QUANSER
INNOVATE·EDUCATE

Concept Review

Motor Modeling

Why model motors?

A motor is a fundamental actuator used in robotics and mechatronics applications requiring both rotational and/or linear actuated degree of mechanical motion. They are rather fundamental in design, and modeling the motor has several benefits from validating controllability to observing motor states that may not be available for sensing. Monitoring the behavior of a physical motor against it's digital twin in simulation can also help with identifying faults. This is also the building block upon which advanced systems are built.

Background

Consider the electromechanical system of a motor as shown in Figure 1. An RL series circuit can be used to model the internal resistance and inductance of the motor.

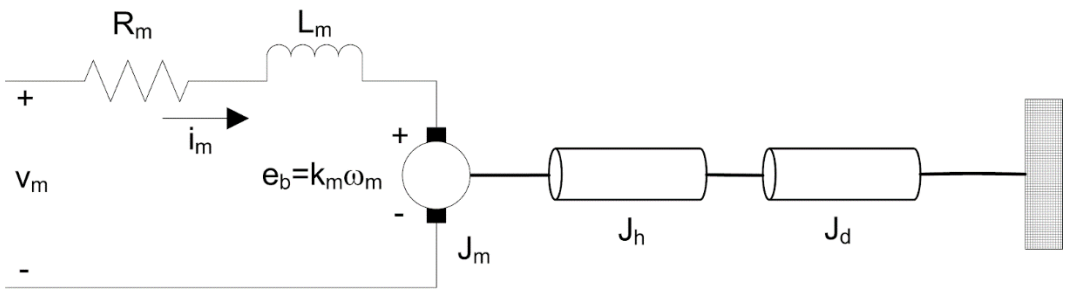

Figure 1. Typical electromechanical motor configuration

Electrical circuit analysis

In the circuit diagram, Kirchoff's laws can dictate the relationship between the command voltage applied to the motor v_m as well as the voltage drops across the resistor v_r, inductor v_l and motor armature e_b. The last term reflects the back-emf generated by the spinning motor shaft and core.

$$v_m = v_r + v_l + e_b \tag{1}$$

The voltage drop across a resistor v_r with resistance R_m relates to the current flow i_m,

$$v_r = i_m R_m \tag{2}$$

The voltage drop across an inductor v_l with inductance L_m relates to the rate of change of current,

$$v_l = \frac{di}{dt} L_m \tag{3}$$

Finally, the back-emf e_b is proportional to the speed of the motor shaft ω_m,

$$e_b = k_m \omega_m \tag{4}$$

Where k_m is the motor speed constant in Vs/rad. Substituting equations 2 to 4 back into 1 yields,

$$v_m = i_m R + \frac{di}{dt} L_m + k_m \omega_m \tag{5}$$

Note that some quantities are functions of time as shown below. The inductance term is also often dropped due to small effects on the overall model. The first relevant equation is,

$$\boldsymbol{v_m(t) = i_m(t)\, R + k_m\ \omega_m(t)} \tag{6}$$

Dynamics

Next, we can consider the dynamic equations related to the mechanical motion of the motor shaft. We start with Newton's law applied to rotational systems, The net motor torque τ_{net} is,

$$\tau_{net} = J_{eq}\dot{\omega} \tag{7}$$

Where J_{eq} is the equivalent moment of inertia of the motor shaft, hub and associated load in kgm^2, and $\dot{\omega}$ is the angular acceleration of the motor. The equivalent moment of inertia can be calculated as the sum of the individual components,

$$J_{eq} = J_m + J_h + J_d \tag{8}$$

The term J_m relates to the mass moment of inertia of motor core, shaft coils etc. The term J_h is an optional term representing a hub or attachment that holds the load to the motor shaft. The final term J_d represents the moment of inertia of the attached load.

The net torque on the motor is given by,

$$\tau_{net} = \tau_{app} - b\omega \tag{9}$$

Where τ_{app} is the applied motor torque and b represents motor damping. Combining 9 with 8 yields the second relevant equation

$$\boldsymbol{\tau_{app} = J_{eq}\dot{\omega} + b\omega} \tag{10}$$

Finally we note that the applied torque τ_{app} is directly proportional to the motor current i_m generated as a result of the applied voltage command v_m, The third relevant equation is,

$$\boldsymbol{\tau_{app} = k_t i_m} \tag{11}$$

Where k_t is the motor torque constant in Nm/A.

© 2022 Quanser Inc., All rights reserved.

Quanser Inc.
119 Spy Court
Markham, Ontario
L3R 5H6
Canada

info@quanser.com
Phone: 19059403575
Fax: 19059403576
Printed in Markham, Ontario.

For more information on the solutions Quanser Inc. offers, please visit the web site at:
http://www.quanser.com

This document and the software described in it are provided subject to a license agreement. Neither the software nor this document may be used or copied except as specified under the terms of that license agreement. Quanser Inc. grants the following rights: a) The right to reproduce the work, to incorporate the work into one or more collections, and to reproduce the work as incorporated in the collections, b) to create and reproduce adaptations provided reasonable steps are taken to clearly identify the changes that were made to the original work, c) to distribute and publicly perform the work including as incorporated in collections, and d) to distribute and publicly perform adaptations. The above rights may be exercised in all media and formats whether now known or hereafter devised. These rights are granted subject to and limited by the following restrictions: a) You may not exercise any of the rights granted to You in above in any manner that is primarily intended for or directed toward commercial advantage or private monetary compensation, and b) You must keep intact all copyright notices for the Work and provide the name Quanser Inc. for attribution. These restrictions may not be waved without express prior written permission of Quanser Inc.