$$| \lambda | [\lambda^{n}] \frac{(1+2\pi)(1+x^{2})^{m}}{(1-x)^{3}} = [\lambda^{n}](1+2\pi)\left[\sum_{k=0}^{m} {m \choose k} \chi^{2k}\right] \left[\sum_{k=0}^{m} {k+2 \choose 2} \chi^{k}\right]$$

$$= [\lambda^{n}](1+2\pi)\sum_{k=0}^{m} \sum_{j=0}^{m} {m \choose j/2} {k-j+2 \choose 2} \chi^{k}$$

$$= \sum_{j=0}^{m} {m \choose j/2} {n-j+2 \choose 2} + 2\sum_{j=0}^{m} {m \choose j/2} {n-j+1 \choose 2}$$

$$= [\lambda^{n}] \frac{1}{(1-\pi)(1-\pi^{m})} = [\lambda^{n}] \left[\sum_{k=0}^{m} \chi^{k}\right] \sum_{k=0}^{m} \chi^{nk}$$

$$= [\lambda^{n}] + 1$$

$$2. \left[\chi^{\text{5000}}\right] = \left(\sum_{k=0}^{50} \chi^{20k}\right) \left[\sum_{k=0}^{40} \chi^{\text{50k}}\right] \sum_{k=0}^{70} \chi^{100k}$$

- The set of all strings where every even block of 1s is followed by an even block of 0s, except for the suffix block of 1s.
 - This is the block decomposition for this set of strings.

c)
$$\overline{\Psi}(x) = \frac{1}{1-x} \left(\frac{1}{1-\left(\frac{x^2}{(1-x^2)^2} - \frac{x^3}{(1-x^2)^2} \right)} \right) \frac{1}{1-x} = \frac{1+2x+x^2}{1-3x^2-5x^3}$$

S' can be described by the recursive expression $T = 0 T 1 \sim 0*01$ S' = 1* T* 0*

$$\overline{Q}_{T}(x) = x^2 \overline{Q}_{T}(x) + \frac{x^2}{1-x}$$

$$\underline{\underline{\sigma}}_{T}(x) = x^{2} \underline{\underline{\sigma}}_{T}(x) + \frac{x^{2}}{1-x}$$

$$\underline{\underline{\tau}}_{T}(x) = \frac{x^{2}}{1-x-x^{2}+x^{3}}$$

$$\underline{\underline{\tau}}_{S}(x) = \frac{1}{1-x} \frac{2}{1-\overline{\underline{\sigma}}_{T}(x)} = \frac{1+x}{1-x-2x^{2}+x^{3}}$$

e)
$$J_{s}(x) = \sum_{n=0}^{\infty} s_{n}x^{n}$$

By theorem 4.8, s_n satisfies the linear recurrence relation with initial conditions given by

$$S_n - S_{h-1} - 2S_{h-2} + S_{h-3} = \begin{cases} 1, & n = 0 \\ 1, & n = 1 \\ 0, & otherwise. \end{cases}$$

Hence we have

$$S_0 = 1$$

 $S_1 = 2$
 $S_2 = 4$
 $S_n = S_{n-1} + 2S_{n-2} - S_{n-3}$, $n \ge 3$

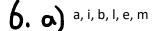
We prove that if G is a tree, the total number of vertices is p = 2n_3 + 2. If G has 1 or 2 vertices, it must have 2 vertices of degree 1. Otherwise, every vertex of degree 1 must be adjacent to a vertex of degree 3.

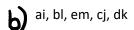
We prove this as follows: suppose there are more than 3 vertices and two vertices of degree 1 are adjacent to each other. Then these two vertices must not be connected to any other vertices, and hence G would not be connected, which is a contradiction.

Hence every vertex of degree 1 is connected to a vertex of degree 3. Then each vertex of degree 3 needs 2 more adjacent vertices. If G has 4 vertices, the other two vertices are degree 1. Otherwise, the other two vertices must be of degree 3, except 2 end-vertices, which are at the end and hence have 2 vertices of degree 1 adjacent. Hence G has $p = 2n_3 + 2$ vertices.

If G has $p = 2n_3 + 2$ vertices, then every vertex of degree 3 must be connected to two other vertices of degree 3 in a line, except for the two ends. The other neighbours must be of degree 1. This is a tree.

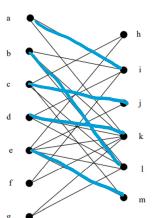
- **b)** 3, 4, 6, 9, 10
- **c)** G is not bipartite. There is an odd cycle 1, 5, 4, 2, 8, 3, 7.
- We can colour the vertices in each level of T all the same colour, and alternate the colouring of levels. The exception are the two vertices that are connected by the one same-level edge we colour these two vertices different colours. Since no tree edges join vertices in the same level, and there is only one non-tree edge that joins vertices of the same level (which we addressed), this means each vertex has no adjacent neighbours of the same colour. Hence H is 3-colourable.





C)
$$X0 = \{f, g\}$$

 $X = \{f, g, d, b, a\}$
 $Y = \{k, l, i\}$



7. **a)**
$$p-q+f=2$$

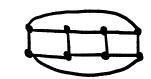
- G has vertices all of the same degree and faces all of the same degree. Hence it is a platonic solid. The platonic solid with vertices of degree 5 is the icosahedron, which has 20 faces.
- c) i.



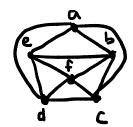
ii.



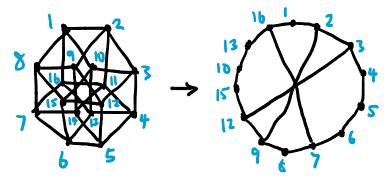
iii



8. a) Planar - this is a planar embedding



b Not planar - this subgraph is an edge subdivision of K3,3:



- If N(x) is not a subset of V(P), then there exists some neighbour of x that is not in P. Then we can extend P to make a longer path by adding that neighbour to the end of the path, which is a contradiction. Hence N(x) is a subset of V(P). The same argument goes for y.
 - Let P be the longest path in G. Suppose P does not have length k. Let x be an endpoint of P. N(x) must be a subset of P. But P has less than k vertices, and x is adjacent to at least k other vertices. This is a contradiction. Hence P must have length at least k.