MARKOV CHAIN ANALYSIS OF THE EHRENFEST URN MODEL

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MOTIVATION

IRREVERSIBILITY VS RECURRENCE

Some thermodynamic processes, such as

- diffusions
- heat transfers $\rightarrow$ irreversible
- phase transitions

but

Newtonian mechanics $\rightarrow$ time-reversible

CONTENTS

THE EHRENFEST URN MODEL

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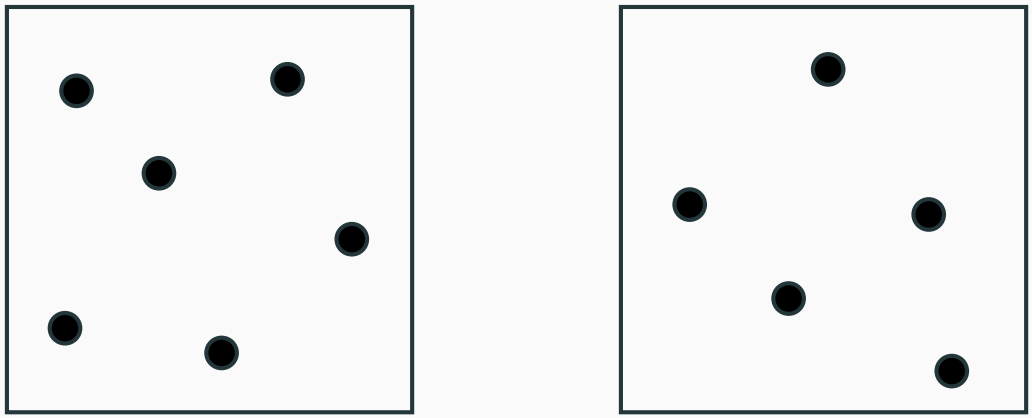
Diffusion of a gas as stochastic process

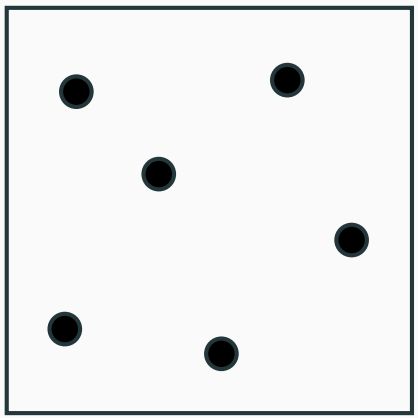
- N particles
- 2 boxes
- discretized time steps



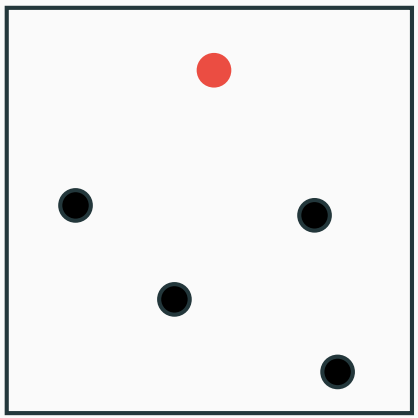
THE EHRENFEST URN MODEL

At each time step:

- a ball is selected at random among the N possible ones



A

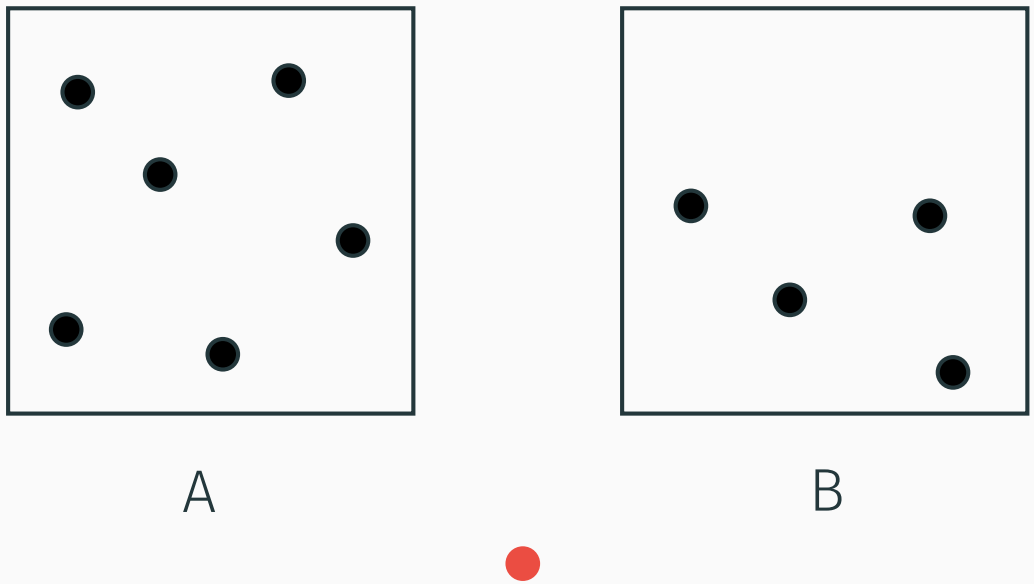


B

THE EHRENFEST URN MODEL

At each time step:

