From Centralized to Decentralized Control of Complex Systems

In the Pursuit of Efficient Control Approaches

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Fall semester 2019

Lecture 4

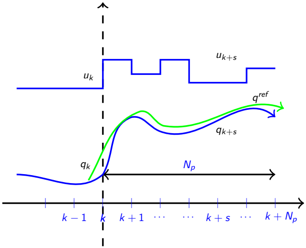
Outline

1. An optimization-based approach for control of complex systems
   - Mixed-integer techniques
   - Flat trajectory

Model Predictive Control (MPC)

Propoi (1963), Cutler et al. (2007), Richalet and O'Donovan (2009)

- Optimization-based control law
- Implicit (on-line) vs. explicit (off-line) implementation
- Constraints handling
- Can be implemented in a distributed fashion



model of the system

reference trajectory

$$\arg\min_{\mathbf{u}} \; V_f(x(k+N_p), q(k+N_p)) + \sum_{s=1}^{N_p-1} V_n(x(k+s), u(k+s), q(k+s))$$

$$\text{subject to: } \begin{cases} x(k+s+1) = f(x(k+s), u(k+s)), & s = 0 : N_p - 1, \\ h(x(k+s), u(k+s), q(k+s)) \leq 0, & s = 1 : N_p - 1. \end{cases}$$

constraint sets representation

optimization solver

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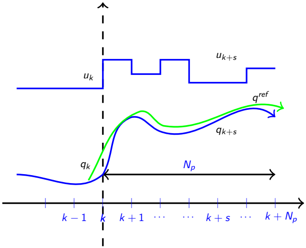

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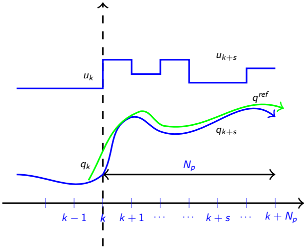

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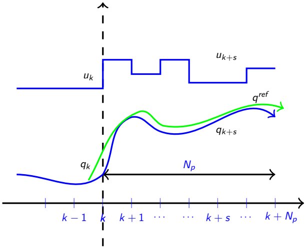

model of the system

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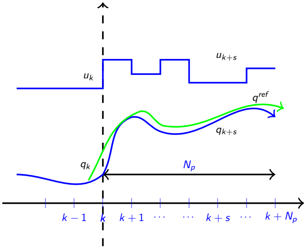

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Mixed-Integer Programming (MIP)

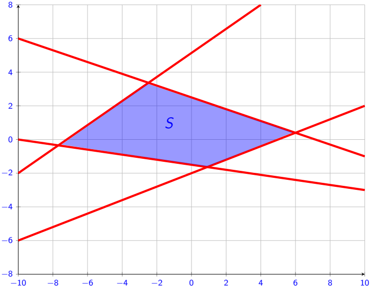
Grundel et al. (2007), Jünger et al. (2009)

- Flexible mathematical model for the formulation of decision and control problems based on optimization
  - combinatorial allocation problem
  - multicast routing problem
- Flexible mathematical model for the formulation of collision avoidance problems involving the control of Multi-Agent Systems
  - path following with obstacle and collision avoidance
  - formation control with collision avoidance
- Fast off-the-shelf solvers available
  - CPLEX, OSL, etc.
- Strong theoretical foundations
  - characterization of tractable special cases
  - NP-hard in general, but can also solve many large problems in practice

MIP – multi-agent context

Consider a bounded polyhedral set

$$S = \left\{x \in \mathbb{R}^n : h_i x \leq k_i,\ i = 1 : N\right\}$$



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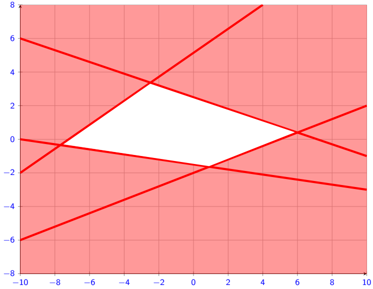
$$S = \left\{ x \in \mathbb{R}^n : h_i x \leq k_i,\ i = 1 : N \right\}$$

Consider the complement of S

$$\mathcal{C}(S) \triangleq cl(\mathbb{R}^n \setminus S) = \bigcup_i \mathcal{R}^-(\mathcal{H}_i), \quad i = 1 : N$$



