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PX505

BioSwarm
Innovation Project

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1. Introduction

Autonomous TurtleBots are compact, versatile robots designed to navigate environments independently while avoiding obstacles and creating detailed maps. Equipped with advanced sensors such as LiDAR, cameras, and ultrasonic sensors, these robots detect and respond to their surroundings in real-time. Using algorithms like Simultaneous Localization and Mapping (SLAM), TurtleBots generate accurate representations of a room's layout, which can be used for navigation and environmental analysis. Their ability to adapt to various scenarios makes them ideal for applications in research, education, and industries requiring dynamic spatial awareness and autonomous operation.

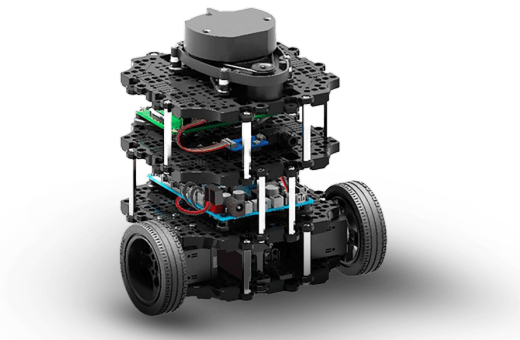


Figure 1.1.: TurtleBot

We implemented the Model Predictive Control (MPC) algorithm that enables precise obstacle avoidance and reference trajectory tracking by predicting the robot's future states and optimizing its movements accordingly. This implementation was carried out using MATLAB for algorithm design and ROS 2 (Robot Operating System) for real-time control and communication with the hardware. The integration of MPC with MATLAB and ROS 2 allows the turtlebots to efficiently navigate through dynamic environments.

2. Model Predictive Control

We have implemented the control algorithm using MATLAB and performed multiple tests, so that we can ensure its stability and performance.

1. Mathematical model

The dynamics of the robot and the vector command vectors are:

$$\zeta = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \quad \dot{\zeta} = \begin{bmatrix} V \cos \theta \\ V \sin \theta \\ \omega \end{bmatrix} \quad u = \begin{bmatrix} V \\ \omega \end{bmatrix} \quad (2.1)$$

Here, V stands for linear velocity and ω stands for angular velocity. This is how the system behaves or reacts under different conditions or inputs. This is a nonlinear function and hence we had to implement a Nonlinear MPC to predict its future behavior.

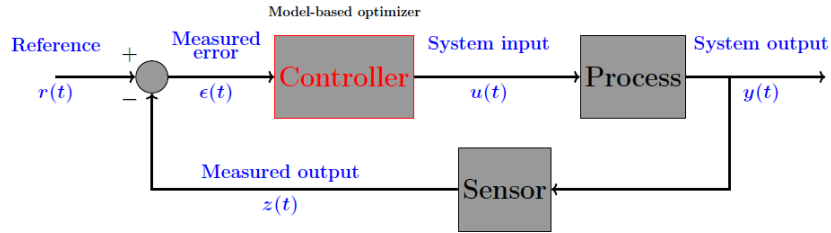


Figure 2.1.: Feedback Loop

2. Model Predictive Control Algorithm

This algorithm uses the model of the system to predict its future behavior and make optimal control decisions based on these predictions. In order to determine the future actions that the robot has to take, we have to look into a set of period of time ahead for which we make predictions about the state of the robot. This future time period is called the *prediction horizon*. We will choose the most desirable control action to input to the robot. The optimal control is the one that best achieves its goals for the system while adhering to certain given constraints.

We can estimate future states from current state as it follows, where we would introduce the *sampling time* period because a continuous based system provides outputs at every moment, while the discrete system provides them at (usually) equally divided samples of

time:

$$x(k+1) = x(k) + T_s * V(k) \cos \theta(k) \quad (2.2)$$

$$y(k+1) = y(k) + T_s * V(k) \sin \theta(k) \quad (2.3)$$

$$\theta(k+1) = \theta(k) + T_s * \omega(k) \quad (2.4)$$

Each future predicted state can be computed as:

$$x(k+1) = x(k) + T_s * f_{dynamics}(x(k), u(k)) \quad (2.5)$$

Where k represents one time step as $t = kT_s$ in the control horizon.

