GAUSS ELIMINATION (Forward Elimination) Step-by-Step

$$\begin{bmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \\ a_{30} & a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

* primes (" ' ") represent the matrix elements replaced value as you go through process.

[index = 0 for $\emptyset$th row ≡ pivot row]

① Eliminate below a_{00} ("the pivot")...

$i = 0$

Ⓐ Normalize row $\emptyset$: i

$$\left[\frac{a_{00}}{a_{00}} \quad \frac{a_{01}}{a_{00}} \quad \frac{a_{02}}{a_{00}} \quad \frac{a_{03}}{a_{00}} \quad \frac{b_0}{a_{00}} \right]$$

$$\left[1 \quad \frac{a_{01}}{a_{00}} \quad \frac{a_{02}}{a_{00}} \quad \frac{a_{03}}{a_{00}} \quad \frac{b_0}{a_{00}} \right]$$

$$\left[1 \quad a_{01}' \quad a_{02}' \quad a_{03}' \quad b_0' \right]$$

Ⓑ Eliminate a_{10}: $j=1$ current row where we are eliminating

(i) multiply row $\emptyset$ by $-a_{10}$

$$-a_{10} \left[1 \quad a_{01}' \quad a_{02}' \quad a_{03}' \quad b_0' \right]$$

$$-a_{10} \quad -a_{10}a_{01}' \quad -a_{02}'a_{00} \quad -a_{10}a_{03}' \quad -a_{10}b_0'$$

(ii) add result of ①-Ⓑ(i) to row 1:

$$-a_{10}x_0 - a_{10}a_{01}'x_1 - a_{10}a_{02}'x_2 - a_{10}a_{03}'x_3 = -a_{10}b_0'$$

$$+ \quad a_{10}x_0 + a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$

$$0 \quad (a_{11} - a_{10}a_{01}')x_1 + (a_{12} - a_{10}a_{02}')x_2 + (a_{13} - a_{10}a_{03}')x_3 = b_1 - a_{10}b_0'$$

"j" "i" "k"   "j" "i" "k"   "j" "i" "k"   "j" "i"

"To calculate or not, that is the question"

(iii) Final form of row 1 (j):

$$0 \quad a_{11} - a_{10}a_{01}' \quad a_{12} - a_{10}a_{02}' \quad a_{13} - a_{10}a_{03}' \quad b_1 - a_{10}b_0'$$

(c) Eliminate a_{20}: $j = 2$ [Recall $i = 0$ right now]

(i) Multiply row 0 [$= i$] by $-a_{20}$

$$-a_{20}\left[\,1 \quad a'_{01} \quad a'_{02} \quad a'_{03} \quad b'_0\,\right]$$

$$\left[\,-a_{20} \quad -a_{20}a'_{01} \quad -a_{20}a'_{02} \quad -a_{20}a'_{03} \quad -a_{20}b'_0\,\right]$$

(ii) add result to row 2 [$= j$]

$$
\begin{array}{rccccc}
 & [\,-a_{20} & -a_{20}a'_{01} & -a_{20}a'_{02} & -a_{20}a'_{03} & -a_{20}b'_0\,] \\
+ & [\,a_{20} & a_{21} & a_{22} & a_{23} & b_2 \\
\hline
 & [\,0 & a_{21}-a_{20}a'_{01} & a_{22}-a_{20}a'_{02} & a_{23}-a_{20}a'_{03} & b_2-a_{20}b'_0\,]
\end{array}
$$

(iii) Restate result showing indices

$$\left[\,0 \quad a_{\underset{j}{2}\overset{k}{1}} - a_{\underset{j}{2}\underset{i}{0}}a'_{\underset{i}{0}\overset{k}{1}} \quad a_{\underset{j}{2}\overset{k}{2}} - a_{\underset{j}{2}\underset{i}{0}}a'_{\underset{i}{0}\overset{k}{2}} \quad a_{\underset{j}{2}\overset{k}{3}} - a_{\underset{j}{2}\underset{i}{0}}a'_{\underset{i}{0}\overset{k}{3}} \quad b_{\underset{j}{2}} - a_{\underset{j}{2}\underset{i}{0}}b'_{\underset{i}{0}}\,\right]$$

D) Eliminate a_{30}: $j=3$ [$i=0$ right now]

(i) Multiply row ~~0~~ (i) by $-a_{30}$

$$-a_{30}\left[\,1 \quad a_{01}' \quad a_{02}' \quad a_{03}' \quad b_0'\,\right]$$

$$\left(\,-a_{30} \quad -a_{30}a_{01}' \quad -a_{30}a_{02}' \quad -a_{30}a_{03}' \quad -a_{30}b_0'\,\right)$$

(ii) add result to row 3 [$j=3$]

$$\begin{array}{rl} & [\,-a_{30} \quad -a_{30}a_{01}' \quad -a_{30}a_{02}' \quad -a_{30}a_{03}' \quad -a_{30}b_0'\,] \\ + & [\,a_{30} \quad a_{31} \quad a_{32} \quad a_{33} \quad b_3\,] \\ \hline & [\,0 \quad a_{31}-a_{30}a_{01}' \quad +a_{32}-a_{30}a_{02}' \quad a_{33}-a_{30}a_{03}' \quad b_3-a_{30}b_0'\,] \end{array}$$

(iii) Restate and show indices:

$$[\,0 \quad a_{31}-a_{30}a_{01}' \quad a_{32}-a_{30}a_{02}' \quad a_{33}-a_{30}a_{03}' \quad b_3-a_{30}b_0'\,]$$

Now repeat w/ $\begin{bmatrix} i=1 \\ j=2,3 \\ k=1,2,3 \end{bmatrix}$ & $\begin{bmatrix} i=2 \\ j=3 \\ k=2,3 \end{bmatrix}$

First elimination w/ $i=0$, $j=1,2,3$, for each j, $k=0,1,2,3$