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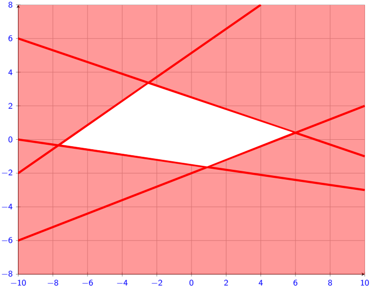
$$\mathcal{C}(S) \triangleq cl(\mathbb{R}^n \setminus S) = \bigcup_i \mathcal{R}^-(\mathcal{H}_i), \quad i = 1 : N$$

Define $\mathcal{C}(S)$ in a linear representation

$$-h_i x \leq -k_i + M\alpha_i, \quad i = 1 :\ N$$

$$\sum_{i=1}^{i=N} \alpha_i \leq N - 1$$

with $(\alpha_1, \ldots, \alpha_N) \in \{0,1\}^{N}$



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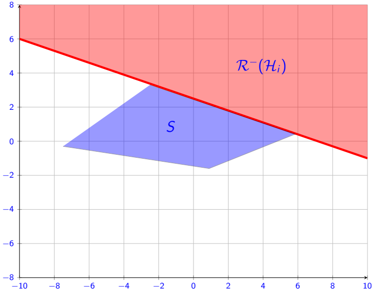


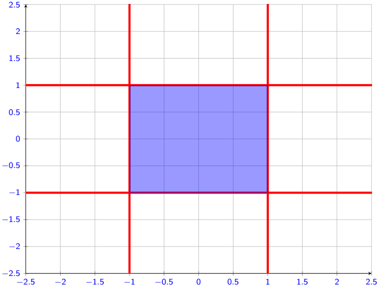
Any of the regions $\mathcal{R}^-(\mathcal{H}_i)$ of $\mathcal{C}(S)$ can be obtained by a suitable choice of binary variables

$$\mathcal{R}^-(\mathcal{H}_i) \longleftrightarrow (\alpha_1, \ldots, \alpha_N)^i \triangleq (1, \ldots, 1, \underbrace{0}_{i}, 1, \ldots, 1)$$

MIP representation – Illustrative example

Consider a polytope $P \subset \mathbb{R}^2$ given by

$$\begin{bmatrix} -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \end{bmatrix} x \leq \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

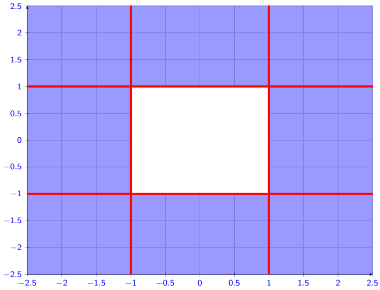


MIP representation – Illustrative example

and its complement $\mathcal{C}(P)$ by

$$\begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} x \leq \begin{bmatrix} -1 + M\alpha_1 \\ -1 + M\alpha_2 \\ -1 + M\alpha_3 \\ -1 + M\alpha_4 \end{bmatrix}$$

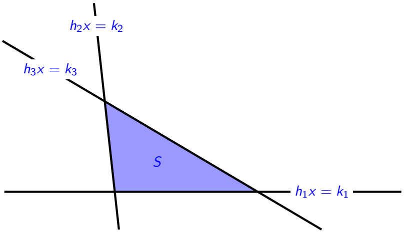
in the classical mixed-integer formulation.



MIP representation – Illustrative example

Consider a triangle from $\mathbb{R}^2$ given by

$$\begin{aligned} h_1 x &\leq k_1 \\ h_2 x &\leq k_2 \\ h_3 x &\leq k_3 \end{aligned}$$

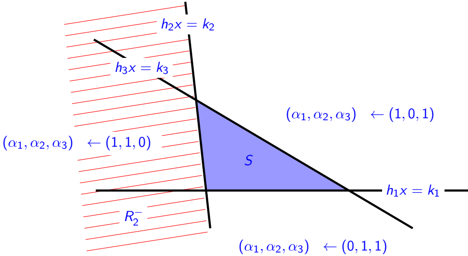


MIP representation – Illustrative example

and its complement

$$\begin{aligned} -h_1 x &\leq -k_1 + M\alpha_1 \\ -h_2 x &\leq -k_2 + M\alpha_2 \\ -h_3 x &\leq -k_3 + M\alpha_3 \end{aligned}$$

in the classical mixed-integer formulation.



