AC555
Obstacle Avoidance

Elena Pîndichi
Simon Abrial
Sinda Hichri

Université Grenoble Alpes Institut Nationale Polytechnique Ecole Supérieure en Systèmes Avancés et Réseaux

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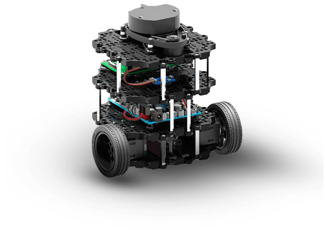
Idea

Improving the Obstacle Avoidance Algorithm:

- Generation of ellipses around the obstacles.
- Creation of moving obstacles by adding constant velocities.
- Implement the MPC controller on the TurtleBot and solving the optimziation problem.



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Obstacles

We defined each obstacle's vertices and created them using an existing library in Python, named *Polygon*.

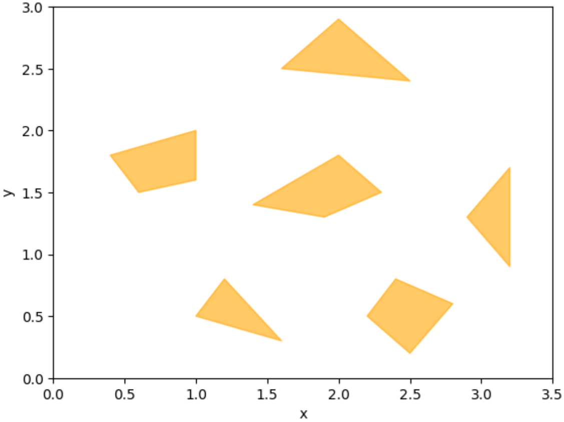
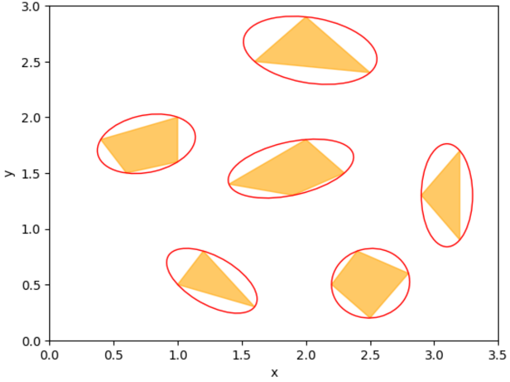


Figure: Polygons

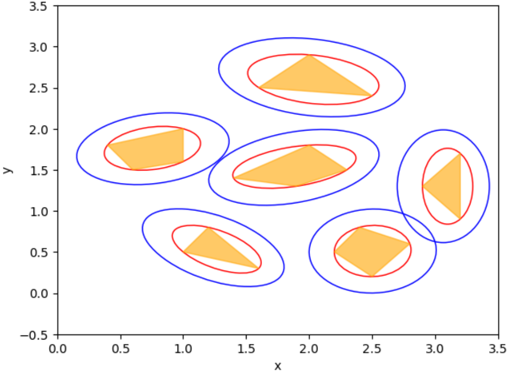
Ellipses

The Minimum Volume Enclosing Ellipsoid (MVEE) algorithm is used to find the smallest ellipsoid that encloses a given set of points in n-dimensional space. This ellipsoid is expressed as:

$$\mathcal{E} = \left\{ \mathbf{x} \in \mathbb{R}^n \mid (\mathbf{x} - \mathbf{c})^T \mathbf{A} (\mathbf{x} - \mathbf{c}) \leq 1 \right\}$$



(a) First Ellipses



(b) Enlarged Ellipses

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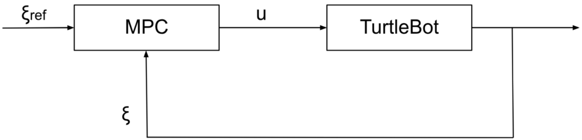
Mathematical Model

The dynamics of the robot with respect to the control input vectors are:

$$\zeta = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \qquad \dot{\zeta} = \begin{bmatrix} V\cos\theta \\ V\sin\theta \\ \omega \end{bmatrix} \qquad u = \begin{bmatrix} V \\ \omega \end{bmatrix}$$

The controller receives data concerning the current system's state and, based on a target point, set by the operator, computes the optimal control inputs.