The predictive control feedback law is computed by minimizing a predicted performance cost, which is defined in terms of the predicted sequences \mathbf{u} , \mathbf{x} . The predicted cost has the general form:

$$J(x_k, \mathbf{u}_k) = \sum_{i=0}^N \left(\|x_{i|k}\|_Q^2 + \|u_{i|k}\|_R^2 \right), \quad (2.6)$$

The optimal control sequence for the problem of minimizing the predicted cost is denoted $\mathbf{u}_N^*(x_k)$ and the optimal value of the cost is $J^*(x_k) = J(x_k, \mathbf{u}_k^*)$ which is often written as J_k^* . We can rewrite the optimization cost function as:

$$\mathbf{u}_N^* = \arg \min_{\mathbf{u}_N} x_N^\top S x_N + \sum_{k=0}^{N-1} \left(x_k^\top Q x_k + u_k^\top R u_k \right), \quad (2.7)$$

$$\text{s.t.} \quad x_{k+1} = x_k + T_s * f(x_k, u_k), \quad (2.8)$$

$$x_{min} \leq x_{k+1} - x_k \leq x_{max}, \quad (2.9)$$

$$u_{min} \leq u_k \leq u_{max}. \quad (2.10)$$

3. MATLAB implementation

Firstly, we implemented the MPC controller with reference tracking, where the reference would be a point. After multiple tests, we have chosen the best weight matrices that should be semi-positive defined, and hence we added values on the main diagonal. The weight matrices are:

$$Q = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 0.0001 \end{bmatrix} R = \begin{bmatrix} 1000 & 0 \\ 0 & 1000 \end{bmatrix} \quad P = \begin{bmatrix} 1000 & 0 & 0 \\ 0 & 1000 & 0 \\ 0 & 0 & 1000 \end{bmatrix} \quad (2.11)$$

The computation time to solve the optimization problem is 0.08 seconds for a prediction horizon of 15 and a simulation horizon of 300. However, reproducing the same tests in Python did not give us the same accuracy, and we have changed the prediction horizon to 25. The reference for the command will be 0 because we do not want the control input to influence the robot.

We have improved our algorithm by creating a complete reference to track. For further tests, we tried to make the robot follow a sin and cos reference so that we ensure that it follows the trajectory. We used the matrix R as the identity matrix in MATLAB, but when we introduced this algorithm in Python, we had to change to the values from above.

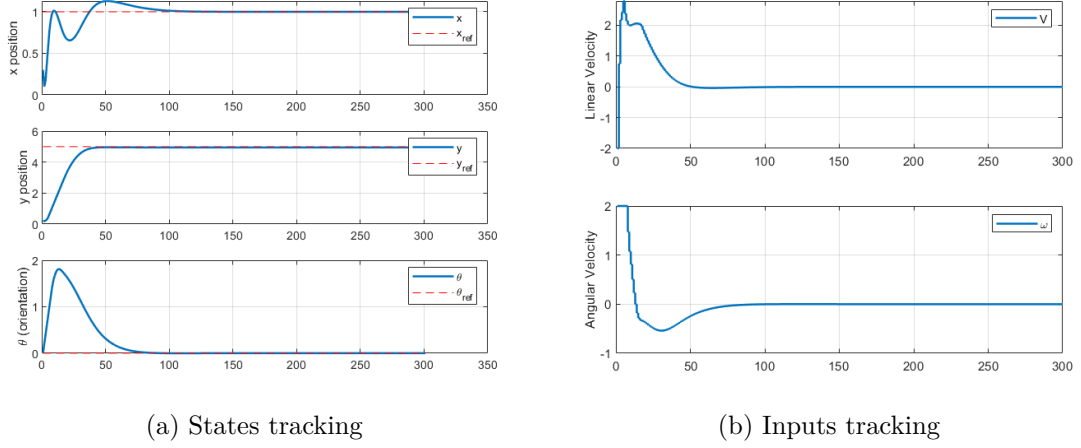


Figure 2.2.: Point tracking

As expected, the main problem encountered while working with the MPC algorithm is the computation time, which in this case is approximately 60 seconds. The results are almost accurate, and the states could follow the trajectory of sin and cos. We have tested the controller on the TurtleBot and we could see that it followed the same trajectory in real-time.