MIP representations

Do we really need N binary variables for representing the complement of a convex region?

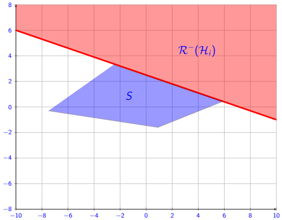
Logarithmic representation

For each region $\mathcal{R}^-(\mathcal{H}_i)$ a unique combination of binary variables $\lambda^i \in \{0,1\}^{\lceil log_2 N \rceil}$ is associated. Then, the affine functions $\alpha_i : \{0,1\}^{\lceil log_2 N \rceil} \to \{0\} \cup [1, \infty)$ are constructed:

$$\alpha_i(\lambda) = \sum_{k=0}^{\lceil log_2 N \rceil} \left(\lambda_k^i + (1 - 2\lambda_k^i) \cdot \lambda_k \right).$$

λ_k denotes the kth component of λ and λ_k^i its value for the tuple associated to region $\mathcal{R}^-(\mathcal{H}_i)$:

$$\alpha_i(\lambda) = \begin{cases} 0, & \text{only if } \lambda = \lambda^i \\ \geq 1, & \text{for any } \lambda \neq \lambda^i \end{cases}$$

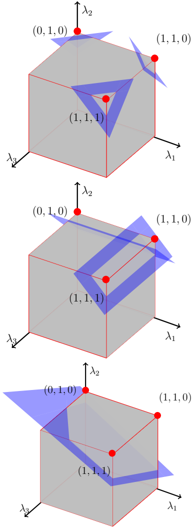


which leads to the compact formulation

$$-h_i x \leq -k_i + M\alpha_i(\lambda), \quad i = 1 : N,$$
$$0 \leq \beta_l(\lambda).$$

Interdicted tuples

In the mixed-integer representation we interdict tuples which describe the obstacle:



- in the classical formulation we force that at least one constraint is active:

$$\sum_{i=1}^{i=N} \alpha_i \leq N - 1$$

- in the logarithmic formulation
  - multiple constraints to interdict tuples Prodan et al. (2012b)

$$0 < \beta_l(\lambda)$$

  - if the allocated tuples are ordered a single constraint suffices Afonso and Galvão (2013)

Illustrative example

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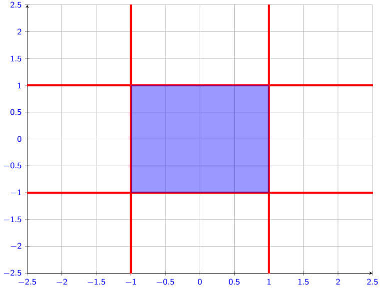
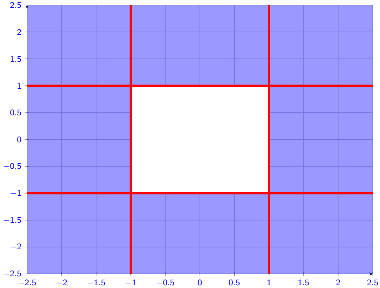


Illustrative example

and its complement $\mathcal{C}(P)$ by

$$\begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} x \leq \begin{bmatrix} -1 + M\alpha_1 \\ -1 + M\alpha_2 \\ -1 + M\alpha_3 \\ -1 + M\alpha_4 \end{bmatrix}$$

in the classical MI formulation.

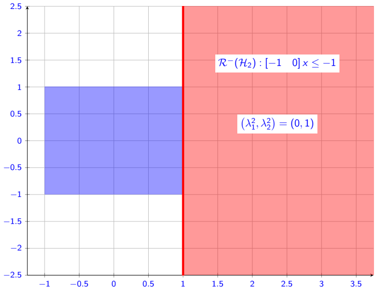


Illustrative example

and its complement $\mathcal{C}(P)$ by

$$\begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} x \leq \begin{bmatrix} -1 + M(\quad \lambda_1 + \lambda_2) \\ -1 + M(1 - \lambda_1 + \lambda_2) \\ -1 + M(1 + \lambda_1 - \lambda_2) \\ -1 + M(2 - \lambda_1 - \lambda_2) \end{bmatrix}$$

in the reduced MI formulation.