Dynamics of the system and Cost

The dynamics are nonlinear and the *sampling period* is used as the discrete system provides the state at equally divided samples of time ($T_s = 0.1$):

$$\begin{aligned} x(k+1) &= x(k) + T_s * V(k)\cos\theta(k) \\ y(k+1) &= y(k) + T_s * V(k)\sin\theta(k) \\ \theta(k+1) &= \theta(k) + T_s * \omega(k) \end{aligned}$$

The predictive control feedback law is computed by minimizing a predicted performance cost, which is defined in terms of the predicted sequences $\mathbf{u}$, $\mathbf{x}$. The predicted cost has the general form:

$$J(x_k, \mathbf{u}_k) = x_N^\top P x_N + \sum_{i=0}^{N-1} \left(||x_{i|k} - x_{ref}||_Q^2 + ||u_{i|k}||_R^2 \right)$$

Optimization cost function for static obstacles

The optimal control sequence for the problem of minimizing the predicted cost is denoted $\mathbf{u}_N^*(x_k)$ and we can rewrite the optimization cost function as:

$$\mathbf{u}_N^\star = \arg\min_{u_N} \left(x_N^\top P x_N + \sum_{k=0}^{N-1} \left((x_k - x_{ref})^\top Q (x_k - x_{ref}) + u_k^\top R u_k \right) \right)$$

$$\begin{aligned} \text{s.t.} \qquad x_{k+1} &= x_k + T_s * f(x_k, u_k), \\ x_{min} \leq x_{k+1} - x_k &\leq x_{max}, \\ u_{min} \leq u_k &\leq u_{max}, \\ (\mathbf{x_{k+1}} - \mathbf{c_{obs_i}})^T \mathbf{A_{obs_i}} (\mathbf{x_{k+1}} - \mathbf{c_{obs_i}}) &> 1, \\ \mathbf{A_{obs_i}} \in \mathbb{R}^{n \times n}, \mathbf{c_{obs_i}} \in \mathbb{R}^n, i &\in N_{obstacles}. \end{aligned}$$

Optimization cost function for moving obstacles

The optimal control sequence for the problem of minimizing the predicted cost while the obstacles are moving is denoted $\mathbf{u}_N^*(x_k)$ and we can rewrite the optimization cost function as:

$$\mathbf{u}_N^\star = \arg\min_{u_N} \left(x_N^\top P x_N + \sum_{k=0}^{N-1} \left((x_k - x_{ref})^\top Q (x_k - x_{ref}) + u_k^\top R u_k \right) \right)$$

$$\begin{aligned}
\text{s.t.} \qquad x_{k+1} &= x_k + T_s * f(x_k, u_k), \\
x_{min} \leq x_{k+1} - x_k &\leq x_{max}, \\
u_{min} \leq u_k &\leq u_{max}, \\
\left(\mathbf{x_{k+1}} - \mathbf{c_{obs_{i|k}}}\right)^T \mathbf{A_{obs_{i|k}}} \left(\mathbf{x_{k+1}} - \mathbf{c_{obs_{i|k}}}\right) &> 1, \\
\mathbf{A_{obs_{i|k}}} \in \mathbb{R}^{n\times n}, \mathbf{c_{obs_{i|k}}} \in \mathbb{R}^n, i &\in N_{obstacles}.
\end{aligned}$$

Weight Matrices

The weight matrices that we have chosen after multiple tests are:

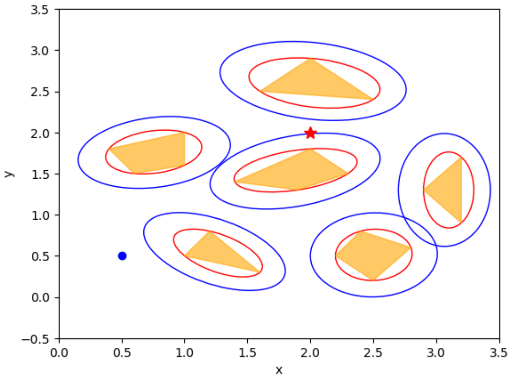
$$Q = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix} \qquad R = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \qquad P = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 100 \end{bmatrix},$$

with $N_{pred} = 100$.

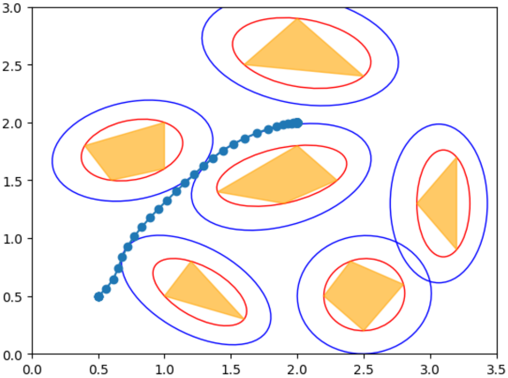
Computation time

The computation time for solving the problem is 10 seconds.

