^n)$, etc. — exponential time (considered "intractable"; these are really, really horrible)

In addition, some programs will never terminate if they get stuck in an infinite loop.

Questions

What is the order of growth for the following functions?

2.1
```
def sum_of_factorial(n):
    if n == 0:
        return 1
    else:
        return factorial(n) + sum_of_factorial(n - 1)
```

2.2
```
def bonk(n):
    total = 0
    while n >= 2:
        total += n
        n = n / 2
    return total
```

2.3
```
def mod_7(n):
    if n % 7 == 0:
        return 0
    else:
        return 1 + mod_7(n - 1)
```

2.4
```
def bar(n):
    if n % 2 == 1:
        return n + 1
    return n

def foo(n):
    if n < 1:
        return 2
    if n % 2 == 0:
        return foo(n - 1) + foo(n - 2)
    else:
        return 1 + foo(n - 2)
```

What is the order of growth of `foo(bar(n))`?

3 Linked Lists

There are many different implementations of sequences in Python. Today, we'll explore the linked list implementation.

A linked list is either an empty linked list, or a Link object containing a `first` value and the `rest` of the linked list.

To check if a linked list is an empty linked list, compare it agains the class attribute `Link.empty`:

```
if link is Link.empty:
    print('This linked list is empty!')
else:
    print('This linked list is not empty!')
```

Implementation

```
class Link:
    empty = ()
    def __init__(self, first, rest=empty):
        assert rest is Link.empty or isinstance(rest, Link)
        self.first = first
        self.rest = rest

    def __repr__(self):
        if self.rest:
            rest_str = ', ' + repr(self.rest)
        else:
            rest_str = ''
        return 'Link({0}{1})'.format(repr(self.first), rest_str)

    @property
    def second(self):
        return self.rest.first

    @second.setter
    def second(self, value):
        self.rest.first = value

    def __str__(self):
        string = '<'
        while self.rest is not Link.empty:
            string += str(self.first) + ' '
            self = self.rest
        return string + str(self.first) + '>'
```

Questions

3.1 Write a function that takes in a Python list of linked lists and multiplies them element-wise. It should return a new linked list.

If not all of the `Link` objects are of equal length, return a linked list whose length is that of the shortest linked list given. You may assume the `Link` objects are shallow linked lists, and that `lst_of_lnks` contains at least one linked list.

```
def multiply_lnks(lst_of_lnks):
    """
    >>> a = Link(2, Link(3, Link(5)))
    >>> b = Link(6, Link(4, Link(2)))
    >>> c = Link(4, Link(1, Link(0, Link(2))))
    >>> p = multiply_lnks([a, b, c])
    >>> p.first
    48
    >>> p.rest.first
    12
    >>> p.rest.rest.rest
    ()
    """
```

3.2 Write a function that takes a sorted linked list of integers and mutates it so that all duplicates are removed.

```
def remove_duplicates(lnk):
    """
    >>> lnk = Link(1, Link(1, Link(1, Link(1, Link(5)))))
    >>> unique = remove_duplicates(lnk)
    >>> unique
    Link(1, Link(5))
    >> lnk
    Link(1, Link(5))
    """
```

4 Midterm Review

4.1 Write a function that takes a list and returns a new list that keeps only the even-indexed elements of lst and multiplies them by their corresponding index.

```
def even_weighted(lst):
    """
    >>> x = [1, 2, 3, 4, 5, 6]
    >>> even_weighted(x)
    [0, 6, 20]
    """

    return [_________________________________________________]
```

4.2 The **quicksort** sorting algorithm is an efficient and commonly used algorithm to order the elements of a list. We choose one element of the list to be the **pivot** element and partition the remaining elements into two lists: one of elements less than the pivot and one of elements greater than the pivot. We recursively sort the two lists, which gives us a sorted list of all the elements less than the pivot and all the elements greater than the pivot, which we can then combine with the pivot for a completely sorted list.

First, implement the `quicksort_list` function. Choose the first element of the list as the pivot. You may assume that all elements are distinct.

```
def quicksort_list(lst):
    """
    >>> quicksort_list([3, 1, 4])
    [1, 3, 4]
    """

    if _____________________________________________________:

        _____________________________________________________

    pivot = lst[0]

    less = __________________________________________________

    greater = _______________________________________________

    return __________________________________________________
```

4.3 Write a function that takes in a list and returns the maximum product that can be formed using nonconsecutive elements of the list. The input list will contain only numbers greater than or equal to 1.

```
def max_product(lst):
    """Return the maximum product that can be formed using lst
    without using any consecutive numbers
    >>> max_product([10,3,1,9,2]) # 10 * 9
    90
    >>> max_product([5,10,5,10,5]) # 5 * 5 * 5
    125
    >>> max_product([])
    1
    """
```

4.4 An **expression tree** is a tree that contains a function for each non-leaf node, which can be either '+' or '*'. All leaves are numbers. Implement `eval_tree`, which evaluates an expression tree to its value. You may want to use the functions `sum` and `prod`, which take a list of numbers and compute the sum and product respectively.

```
def eval_tree(tree):
    """Evaluates an expression tree with functions the root.
    >>> eval_tree(tree(1))
    1
    >>> expr = tree('*', [tree(2), tree(3)])
    >>> eval_tree(expr)
    6
    >>> eval_tree(tree('+', [expr, tree(4), tree(5)]))
    15
    """
```

4.5 Complete `redundant_map`, which takes a tree `t` and a function `f`, and applies `f` to the node (2^d) times, where d is the depth of the node. The root has a depth of 0.

```
def redundant_map(t, f):
    """
    >>> double = lambda x: x*2
    >>> tree = Tree(1, [Tree(1), Tree(2, [Tree(1, [Tree(1)])])])
    >>> print_levels(redundant_map(tree, double))
    [2] # 1 * 2 ˆ (1) ; Apply double one time
    [4, 8] # 1 * 2 ˆ (2), 2 * 2 ˆ (2) ; Apply double two times
    [16] # 1 * 2 ˆ (2 ˆ 2) ; Apply double four times
    [256] # 1 * 2 ˆ (2 ˆ 3) ; Apply double eight times
    """
    t.label = ________________________________________________

    new_f = __________________________________________________

    t.branches = _____________________________________________

    return t
```