In the reduced representation only $N_0 = \lceil \log_2 4 \rceil = 2$ binary variables are needed.

For region $\mathcal{R}^-(\mathcal{H}_2)$ associate tuple $(\lambda_1^2, \lambda_2^2) = (0, 1)$ which leads to the mapping

$$\alpha_2 = 1 + \lambda_1 - \lambda_2$$

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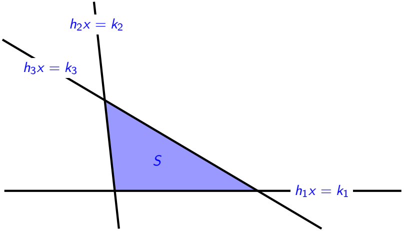

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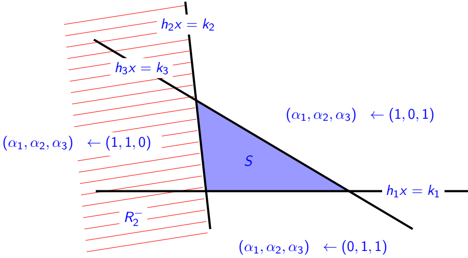

Illustrative example

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$$\begin{aligned} -h_1x &\leq -k_1 + M(\lambda_1 + \lambda_2) \\ -h_2x &\leq -k_2 + M(1 - \lambda_1 + \lambda_2) \\ -h_3x &\leq -k_3 + M(1 + \lambda_1 - \lambda_2) \end{aligned}$$

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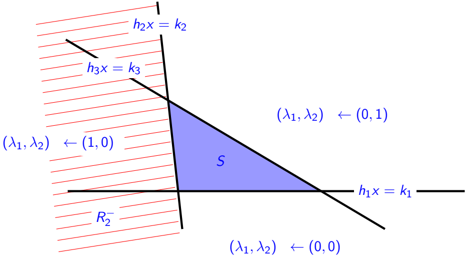

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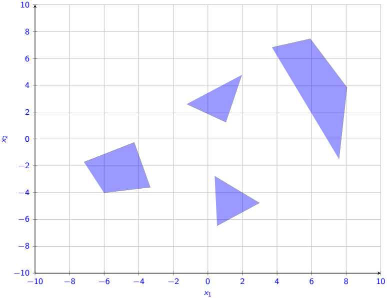
$$\alpha_2(\lambda) = 1 - \lambda_1 + \lambda_2$$

Non-connected and non-convex regions

Consider the complement $\mathcal{C}(\mathbb{S}) = cl(\mathbb{R}^n \setminus \mathbb{S})$ of a union of polyhedral sets $\mathbb{S} = \bigcup_l S_l$

$$\mathcal{A}(\mathbb{H}) = \bigcup_{l=1,\ldots,\gamma(N)} \underbrace{\left(\bigcap_{i=1}^{N} R^{\sigma_l(i)}(\mathcal{H}_i) \right)}_{A_l}$$



[0] Prodan I., Stoican F., Olaru S. and Niculescu S-I. (2016): Mixed-Integer Representations in Control Design, SpringerBriefs in Control, Automation and Robotics Series, Springer.

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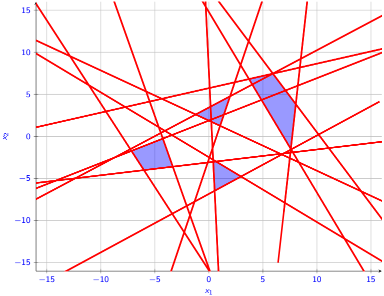
$$
\begin{aligned}
&\vdots\\
A_l &\begin{cases} \sigma_l(1)h_1 x & \leq \sigma_l(1)k_1 + M\alpha_l(\lambda) \\ & \vdots \\ \sigma_l(N)h_N x & \leq \sigma_l(N)k_N + M\alpha_l(\lambda) \end{cases}\\
&\vdots\\
&0 \leq \beta_l(\lambda)
\end{aligned}
$$



Using the hyperplanes $\mathcal{H}_i$ we partition the space into disjoint cells A_l and we associate a linear combination of binary variables $\alpha_l(\lambda)$ to each cell.

[0] Prodan I., Stoican F., Olaru S. and Niculescu S-I. (2016): Mixed-Integer Representations in Control Design, SpringerBriefs in Control, Automation and Robotics Series, Springer.

