Control rod calibration experiment: reactor period method

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Abstract

This experiment, performed at the LENA TRIGA in Pavia, aims to calibrate the regulating control rod through the reactor period method. The differential and integral reactivities will be calculated by inserting the control rods in steps. The final result for the integral reactivity showed a good accordance with theoretical expectation, as well as the obtained total worth of the rod (1.19 \$) in comparison to the one found by LENA's operators. Both the 6-groups and 1-group approximation were performed, the former showing a better accuracy while the latter overestimates the control rod worth.

1 Introduction and theoretical background

Training, research, isotopes, General Atomics (TRIGA) is a pool-type nuclear reactor built by General Atomics around the 1960s for mainly training and research, which developed applications also in the medical field (like BNCT and $Tc55^m$ production), detector testing and neutron activation analysis [AGE16].

Its leading feature is its **inherent safety**: it avoids risks by design. This feature is reached thanks to containment, cooling and criticality control. A clear example is the fuel which is blended together with the moderator, creating crystals of uranium zirconium hydride (UZrH). U235 is at 20% enrichment, H grants moderation while Zr brings benefits in chemical and temperature stability and neutron economy. Notably, it also gives the reactor a strongly negative temperature coefficient.

Other interesting features for safety are an open pool setup, with an unpressurized configuration, and natural circulation that requires no pumps (for <2 MW), for $\geq$ 100 keV or long term we will need heat removal in the reactor tank.

The power for TRIGA Mark II reactors is higher than the former generation, suitable for pulsed regimes and irradiation experimentations. Indeed, another difference to Mark I is the above ground configuration which grants access to the many irradiation ports.

Furthermore, TRIGA's core is composed by concentric rings, with a 60-100 positions for fuel-moderator elements, dummy elements, control rods, irradiation channels. The core differs between Mark I, II and its further generations: to reach higher powers, we take away dummy elements and increase the fuel elements. Pavia's TRIGA is an earlier type of Mark II and operates at a power of 250 kW.

1.1 Control rods

For the purpose of our work we will focus the discussion on the **control rods** (CRs). In a TRIGA not used for pulsed operation, they are the shim, the regulating and the transient rods.

Control Rod	Material	Purpose	ρ worth [\$]	Movement	IC [cm]	OC [cm]
Shim	B_4C	Coarse ρ adjustment	3-3.5	motorized	2.85	3.18
Reg	B_4C	Fine ρ adjustment	1-1.5	motorized	1.93	2.22
Trans	Borated graph.	Safety	2-2.5	pneumatic	2.21	2.54

Table 1: TRIGA's control rods characteristics. ρ indicates the reactivity.

Overall, the control rods serve as absorbers, to vary reactivity. Reactivity is a balance between capture, fission and loss, and changing the capture cross section is a convenient way to act on this balance. The choice of their material is fundamental when it comes to the neutron absorption capability of the CRs, while the choice of position influences the flux.

Some instances that underline the functions of the CR are safety, compensation of reactivity during fuel cycle and startup/shutdown [Lam83]. To design the reactor we need to precisely know these effects.

Furthermore, the design and number of CRs has to satisfy the behavior of reactivity during fuel lifetime. In fact, at the beginning of life (BOL) we need a high reactivity excess for compensating for the burnup, poisons and temperature effects. As we go towards end of life (EOL), at zero power condition, we need to add negative reactivity to compensate for the excess, plus some more negative reactivity to guide the reactor from a critical to a sub-critical state.

Thus, to design CRs we need to know in detail both quantitatively and qualitatively their action.

The total control requirements in [pcm], or [$] are given by the sum of the burnup (Bu), the poisons, the temperature effects, and shutdown margin.

Its not so easy to know how many [pcm] a CR can insert because of the so called *shadowing effect*: CR worth depends from its position and from the reciprocal distance between it and the others. This effect influences the neutron flux and thus the effectiveness of the CR itself.

The total CR worth is the objective of our investigation: we need to find the **differential** and **integral** worth.

The differential CR worth is the reactivity change per unit movement of the CR within the reactor core. Thus, it indicates how sensitive the reactor is to small movements of the control rod at specific positions within the core (it helps in fine-tuning the reactor's power level by showing how much reactivity changes with small adjustments of the control rods).

The integral CR worth is the total reactivity change introduced by a CR when it is inserted into or withdrawn from the reactor core for a certain overall depth. It provides a measure of the total reactivity that can be controlled by inserting or extracting a single rod or a group of rods. This is crucial for understanding the overall capability of the control rods to shut down the reactor or adjust its power level.

The total worth of the rector is the sum of all the integral worths.

If we want to take an analytical approach to find the CR worth, some important hypothesis are needed:

- Homogeneous reactor;
- Perturbation theory;
- One energy group;
- One rod;
- No scattering.

The theoretical characterization could be considered sound but the hypothesis might be too strong. Furthermore, it cannot take into account shadowing effect or the neutronics changes during irradiation time.

A better way to find the CR worth is thus *experimental*, with a calibration procedure.

Control rod calibration is fundamental for the operational point of view of the reactor, making it possible to determine the needed core excess and shutdown margin. It can also give the reactivity value, for example in [pcm], and its rate, in [pcm/s], as the CR changes position.

1.1.1 Core excess

The core excess (CE) is a surplus of positive reactivity that can be added by extracting the CRs, starting from critical condition.

It is a safety parameter (complementary to the shut down margin) and an operational parameter (to grant the possibility of reaching criticality after shutdown).

Since the core excess changes with power level and burnup, it is important to evaluate it at different times during operational lifetime.

