Delft University of Technology

MDO for aerospace Applications
AE4205

Assignment Part 1: Wing parametrization and XDSM

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Group Number: 42

Reference Aircraft: B737-800

November 27, 2022

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Nomenclature

Λ_1 Leading edge sweep angle for the first trapezoid [°]

λ_1 Taper ratio for the first trapezoid

Λ_2 Leading edge sweep angle for the second trapezoid [°]

λ_2 Taper ratio for the second trapezoid

ρ_{fuel} Fuel density [$\frac{kg}{m^3}$]

θ_1 Twist angle in the kink [°]

θ_2 Twist angle in the tip [°]

b_1 Semi-span of the inboard trapezoid [m]

b_2 Semi-span of the outboard trapezoid [m]

c_0 Chord at the fuselage [m]

c_1 Chord at the kink [m]

c_2 Chord at the tip [m]

CST_1 Class Shape function Transformation of the root airfoil

CST_2 Class Shape function Transformation of the tip airfoil

E Aerodynamic efficiency of the entire aircraft

f_{tank} factor accounting for the volume of the tank available for the fuel

g Gravity constant equal to 9.81 [$\frac{m}{s^2}$]

S Planform area of the wing [m^2]

V_{tank} Volume of the fuel tank [m^3]

W_{A-W} Weight of the entire aircraft excluding the contribution of the fuel and the wing [N]

W_{fuel} Fuel weight [N]

$W_{str,wing}$ Wing structural weight [N]

W_{TOmax} Maximum take-off weight [N]

1 Introduction

The aim of this assignment is to optimize the wing of the reference aircraft *B737-800*, in order to minimize the aircraft maximum takeoff weight. For doing so a optimization problem is set up, where an hybrid version of the IDF architecture is defined. A disciplinary decomposition of the system, that is to say, the aircraft, is made in order to obtain a manageable structure. 4 disciplines are defined, *LOADS, STRUCTURES, AERODYNAMICS and PERFORMANCE*, with a coupling between the disciplines *LOADS and STRUCTURE*.

For the disciplines blocks, solvers are provided in Brightspace, *Q3D* for *AERODYNAMIC and LOADS*, setting a viscous analysis for the former and an inviscid for the latter. In addition, a structural sizing tool called *EMWET* is used for the *STRUCTURE* discipline. According to the *PERFORMANCE* discipline, the fuel fraction method based on Breguet equation is defined.

In this first deliverable, the parametrization and Optimization problem specification is carried out. In section 2 the wing parametrization is done, defining the parameters that will define the planform of the wing. In section 3 the optimization problem specification is set up. For doing so, the design vector, bounds of the design variables, objective function, inequality constraint and consistency constraints are defined. In section 4 an *Extended Design Structure Matrix* is implemented to provide a detailed formalization of our MDO architecture. A nomenclature table is provided to specify the description and units of all the symbols.

2 Wing parametrization

The planform shape is defined by two trapezoids sharing one side (c_1). A direct way of representing the planform parameters can be seen in Figure 1. The parameters shown there are:

- c_0: Chord at the fuselage
- c_1: Chord at the kink
- c_2: Chord at the wing tip
- b_1: Span for the first trapezoid
- b_2: Span for the second trapezoid
- Λ_1: Leading edge sweep angle for the first trapezoid
- Λ_2: Leading edge sweep angle for the second trapezoid

These completely determines the geometry, since two trapezoids sharing a side have seven *DOF* $(2 \times 4 - 1)$, and there are 7 parameters.

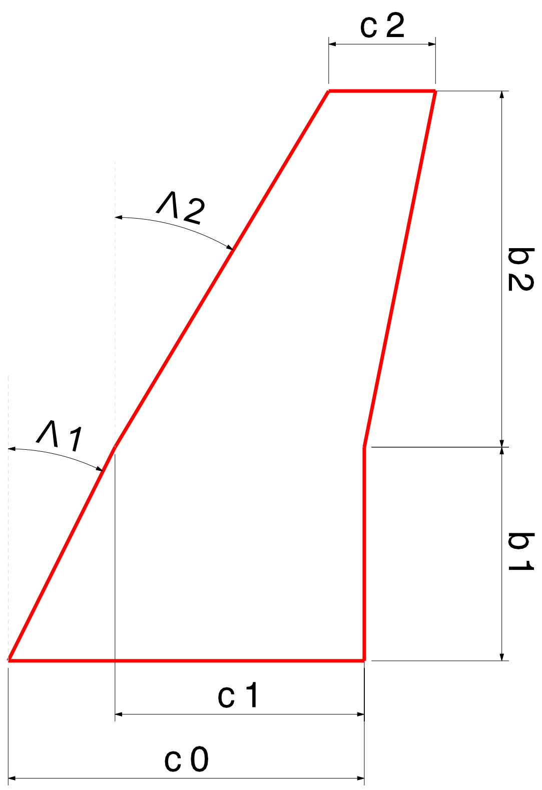


Figure 1: *Planform diagram.*

As to what parameters will be used for the optimization, a few ideas must be noted:

1. The span for the first trapezoid must remain constant.
2. The trailing edge for the first trapezoid must remain constant.
3. It is more desirable to have non-dimensional parameters with clear bounds.

