Exercises 1

1. a) The largest number that can be represented by a floating point system (2, 5, -10, 10) is 0.11111 x 2^10 which is equal to 992 (base 10). The largest value of n where n! <= 992 is 6 (since 6! = 720). 6! = 0.101101 x 2^10, which needs 6 digits. 5! = 0.1111 x 2^7. Hence the largest number we can represent is **n=5.**

b) 7 has exponent 3 in the base-2 system. Hence we have

$$2^{-12} = 2^{3-t} \longrightarrow t = 15$$

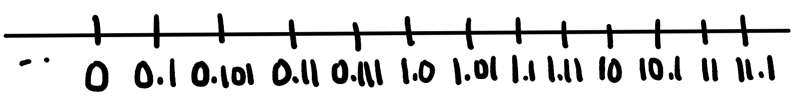
70 has exponent 7. Hence its distance from the next largest floating point number is

$$2^{7-15} = 2^{-8}$$

c) 8 (base 10) = 0.1 x 2^4. Since 8 is a number in this floating point system and we have x < 8 < y, to have the smallest y-x we must have that x, 8, and y are consecutive numbers. The distance from x to 8 is 2^(3-t). The distance from y to 8 is 2^(4-t). Hence we have that the smallest y-x is

$$2^{4-t} + 2^{3-t}$$

2. a)



Negative numbers are a mirror image of the positive ones (didn't want to draw them lol)
**Not to scale

b) There are 25 elements in S.
OFL = largest positive number = 11.1
UFL = smallest positive number = 0.1

$$|f(x) - x| \leq 0.1$$

$$|x| \geq 0.1$$

$$\frac{|f(x) - x|}{|x|} \leq 1 \longrightarrow \text{machine epsilon}$$

3. 1.5 x 10^8 = 0.100011110000110100011 x 2^28
The length on the last bit of this mantissa represents 2^8.

75 (base 10) = 0.1001011 x 2^7
The length on the last bit of this mantissa represents 2.