To calculate core excess, we evaluate every rod as in the following equation

$$\Delta\rho_{CE,CRi}(z_{critical} \rightarrow z_{extracted}) = \rho_i(z_{extracted}) - \rho_i(z_{critical}) \tag{1}$$

where $\rho_{CE,CRi}$ is the reactivity core excess for the i-th CR. Thus,

$$\Delta\rho_{CE,TOT} = \Delta\rho_{CE,CR1} + \Delta\rho_{CE,CR2} + \Delta\rho_{CE,CR3} \tag{2}$$

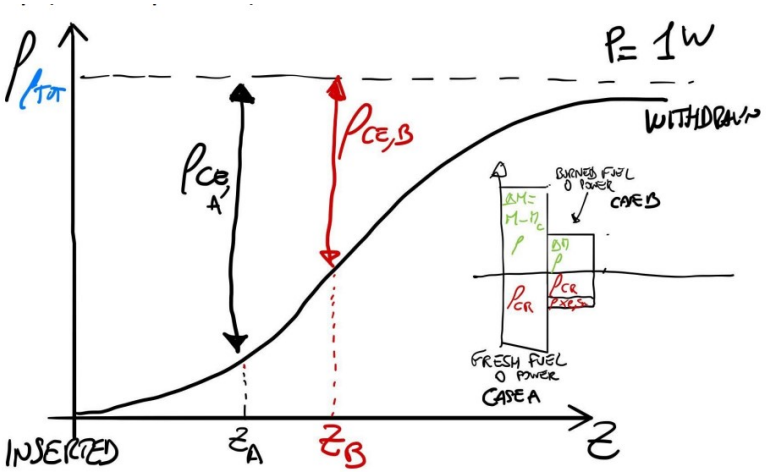

Figure 1: The core excess of a reactor is changing according to the power level and burnup.

In Pavia's TRIGA we have

$$\Delta\rho_{CE,TOT} = \Delta\rho_{CE,Shim} + \Delta\rho_{CE,Reg} + \Delta\rho_{CE,Trans} \tag{3}$$

1.1.2 Shutdown margin

It is the negative reactivity resulting from a full insertion of all the CRs starting from critical condition, except for the highest reactivity worth CRs that are assumed to be completely withdrawn (stuck rod situation, to be evaluated for safety).

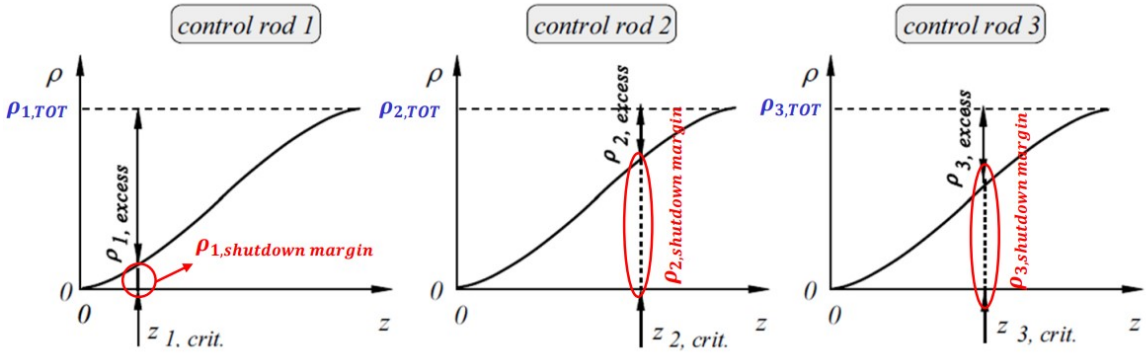

Figure 2: The shutdown margin complements the core excess in the calibration ρ - z plot.

$$\Delta\rho_{SM,CRi}(x_{critical} \rightarrow x_{inserted}) = -(\rho_i(x_{inserted}) - \rho_i(x_{critical})) = \rho_i(x_{critical}) \tag{4}$$

with i=1,2,3 representing the three CRs and SM meaning shut down margin.

$$\Delta\rho_{SM,TOT} = \Delta\rho_{SM,Shim} + \Delta\rho_{SM,Reg} + \Delta\rho_{SM,Trans} \tag{5}$$

2 Control rod calibration with reactor period

The experimental procedure must be performed at zero power and the CR reactivity will be calculated indirectly through the dependence of power from time.

There are different methods to calibrate CRs but in our study we will focus on the **reactor period method**.

2.1 Point kinetics and reactor period

For our aim, the time dependent behavior of a reactor can be evaluated with point kinetics under the assumptions of

- No thermal feedback, since we are at zero power;
- No spatial effect;
- One energy group, since we care just for the power and not its distribution.

The one energy group point kinetics equations are as follows:

$$\begin{cases} \frac{dP}{dt} = \frac{\rho-\beta}{\Lambda}P + \lambda c \\ \frac{dc}{dt} = \frac{\beta}{\Lambda}P - \lambda c \end{cases} \tag{6}$$

