A2 F2022

1. a) [I'm not drawing the tree sorry]. At level k, the amount of work done is

$$5^k \cdot \frac{n}{3^k}\sqrt{\frac{n}{3^k}} = \left(\frac{5}{3}\right)^k \cdot \frac{1}{3^{k/2}} n\sqrt{n}$$

There are log_5(n) levels in the tree. Hence the total amount of work done is

$$n\sqrt{n} \sum_{k=1}^{\log_5 n} \left(\frac{5}{3}\right)^k \cdot \frac{1}{3^{k/2}} \in \Theta(n\sqrt{n})$$

Master's Theorem: 5 < 3^1.5 -> T(n) in O(nsqrt(n))

b) At level k, the amount of work done is

$$6^k \cdot \left(\frac{3}{7}\right)^{2k} n^2 = \left(\frac{54}{49}\right)^k n^2$$

There are log_(7/3)n levels in the tree. Hence the total amount of work done is

$$n^2 \sum_{k=1}^{\log_{7/3} n} \left(\frac{54}{49}\right)^k \in \Theta\left(\left(\frac{54}{49}\right)^{\log_{7/3} n} n^2\right) \in \Theta\left(n^{\log_{7/3} 6}\right)$$

2. a) Base case: T(3) is constant.

Inductive hypothesis:

$$T(k) \le ck\log k \quad , \quad k < n$$

Inductive step:

$$\begin{aligned} T(n) &= 3T(\lfloor n/3 \rfloor) + 2n \\ &\le 3T(n/3) + 2n \\ &\le 3c\frac{n}{3}\log\left(\frac{n}{3}\right) + 2n \\ &= cn\log n + n(2 - c\log 3) \\ &\le cn\log n \quad , \quad 2 < c\log 3 \end{aligned}$$

$\in O(n \log n)$

b) Base case: T(3) is constant.

Inductive hypothesis:

$$T(k) \leq ck - \frac{10}{3},\ k < n$$

Inductive step:

$$T(n) = 3T(\lfloor n/3 \rfloor) + 10$$

$$\leq 3T(n/3) + 10$$

$$\leq 3\left(c\frac{n}{3} - \frac{10}{3}\right) + 10$$

$$= cn$$

$$\in O(n)$$

3. [Dynamic programming]

4. [Dynamic programming]