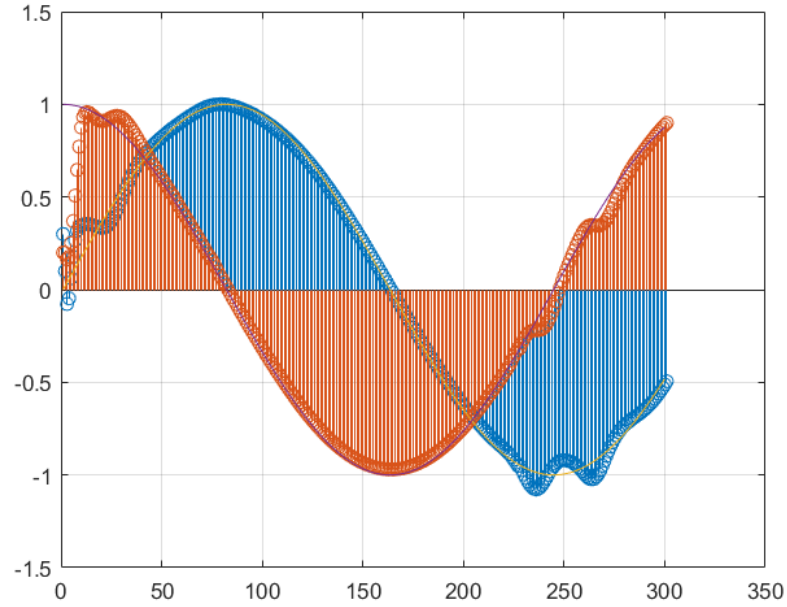


Figure 2.3.: X and Y trajectory tracking

3. ROS2 environment and implementation

We had to get used to working with the ROS2 Humble environment and prepare our setup to work with the robot.

1. Connecting with the TurtleBot

We installed the ROS2 Humble for Linux distribution 22.04 and we created our own workspace where we could add our nodes. For the installations, we followed the tutorial on ROS, and hence we got in depth with the operating system.

In our setup, we utilized a Virtual Machine (VM) and connected to the TurtleBot via SSH. To enable network connectivity, configure the VM's network as NAT (Network Address Translation). This setup allows the VM to access external networks by routing traffic through the host machine's network interface. On the host machine, create a hotspot using the name and password specified in the TurtleBot's network configuration file located at `/etc/netplan/50-cloud-init.yaml`. For our configuration, the credentials are: name = ubuntu and password = turtlebot. Establish an SSH session to the TurtleBot using the command: `ssh ubuntu@<ip_of_turtlebot>`. The TurtleBot's IP address can either be fixed or dynamically assigned and can be identified through the hotspot connections.

2. Reference tracking

2.1. Creating the map

As mentioned previously, we have implemented the MPC controller with reference tracking. The trajectory was created based on a world map of the TurtleBot from gazebo environment. The map would be created thorough SLAM algorithm. In order to create and save the map, the robot had to be moved through every corner of it and we could see the dimensions and obstacles of the map in the RVIZ application. The commands to map the world and to save it are:

```
1 # In the first terminal, launch the world that you want to map, for
   # example, we will use the world launch:
2 $ ros2 launch turtlebot3_gazebo turtlebot3_world.launch.py
3
4 # In the 2nd terminal start the Navigation 2 stack. Note: if you are
   # using Gazebo, add "use_sim_time:=True" to use the Gazebo time. If
   # using the real robot, skip this argument.
5 $ ros2 launch nav2_bringup navigation_launch.py use_sim_time:=True
6
7 # In the 3rd terminal, start the slam_toolbox. Note: same as previously,
   # if using Gazebo, add the argument to use the simulation time.
8 $ ros2 launch slam_toolbox online_async_launch.py use_sim_time:=True
9
10 # In the 4th terminal strat RViz:
```

```

11 $ ros2 run rviz2 rviz2 -d /opt/ros/humble/share/nav2_bringup/rviz/
    nav2_default_view.rviz
12 # To make the robot run, use the teleop command in a 5th terminal
13
14 # To save the map, you can use:
15 $ ros2 run nav2_map_server map_saver_cli -f my_map

```

After saving the map, we refined its contours to make it compatible with the OpenCV library in Python. Using the GIMP application, we adjusted the image and defined polytopes for the obstacles. This allowed us to generate a trajectory that avoided these obstacles. We then implemented the Dijkstra algorithm to calculate the shortest path from the starting point to the goal, with the results displayed in the compiled image. Additionally, the map's resolution was adjusted to align with real-world dimensions, enabling us to modify point values accurately.

2.2. First map and trajectory

We have updated an existing version of the code by creating the trajectory through the midpoints of the common edge between two consecutive polytopes generated from the Dijkstra algorithm. When we first implemented the polytopes, they were square shaped. We used the *CubicSpline* to create the smooth trajectory:

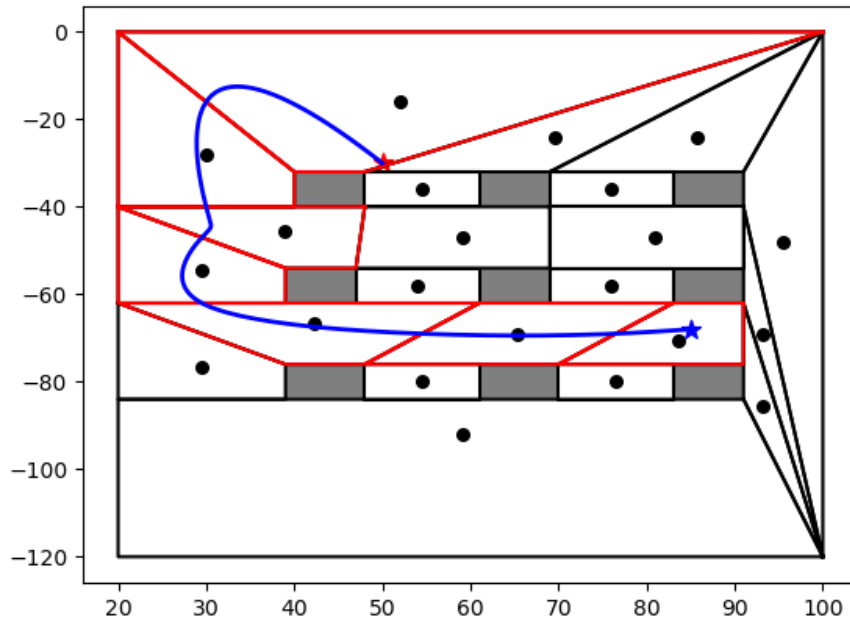


Figure 3.1.: First implementation in TurtleBot world

2.3. Developing the code

We developed the code so that we would not miss too much space near the obstacles and hence we created hexagons around the objects and we used OpenCV, as explained previously, to get the real map. The polytopes were created by getting the middle points of the circles and draw the hexagons by the contour of the objects.

As explained previously, the Dijkstra algorithm would create the fastest path between the 2 points and the path would be highlighted in red.

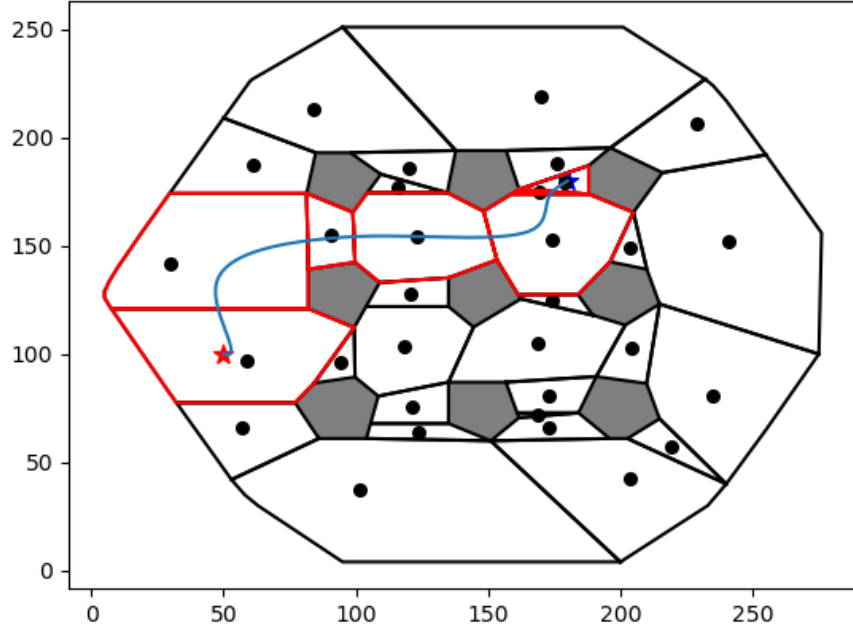


Figure 3.2.: Computing the Trajectory

2.4. ROS2 nodes

We have implemented our own node in which we call the MPC function that follows the trajectory generated by the pre-created function. This code was developed so that we can update the position with the current goal point and give it another goal point to reach. It would have to compute the algorithm in real-time and create the trajectory at each reached goal point.

2.5. Gazebo test

The first test was implemented in the TurtleBot world where we gave it the trajectory created in Fig: 3.2 and we were plotting, as the robot was moving, the states following the reference, but the computation time was affecting the decisions.