Under the condition where the reactivity insertion/extraction in a formerly critical reactor is constant and equal to a step of value ρ_0, we can express the time behavior of power P(t) as the sum of two exponential terms

$$P(t) = P_1 e^{s_1 t} + P_2 e^{s_2 t} \tag{7}$$

We will skip the derivation and just look at the solution for $|\rho_0| \ll \beta$ (a small reactivity insertion), knowing s_1 is the time-dominant solution :

$$P(t) \cong P_0 \left[\frac{\beta}{\beta-\rho_0} e^{\frac{\lambda\rho_0}{\beta-\rho_0}} - \frac{\rho_0}{\beta-\rho_0} e^{-\frac{\beta-\rho_0}{\Lambda}t} \right] \sim e^{\frac{t}{T}} \tag{8}$$

where T is the period of the reactor: the time required for the power of to vary by a factor e

$$T = \frac{1}{s_1} = \frac{\beta-\rho_0}{\lambda\rho_0} \tag{9}$$

Looking at the result obtained by setting the determinant of the system of equations 6 as zero, we obtain

$$\Lambda s^2 + \rho_0(\beta - \rho_0 + \lambda\Lambda)s - \rho_0\lambda = 0 \tag{10}$$

and substituting

$$\rho_0 = s\Lambda + \frac{s\beta}{s+\lambda} \tag{11}$$

where

$$\Lambda = \frac{l}{k} \tag{12}$$

and

$$k = \frac{1}{1+\rho_0} \tag{13}$$

With these ingredients, we can finally find the **inhour equation** (one group):

$$\rho_0 = \frac{sl}{sl+1} + \frac{1}{sl+1}\frac{s\beta}{s+\lambda} \tag{14}$$

Notably, the graphical solution lets me calculate T given both by a positive and a negative ρ insertion.

More in general, for the six energy groups we will obtain

$$\rho_0 = \frac{\Lambda}{T} + \sum_{i=1}^{6} \frac{\beta_i}{1+\lambda_i T} \tag{15}$$

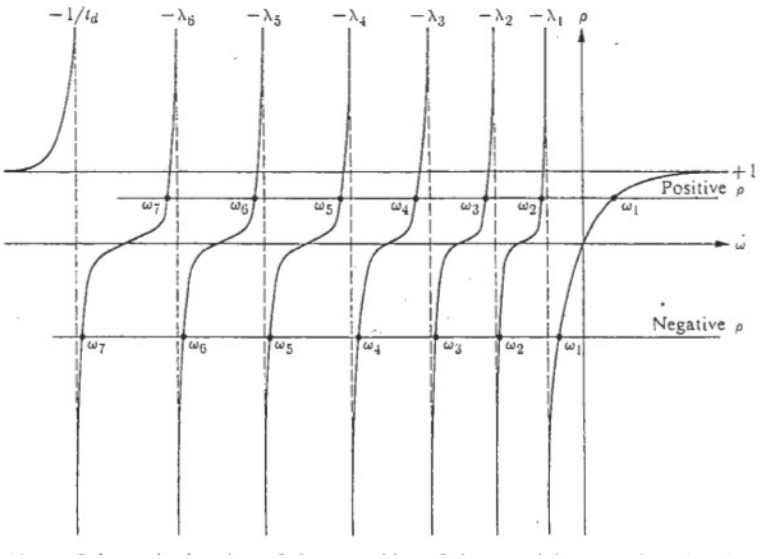

Figure 3: Schematic drawing of the two sides of the reactivity equation showing the seven roots for positive and negative reactivity. The figure is not drawn to scale.

2.2 Experimental procedure of CR calibration with reactor period

The steps that must be followed in the procedure are:

- Start from cold, clean conditions with no neutron source inserted and all CRs inserted, with the nuclear reactor in subcritical conditions;
- Extract the Trans, injecting in the system a reactivity of about 2 $ (TRIGA has SM of about 4 $ so we remain in subcriticality);
- Move the Shim CR, until criticality is reached, leaving the regulating rod completely inserted;
- Extract Reg CR to introduce a step reactivity insertion equal to the differential Reg CR worth: the power will increase and we will be in supercriticality;
- Leave all the higher solutions of 8 equations to decay and find T;
- Find ρ from T with the inhour equation. The procedure will be done with a power rise from 3 to 4.5 W and from 4 to 6 W;
- Insert again the Shim CR and bring back the reactor to critical conditions;
- Repeat the procedure until the Reg is completely extracted.

The reactor criticality was achieved at a power of 1W, since the calibration procedure must be performed at *zero power conditions* ($< 10W$), so that the effects of thermal feedback on reactivity are negligible.

The advantages of this type of method are that it is absolute and accurate, while the disadvantages are that it needs a long time to complete the measurement and that the reactor will be supercritical.

3 Data analysis assignments

- Check the values inserted and make judgment on accepted/rejected time measurements;
- Calculate an interpolation for the integral and differential reactivity worth vs z-position and plot them;
- Estimate the possible sources of uncertainty and calculate their impact on the CR worth;

- Calculate the integral/differential reactivity worth relying on one-group precursor approximation;
- Calculate the total regulating CR reactivity worth.

3.1 Acquisition of reactor period

The data was collected at the TRIGA of the Laboratorio Energia Nucleare Applicata (LENA) of Pavia. Before starting to evaluate the experimental results, we need to know the kinetic parameters for the point kinetics equation, which were tabulated.

Kinetics parameters TRIGA							
Parameter	Precoursor group						TOTAL
	1	2	3	4	5	6	
λ_i (s^{-1})	3,01	1,14	0,301	0,111	0,0305	0,0124	0,0767
β_i/β (-)	0,042	0,115	0,395	0,196	0,219	0,033	
β_i (-)	3,07E-04	8,40E-04	2,88E-03	1,43E-03	1,60E-03	2,41E-04	7,30E-03
β_i (pcm)	30,7	84,0	288	143	160	24,1	730
β (-)							7,30E-03
β (pcm)							730
Λ (s)							5,0E-05

Figure 4: Tabulated kinetics parameters for TRIGA, for 1 and 6 energy groups.

