MATH213 Final W2022

1. A

Using KVL, we obtain

$$x(t) = V_L + V_R + V_C$$

$$= Li' + Ri + \frac{1}{C}\int i\,dt$$

$$= LCy'' + RCy' + y \quad \text{since } i = C\frac{dv}{dt}$$

$$= y'' + y' + y$$

2. C

$$V = LI_L' = I_L'$$

$$I_R = \frac{V}{R} = V = I_L'$$

Using KCL, we obtain

$$x(t) = I_R + I_L$$

$$= y'(t) + y(t)$$

Taking the Laplace transform, we obtain

$$X(s) = sY(s) + Y(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+1}$$

$$H(j\omega) = \frac{1}{j\omega+1}$$

3. $|n| > 8$

We have that

$$y(t) = \sum_{n=-\infty}^{\infty} c_n H\left(-j\frac{2\pi n}{T}\right) e^{-j\frac{2\pi n}{T}t}$$

$$= \sum_{n=-\infty}^{\infty} c_n H(-12jn)\, e^{-12jnt}$$

Since H(jw) = 0 for |w| > 100, the largest value of |n| for which c_n is nonzero is

$$|12n| \leq 100$$

which implies $|n| \leq 8$. Then for $|n| > 8$, c_n is guaranteed to be 0.

4. $R \geq 2\sqrt{L/C}$

Using KVL, we obtain

$$x(t) = V_L + V_R + V_C$$

$$= Li' + Ri + \frac{1}{C}\int i\,dt$$

$$= LCy'' + RCy' + y \qquad \text{since } i = C\frac{dv}{dt}$$

$$\frac{x}{LC} = y'' + \frac{R}{L}y' + y$$

Taking the Laplace transform, we obtain

$$\frac{X(s)}{LC} = s^2Y(s) + s\frac{R}{L}Y(s) + Y(s)$$

The transfer function is

$$\frac{Y(s)}{X(s)} = \frac{1}{LC\left(s^2 + s\frac{R}{L} + \frac{1}{LC}\right)}$$

The poles are

$$-\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \quad \begin{cases} R > 2\sqrt{\frac{L}{C}} & \text{2 different real poles: overdamped} \\ R = 2\sqrt{\frac{L}{C}} & \text{2 identical real poles: critically damped} \\ R < 2\sqrt{\frac{L}{C}} & \text{Complex conjugate poles: underdamped (oscillation)} \end{cases}$$

5. D

Taking the Laplace transform, we obtain

$$sY(s) + 7Y(s) = 3X(s)$$

The transfer function is

$$H(s) = \frac{Y(s)}{X(s)} = \frac{3}{s+7}$$

The Laplace of the step response is

$$\frac{1}{s}H(s) = \frac{3}{s(s+7)}$$

It has two poles, one of which is negative and real and the other at zero. Hence by the final value theorem, the final value of the step response is

$$\lim_{s\to 0} \frac{3s}{s(s+7)} = \lim_{s\to 0} \frac{3}{s+7} = \frac{3}{7}$$

6. A

$$y(t) = \sum_{n=-\infty}^{\infty} c_n H\left(-j\frac{2\pi n}{T}\right) e^{-j\frac{2\pi n}{T}t}$$

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{-j\frac{2\pi n}{T}t}$$

$$= (4-4)e^{-4t}u_{-1}(t) - (4-5)e^{-5t}u_{-1}(t)$$

$$= (4-4)e^{-4t}u_{-1}(t) - (4-5)e^{-5t}u_{-1}(t)$$

$$= e^{-5t}u_{-1}(t)$$

7. B

We take the Laplace transform to obtain

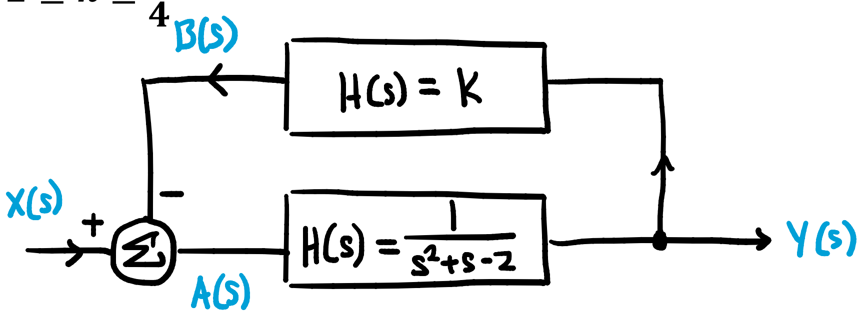
$$s^2Y(s) + sY(s) - 2Y(s) = X(s)$$

The transfer function is

$$\frac{Y(s)}{X(s)} = \frac{1}{s^2+s-2} = \frac{1}{(s+2)(s-1)}$$

The system is BIBO stable if its transfer function is stable and proper: when all of the poles of its transfer function lie strictly to the left of the imaginary axis (have negative real parts). The transfer function has two poles -2 and 1. Hence the system is not BIBO stable.

8. $2 \leq k \leq \frac{9}{4}$



$$A(s) = X(s) - B(s)$$

$$B(s) = KY(s)$$

$$Y(s) = \frac{A(s)}{s^2+s-2} = \frac{X(s) - KY(s)}{s^2+s-2}$$

The transfer function is

$$\frac{Y(s)}{X(s)} = \frac{1}{(s^2+s-2)\left(1+\frac{K}{s^2+s-2}\right)} = \frac{1}{s^2+s-2+K}$$

If $k < 2$, the transfer function has one positive and one negative real pole.
If $k = 2$, the transfer function has 2 poles: 0 and -1
If $2 < k < 2.25$, the transfer function has two negative real poles.
If $k = 2.25$, the transfer function has 1 pole at -1/2
If $k > 2.25$, the transfer function has complex poles.

9. $y[n] = \alpha y[n-1] + (1-\alpha)x[n]$

$$= (1-\alpha)x[n] + \alpha((1-\alpha)x[n-1] + \alpha((1-\alpha)x[n-2] \cdots$$

$$= (1-\alpha)\sum_{k=0}^{n} \alpha^k x[n-k]$$

Assume initial rest conditions n0 = 0 and let

$$x[n] = \delta[n]$$

Then we have

$$y[n] = \alpha^n(1-\alpha)$$

Thus, the impulse response is

$$\alpha^n(1-\alpha)u_{-1}[n]$$

10. $\boldsymbol{S_1, S_3, S_4}$

S_1: not time-invariant
S_2: linear, time-invariant
S_3: not linear?
S_4: not time-invariant
S_5: linear, time-invariant