Thus, the following list of parameters was developed:

- c_0: Chord at the fuselage (non-dimensionalized using the reference value)
- $\lambda_1 = \frac{c_1}{c_0}$: Taper ratio for the first trapezoid
- $\lambda_2 = \frac{c_2}{c_1}$: Taper ratio for the second trapezoid
- b_2: Span for the second trapezoid (non-dimensionalized using the reference value)
- Λ_2: Leading edge sweep angle for the second trapezoid (divided by a reference value)

As expected, the first restriction simply removed b_1. The second one imposes a relation between λ_1 and Λ_1, so one of them can be chosen over the other.

The taper ratios are chosen over other non-dimensional parameters as setting boundaries for them is usually easier.

The following reference values where obtained from the B737-800 blueprints:

- $b_1 = 3.162m$
- $c_{0,ref} = 6.4823m$
- $b_{2,ref} = 11.5252m$
- $\Lambda_{2,ref} = 27.8572°$

To this, additional parameters regarding the airfoil and wing's twist angle must be added. Three control points are again used (fuselage, kink and wing tip) where the key airfoils and twist angles are defined.

As instructed by the problem description, the twist angle at the fuselage is assumed to be 0, and the airfoil at the kink is obtained by interpolating the fuselage and wing tip ones.

This means there are 4 additional parameters to consider:

- θ_1: Twist angle at the kink
- θ_2: Twist angle at the wing tip
- AF_1: Airfoil at the fuselage
- AF_2: Airfoil at the wing tip

10° will be used as a reference value for the twist angle ($\theta_{1,ref} = \theta_{2,ref} = 10°$).

3 Problem statement

3.1 Design vector

Following the conclusions from section 2, the design vector is derived.

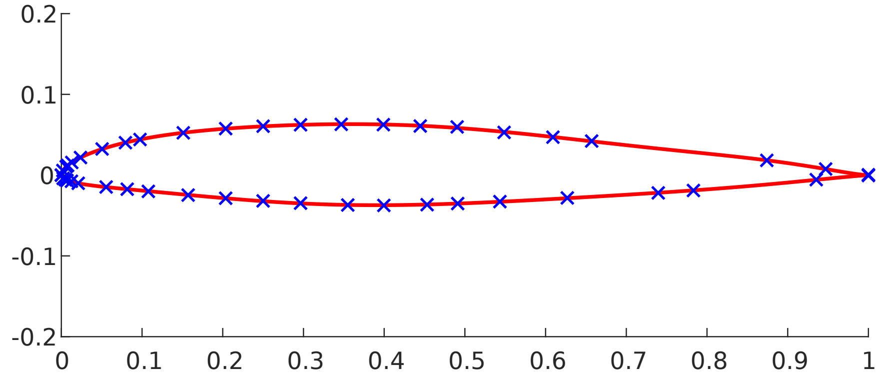
The objective of the optimization is to minimize the maximum take-off weight for a given aircraft. For this, different wing geometries are tested via the optimizer, and each of the geometries is represented using the design vector, which contains all the chosen parameters, in addition to the extra design variables.

Regarding the airfoil design, it is implicitly defined by the CST coefficients. In addition, the position of the spars is going to be assumed constant, so they are not going to be defined as design variables. The front spar will be positioned at the 17.5% of the local chord and the back spar at the 57.5%.

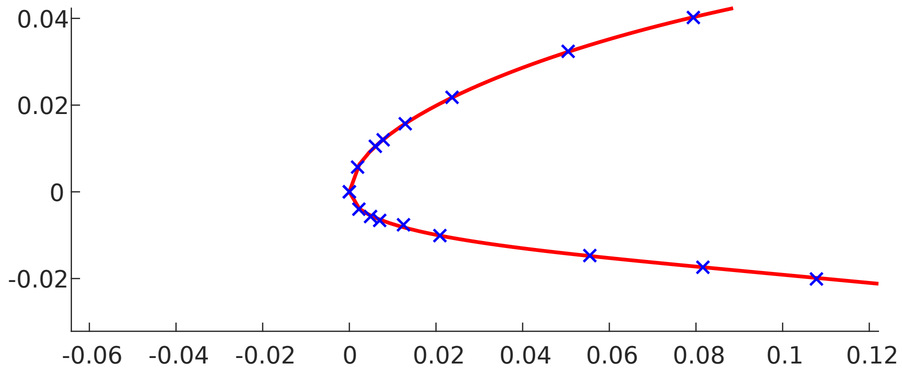
$$x = \left\{ \begin{array}{c} {}^{c_0}/_{c_{0,ref}} \\ \lambda_1 \\ \lambda_2 \\ {}^{b_2}/_{b_{2,ref}} \\ {}^{\Lambda_2}/_{\Lambda_{2,ref}} \\ CST_1 \\ CST_2 \\ {}^{\theta_1}/_{\theta 1,ref} \\ {}^{\theta_2}/_{\theta 2,ref} \\ {}^{\hat{W}_{fuel}}/_{\hat{W}_{fuel,ref}} \\ {}^{\hat{W}_{str,wing}}/_{\hat{W}_{str,wing,ref}} \\ {}^{\hat{E}}/_{\hat{E}_{ref}} \end{array} \right\} \tag{1}$$

