PROGRAM LOGICS VIA DISTRIBUTIVE MONOIDAL CATEGORIES

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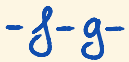
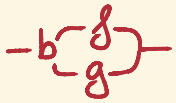
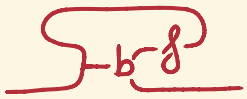
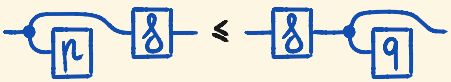
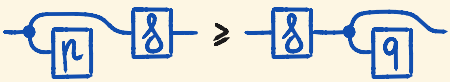
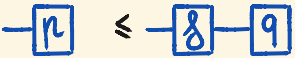
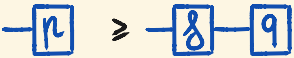
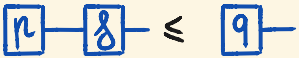
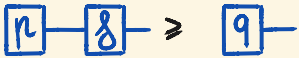
- from categorical structure, derive verification axioms
- general categorical semantics for program logics

CATEGORICAL STRUCTURE FOR pGCL

$f;g$	composition	
skip	identity	
if b then f else g	coproducts	
while b do f	trace	
$x \leftarrow e$, $x \xleftarrow{\$} \mu$	copy-discard monoidal	

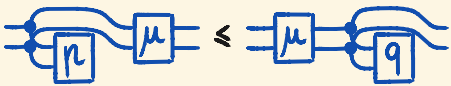
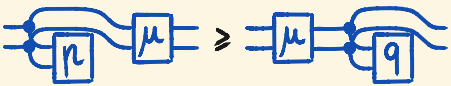
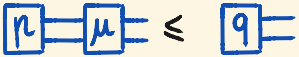
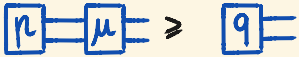
PROGRAM TRIPLES

	CORRECTNESS	INCORRECTNESS
ASSERT		
	Hoare logic	
PREDICATE		
	Outcome logic	
STATE		
		Incorrectness logic

THEOREM. The rules of program logics derive from the axioms of imperative categories.

RELATIONAL PROGRAM TRIPLES

There is a coupling $\mu \triangleright f\&g$ such that

	CORRECTNESS	INCORRECTNESS
ASSERT		
	Relational Hoare logic	
PREDICATE		
		Quantitative probabilistic relational Hoare logic
STATE		

THEOREM. The rules of relational program logics derive from the axioms of imperative categories.

EXAMPLES

- Par , deterministic semantics
- Rel , relational semantics
- $\mathrm{Stoch}_{\leq}$, discrete probabilistic semantics
- $\mathrm{BorelStoch}_{\leq}$, measurable probabilistic semantics
- Imp , discrete imprecise probabilistic semantics