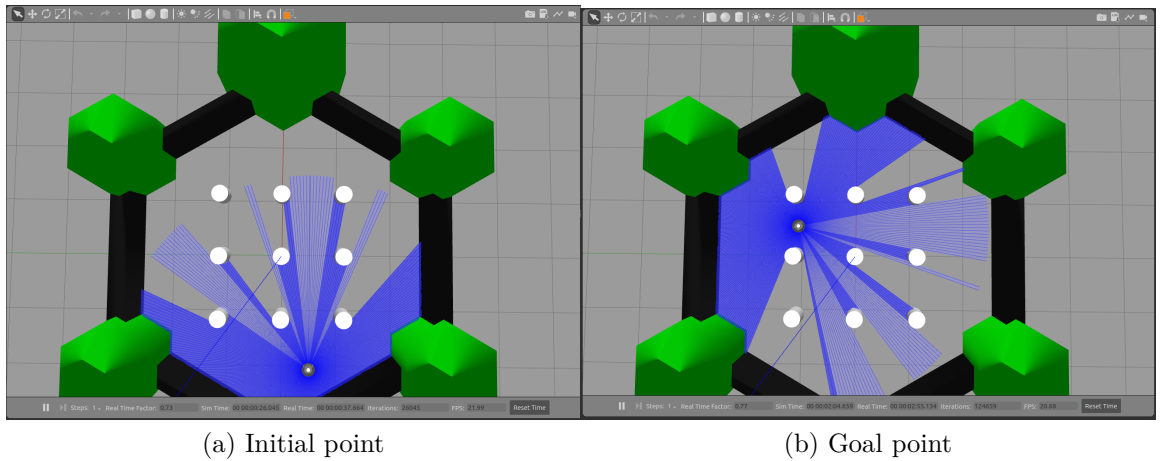


Figure 3.3.: Trajectory tracking

4. Conclusion

1. Future Directions

We used the TurtleBot to create a real-world map; however, limitations in the LiDAR sensor precision and time constraints prevented us from successfully mapping a single room. Our attempts to do so consistently resulted in errors while generating the room's contours and updating the current positions. Additionally, we encountered issues with the odometry queue, which further hindered our ability to compute the map within the available time left.

Moreover, we would want to create the trajectory using the flatness algorithm, so that we could get a better precision in obstacle avoidance.

2. Conclusion

The implementation of an MPC (Model Predictive Control) algorithm with reference tracking for obstacle avoidance on the TurtleBot demonstrated the potential of advanced control strategies in robotic navigation. By integrating reference tracking, the TurtleBot was able to dynamically adjust its trajectory to follow a desired path while effectively avoiding obstacles in its environment. The use of MPC allowed for predictive decision-making, ensuring smooth and efficient navigation by anticipating future states based on the robot's dynamics and sensor data. Despite challenges such as computational demands and real-time tuning, the approach proved robust and adaptable. This work highlights the effectiveness of MPC in achieving precise and safe navigation, laying the groundwork for further enhancements in autonomous robotics applications.

Appendices

A. MATLAB code

```
1 clc; close all; clear;
2
3 % Load CasADi
4 import casadi.*;
5
6 %% Parameters
7 N_pred_val = 25;      % Prediction horizon value
8 Q_val = 100;          % State weight value
9 R_val = 1;            % Input weight value
10 P_val = 1000;        % Terminal state value
11
12 % Define prediction and simulation steps
13 Te = 0.1;             % Sampling time
14 Npred = N_pred_val;   % Prediction horizon
15 Nsim = 300;           % Number of simulation steps
16
17 % Physical parameters
18 r = 33e-3;            % Radius of wheels
19 L = 160e-3;           % Distance between wheels
20
21 % Define system dimensions
22 dx = 3;               % State dimensions: x, y, theta
23 du = 2;               % Control dimensions: V, omega
24
25 % Initial condition
26 x0 = [0.3; 0.2; 0];   % Initial state [x; y; theta]
27 u0 = zeros(du, 1);    % Initial input [V; omega]
28
29 % Reference
30 t = 0 : 2 * pi / (Nsim + Npred) : 2 * pi;
31 xref = [sin(t); cos(t); zeros(1, length(t))];
32
33 % Constraints
34 Vmin = -10; Vmax = 10; % Velocity limits
35 omegamin = -10; omegamax = 10; % Angular velocity limits
36 xmin = -1 * 0.2 * ones(3, 1); % State limits
37 xmax = 1 * 0.2 * ones(3, 1);
38
39 % Weights for cost function
40 % Q = Q_val * eye(dx);
41 Q = diag([100, 100, 0.0001]);
42 R = R_val * eye(du);
43 P = P_val * Q;
44
45 %% CasADi nonlinear optimization
46 solver = casadi.Opti();
47
48 % Define optimization variables
49 x = solver.variable(dx, Npred + 1);
50 u = solver.variable(du, Npred);
51
52 % Define initial state as a parameter
```

```

53 xinit = solver.parameter(dx, 1);
54
55 % Nonlinear dynamics
56 f_dynamics = @(x, u) [ u(1) * cos(x(3));
57                        u(1) * sin(x(3));
58                        u(2) ];
59
60 % Add initial state constraint
61 solver.subject_to(x(:, 1) == xinit);
62
63 % Add constraints and dynamics for each prediction step
64 for k = 1 : Npred
65     % State update constraint using discretized nonlinear dynamics
66     solver.subject_to(x(:, k+1) == x(:, k) + Te * f_dynamics(x(:, k), u
(:, k)));
67     solver.subject_to(xmin <= x(:, k + 1) - x(:, k) <= xmax)
68
69     % Control input constraints
70     solver.subject_to(Vmin <= u(1, k) <= Vmax);
71     solver.subject_to(omegamin <= u(2, k) <= omegamax);
72 end
73
74 %% Define the objective function
75 % Define the solver
76 options.ipopt.print_level = 0;
77 options.ipopt.sb= 'yes';
78 options.print_time = 0;
79
80 % simulation loop
81 usim = zeros(du, Nsim);
82 xsim = zeros(dx, Nsim);
83 xsim(:, 1) = x0;
84 usim_init = u0;
85
86 t1 = tic();
87 for i = 1 : Nsim
88     objective = 0;
89     for k = 1 : Npred
90         if k ~= 1
91             objective = objective + (x(:, k) - xref(:, i + k - 1))' * Q
* (x(:, k) - xref(:, i + k - 1)) ...
92
+ (u(:, k) - u(:, k - 1))' * ...
93
R * (u(:, k) - u(:, k - 1));
94         else
95             objective = objective + (x(:, k) - xref(:, i + k - 1))' * Q
* (x(:, k) - xref(:, i + k - 1)) ...
96
+ u(:, k)' * R * u(:, k);
97         end
98     end
99
100     objective = objective + (x(:, Npred + 1) - xref(:, i + Npred))' * P
* (x(:, Npred + 1) ...
101
- xref(:, i + Npred));
102     solver.minimize(objective)
103     t2 = toc(t1);
104     solver.solver('ipopt', options)

