

CS370 Final F2002

1. No.

2. a) $t=4, L=-3, U=2$

b) $E = \frac{1}{2} \cdot 10^{-4} = \frac{1}{20000}$

c) i. $\pi_1 \oplus \pi_2 = 0.7612 \cdot 10^2$
 $\pi_1 \ominus \pi_2 = 0.1000 \cdot 10^{-1}$

ii. ans2

iii. $\Delta = 10^{-1-4} = 10^{-5}$ *interpreting as numbers in $[0.01000, 0.01046)$

$$\frac{0.01046 - 0.01}{\Delta} = 46$$

3. a) i. $n=5$

ii. Same for both:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0.25 & 0.25^2 & 0.25^3 & 0.25^4 \\ 1 & 0.5 & 0.5^2 & 0.5^3 & 0.5^4 \\ 1 & 0.75 & 0.75^2 & 0.75^3 & 0.75^4 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

iii. $p_x(t): [-1, 0, 1, 0, 1]$
 $p_y(t): [0, 1, 0.5, 0, -1]$

iv. [Not covered]

b) [Not covered]

4. A) False (A singular matrix has an LU factorization if and only if all leading nonprincipal submatrices are nonsingular [not covered]).
True. False.
B) [Not covered]

5. [Not covered?]
Spectrum 1: B. Tbh I have no idea
Spectrum 2: A. See reasoning above
Spectrum 3: C. Idk the data sequence graph is rounder
Spectrum 4: None

6. a) $W^{N/2} = e^{\frac{i2\pi}{N} \frac{N}{2}} = e^{i\pi} = -1$

b) $W^{N-k} = e^{\frac{2\pi i (N-k)}{N}} = e^{\frac{2\pi i}{N}} e^{-\frac{2\pi i k}{N}} = W^{-k} \quad \text{for } 1 \leq k \leq N-1$

$$F_{N-k} = \sum_{n=0}^{N-1} f_n W^{-(N-k)n} = \sum_{n=0}^{N-1} f_n W^{nk} = \sum_{n=0}^{N-1} \overline{f_n W^{-nk}} = \overline{F_k}$$

- c) $C = 1$
There are $2N$ multiplications in each summation (real*complex = 2 real multiplications). There are $N/2$ even coefficients.

- d) Since $F_{N-k} = \overline{F_k}$, we can compute the first half of the Fourier coefficients, then just take the conjugate of them to get the second half. We get that the recurrence relation is $T(n) = T(n/2) + n$, so the runtime is $O(N \log N)$.

7. A) 4 B) 5 C) 2 D) 6

8. a) Let $y_1 = y(t)$

$$y_1' = y_2$$

$$x'(t) = 1 - (x^2(t) + y^2(t))$$

$$y_2' = 2(y_1 - x(t))$$

$$y_2' = 2(y_1 - x(t))$$

$$\begin{bmatrix} x'(t) \\ y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} 1 - (x^2(t) - y(t)) \\ y_2 \\ 2(y_1 - x(t)) \end{bmatrix}$$

b) ?? if $y(t) > 2$ return 1 else -1

c) [Not covered]

9. a) $y_1 = y_0 + hf(t_0, y_0)$

$$= 1.5 + 0.2(2 \cdot 1 - 1.5)$$

$$= 1.6$$

$$y_2 = 1.6 + 0.2(2 \cdot 1.2 - 1.6)$$

$$= 1.76$$

b) [I have no idea what A and B are but here's what our notes say for adaptive time stepping]
If the error is greater than the tolerance, we halve h_k for the next step. If it is too small, we double h_k for the next step.