We will experimentally collect the time it takes to have a 50% power increase when moving from a certain starting position of the Reg control rod to another. After the measurement of T we restore criticality by inserting the Shim. Thus, the aim is to find the calibration for the Reg CR, but with that we will obtain "for free" the calibration for the central part of the Shim (the Shim's positive reactivity to be inserted is equal to the negative reactivity of the Reg movement), since it is moved in order to compensate the reactivity inserted with the Reg, restoring critical conditions in the core. The Shim calibration goes beyond the scope of out experiment.

Data for the differential calibration of the CR								
Shim position (digit)	Reg starting position (digit)	Reg final position (digit)	Registred time for the 50% power increase (s)					
			from 3 W to 4.5 W			from 6 to 9 W		
564	128	234	36,68	38,4	37	35	41,15	41,44
547	234	303	33,77	33,51	33,77	34,45	35,79	36,55
528	303	370	20	22,12	20,15	22,91	24,25	24,28
506	370	420	31,92	31,54	31,59	33,48	33,4	33,49
487	420	497	13,28	13,22	12,55	15,07	14,89	13,89
456	497	581	13,3	13,36	13,37	14,99	14,83	14,79
425	581	679	16,13	16,02	15,86	18,17	17,81	18,2
	679	818	36,64	36,39		39,19	39,8	

Figure 5: Data gathered at LENA for the calibration by moving the control rods and taking the time measurement for different power ranges.

The CRs positions are indicated in [digits], and the conversion in [cm] can be performed by the following conversion

$$[cm] = ([digits] - 124)\frac{38.1}{700} \tag{16}$$

The purpose of getting different values for the CR positions (and thus different times) is to

- Have different measurements of time to assess the uncertainty of this measurement (standard deviation);
- See the difference between initial and final part of the transient.

Position (cm)	Shim position	Reg starting position (digit)	Reg final position (digit)	Time for 50% power increase (s)	Estimated period (s)
0,0		128	128		
5,9	564	128	234	38,28	94,41
9,7	547	234	303	34,64	85,43
13,4	528	303	370	22,29	54,96
16,1	506	370	420	32,57	80,33
20,4	487	420	497	13,82	34,08
25,0	456	497	581	14,11	34,79
30,4	425	581	679	17,03	42,01
38,1	0	679	818	38,01	93,73

Figure 6: The estimated time for the 50% power increase was calculated with the mean of the measured times at different power ranges and different rod positions.

We will then calculate the mean value for the time, both for the sets of measurements at 3 W - 4.5 W and 6 W - 9 W. The power will behave with the law described in equation 7.

Given the estimated time t_M (fifth column of figure 6) we can find the estimated period with

$$ln(1.5) = \frac{t_M}{T} \tag{17}$$

From this equation we can extract T and place it in the inhour equation 15. Using figure 4 we can substitute the necessary kinetic parameters. We calculate $\frac{\Lambda}{T}$ and $\frac{\beta_i}{1+\lambda_i T}$ for each energy group and position.

Position (cm)	Shim position (digit)	Reg starting position (digit)	Reg final position (digit)	Λ/T	$\beta_1/(1+l_1*T)$	β2/(1+l2*T)	β3/(1+l3*T)	β4/(1+l4*T)	β5/(1+l5*T)	β6/(1+l6*T)
0,0		128	128	0	0	0	0	0	0	0
5,9	564	128	234	5,3E-07	1,08E-06	7,73E-06	9,80E-05	1,25E-04	4,12E-04	1,11E-04
9,7	547	234	303	5,9E-07	1,19E-06	8,53E-06	1,08E-04	1,36E-04	4,43E-04	1,17E-04
13,4	528	303	370	9,1E-07	1,84E-06	1,32E-05	1,64E-04	2,02E-04	5,97E-04	1,43E-04
16,1	506	370	420	6,2E-07	1,26E-06	9,07E-06	1,15E-04	1,44E-04	4,63E-04	1,21E-04
20,4	487	420	497	1,5E-06	2,96E-06	2,11E-05	2,56E-04	2,99E-04	7,84E-04	1,69E-04
25,0	456	497	581	1,4E-06	2,90E-06	2,06E-05	2,51E-04	2,94E-04	7,76E-04	1,68E-04
30,4	425	581	679	1,2E-06	2,41E-06	1,72E-05	2,11E-04	2,53E-04	7,01E-04	1,58E-04
38,1	0	679	818	5,3E-07	1,08E-06	7,78E-06	9,87E-05	1,25E-04	4,14E-04	1,11E-04

Figure 7: Experimental and derived data necessary to calculate the reactivity.

Summing the contributions over all the energy groups shown in figure 7, we can finally obtain the measurement for differential reactivity in [pcm], by multiplying what we obtained directly from the inhour formula by 100000, and we shift to [$] by converting $\rho[\$] = \frac{\rho[pcm]}{\beta[pcm]}[\$]$.

To compute the integral reactivity at a certain position, we need to sum the differential reactivity to the integral reactivity of the former position, starting from zero.

$$\rho_z^{integral} = \rho_z^{differential} + \rho_{z-1}^{integral} \tag{18}$$

Where z refers to the position of the CR. The result of these calculations can be seen in figure 8.

3.2 Total reactivity worth of Reg control rod

The total reactivity worth of the Reg CR is 1.19 \$ $\pm 0,05$, where the uncertainty can be found in section 3.4. We can compare this value with the one obtained at LENA the 27 July 2015, represented in figure 9, where we can clearly see that the total value is 1.26\$. This is acceptable since 9 years passed, and reactivity changes with time, like we can see in figure 1.