Results



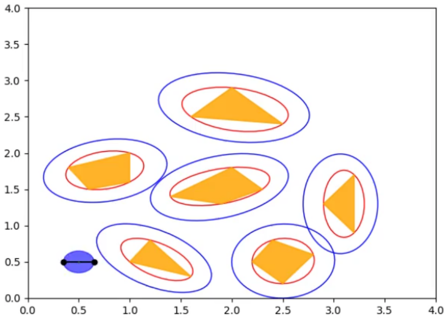
(a) Initial and Goal points



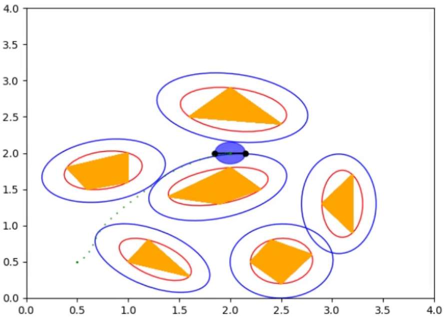
(b) Computed Trajectory

Figure: Static Obstacles

Results



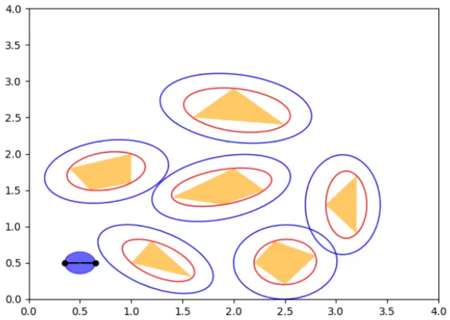
(a) Initial position



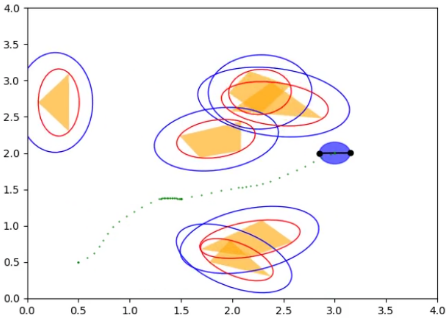
(b) Final Position

Figure: Static Obstacles

Results



(a) Initial position



(b) Final Position

Figure: Dynamic Obstacles

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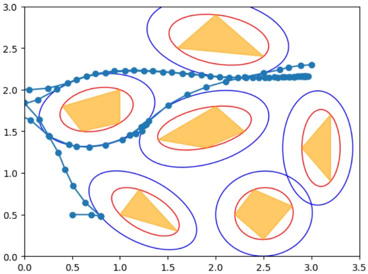
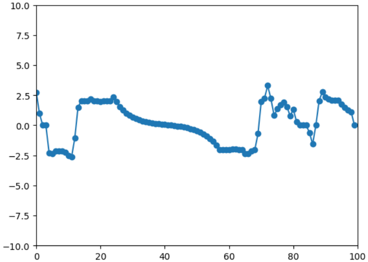
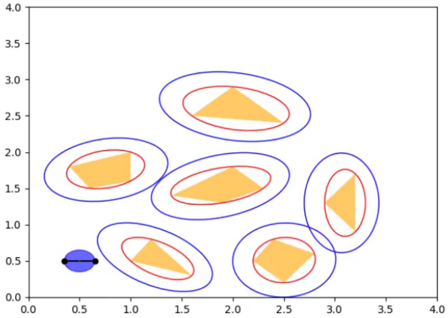
Perturbations

- Addition of noisy perturbations on the first state of x_k.
- ξ_{ref}
- Relaxing the input constraints.
- Random perturbations between [-0.1, 0.1].

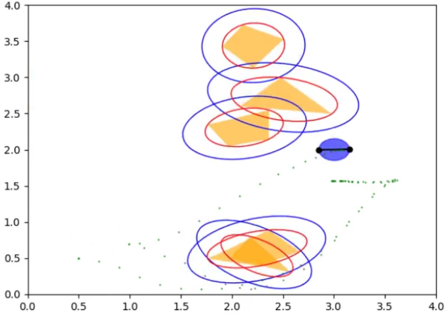




Results



(a) Initial position



(b) Final Position

Figure: Dynamic Obstacles

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Conclusion and Future Directions

Future Directions:

- Implementation of two robots that go to the same goal point.
- Making one robot follow another one while avoiding obstacles.

Conclusions:

- Implementation of two MPC problems.
- Creating an obstacle avoidance algorithm for moving objects.
- Understanding of how random perturbations might affect the behavior of the robot.

Thank you for your time!