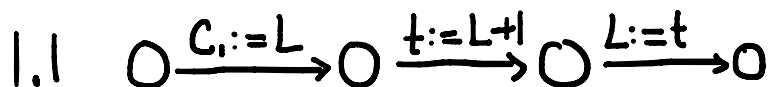


Final F2007 Problem 1



1.2 H holds initially since $L = 0$ and $R_1, R_2 = \text{false}$. The only potentially dangerous actions are a_i and b_i for $i = 1, 2$. We prove that actions a_1 and b_1 preserve H. The proof is symmetric for a_2 and b_2 .

a_1 : Before a_1 is executed, we have that $|R_1| = 0$, hence $L = |R_2|$. After a_1 , we have that we are at w_1 , so $|R_1| = 1$. a_1 increments L. Hence we have that $L = 1 + |R_2| = |R_1| + |R_2|$. Hence a_1 preserves H.

b_1 : Before b_1 is executed, we have that we are at b_1 , hence $|R_1| = 1$, and $L = 1 + |R_2|$. After b_1 , we have that we are at d_1 , hence $|R_1| = 0$. Hence we have that $L = 0 + |R_2| = |R_1| + |R_2|$.

1.3 a) $L = 1$, since $|R_2| = 1$ and $|R_1| = 0$.
 $C_1 = 0$
 $C_2 = 0$

b) $R_1 \uparrow R_2 \Rightarrow C_1 + C_2 = 1$

I holds initially since R_1 and R_2 are both false. The statement $R_1 \uparrow R_2$ becomes true when P_1 enters its R-section when R_2 is true, or vice versa. The logic is symmetrical.

Before P_1 enters its R-section, we have that we are in a state where $a_1 \wedge R_2$ holds, hence we have $L = 1, C_1 = 0, C_2 = 0$ (from part a). Hence after executing a_1 , we have that $C_1 = 1, C_2 = 0$. C_1 and C_2 are not modified again until one of the processes exits their R-section, so I holds.

c) Assume both processes are in their critical sections, ie $\text{in } cs_1 \wedge \text{in } cs_2$. Hence we have that R_1 and R_2 are both true. By invariant I, we must have that $C_1 + C_2 = 1$, in other words, one of C_1, C_2 is equal to 1 (by F). Suppose $C_1 = 1$ (the proof is symmetric for $C_2 = 1$). Then by G_1 , we must have that we are at w_1 . This is a contraction to our initial statement $\text{in } cs_1 \wedge \text{in } cs_2$. Hence mutual exclusion is ensured.

1.4

- a) some i . in $w_i \rightarrow$ some j . at cs_j (pretend it is a squiggly arrow)
- b) Suppose we have that we are in w_1 (the logic is symmetric for w_2). We have that C_1 is either equal to 0 or 1. If $C_1=0$, we proceed to the critical section, and we are done. Otherwise, we must have that $C_1=1$. By invariant G_1 , we have that we are in R_2 or at d_2 . If R_2 is true, there are 3 cases. If we are in cs_2 , we are done. If we are at w_2 , by invariant I, we have that $C_2=0$, so process 2 can proceed to cs_2 and we are done. If we are at b_2 or at d_2 , process 2 will proceed to execute d_2 , setting $C_1=0$, and process 1 can proceed to cs_1 .
- c) Starvation does not occur. We show that in $w_i \rightarrow$ at cs_i (pretend it is a squiggly arrow)

We show this for process 1 (process 2 is symmetrical). From the proof in part b, we have shown that if we are in w_1 , we proceed to cs_1 for all cases except if we are in cs_2 or at w_2 . If this is the case, process 2 will proceed until it executes d_2 . Then we will have $C_1=0$. If process 2 continues executing, it will reach a_2 , where it finds that $L=1$, hence setting $C_2=1$ and must await, allowing process 1 to proceed to cs_1 .