3.3 Calibration curve fit

If we want to fit the curves we obtained experimentally, we can think about the physical meaning of reactivity. Indeed, $\delta\rho$ (infinitesimal reactivity) will be proportional to an absorption term and to the

Differential reactivity (-)	Differential reactivity (pcm)	Differential reactivity ($)	Integral reactivity ($)
0	0	0	0,00
7,55E-04	76	0,10	0,10
8,15E-04	82	0,11	0,22
1,12E-03	112	0,15	0,37
8,54E-04	85	0,12	0,49
1,53E-03	153	0,21	0,70
1,51E-03	151	0,21	0,90
1,34E-03	134	0,18	1,09
7,59E-04	76	0,10	1,19

Figure 8: Worth of differential and integral reactivity, relative to the positions in figure 5.

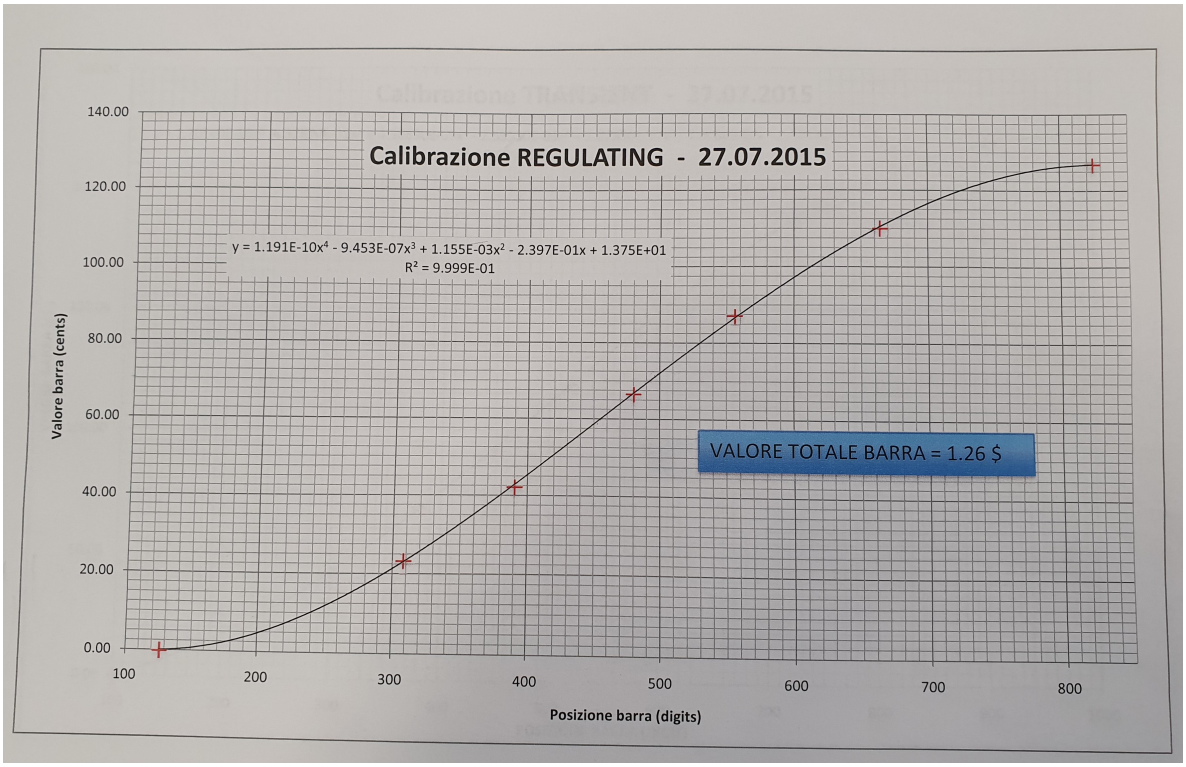

Figure 9: The curve of Reg calibration performed at LENA can give us a benchmark for our result.

flux distribution, thus

$$\delta\rho \sim \Sigma_a \phi_z^2 dz \tag{19}$$

Taking into account that the reactor is cylindrical, the flux in the axial direction, meaning the z axis, will be represented by a sinusoidal function

$$\phi(z) = \phi_{z,max} sen(\frac{\pi z}{H}) \tag{20}$$

Where H is the height of the reactor. $\delta\rho$ is the infinitesimal reactivity and if we wanted to obtain the differential or integral we could simply integrate it along the needed length of the CR along the z axis, e.g. between generic z_1 and z_2 for the differential and between 0 and z_3 for integral, or between 0 and H for the total.

We can think that the fit function to be used is a sinusoidal function, like the one described below

$$y = C + asen(bx + p) \tag{21}$$

Where y is the reactivity and x is the CR position, while the other parameters are to be obtained with the python code contained in section 6.

3.4 Uncertainty evaluation

Since the whole structure and application of science depends on measurements, the ability to evaluate these uncertainties and keep them to a minimum is crucially important [Tay82].

Errors and uncertainties are used to confirm the consistency of data, reject outliers, and compare with analytical results or other numerical/experimental results. In the case of this experiment, where we have repeated measurements, we can start by finding the standard deviations of t_M, T and $\frac{\Lambda}{T}$. The other parameters are tabulated and thus we can hypothesize that they need no uncertainty evaluation.