The airfoils are parametrized using CST curves. CST_1 and CST_2 contain all the coefficients for the CST curves of both the intrados and extrados.

For each CST curve, a total of 8 coefficients are used, two for the class function and 6 for the shape function. This structure was tested using one of the aircraft original airfoils[1] (the least smooth one), and the results were satisfactory, as shown in Figure 2.



(a) *General view*



(b) *Leading edge detail*

Figure 2: *Original Point Cloud vs. CST Parametrization.*

3.2 Bounds of the design variables

In order to define and restrict the design space of the variables lower and upper boundaries are set. Most of these boundaries are defined allowing $\pm 20\%$ variation around the reference values. Exceptions include the twist angle, which are going to be defined from $-5°$ to $10°$, as well as the taper ratios, which all vary from 0 to 1, and CST coefficients, which vary from -1 to 2. Taper ratios obviously can not be lower than 0, and could be larger than 1, but in commercial aerodynamics this is extremely unusual.

A default variation of $\pm 20\%$ was chosen as a compromise between giving the optimizer enough freedom to do its job and trying to keep a final design close enough to the original one.

	Lower Boundary	**Upper Boundary**
$c_0/c_{0,ref}$	0.8	1.2
λ_1	0	1
λ_2	0	1
$b_2/b_{2,ref}$	0.8	1.2
$\Lambda_2/\Lambda_{2,ref}$	0.8	1.2
CST_1[1]	-1	2
CST_2[2]	-1	2
$\theta_1/\theta 1, ref$	-0.5	1
$\theta_2/\theta 2, ref$	-0.5	1
$\hat{W}_{fuel}/\hat{W}_{fuel,ref}$	0.8	1.2
$\hat{W}_{str,wing}/\hat{W}_{str,wing,ref}$	0.8	1.2
$\hat{E}/\hat{E}_{ref}$	0.8	1.2

Table 1: *Lower and upper boundary values for the design variables.*

3.3 Objective function

The figure of merit to minimize is defined by the objective function. The maximum take-off weight is composed by the sum of three different weights, which are the required fuel weight to fulfill the mission W_{fuel}, the weight of the entire aircraft excluding the contribution of the fuel and the wing, W_{A-W}, constant along the different iterations, and the wing structural weight, $W_{str,wing}$.

$$W_{TOmax}(x) = W_{A-W} + W_{fuel}(x) + W_{str,wing}(x) \tag{2}$$

3.4 Inequality constraints

The first constraint stems from the physical space available in the wing integral fuel tanks. The volume available for the fuel is considered as a fraction of the tank ($f_{tank} = 0.93$), which accounts for structural limitations. The chordwise area of the tank is defined by the spar position mentioned above, the spanwise position will vary from the winf-fuselage intersection to the 85% of the semispan.

$$\frac{W_{fuel}(x)}{\rho_{fuel}\ g} \leq V_{tank}(x) f_{tank} \tag{3}$$

[1] These limits apply to all the vector components. The upper bound is 2 to allow for clearance for the second coefficient of the class function (the reference airfoil has a value of $\tilde{1}.5$ for this coefficient). The lower bound is negative to allow for clearance for the shape coefficients, it would be extremely unusual for the shape coefficients to have a norm larger than 1 given the natural (thin) shape of airfoils.

[2] See previous footnote

In order to fulfill with take-off and landing requirements without changing the high lift devices a maximum wing-loading is defined.

$$\frac{W_{TOmax}}{S} \leq \left.\frac{W_{TOmax}}{S}\right|_{ref} \tag{4}$$

Inequality constraint equations are going to be modified when using the function *fmincon*, as they need to be expressed as a statement lower than zero.

