MARKOV CHAIN ANALYSIS OF THE EHRENFEST URN MODEL

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MOTIVATION

Some thermodynamic processes, such as

- diffusions
- heat transfers $\quad\rightarrow\quad$ irreversible
- phase transitions

but

Newtonian mechanics $\quad\rightarrow\quad$ time-reversible

CONTENTS

The Ehrenfest urn model

Analysis via Markov chains

- limiting distribution
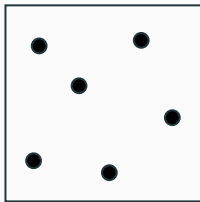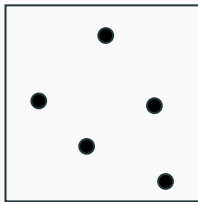- mean recurrence time

Simulation

THE EHRENFEST URN MODEL

THE EHRENFEST URN MODEL

Diffusion of a gas as stochastic process

- N particles
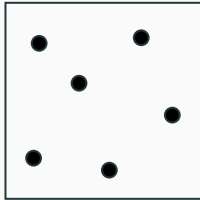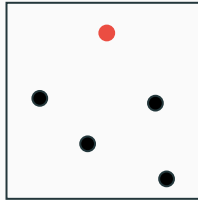- 2 boxes
- discretized time steps

A   B

At each time step:

- a ball is selected at random among the *N* possible ones
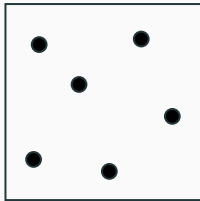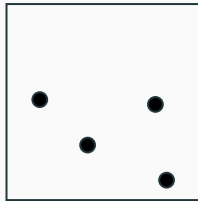

A   B

At each time step:

- a ball is selected at random among the *N* possible ones
- it is extracted from its box

A   B

At each time step:

- a ball is selected at random among the N possible ones
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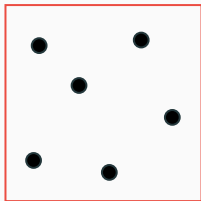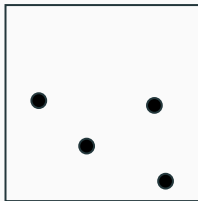- a box is selected at random

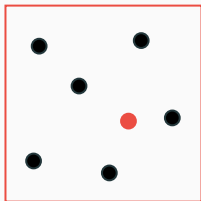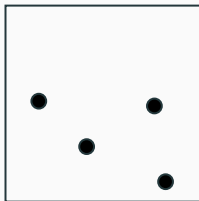

A   B

THE EHRENFEST URN MODEL

At each time step:

- a ball is selected at random among the *N* possible ones
- it is extracted from its box
- a box is selected at random
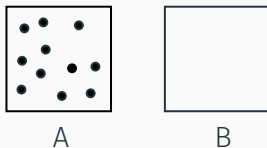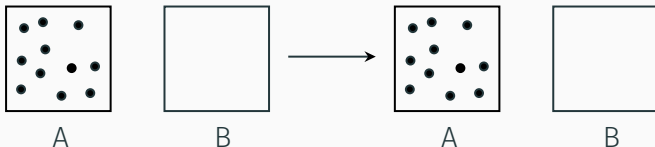- the particle is put in the selected box

A   B

Meaningful questions about equilibrium

- probability of having all the particles in box A?

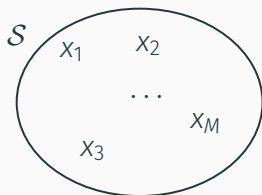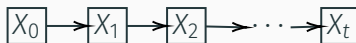- time needed for recurrence?

MARKOV CHAIN ANALYSIS

Discrete Markov chains

A mathematical theory for stochastic processes

State space

Stochastic process

Markov property

Memorylessness: only the current state influences the next transition