In Memory of MARIA KORKINA

EXACT ANALYTICAL SOLUTIONS FOR MODELING COSMOLOGICAL BLACK HOLES

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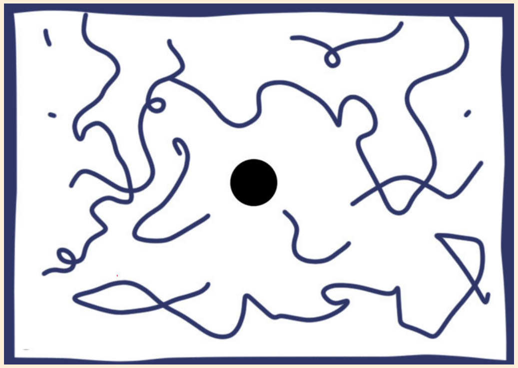
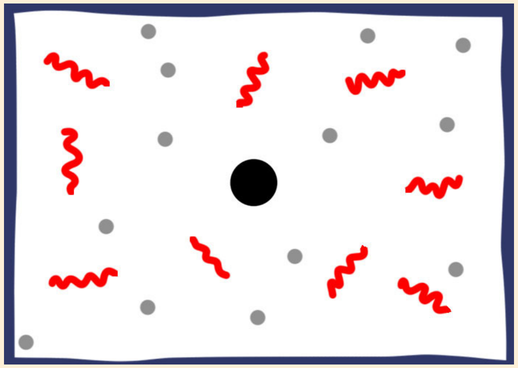
Motivation



Cosmological Evolution

Primordial Black Holes



Black Holes in Early Universe

Gravitational Wave Observations

Misner-Sharp mass and Einstein equations

$$\mathcal{R}_\mu^\nu - \frac{1}{2}\delta_\mu^\nu \mathcal{R} = \mathrm{T}_\mu^\nu$$

$$ds^2 = e^{\nu(R,t)}dt^2 - e^{\lambda(R,t)}dR^2 - r^2(R,t)d\sigma^2$$

$$m(R,t) = r(R,t)\left(1 + e^{-\nu(R,t)}\dot{r}^2 - e^{-\lambda(R,t)}r'^2\right)$$

Additive for noninteracting sources.
Invariant with respect to the transformation $\{R,t\} \rightarrow \{\tilde{R},\tilde{t}\}$

$$\mathrm{T}_0^0: \quad m' = \rho r^2 r'$$

$$\mathrm{T}_1^1: \quad \dot{m} = -p_r r^2 \dot{r}$$

$$\mathrm{T}_0^1: \quad 2\dot{r}' = \nu'\dot{r} + \dot{\lambda}r'$$

$$\mathrm{T}_2^2: \quad 2\dot{m}' = m'\frac{\dot{r}}{r'}\nu' + \dot{m}\frac{r'}{\dot{r}}\dot{\lambda} - 4r\dot{r}r'p_r$$

ρ energy density
$p_\perp$ tangential pressure
p_r radial pressure

Cahill, M. E. & McVittie, G. C., 1970, J. Math. Phys., 11, 1382–1401.

Einstein equations and matter content

$$\mathcal{R}_{\mu}^{\nu} - \frac{1}{2}\delta_{\mu}^{\nu}\mathcal{R} = \mathrm{T}_{\mu}^{\nu}$$

Anisotropic perfect fluid

$$T_{\mu}^{\nu} = (\rho + p_{\perp})\, u_{\mu}u^{\nu} - p_{\perp}\delta_{\mu}^{\nu} + (p_r - p_{\perp})X_{\mu}X^{\nu}$$

ρ Energy density

$p_{\perp}$ Tangential pressure

p_r Radial pressure

u^{ν} 4-velocity

X^{ν} Unit vector along radial direction

Effective equation of state

$$\frac{p_r + 2p_{\perp}}{3} = w\rho$$

w	Source
0	Dust
1/3	Radiation
-1	Vacuum
-1/3	Strings
-2/3	Domain Walls

Charged black hole in the dust + radiation mixture

Isotropic perfect fluids, non-static spacetime (LTB class)

$$ds^2 = dt^2 - r'^2(R,t)dR^2 - r^2(R,t)d\sigma^2$$

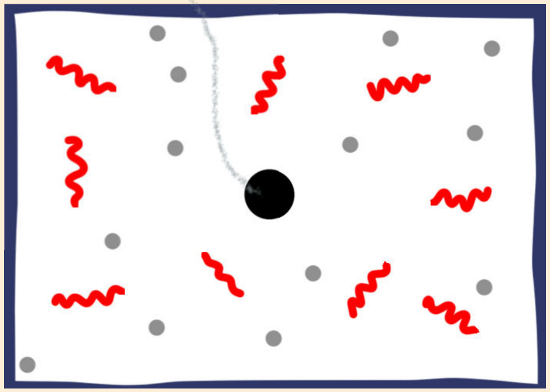
$$m(R,t) = r(R,t)\dot{r}^2(R,t)$$

$$\rho = \rho_{dust} + \rho_{rad} + \rho_q$$

$$m = r_g - \frac{Q^2}{r} + a_{dust}R^3 + \frac{a_{rad}^2 R^4}{r}$$

$$t - t(R) = \frac{2}{3}\frac{r^{3/2}\sqrt{m}}{\left(r_g + a_{dust}R^3\right)^2}\left(m + \frac{3}{r}\left(Q^2 - a_{rad}^2 R^4\right)\right)$$

