CS370 Final F2002

1. No.

2. a) t=4, L=-3, U=2

b) $E = \frac{1}{2} \cdot 10^{-4} = \frac{1}{20000}$

c) i. $x_1 \oplus x_2 = 0.7612 \cdot 10^2$

$x_1 \ominus x_2 = 0.1000 \cdot 10^{-1}$

ii. ans2

iii. $\Delta = 10^{-1-4} = 10^{-5}$ *interpreting as numbers in $[0.01000, 0.01046)$

$$\frac{0.01046 - 0.01}{\Delta} = 46$$

3. a) i. $n = 5$

ii. Same for both:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0.25 & 0.25^2 & 0.25^3 & 0.25^4 \\ 1 & 0.5 & 0.5^2 & 0.5^3 & 0.5^4 \\ 1 & 0.75 & 0.75^2 & 0.75^3 & 0.75^4 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

iii. px(t): [-1, 0, 1, 0, 1]
py(t): [0, 1, 0.5, 0, -1]

iv. [Not covered]

b) [Not covered]

4. A) False (A singular matrix has an LU factorization if and only if all leading nonprincipal submatrices are nonsingular [not covered]).
True. False.

B) [Not covered]

5. [Not covered?]
Spectrum 1: B. Tbh I have no idea
Spectrum 2: A. See reasoning above
Spectrum 3: C. Idk the data sequence graph is rounder
Spectrum 4: None

6. a) $W^{N/2} = e^{\frac{i2\pi}{N}\frac{N}{2}} = e^{i\pi} = -1$

b) $W^{N-k} = e^{2\pi i(N-k)/N} = e^{2\pi i} e^{-2\pi i k/N} = W^{-k}$ for $1 \le k \le N-1$

$$F_{N-k} = \sum_{n=0}^{N-1} f_n W^{-(N-k)n} = \sum_{n=0}^{N-1} f_n W^{nk} = \sum_{n=0}^{N-1} \overline{f_n W^{-nk}} = \overline{F_k}$$

c) C = 1/2
There are N multiplications in each summation. There are N/2 even coefficients.

d) Since $F_N - k = \overline{F_k}$, we can compute the first half of the Fourier coefficients, then just take the conjugate of them to get the second half. We get that the recurrence relation is T(n)=T(n/2)+n, so the runtime is O(NlogN).

7. A) 4 B) 5 C) 2 D) 6

8. a) Let $Y_1 = y(t)$

$Y_1' = Y_2$

$x'(t) = 1 - (x^2(t) + y^2(t))$

$Y_2' = 2(Y_1 - x(t))$

$$Y_2' = 2(Y_1 - x(t))$$

$$\begin{bmatrix} x'(t) \\ Y_1' \\ Y_2' \end{bmatrix} = \begin{bmatrix} 1-(x^2(t) - y(t)) \\ Y_2 \\ 2(Y_1 - x(t)) \end{bmatrix}$$

b) ?? if y(t) > 2 return 1 else -1

c) [Not covered]

9. a) $y_1 = y_0 + hf(t_0, y_0)$

$$= 1.5 + 0.2(2 \cdot 1 - 1.5)$$

$$= 1.6$$

$$y_2 = 1.6 + 0.2(2 \cdot 1.2 - 1.6)$$

$$= 1.76$$

b) [I have no idea what A and B are but here's what our notes say for adaptive time stepping]
If the error is greater than the tolerance, we halve h_k for the next step. If it is too small, we double h_k for the next step.