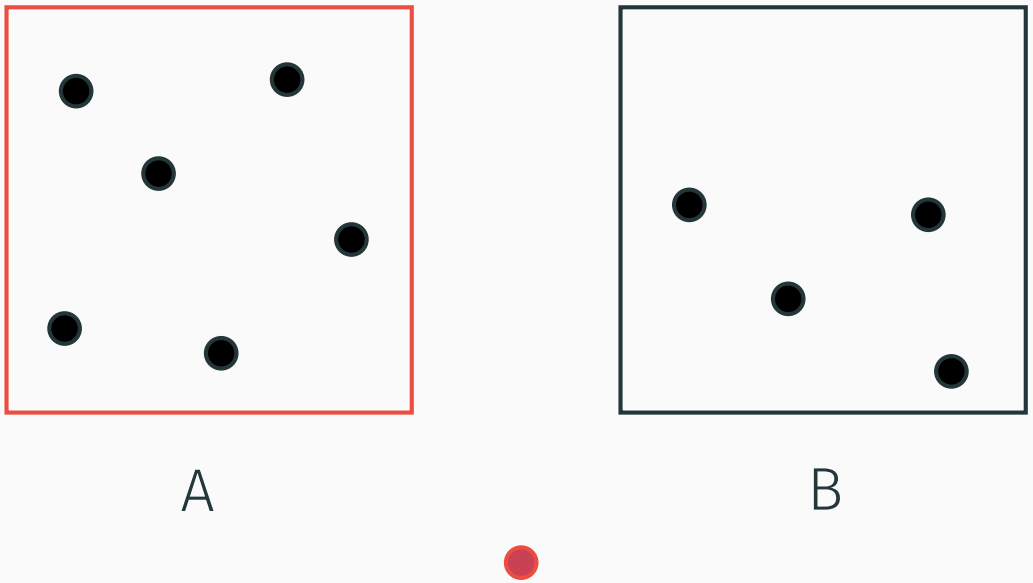
- a ball is selected at random among the N possible ones
- it is extracted from its box



THE EHRENFEST URN MODEL

At each time step:

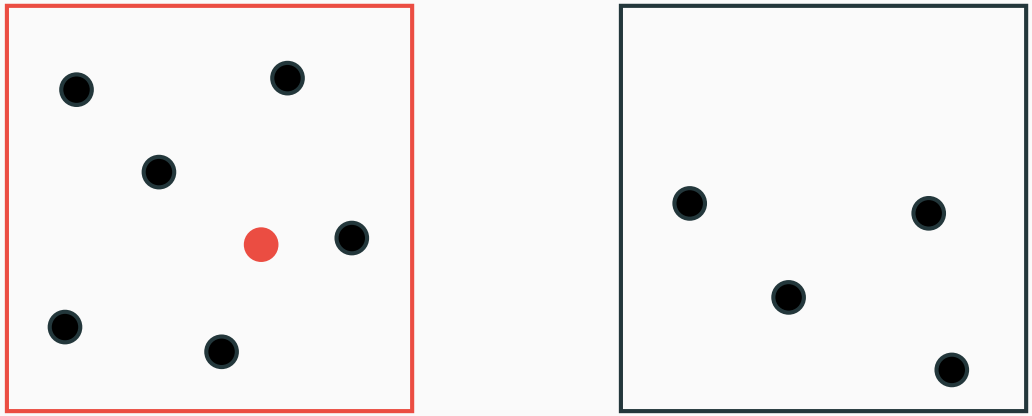
- a ball is selected at random among the N possible ones
- it is extracted from its box
- a box is selected at random



THE EHRENFEST URN MODEL

At each time step:

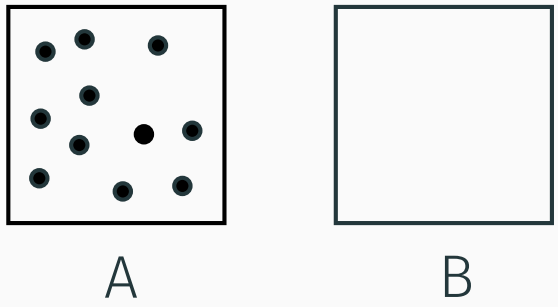
- a ball is selected at random among the N possible ones
- it is extracted from its box
- a box is selected at random
- the particle is put in the selected box



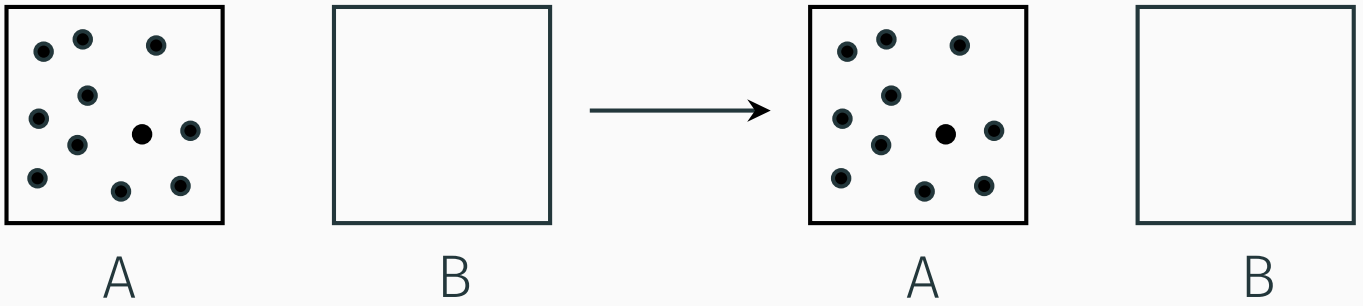
A B

Meaningful questions about equilibrium

- probability of having all the particles in box A?



- time needed for recurrence?

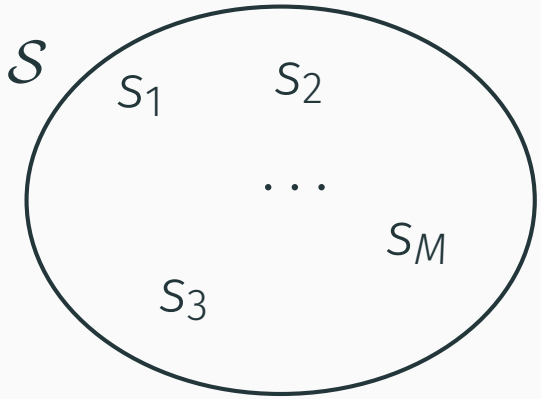


MARKOV CHAIN ANALYSIS

Discrete Markov chains

A mathematical theory for stochastic processes

The state of the system X is a random variable



State space

Stochastic process

Markov property

Memorylessness: only the current state influences the next transition

Distributions

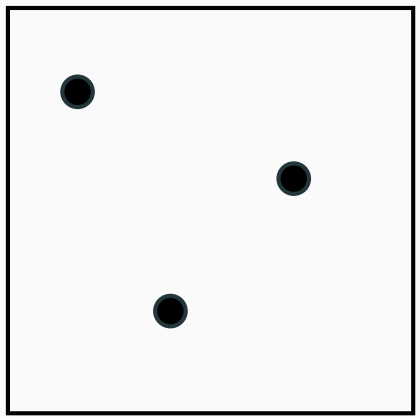
X = state of the system = random variable

Distribution $\rightarrow$ probabilities for the system to be in the various possible states

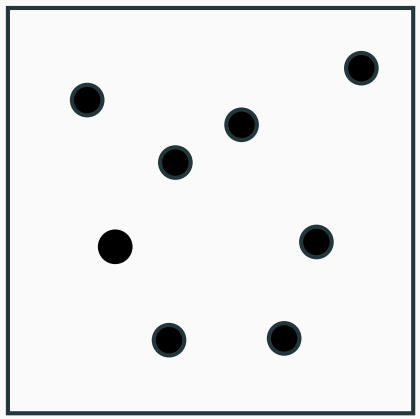
Analogy with the probability distribution in statistics

In the Ehrenfest model:

- state = number of particles in box A
- possible values: $0, \ldots, N$
- example



A



B

$X = 3$

A sequence of states of the Ehrenfest model is a Markov chain

ASYMPTOTIC BEHAVIOUR

LIMITING DISTRIBUTION