```

```

105     solver.set_value(xinit, xsim(:, i))
106     sol = solver.solve();
107     usol = sol.value(u);
108     usim(:, i) = usol(:, 1);
109     xsim(:, i + 1) = xsim(:, i) + Te * f_dynamics(xsim(:, i), usim(:, i)
110 );
111 end
112
113 %% Plot results
114 figure
115 stem(xsim(1, :));
116 hold on
117 stem(xsim(2, :));
118 hold on
119 plot(xref(1, 1:Nsim))
120 hold on
121 plot(xref(2, 1:Nsim))
122 grid

```

B. Reference generating (taken from previous student)

```
1 import cv2
2 import numpy as np
3
4 import matplotlib.pyplot as plt
5 import gdspy
6 from poly_decomp import poly_decomp as pd
7 from matplotlib.patches import Circle, Wedge, Polygon
8 from numpy import linalg as LA
9 #import win32clipboard as clipboard
10 from scipy.sparse.csgraph import shortest_path
11 from scipy.interpolate import BSpline
12
13 def compare_2_areas_get_1(contour_1, contour_2):
14     area_1 = cv2.contourArea(contour_1)
15     area_2 = cv2.contourArea(contour_2)
16     max_ = max(area_1, area_2)
17     return max_
18
19
20 def plot_poly_map(ws, ax, col):
21     # plot the whole polytope map in subplot ax with color col
22     for i in range(len(ws)):
23         x_de = [ws[i][k][0] for k in range(len(ws[i]))]
24         x_de.append(ws[i][0][0])
25         y_de = [ws[i][k][1] for k in range(len(ws[i]))]
26         y_de.append(ws[i][0][1])
27         ax.plot(x_de, y_de, color=col)
28     return 0
29
30 # Check if two decomposed polytopes lies next to each other
31 # if the there are exactly 2 vertices of one inside the other ==> true
32 def check_consecutive_polytopes(p1, p2):
33     check = False
34     s = 0
35     check_list = gdspy.inside(p1, gdspy.Polygon(p2))
36     for i in range(len(check_list)):
37         if check_list[i]:
38             s = s + 1
39     if s == 2:
40         check = True
41     return check
42
43
44 def center(polyp):
45     xc = np.matrix(polyp)
46     return xc.mean(0)
47
48
49 def find_polyp(workspace, points): ##### check point in polygon
```

```

50     polyp_index = -1
51     for i in range(len(workspace)):
52         if (gdspy.inside([points], gdspy.Polygon(ws[i])))[0]:
53             polyp_index = i
54     return polyp_index
55
56
57 def generate_sequence(workspace, init_pts, goal_pts):
58     init_polyp = find_polyp(workspace, init_pts)
59     goal_polyp = find_polyp(workspace, goal_pts)
60     seq = [init_polyp, goal_polyp]
61     return seq
62
63 def get_path(Pr, i, j):
64     path = [j]
65     k = j
66     while Pr[i, k] != -9999:
67         path.append(Pr[i, k])
68         k = Pr[i, k]
69     return path[::-1]
70
71
72 print(cv2.__version__)
73 resolution = 0.1 # resolution in file .yaml
74 image = cv2.imread('/home/truontmh/python_code/binary_map_after_gimp.png')
75 # convert image to gray image
76 image = cv2.cvtColor(image, cv2.COLOR_BGR2GRAY)
77
78 height, width = image.shape
79 print(f'Height in real environment: {height * resolution} (m)')
80 print(f'width in real environment: {width * resolution} (m)')
81
82 # set a threshold, convert image to binary image (0 and 255)
83 # 0 is black, 255 is white
84 threshold = 127
85 _, binary_image = cv2.threshold(image, threshold, 255, cv2.THRESH_BINARY)
86 image = binary_image
87
88 ##### find contour of black pixels #####
89
90 inverted_image = cv2.bitwise_not(image) # function cv2.findContours will
    find the contours of white pixels
91 contours, _ = cv2.findContours(inverted_image, cv2.RETR_LIST, cv2.
    CHAIN_APPROX_SIMPLE)
92 print("len of contours: ", len(contours))
93
94 ##### compute areas of contours #####
95 area_list = []
96 for contour in contours:
97     area = cv2.contourArea(contour)
98     area_list.append(area)
99
100 max_area = max(area_list)
101 max_area_index = area_list.index(max_area)
102
103 area_list[max_area_index] = 0 # convert max value to 0, -> find the
    second largest value
104 second_max_area = max(area_list)
105 second_max_area_index = area_list.index(second_max_area)