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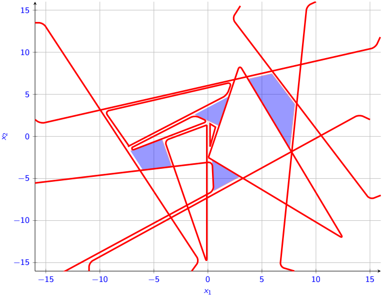
\end{array}
$$



The number of cells can be reduced through merging procedures.

[0] Prodan I., Stoican F., Olaru S. and Niculescu S-I. (2016): Mixed-Integer Representations in Control Design, SpringerBriefs in Control, Automation and Robotics Series, Springer.

Obstacle avoidance problems

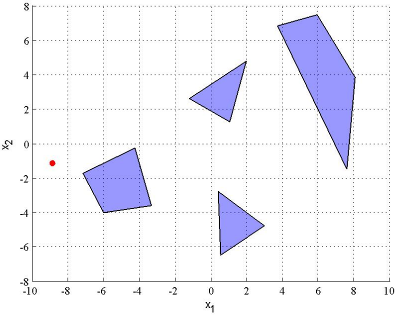
Consider a dynamical agent characterized by the LTI dynamics:

$$x(k+1) = Ax(k) + Bu(k), \quad y(k) = Cx(k),$$

with $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$ and $y(k) \in \mathbb{R}^p$ the agent state, input and output, respectively.

Collision avoidance condition:
For any obstacle S_l and an agent characterized by its dynamical state $x(k)$ we have:

$$\{x(k)\} \cap S_l = \emptyset, \quad \forall l = 1 \ldots N_o.$$



[0] (Stoican et al., ECC'13)

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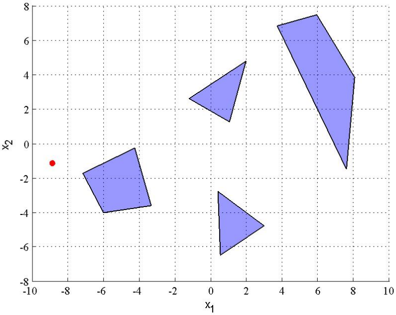
MIP representation of the feasible space:

- 14 hyperplanes
- 106 regions obtained with hyperplane arrangements
- 10 cells describing the interdicted regions
- 96 cells describing the feasible region
- $N_0 = 4$ the number of the binary variables



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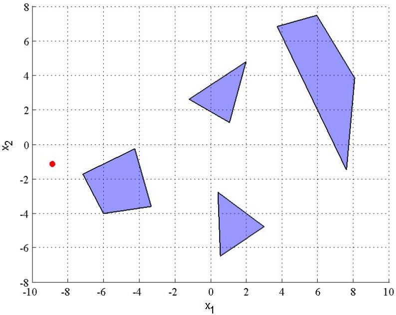
Solve the MIQP optimization problem over a finite prediction horizon:

$$u^* = \arg \min_{u(k),\dots u(k+N_p-1)} \sum_{i=0}^{N_p-1} \|x(k+i+1)\|_Q + \|u(k+i)\|_R,$$

$$\begin{aligned} \text{s.t. } & x(k+i+1) = Ax(k+i) + Bu(k+i), \\ & y(k+i) \in \mathcal{Y}, \; u(k+i) \in \mathcal{U}, \\ & x(k+i+1) \notin \mathbb{S}, \quad l = 1 \dots N_o. \end{aligned}$$



Conclusion: 72% complexity reduction of binary variables.

[0] (Stoican et al., ECC'13)

Obstacle and collision avoidance example

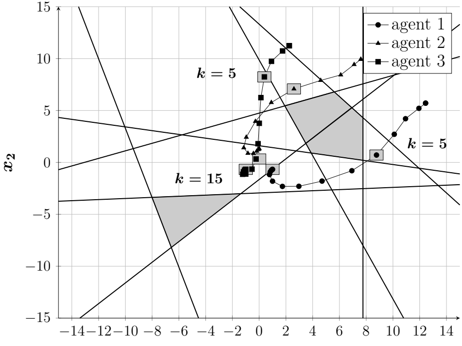
Collision avoidance conditions:

1. for any obstacle S_l and any agent characterized by its dynamical state $x_i(k)$ and the associated safety region S_i^a, the collision avoidance conditions are:

$$(\{x_i(k)\} \oplus S_i^a) \cap S_l = \emptyset, \quad \forall i = 1 \dots N_a, \ \forall l = 1 \dots N_o.$$

2. for any two agents characterized by their dynamical states $x_i(k)$, $x_j(k)$ and their associated safety regions S_i^a, S_j^a, the collision avoidance conditions are:

$$(\{x_i(k)\} \oplus S_i^a) \cap \left(\{x_j(k)\} \oplus S_j^a\right) = \emptyset, \quad \forall i,j = 1 \dots N_a, \ i \neq j.$$



Obstacle and collision avoidance example

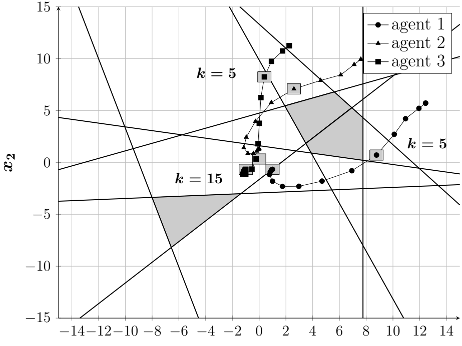
Collision avoidance conditions:

1. for any obstacle S_l and any agent characterized by its dynamical state $x_i(k)$ and the associated safety region S_i^a, the collision avoidance conditions are:

$$x_i(k) \notin (\{-S_i^a\} \oplus S_l), \quad \forall i = 1 \dots N_a, \ \forall l = 1 \dots N_o,$$

2. for any two agents characterized by their dynamical states $x_i(k)$, $x_j(k)$ and their associated safety regions S_i^a, S_j^a, the collision avoidance conditions are:

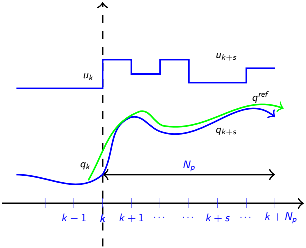
$$x_i(k) - x_j(k) \notin \left(\{-S_i^a\} \oplus S_j^a\right), \quad \forall i, j = 1 \dots N_a, \ i \neq j.$$



Model Predictive Control (MPC)

Propoi (1963), Cutler et al. (2007), Richalet and O'Donovan (2009)

- Optimization-based control law
- Implicit (on-line) vs. explicit (off-line) implementation
- Constraints handling
- Can be implemented in a distributed fashion



model of the system

reference trajectory

$$\arg\min_{\mathbf{u}} \; V_f(x(k+N_p), q(k+N_p)) + \sum_{s=1}^{N_p-1} V_n(x(k+s), u(k+s), q(k+s))$$

$$\text{subject to: } \begin{cases} x(k+s+1) = f(x(k+s), u(k+s)), & s = 0 : N_p-1, \\ h(x(k+s), u(k+s), q(k+s)) \leq 0, & s = 1 : N_p-1. \end{cases}$$

constraint sets representation

optimization solver

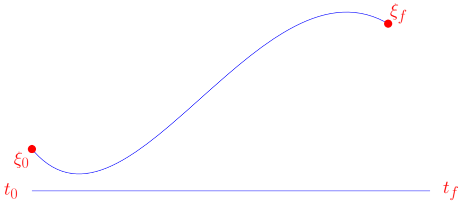
Reference trajectory generation

Consider the system

$$\dot{x}(t) = f(x(t), u(t)),$$

with $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$.

How we can generate a feasible reference trajectory $(\xi^{ref}(t), u^{ref}(t))$ that steers the model from an initial state $\xi^{ref}(t_0)$ to a final state $\xi^{ref}(t_f)$, over a fixed time interval $[t_0, t_f]$?