Then, to obtain the uncertainties of derived values like the reactivity, we can use *error propagation*, since the relative uncertainties involved are small. Given a variable x depending on other variables a, b and c we can calculate the standard deviation as follows [Mof88]

$$x = f(a, b, c) \tag{22}$$

$$dx_i = f(da_i, db_i, dc_i) \tag{23}$$

$$\sigma_x^2 = \left(\frac{\delta x}{\delta a}\right)^2 \sigma_a^2 + \left(\frac{\delta x}{\delta b}\right)^2 \sigma_b^2 + \left(\frac{\delta x}{\delta c}\right)^2 \sigma_c^2 \tag{24}$$

Taking into account that the differential (and thus integral) reactivity uncertainty is depending on the uncertainty of the period, like shown in the inhour equation 15. Since we have 6 different groups, the total uncertainty will be the sum of the uncertainties over all the six groups calculated with the equation shown in 24.

So, the overall formula to calculate the error will be represented below

$$\sigma_{\rho diff} = \left(\frac{\Lambda}{T^2} + \sum_{i=1}^{6} \frac{\beta_i \lambda_i}{(1+\lambda_i T)^2}\right) \sigma_T \tag{25}$$

This uncertainty can be added as error bars in the graph showcasing the experimental data. The results will be shown in section 4.

stdev tM	stdev T	stdev Λ/T	stdev group1	stdev group2	stdev group3	stdev group4	stdev group5	stdev group6	std sum err propag
2,58	6,35	5,80	1,13E-08	8,11E-08	1,00E-06	1,21E-06	3,24E-06	6,34E-07	6,17E-06
1,25	3,08	2,81	1,38E-08	9,89E-08	1,22E-06	1,45E-06	3,75E-06	7,04E-07	7,23E-06
1,90	4,68	4,28	3,33E-08	2,36E-07	2,82E-06	3,15E-06	6,81E-06	1,06E-06	1,41E-05
0,98	2,42	2,21	1,57E-08	1,12E-07	1,37E-06	1,62E-06	4,10E-06	7,50E-07	7,96E-06
1,00	2,46	2,25	8,60E-08	6,03E-07	6,85E-06	6,94E-06	1,17E-05	1,48E-06	2,77E-05
0,84	2,07	1,89	8,26E-08	5,79E-07	6,59E-06	6,72E-06	1,15E-05	1,46E-06	2,69E-05
1,14	2,81	2,56	5,68E-08	4,00E-07	4,66E-06	4,95E-06	9,37E-06	1,29E-06	2,07E-05
1,74	4,29	3,72	1,15E-08	8,23E-08	1,02E-06	1,22E-06	3,27E-06	6,39E-07	6,25E-06

Figure 10: Results of the standard deviations and step-by-step calculations for the error propagation.

3.5 One group approximation

To simplify the problem, one could evaluate just the one group approximation. We thus take equation 15 and simplify it to

$$\rho_0 = \frac{\Lambda}{T} + \frac{\beta}{1+\lambda T} \tag{26}$$

The results appear in figure 12 and the plots will be shown in the section 4.

std	std	std	std
diff react (-)	diff react (pcm)	diff react ($)	int react ($)
3,93E-05	3,93	5,38E-03	5,38E-03
2,23E-05	2,23	3,05E-03	8,43E-03
6,61E-05	6,61	9,06E-03	1,75E-02
1,93E-05	1,93	2,64E-03	2,01E-02
6,82E-05	6,82	9,35E-03	2,95E-02
5,58E-05	5,58	7,64E-03	3,71E-02
5,83E-05	5,83	7,98E-03	4,51E-02
2,68E-05	2,68	3,68E-03	4,88E-02

Figure 11: Uncertainties calculated for the reactivity.

one group									
β/(1+ρT)	differential react (-)	differential react (pcm)	differential react ($)	integral react ($)	std err propag	std diff react (-)	std diff react (pcm)	std diff react ($)	std int react ($)
0,00E+00	0,00E+00	0,00	0,00	0,00	0,00E+00	0,00E+00	0,00	0,00E+00	0,00E+00
8,86E-04	8,86E-04	88,62	0,12	0,12	8,24E-06	5,24E-05	5,24	7,18E-03	7,18E-03
9,66E-04	9,67E-04	96,69	0,13	0,25	9,81E-06	3,02E-05	3,02	4,14E-03	1,13E-02
1,40E-03	1,40E-03	140,03	0,19	0,45	2,06E-05	9,65E-05	9,65	1,32E-02	2,45E-02
1,02E-03	1,02E-03	101,98	0,14	0,59	1,09E-05	2,64E-05	2,64	3,62E-03	2,82E-02
2,02E-03	2,02E-03	202,12	0,28	0,86	4,29E-05	1,06E-04	10,56	1,45E-02	4,26E-02
1,99E-03	1,99E-03	199,10	0,27	1,13	4,16E-05	8,62E-05	8,62	1,18E-02	5,44E-02
1,73E-03	1,73E-03	173,00	0,24	1,37	3,14E-05	8,82E-05	8,82	1,21E-02	6,65E-02
8,91E-04	8,92E-04	89,17	0,12	1,49	8,35E-06	3,59E-05	3,59	4,91E-03	7,14E-02

Figure 12: Results from the one group approximation and their uncertainty.

4 Results

We calculate the fits with the Python script shown in section 6, to see if the experimental data aligns. The theoretical reasoning for the fit is explained in section 3.3.

4.1 Six groups approximation

Let us begin evaluating the results with the six group approximation.

The fit curve in figure 13 is printed by the Python as `Fitted function: y = 0.0649 -0.1310 * sin(0.0852 * x + 2.6903)`.

The fit curve in figure 14 is printed by the Python as `Fitted function: y = 0.5942 -0.5968 * sin(0.0798 * x + 1.6873)`.