M, Q, a_{dust}, a_{rad}



Kopteva, E. et al ApJ 866, 98 (2018) - particular case of dust only

Charged black hole in the mixture of anisotropic fluids

Anisotropic perfect fluids, static spacetime (Kiselev class)

$$ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2 d\sigma^2$$

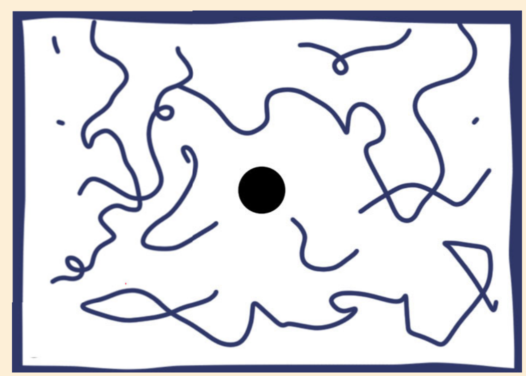
$$f(r) = 1 - \frac{r_g}{r} - \sum_i \frac{K_i}{r^{3w_i+1}}$$

w	Source
1/3	Radiation
-1	Vacuum
-1/3	Strings
-2/3	Domain Walls

Particular cases:
Schwarzschild solution $K = 0$
Reissner-Nordstrom solution $K=-Q^2$
Kottler (Schwarzschild-de Sitter solution) $K=\Lambda/3$
Reissner-Nordstrom-de Sitter solution $K_1=-Q^2$, $K_2=\Lambda/3$

Superradiance in Cosmology
(ongoing project)

M, Q, K, w



Conclusions

Found new exact solutions that will be used for modeling primordial black holes and black holes in the early universe.

Analyzed Kiselev-type solutions for charged black holes immersed in anisotropic dark fluid. Built the complete horizon structure and parameter space, defined generally for all possible models within this type.

For particular cases of a gas of strings and a network of domain walls, superradiant scattering of a charged scalar field was analyzed, and quasinormal modes were calculated.

THANK YOU

LTB solution for the dust matter

In comoving coordinates ***u^i*=0**

$$ds^2 = d\tau^2 - \frac{r'^2(R,\tau)}{f^2(R)}dR^2 - r^2(R,\tau)d\sigma^2$$

$$m(R) = r(R,\tau)\left(1 + \dot{r}^2(R,\tau) - f^2(R)\right)$$

Flat space case $f(R) = 1$

$$r(R,\tau) = \left[\pm\frac{3}{2}\sqrt{m(R)}(\tau - \tau_0(R))\right]^{\frac{2}{3}}$$

Lemaitre

$$m(R) = r_g$$

$$\tau_0(R) = R$$

Friedman (flat)

$$m(R) = a_{dust}R^3$$

$$f(R) = 1$$

$$\tau_0(R) = 0$$

Arbitrary functions of the solution

$f(R = const) = \dfrac{E}{mc^2}$ Total energy of the particle in the shell

$m(R) = \int_0^R \varepsilon\, r^2 r'\, dR$ Total mass of the dust in the shell

$\tau_0(R)$ Time of Big Bang or collapse

Reissner-Nordstrom solution

$$ds^2 = \left(1 - \frac{r_g}{r} + \frac{q^2}{r^2}\right)dt^2 - \left(1 - \frac{r_g}{r} + \frac{q^2}{r^2}\right)^{-1} dr^2 - r^2 d\sigma^2$$

$$m = r_g - \frac{q^2}{r}$$ Invariant with respect to the transformation $\{t,r\} \longrightarrow \{\tau,R\}$

$$ds^2 = d\tau^2 - r'^2(R,\tau)dR^2 - r^2(R,\tau)d\sigma^2$$

$$m(R,\tau) = r(R,\tau)\dot{r}^2(R,\tau) = r_g - \frac{q^2}{r(R,\tau)}$$

$$\tau - \tau_0(R) = \frac{2}{3}\frac{r^{3/2}(R,\tau)}{r_g^2}\sqrt{m(R,\tau)}\left(m(R,\tau) + \frac{3q^2}{r(R,\tau)}\right)$$

Chernin solution for the universe with dust and radiation

$$ds^2 = d\tau^2 - r'^2(R,\tau)dR^2 - r^2(R,\tau)d\sigma^2$$

$$\tau - \tau_0 = \frac{2}{3}a_{dust}\left(\frac{r(R,\tau)}{a_{dust}R} - 2\frac{a_{rad}^2}{a_{dust}^2}\right)\sqrt{\frac{r(R,\tau)}{a_{dust}R} + \frac{a_{rad}^2}{a_{dust}^2}}$$

$$m(R,\tau) = a_{dust}R^3 + \frac{a_{rad}^2 R^4}{r(R,\tau)}$$

$$m(R,\tau) = r(R,\tau)\dot{r}^2(R,\tau)$$

$$\varepsilon = \frac{a_{dust}R^3}{r^3} + \frac{a_{rad}^2 R^4}{r^4}$$

$$m' = \varepsilon r^2 r'$$

$$r = R\left(\frac{a_{dust}}{4}\eta^2 + a_{rad}\eta\right)$$

$$t - t_0 = \frac{a_{dust}}{12}\eta^3 + \frac{a_{rad}}{2}\eta^2$$

Chernin, A.D. A Model of a Universe Filled by Radiation and Dustlike Matter, Soviet Astronomy A.J., Vol. 9, No. 5., P. 871-872. (1966).