```



```

106 ##### remove largest and second largest contour #####
107 filter_contours = [x for i,x in enumerate(contours) if i not in (
108     max_area_index, second_max_area_index)]
109 filter_area = [x for i,x in enumerate(area_list) if i not in (
110     max_area_index, second_max_area_index)]
111
112 ##### compute moments of contours #####
113 centroid_list = []
114 for contour in filter_contours:
115     moment = cv2.moments(contour)
116     # Compute centroids from moments
117     if moment['m00'] != 0:
118         cX = int(moment['m10']/moment['m00'])
119         cY = int(moment['m01']/moment['m00'])
120     else:
121         cX, cY = 0,0 # if the contour doesn't have area
122     centroid_list.append([cX,cY])
123
124 ##### Find contour pairs with close center points. After that,
125     chose one which have larger area.
126 thresh_distance = 5
127 get_contour_index_list = []
128 for i in range(len(centroid_list)):
129     for j in range(len(centroid_list)):
130         if i != j:
131             point_1 = np.array(centroid_list[i])
132             point_2 = np.array(centroid_list[j])
133             distance = np.linalg.norm(point_2 - point_1)
134             if distance < thresh_distance:
135                 max_ = compare_2_areas_get_1(filter_contours[i],
136 filter_contours[j])
137                 if max_ == filter_area[i] and i not in
138 get_contour_index_list:
139                     get_contour_index_list.append(i)
140                 elif max_ == filter_area[j] and j not in
141 get_contour_index_list:
142                     get_contour_index_list.append(j)
143
144 ##### find convex for contours
145 hull = []
146 hull.append(cv2.convexHull(contours[max_area_index]))
147 for index in get_contour_index_list:
148     hull.append(cv2.convexHull(filter_contours[index]))
149 ##### reduce the number of point of each hull -> reduce the
150     computations
151 approx_hull_list = []
152 for convex_hull in hull:
153     convex_hull = convex_hull.reshape(-1,1,2)
154     desired_vertices = len(convex_hull)//1.67
155     epsilon = 0.01 * cv2.arcLength(convex_hull, True)
156     approx_hull = cv2.convexHull
157     while len(approx_hull) > desired_vertices:
158         epsilon += 0.01 * cv2.arcLength(convex_hull, True)
159         approx_hull = cv2.approxPolyDP(convex_hull, epsilon, True)