While the fit for the integral reactivity, figure 14, confirms the findings, the one for the differential reactivity, figure 13, points to some issues, since many error bars do not overlap with the fit curve. Actually, the CR effect varies according to the position: it produces larger variations in reactivity close

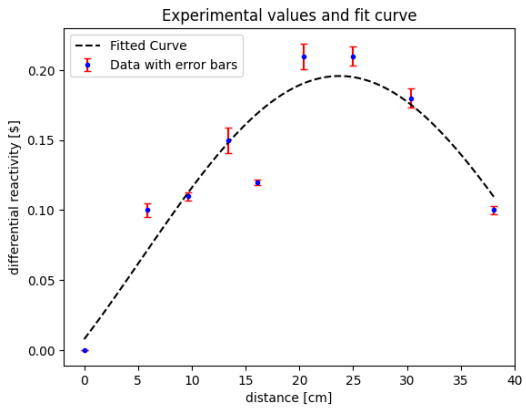

Figure 13: Fit for the 6-group differential ρ.

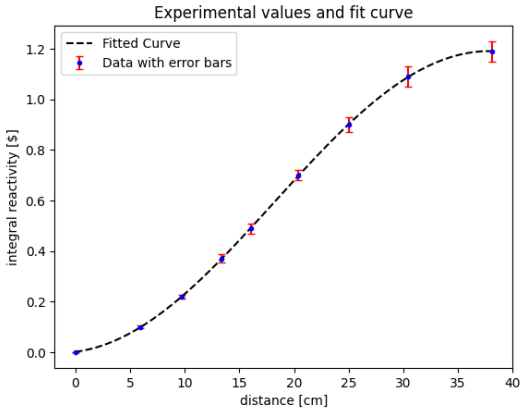

Figure 14: Fit for the 6-group integral ρ.

to the equator of the core, where the neutron flux is higher, because the solution of the axial direction is a sinusoidal. This does not justify the experimental points at 7 cm, 16 cm and 20 cm and it would be advisable to re-take the period measurements.

4.2 One group approximation

Now let us analyze the result for the 1-group approximation, that we overviewed in section 3.5.

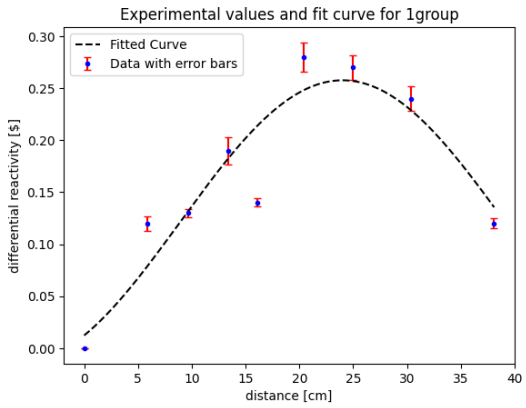

Figure 15: The fit for the 1-group differential ρ.

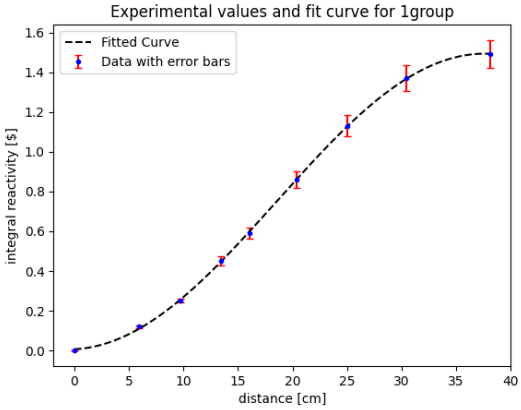

Figure 16: The fit for the 1-group integral ρ.

The fit curve in figure 15 is printed by the Python as `Fitted function: y = 0.1215 -0.1362 * sin(0.1041 * x + 2.2119)`.

The fit curve in figure 16 is printed by the Python as `Fitted function: y = 0.7488 -0.7443 * sin(0.0822 * x + 1.6176)`.

The conclusions that can be deduced from figure 15 and 16 are the same as the ones we explained for the 6-group approximation in section 4.1.

4.3 Six and one group approximation comparison

Let us now look at how the 1-group and 6-groups compare to each other.

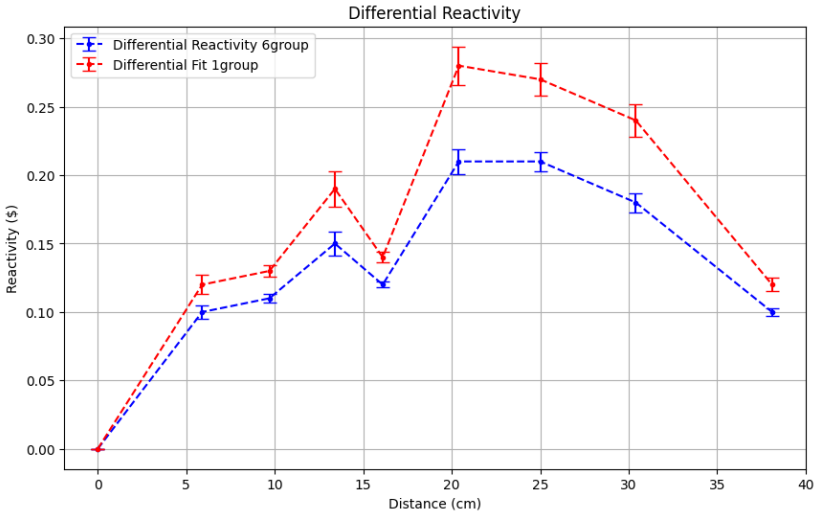

Figure 17: The comparison between the differential reactivity obtained for the 6 groups and one group approximation.