Reference trajectory generation – differential flatness

Consider the continuous nonlinear system

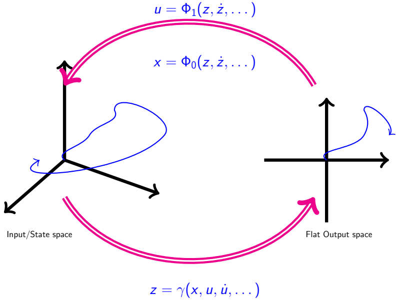
$$\dot{x}(t) = f(x(t), u(t)),$$

it is called differentially flat if there exist $z(t)$ s.t. the states and inputs can be algebraically expressed in terms of $z(t)$ and a finite number of its derivatives (Fliess et al. (1995)):

$$x(t) = \Phi_0(z(t), \dot{z}(t), \cdots, z^{(q)}(t)),$$
$$u(t) = \Phi_1(z(t), \dot{z}(t), \cdots, z^{(q+1)}(t)),$$

where

$$z(t) = \gamma(x(t), u(t), \dot{u}(t), \cdots, u^{(q)}(t)).$$



- For any linear and nonlinear flat system, the number of flat outputs equals the number of inputs Lévine (2009)
- For linear systems, the flat differentiability is implied by the controllability property Sira-Ramírez and Agrawal (2004)

Flat trajectory – Constructive details

Differentially flat systems are well suited to problems requiring trajectory planning $\Rightarrow$ it reduces the problem of trajectory generation to finding a trajectory of the flat outputs:

- assume an interval $t \in [0, T]$
- boundary conditions $x(0) = x_0, x(T) = x_f$, $u(0) = u_0, u(T) = u_f$
- choose a basis function $\Lambda(t) = [\ldots \Lambda^i(t) \ldots]$
- parametrize the flat output $z(t) = \sum_{i=1}^{N_\alpha} \alpha_i \Lambda^i(t)$
- and its derivatives $z^{(q)}(t) = \sum_{i=1}^{N_\alpha} \alpha_i \Lambda^{(q)}(t)$
- obtain coefficients α_i from the boundary conditions
- go back to $x(t)$ and $u(t)$

Constructive details – II

There are several issues:

- there are many basis functions but not all of them all well-suited
  - polynomials (t^i): poor numerical performance, their dimension depends on the number of conditions imposed on the inputs, states and their derivatives
  - Bésier basis functions: their dimension depends on the number of conditions imposed on the inputs, states and their derivatives
  - B-spline basis functions: their degree depends only up to which derivative is needed to ensure continuity
- state and input constraints are not enforced: we impose constraints at the boundaries (or even in intermediate points) but what happens “in-between” ?
- shortcomings which are not usually taken into account:
  - imposition of avoidably strict constraints;
  - the trajectory passing through two consecutive way-points intersects one (or more) obstacles.

Example for and UAV dynamics

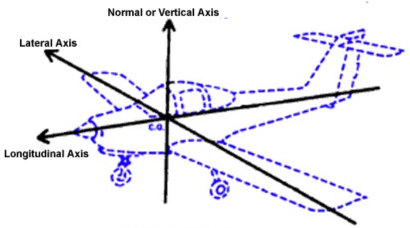
Bencatel et al. (2011)

Consider the 3-DOF kinematic model

$$\dot{x}(t) = V_a(t)\cos\Psi(t) + W_x$$
$$\dot{y}(t) = V_a(t)\sin\Psi(t) + W_y$$
$$\dot{\Psi}(t) = \frac{g\ \tan\Phi(t)}{V_a(t)}$$

described generally by

$$\dot{\xi}(t) = f(\xi(t), u(t))$$



The Axes of an Airplane

with $\xi(t) = \left[x^T(t)\ y^T(t)\ \Psi^T(t)\right]^T \in \mathbb{R}^3$ the state vector, $u = \left[V_a^T(t)\ \Phi^T(t)\right]^T \in \mathbb{R}^2$ the input vector and W_x, W_y the wind velocity components.

Specifications:

- air relative velocity $V_a \in [18, 25]$ m/s;
- bank angle $\Phi \in [-0.43, 0.43]$ rad;
- rate of change of V_a is limited to the maximum acceleration the aircraft can produce, i.e., $0.1 \sim 0.2$ m/s^2;
- variation of Φ is limited to $0.5 \sim 1.1$ rad/s.

