

MATH213 Final W2022

1. A

Using KVL, we obtain

$$\begin{aligned}x(t) &= V_L + V_R + V_C \\&= L i' + R i + \frac{1}{C} \int i dt \\&= LC y'' + RC y' + y \quad \text{since } i = C \frac{dv}{dt} \\&= y'' + y' + y\end{aligned}$$

2. C

$$\begin{aligned}V &= L I_L' = I_L' \\I_R &= \frac{V}{R} = V = I_L'\end{aligned}$$

Using KCL, we obtain

$$\begin{aligned}x(t) &= I_R + I_L \\&= y'(t) + y(t)\end{aligned}$$

Taking the Laplace transform, we obtain

$$X(s) = sY(s) + Y(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+1}$$

$$H(j\omega) = \frac{1}{j\omega+1}$$

3. $|n| > 8$

We have that

$$\begin{aligned} y(t) &= \sum_{n=-\infty}^{\infty} c_n H(-j \frac{2\pi n}{T}) e^{-j \frac{2\pi n}{T} t} \\ &= \sum_{n=-\infty}^{\infty} c_n H(-j 2\pi n) e^{-j 2\pi n t} \end{aligned}$$

Since $H(j\omega) = 0$ for $|\omega| > 100$, the largest value of $|n|$ for which c_n is nonzero is

$$|2\pi n| \leq 100$$

which implies $|n| \leq 8$. Then for $|n| > 8$, c_n is guaranteed to be 0.

4. $R \geq 2\sqrt{L/C}$

Using KVL, we obtain

$$\begin{aligned} x(t) &= V_L + V_R + V_C \\ &= L i' + R i + \frac{1}{C} \int i dt \\ &= LC y'' + RC y' + y \quad \text{since } i = C \frac{dv}{dt} \end{aligned}$$

$$\frac{x}{LC} = y'' + \frac{R}{L} y' + y$$

Taking the Laplace transform, we obtain

$$\frac{X(s)}{LC} = s^2 Y(s) + s \frac{R}{L} Y(s) + Y(s)$$

The transfer function is

$$\frac{Y(s)}{X(s)} = \frac{1}{LC(s^2 + s \frac{R}{L} + \frac{1}{LC})}$$

The poles are

$$-\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \quad \left\{ \begin{array}{l} R > 2\sqrt{\frac{L}{C}} \\ R = 2\sqrt{\frac{L}{C}} \\ R < 2\sqrt{\frac{L}{C}} \end{array} \right.$$

2 different real poles: overdamped

2 identical real poles: critically damped

Complex conjugate poles: underdamped (oscillation)

5. D

Taking the Laplace transform, we obtain

$$sY(s) + 7Y(s) = 3X(s)$$

The transfer function is

$$H(s) = \frac{Y(s)}{X(s)} = \frac{3}{s+7}$$

The Laplace of the step response is

$$\frac{1}{s} H(s) = \frac{3}{s(s+7)}$$

It has two poles, one of which is negative and real and the other at zero. Hence by the final value theorem, the final value of the step response is

$$\lim_{s \rightarrow 0} \frac{3s}{s(s+7)} = \lim_{s \rightarrow 0} \frac{3}{s+7} = \frac{3}{7}$$

6. A

$$y(t) = \sum_{n=-\infty}^{\infty} c_n H(-j\frac{2\pi n}{T}) e^{-j\frac{2\pi n}{T} t}$$

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{-j\frac{2\pi n}{T} t}$$

$$= (4-4) e^{-4t} u_{-1}(t) - (4-5) e^{-5t} u_{-1}(t)$$

$$\begin{aligned}
 &= (4-4)e^{-4t}u_{-1}(t) - (4-5)e^{-5t}u_{-1}(t) \\
 &= e^{-5t}u_{-1}(t)
 \end{aligned}$$

7. B

We take the Laplace transform to obtain

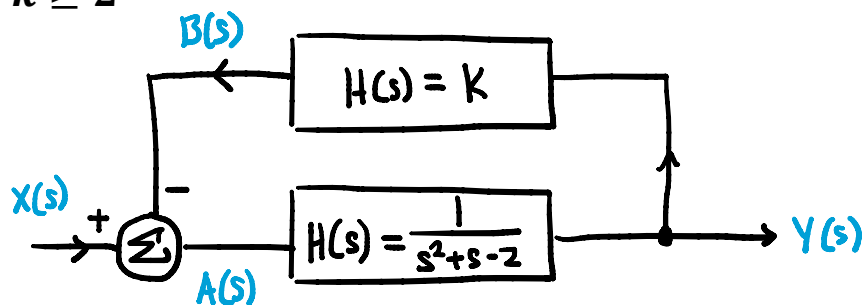
$$s^2Y(s) + sY(s) - 2Y(s) = X(s)$$

The transfer function is

$$\frac{Y(s)}{X(s)} = \frac{1}{s^2 + s - 2} = \frac{1}{(s+2)(s-1)}$$

The system is BIBO stable if its transfer function is stable and proper: when all of the poles of its transfer function lie strictly to the left of the imaginary axis (have negative real parts). The transfer function has two poles -2 and 1. Hence the system is not BIBO stable.

8. $k \geq 2$



$$A(s) = X(s) - B(s)$$

$$B(s) = KY(s)$$

$$Y(s) = \frac{A(s)}{s^2 + s - 2} = \frac{X(s) - KY(s)}{s^2 + s - 2}$$

The transfer function is

$$\frac{Y(s)}{X(s)} = \frac{1}{(s^2+s-2)(1+\frac{k}{s^2+s-2})} = \frac{1}{s^2+s-2+k}$$

If $k < 2$, the transfer function has one positive and one negative real pole.

If $k = 2$, the transfer function has 2 poles: 0 and -1

If $2 < k < 2.25$, the transfer function has two negative real poles.

If $k = 2.25$, the transfer function has 1 pole at -1/2

If $k > 2.25$, the transfer function has complex poles with negative real parts (-1/2).

$$\begin{aligned} 9. \quad y[n] &= \alpha y[n-1] + (1-\alpha)x[n] \\ &= (1-\alpha)x[n] + \alpha((1-\alpha)x[n-1] + \alpha((1-\alpha)x[n-2] \dots \\ &= (1-\alpha) \sum_{k=0}^n \alpha^k x[n-k] \end{aligned}$$

Assume initial rest conditions $y[0] = 0$ and let

$$x[n] = \delta[n]$$

Then we have

$$y[n] = \alpha^n (1-\alpha)$$

Thus, the impulse response is

$$\alpha^n (1-\alpha) u_{-1}[n]$$

$$10. \quad s_1, s_3, s_4$$

S_1 : not time-invariant

S_2 : linear, time-invariant

S_3 : not linear?

S_4 : not time-invariant

S_5 : linear, time-invariant