Concurrency E07

1.1

$$\circ \xrightarrow{C_1 := L} \circ \xrightarrow{t := L+1} \circ \xrightarrow{L := t} \circ$$

1.2

H holds initially since L = 0 and R_1, R_2 = false. The only potentially dangerous actions are a_i and b_i for $i = 1,2$. We prove that actions a_1 and b_1 preserve H. The proof is symmetric for a_2 and b_2.

a_1: Before a_1 is executed, we have that $|R_1|$ = 0, hence L = $|R_2|$. After a_1, we have that we are *at w_1*, so $|R_1|$ = 1. a_1 increments L. Hence we have that L = 1 + $|R_2|$ = $|R_1|$ + $|R_2|$. Hence a_1 preserves H.

b_1: Before b_1 is executed, we have that we are *at* b_1, hence $|R_1|$ = 1, and L = 1 + $|R_2|$. After b_1, we have that we are at d_1, hence $|R_1|$ = 0. Hence we have that L = 0 + $|R_2|$ = $|R_1|$ + $|R_2|$.

1.3

a) L = 1, since $|R_2|$ = 1 and $|R_1|$ = 0.
   C_1 = 0
   C_2 = 0

b) R_1 ^ $R_2 \Rightarrow C_1 + C_2 = 1$

   I holds initially since R_1 and R_2 are both false. The statement R_1 ^ R_2 becomes true when P_1 enters its R-section when R_2 is true, or vice versa. The logic is symmetrical.

   Before P_1 enters its R-section, we have that we are in a state where *at* a_1 ^ R_2 holds, hence we have L = 1, C_1 = 0, C_2 = 0 (from part a). Hence after executing a_1, we have that $C_1 = 1$, C_2 = 0. C_1 and C_2 are not modified again until one of the processes exits their R-section, so I holds.

c) Assume both processes are in their critical sections, ie *in* cs_1 ^ *in* cs_2. Hence we have that R_1 and R_2 are both true. By invariant I, we must have that $C_1 + C_2 = 1$, in other words, one of C_1, C_2 is equal to 1 (by F). Suppose C_1= 1 (the proof is symmetric for C_2= 1). Then by G_1, we must have that we are *at* w_1. This is a contraction to our initial statement *in* cs_1 ^ *in* cs_2. Hence mutual exclusion is ensured.

1.4

a) some *i* . in w_i -> some *j* . at cs_j (pretend it is a squiggly arrow)

b) Suppose we have that we are *in* w_1 (the logic is symmetric for w_2). We have that C_1 is either equal to 0 or 1. If C_1=0, we proceed to the critical section, and we are done. Otherwise, we must have that C_1=1. By invariant G_1, we have that we are in R_2 or *at* d_2. If R_2 is true, there are 3 cases. If we are *in* cs_2, we are done. If we are *at* w_2 , by invariant I, we have that C_2=0, so process 2 can proceed to cs_2 and we are done. If we are *at* b_2 or *at* d_2, process 2 will proceed to execute d_2, setting C_1=0, and process 1 can proceed to cs_1.

c) Starvation does not occur. We show that
in w_i -> at cs_i (pretend it is a squiggly arrow)

We show this for process 1 (process 2 is symmetrical). From the proof in part b, we have shown that if we are *in* w_1, we proceed to cs_1 for all cases except if we are *in* cs_2 or *at* w_2. If this is the case, process 2 will proceed until it executes d_2. Then we will have C_1=0. If process 2 continues executing, it will reach a_2, where it finds that L=1, hence setting C_2=1 and must await, allowing process 1 to proceed to cs_1.