From figure 17 and 18 we can clearly see how the 1-group gives us an overestimation of the results obtained with the 6-groups approach. Furthermore, the uncertainties are higher in the 1-group case.

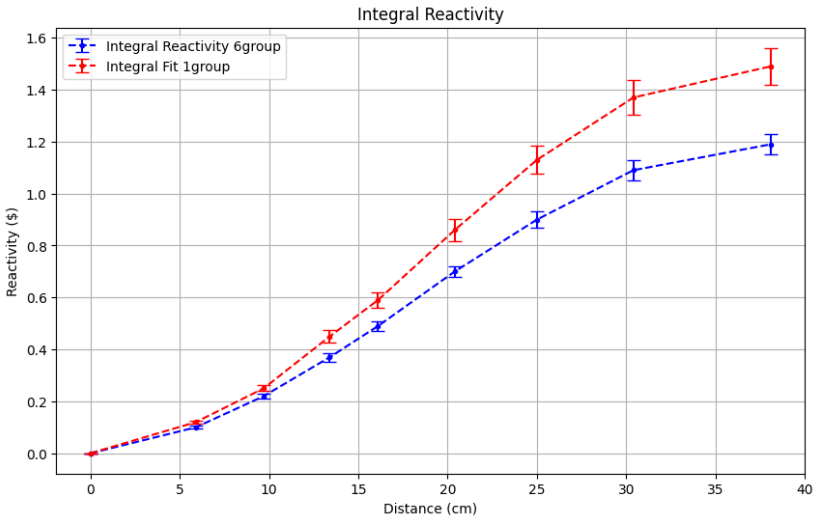

Figure 18: The comparison between the integral reactivity obtained for the 6 groups and one group approximation.

In nuclear engineering, a lot of the times is better to be *conservative*. This concept takes different meanings according to the situation but for safety, for example, it is better to consider that the CR worth is lower than it actually is. The problem is that this might go against the will to optimize other design parameters, since CRs are not only used for safety, as explained in section 1.

Furthermore, we might not have access to the kinetic parameters for every energy group, or we might want to do a quicker evaluation of the data.

To conclude, the usage of either of the approximations depends on how much time we have, which data and which purpose, but overall the 6-groups gives us a more accurate estimate, as expected.

5 Comparison with other experimental labs

The reactor period method is not the only method to perform CR calibration. Indeed, there notably exists also the **subcritical method**.

The subcritical method, which uses a neutron source in a subcritical reactor, is a simple and quick way to calculate CR worth or first core loading through the so called 1/M plot (where M is the subcritical multiplication factor, that relates the source level to the steady state level). It can also give Shim calibration and does not require the nuclear reactor to return to criticality. It is very susceptible to source, instrumentation, burnup, positioning and is less accurate than the reactor period method. Furthermore, it is a *comparative* method: k and M are found through the ratio of the detector's counting rate at a given condition to the counting rate of a reference, which might for example be given by the period method and the total CR worth obtained (e.g. 1.19 $).

Overall, the subcritical method can be used for a quick estimation of CR worth and the period method for a more precise evaluation.

6 Python code

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit

# integral fit:   #run this cell to calculate integral reactivity
x_data = np.array([0,5.9,9.7,13.4,16.1,20.4,25,30.4,38.1]) #cm
y_data = np.array([0,0.1,0.22,0.37,0.49,0.7, 0.9,1.09,1.19]) #dollar

#integral error   #run this cell to calculate integral reactivity error
interr= np.array([0,0.005,0.008,0.017,0.02,0.02,0.03,0.04,0.04]) #dollar

#differential fit   #run this cell to calculate differential reactivity
x_data = np.array([0,5.9,9.7,13.4,16.1,20.4,25,30.4,38.1]) #cm
y_data = np.array([0,0.1,0.11,0.15,0.12,0.21,0.21,0.18,0.1]) #dollar

#diff error   #run this cell to calculate differential reactivity error
differr= np.array[0.005,0.003,0.009,0.002,0.009,0.007,0.007,0.003] #dollar

#define the sinusoidal function for the fit
def fit_func(x, C, a, b, p):
    return C + a * np.sin(b * x + p)

#fit the sinusoidal function to the data
initial_guess = [np.mean(y_data), np.max(y_data)-np.min(y_data), 2*np.pi/np.ptp(x_data
                                                ), 0]  # Initial guess for C, a, b, p
params, _ = curve_fit(fit_func, x_data, y_data, p0=initial_guess, maxfev=10000)

#extract the parameters for the fit
C, a, b, p = params

#print the best fitting function with its parameters
print(f"Fitted function: y = {C:.4f}  {a:.4f} * sin({b:.4f} * x + {p:.4f})")

#plot the data points
plt.errorbar(x_data, y_data, yerr=interr, fmt='.', label='Data with error bars', color
                                               ='blue', ecolor='red', capsize=3)   #
                                               specify either interr or differr

#plot the fitted curve
x_fit = np.linspace(min(x_data), max(x_data), 1000)
y_fit = fit_func(x_fit, *params)
plt.plot(x_fit, y_fit, label='Fitted Curve', color='black', linestyle='--')

plt.xlabel('distance [cm]')
plt.ylabel('integral reactivity [$]')   #write either integral or differential
plt.title('Experimental values and fit curve')
plt.legend()
plt.show()
```

References

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