```

```

159     while len(approx_hull) < desired_vertices:
160         epsilon = epsilon/1.01
161         approx_hull = cv2.approxPolyDP(convex_hull, epsilon, True)
162         approx_hull_list.append(approx_hull)
163
164
165
166
167
168 ##### Create polytope
169 obstacles_list = []
170 hole_list = []
171 hole_large_list = []
172 safety_offset = 5
173
174
175 for i in range(len(approx_hull_list)):
176     if i == 0:
177         ws_limit = np.array(approx_hull_list[0])
178         ws_limit = [tuple(x[0]) for x in ws_limit]
179     if i!=0:
180         obstacle = approx_hull_list[i]
181         obstacle = np.array(obstacle)
182         obstacle = [tuple(x[0]) for x in obstacle]
183         obstacles_list.append(obstacle)
184
185
186
187
188 ws_poly = gdsapy.Polygon(ws_limit)
189 for i in range(len(obstacles_list)):
190     hole_list.append(gdsapy.Polygon(obstacles_list[i]))
191
192 # enlarge obstacle a little bit
193 for i in range(len(hole_list)):
194     hole_large_list.append(gdsapy.offset(hole_list[i],safety_offset))
195
196 # subtraction
197 poly_with_hole = gdsapy.boolean(ws_poly, hole_large_list, "not")
198
199 # decomposition
200 ws = pd.polygonQuickDecomp(poly_with_hole.polygons[0])
201
202 # initial point and goal
203 init = [100, 50]
204 goal = [186,182]
205 n_polyp = len(ws)
206 init_goal_idx = (generate_sequence(ws, init, goal))
207 center_list = []
208 for j in range(n_polyp):
209     center_list.append(center(ws[j]).A1)
210
211
212 # print(center_list[0])
213 N = np.empty((0, n_polyp), float)
214 #print(f'ahihihhi {N.shape}')
215 N_temp = np.array([])
216 for i in range(n_polyp):
217     N_temp = []
218     for j in range(n_polyp):

```

```

219         if check_consecutive_polytopes(ws[i], ws[j]) and (i != j):
220             temp = [center_list[i], center_list[j]]
221             N_temp.append(LA.norm(temp))
222         else:
223             N_temp.append(0)
224     N = np.append(N, [N_temp], axis=0)
225     #print(N)
226     # Dijkstras algorithm
227     D, Pr = shortest_path(N, directed=False, method='D', return_predecessors
228                          =True)
229     sequence = get_path(Pr, init_goal_idx[0], init_goal_idx[1])
230     # create b_spline curve
231     sequence_point = []
232     sequence_point.append(init)
233     for i in sequence:
234         sequence_point.append(center_list[i])
235     sequence_point.append(goal)
236
237     points = np.array(sequence_point)
238     degree = 3
239     t = np.linspace(0, 1, len(points) + degree + 1 - 2*degree)
240     knots = np.concatenate(([0]*degree, t, [1]*degree))
241     bspline = BSpline(knots, points, degree)
242     t_eval = np.linspace(0, 1, 1000)
243     spline_points = bspline(t_eval)
244
245
246
247
248
249
250     fig = plt.figure()
251     ax = fig.add_subplot(111)
252     plot_poly_map(ws, ax, 'black')
253     # plot_poly_map([obstacle1, obstacle2, obstacle3], ax, 'blue')
254     plt.scatter(init[0], init[1], color='red', s=80, marker='*')
255     plt.scatter(goal[0], goal[1], color='blue', s=80, marker='*')
256     plt.scatter([center_list[i][0] for i in range(len(center_list))], [
257                 center_list[i][1] for i in range(len(center_list))],
258                 color='k', s=30)
259     plot_poly_map([ws[i] for i in sequence], ax, 'red')
260     for hole_large in hole_large_list:
261         p = Polygon(hole_large.polygons[0], facecolor = 'k', alpha = 0.5)
262         plt.gca().add_patch(p)
263     # plt.plot(x_fine, y_fine, label='Cubic Spline Trajectory', color='blue
264     #          ') # trajectory
265     plt.plot(spline_points[:, 0], spline_points[:, 1], label='B-spline
266             trajectory')
267
268
269
270     plt.show()
271
272
273
274     # Create a white image
275     white_image = image.copy()
276     for i in range(height):
277         for j in range(width):
278             white_image[i][j] = 255

```

```

275 white_image = cv2.cvtColor(white_image, cv2.COLOR_GRAY2BGR)
276
277 white_image_2 = white_image.copy()
278
279 # Show
280 cv2.drawContours(white_image, hull, -1, (255,0,0), thickness=2)
281 cv2.drawContours(white_image_2, approx_hull_list, -1, (0,0,255), thickness
    =2)
282 cv2.imshow('Image', white_image)
283 #cv2.imshow('original_image', image)
284 cv2.imshow('reduce_convex', white_image_2)
285 cv2.waitKey(0)
286 cv2.destroyAllWindows()

```