3.5 Equality and Consistency constraints

No equality constraint are defined for the optimization problem. According to the consistency constraints, they are defined in the IDF architecture in order to keep the discipline consistency. They are modelled as extra equality constraints for the extra design variables defined when decoupling the problem. Consistency is achieved when the initial guessed value and the actual value difference is lower than a predefined residual.

$$\hat{W}_{fuel} - W_{fuel} = 0 \tag{5}$$

$$\hat{E} - E = 0 \tag{6}$$

$$\hat{W}_{str,wing} - W_{str,wing} = 0 \tag{7}$$

4 XDSM

The *Extended Design Structure Matrix* is defined to describe the data exchanged among the disciplinary analyses of the given IDF architecture, as it could be seen in Figure 3. The grey connectors indicate the data exchange channels, whereas the black lines indicate the process sequence. As for the equations that govern the blocks that correspond to the objective function and inequality and consistency constraints they are given in section 3. The objective function corresponds to Equation 2, inequality constraint of the fuel is given by Equation 3, and the one for the wing loading in Equation 4. The consistency constraint for the extra design variables, fuel weight, aerodynamic efficiency and wing weight are given in Equation 5, Equation 6 and Equation 7, respectively.

The vector x_{wing} that can be see in Figure 3 refers to the parameters that define the wing.

To avoid possible convergence problems derived from difference in scales of the parameters, dimensionless parameters will be used. For doing so, the actual value of the design variables are going to be divided by the reference values of the corresponding parameter, obtained from the reference aircraft. Although dimensional parameters are represented in the *XDSM* diagram, the data used in *MATLAB* will be dimensionless.

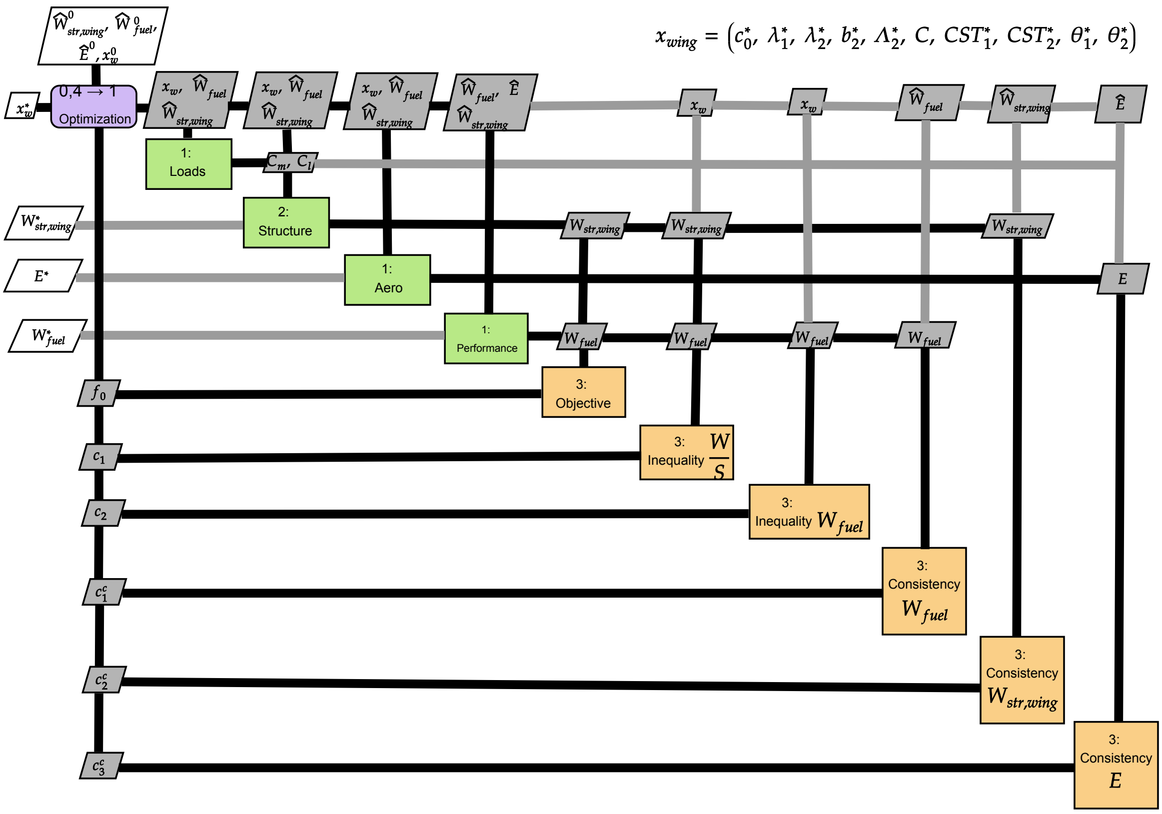


Figure 3: *XDSM diagram for the IDF architecture.*

References

[1] *BOEING 737 OUTBOARD AIRFOIL*. URL: http://airfoiltools.com/airfoil/details?airfoil=b737d-il.