Illustrative example

3-DOF model of an airplane in which the autopilot forces coordinated turns (zero side-slip) at a fixed altitude:

$$\dot{x}(t) = V_a(t)\cos\Psi(t),$$
$$\dot{y}(t) = V_a(t)\sin\Psi(t),$$
$$\dot{\Psi}(t) = \frac{g \ \tan\Phi(t)}{V_a(t)}$$

$$\Psi(t) = \arctan\left(\frac{\dot{z}_2(t)}{\dot{z}_1(t)}\right),$$
$$V_a(t) = \sqrt{\dot{z}_1^2(t) + \dot{z}_2^2(t)},$$
$$\Phi(t) = \arctan\left(\frac{1}{g}\frac{\ddot{z}_2(t)\dot{z}_1(t) - \dot{z}_2(t)\ddot{z}_1(t)}{\sqrt{\dot{z}_1^2(t) + \dot{z}_2^2(t)}}\right).$$

with

- states are the position $(x(t), y(t))$ and the heading (yaw) angle $\Psi(t) \in [0, 2\pi]$ rad
- inputs signals are the airspeed velocity $V_a(t)$ and the bank (roll) angle $\Phi(t)$
- $z(t) = \begin{bmatrix} z_1(t) & z_2(t) \end{bmatrix}^T = \begin{bmatrix} x(t) & y(t) \end{bmatrix}^T$ is the flat output

Flat trajectory generation

Find a reference trajectory $(\xi^{ref}(t), u^{ref}(t))$ that steers the model from an initial state $\xi^{ref}(t_0)$ to a final state $\xi^{ref}(t_f)$, over a fixed time interval $[t_0, t_f]$:

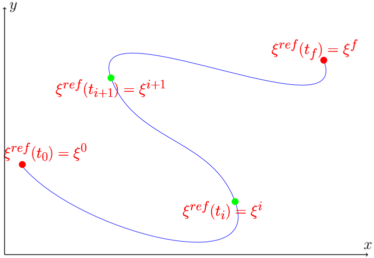
$$\begin{cases} \xi^{ref}(t) &= \eta_0(z(t), \dot{z}(t)), \\ u^{ref}(t) &= \eta_1(z(t), \dot{z}(t), \ddot{z}(t)), \end{cases} \quad \text{with } z(t) = [z_1(t)\ z_2(t)]^T \in \mathbb{R}^2 \text{ the flat output.}$$

The corresponding reference state and input for the system are:

$$\begin{cases} \xi^{ref}(t) &= \begin{bmatrix} z_1(t) & z_2(t) & \arctan\left(\frac{\dot{z}_2(t)}{\dot{z}_1(t)}\right) \end{bmatrix}^T, \\ u^{ref}(t) &= \begin{bmatrix} \sqrt{\dot{z}_1^2(t) + \dot{z}_2^2(t)} & \arctan\left(\frac{1}{g}\frac{\ddot{z}_2(t)\dot{z}_1(t) - \dot{z}_2(t)\ddot{z}_1(t)}{\sqrt{\dot{z}_1^2(t) + \dot{z}_2^2(t)}}\right) \end{bmatrix}^T, \end{cases}$$

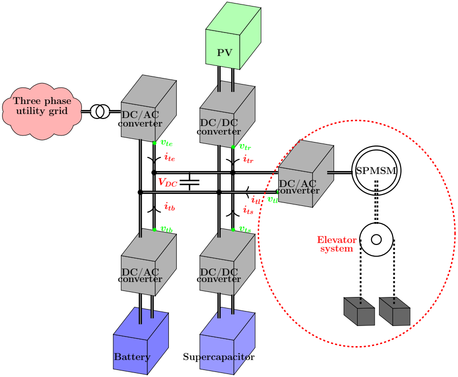
Introduce a set of way-points through which the vehicle must pass:

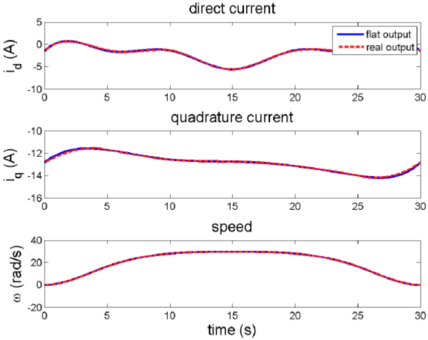
$$\mathbb{P} \triangleq \{p^i = (\xi^i, u^i), \quad i = 0 : N_w\}.$$



Example for the DC microgrid system

Generation of a speed profile for the elevator system Hung Pham et al. (2015):





[0] Pham, T., I. Prodan, D. Genon-Catalot et L. Lefèvre: *Efficient energy management for an elevator system under a constrained optimization framework*, in Proceedings of the 19th IEEE International Conference on System Theory, Control and Computing, pp. 613–618, 2015.

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