Taylor Series

Taylor Series Expansions (assume equally spaced points)

$$f(x)=f(x_0)+f'(x_0)h+\frac{f''(x_0)}{2!}h^2+\frac{f'''(x_0)}{3!}h^3+\frac{f^{(4)}(x_0)}{4!}h^4+\dots$$

Taylor series can be used to estimate the value of f(x) when you only actually know f(x_0). The value x_0 is like a “base” point and you are going to project from x_0 to x to estimate the value of f(x).

Here is a video that might help visualize Taylor Series. Note they use a different notation. In the video c = x_0. https://youtu.be/AFMXixBVP-0

It convenient to use the difference between x and x_0.

$$h=x-x_0$$

The value of h is just the distance you are projecting from x_0. The smaller h is, in general, the better your estimate is.

There is a value between x_i and x_{i+1}, ξ such that (this is from mean value theorem in calculus):

$$f(x)=f(x_0)+f'(x_0)h+\frac{f''(\xi)}{2!}h^2$$

Sometimes that last part is called the remainder, R_n

$$f(x)=f(x_0)+f'(x_0)h+R_n$$

Note that *n* is the number of terms. In the above example n = 2.

Example: Find $f(x)=\cos(x)$ at x = π/12 using a base point of x_0 = 0 use n = 2 and n = 4

$$f'(x)=-\sin(x)=-\sin(0)=0$$
$$f''(x)=-\cos(x)=-\cos(0)=-1$$
$$f'''(x)=\sin(x)=\sin(0)=0$$

So for n = 2:

$$f(\pi/12)\approx f(0)+f'(0)h$$
$$f(\pi/12)\approx 1+0\,x(\pi/12-0)=1$$

Real answer is 0.965925826289068

For n = 4:

$$f(\pi/12) \approx f(0) + f'(0)h + \frac{f''(0)h^2}{2} + \frac{f'''(0)h^3}{6}$$

$$f(\pi/12) \approx 1 + 0\,x(\pi/12 - 0) + \frac{(-1)\,x(\pi/12)^2}{2} + \frac{(0)\,x(\pi/12)^3}{6} = 0.966$$