CS370 Midterm F2005

1. a) $L_1(x) = \frac{(x-1)(x-2)}{(-1)(-2)}$

$L_2(x) = \frac{x(x-2)}{(1-2)}$

$L_3(x) = \frac{x(x-1)}{2(2-1)}$

$p_1(x) = 5L_1(x) + \frac{5}{2}L_2(x) + L_3(x)$

$= \frac{5}{2}(x-1)(x-2) - \frac{5}{2}x(x-2) + \frac{1}{2}x(x-1)$

$= 5 - 3x + \frac{1}{2}x^2$

b) $p(0) = 5 \rightarrow c_0 = 5$

$p(1) = 5/2 \rightarrow c_0 + c_1 + c_2 = 5/2$

$p(2) = 1 \rightarrow c_0 + 2c_1 + 4c_2 = 1$

$\left.\begin{array}{l} c_0 = 5 \\ c_1 = -3 \\ c_2 = 1/2 \end{array}\right\}$

$p_2(x) = 5 - 3x + \frac{1}{2}x^2$

c) p1(x) and p2(x) are the same polynomial.

2. a) i. $S_1'(1) = S_2'(1)$

ii. $S_1''(1) = S_2''(1)$

b) $S_1(1) = S_2(1) \rightarrow 0.917 - 5.917 + a_1 = b_3 + b_2 + b_1 + 5.375$

$S_1'(1) = S_2'(1) \rightarrow 2.751 - 5.917 = 3b_3 + 2b_2 + b_1$

$S_1''(1) = S_2''(1) \rightarrow 5.502 = 6b_3 + 2b_2$

$$S''(3) = 0 \quad \rightarrow 2b_2 = 0$$

3. [Not covered on midterm]

4. [Not covered]

5. a) $0.00001 \cdot 10^{-5}$

b) $0.25790 \cdot 10^{4}$

c) $5912.32 \oplus 4087.50$

$= 0.59123 \cdot 10^{4} + 0.40875 \cdot 10^{4}$

$= 0.99998 \cdot 10^{4}$

d) The distance between consecutive floating point numbers in the interval [1, 2) is

$\Delta = 10^{1-5} = 10^{-4}$

Hence there are 1/10^-4 = 10^4 numbers in the interval [1, 2).

6. a) $2^{4} - 2^{-23} \otimes 2^{5} + 2^{-22}$

$= fl(2^{9} + 2^{-18} - 2^{-18} - 2^{-45})$

$= 2^{9} - 2^{-45}$

b) $$\frac{|a \otimes b - ab|}{|ab|} = \frac{|ab(1+\delta) - ab|}{|ab|}$$

$$= \frac{|\delta|}{|ab|} \leq \frac{eps}{|ab|}$$

$$\frac{\text{Rel Error}}{eps} \approx \frac{1}{|cb|}$$

7. [Not covered]

8. [Not covered]