MATH 239 W2012 Midterm Practice

1. Both graphs have one vertex with degree 3 - vertex B and vertex V, respectively. Vertex B is only contained in one triangle, while vertex V is contained in two distinct triangles. Hence the two graphs are not isomorphic.

2. a) This graph cannot exist. Suppose such a graph does exist. We can delete a node with degree 8 and its edges, and its degree sequence must also be a graph. Assuming it is connected to the next vertices with the most amount of edges, we obtain
(7, 7, 7, 5, 4, 4, 2, 2, 1). We can iterate this, repeatedly removing the next vertex:
(6, 6, 4, 3, 3, 1, 1, 1)
(5, 3, 2, 2, 1, 0, 0)
(2, 1, 1, 0, -1, 0)

This last iteration is clearly not a graph, since it has a vertex with degree -1. Hence this is not a graph.

b) Using the same logic as part a, removing vertices obtains
(7, 7, 7, 5, 4, 4, 2, 1, 1)
(6, 6, 4, 3, 3, 1, 1, 1)
(5, 3, 2, 2, 1, 0, 0)
(2, 1, 1, 0, -1, -1)

This last iteration is clearly not a graph, since it has vertices with degree -1. Hence this is not a graph.

3. There are (n choose 3) vertices. For each vertex string, there are 3 ways to choose 2 positions for the two 1s in the same position, and 0 in the other. The other 1 can hence be in ((n choose 3) - 3) possible positions. Hence each vertex has degree 3((n choose 3) - 3).

$$|E(G)| = \frac{1}{2}\sum_{v \in V} \deg(v) = \frac{1}{2}\binom{n}{3}3\left(\binom{n}{3}-3\right)$$

4. Since G is a 237-cube, it is bipartite, with bipartition (X, Y) where X is the set of strings containing an even number of ones, and Y the strings with an odd number of ones. If vertices x and y are adjacent, then the strings x and y differ in exactly one position, and exactly one of x and y contain an even number of ones.

Let H be a subgraph of G. Then (X intersect V(H), Y intersect V(H)) partitions the set of vertices in H. If X intersect V(H) is empty, then there is no edge in H, since there are no edges between vertices in Y, and H is bipartite, and vice versa for if Y intersect V(H) is empty. Otherwise, since every edge in G has endpoints in both X and Y, no edge in H can have endpoints in both X intersect V(H) and Y intersect V(H). Hence H is bipartite.