20171208 Math180a Lecture Notes Friday

Cabinet Math180a Lecture Notes

Midterm problem

Given Random Variable \(X\) with pdf $f_X(x)$

$$f_X(x) \begin{cases} \frac{1}{2} & -1 < x < 1 \\ 0 & otherwise \end{cases}$$

And $Y = X^2$

1. $E[Y] = E[X^2]$

$$\begin{aligned} \int_{-\infty}^{\infty} x^2 \cdot f_X(x)dx &= \int_{-1}^{1} \frac{1}{2} x^2 \cdot f_X(x)dx \\ &= \frac{1}{6} x^3 \Big|_{-1}^{1} = \frac{1}{3} \end{aligned}$$

2. Find pdf $f_Y(y)$ of Y
   Possible value for $Y \ldots X \in [-1, 1] \rightarrow Y \in [0, 1]$

Now fix $y \in [0, 1]$

$$\begin{aligned} F_Y(y) &= P(Y \leq y) \\ &= P(X^2 \leq y) \\ &= P\left(-\sqrt{y} \leq X \leq \sqrt{y}\right) \\ &= \int_{-\sqrt{y}}^{\sqrt{y}} f_X(x)dx \\ &= \int_{-\sqrt{y}}^{\sqrt{y}} \frac{1}{2} dx \\ &= \frac{2}{2} \sqrt{y} = \sqrt{y} \end{aligned}$$

$$f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{1}{2\sqrt{y}}$$

$$f_Y(y) = \begin{cases} \frac{1}{2\sqrt{y}} & 0 < y < 1 \\ 0 & otherwise \end{cases}$$

Homework problem

X : grade of a random student in class.

- Half of the students got about 80
- 10% of the students got about 95

Suppose X has a normal distribution.

find $P(X < 70)$

1. $\mu = 80$
2. $P(X > 95) = 0.1 \Rightarrow 1 - \Phi(\frac{15}{\sigma}) = 0.1 \Rightarrow \Phi(\frac{15}{\sigma}) = .9 \Rightarrow \frac{15}{\sigma} = 1.28 \Rightarrow \sigma = 11.72$

Bounds

Suppose X is the weight of a randomly selected apple

1. $E[X] = 75$, what can you say about $P(X > 85)$?

> For a non-negative random variable with a known expected value, use Markov's Inequality

$$P(X > 85) \leq \frac{E[X]}{85} = \frac{75}{85} = 0.88$$

2. Now suppose we know the standard deviation of X is 5.
   Question: What can you say about $P(65 \leq X \leq 85)$?

> By knowing standard deviation, we can use Chebyshev's

$$P(|X - \mu| \geq a) \leq \frac{Var(X)}{a^2}$$

$$P(|X - 75| \geq 10) = 1 - P(|X - 75| > 10) \leq \frac{5^2}{10^2} = 0.25$$

$$P(|X - 75| > 10) \geq 1 - 0.25 = 0.75$$

3. Suppose we now have 100 apples. What can you say about $P(\bar{S}_{100} \geq 80)$, When $\bar{S}_{100}$ is the average weight of 100 apple.

> By the central limit theorem…

$$E[\bar{S}_{100}] = 75$$

$$Var(\bar{S}_{100}) = \frac{Var(X)}{100} = 0.25$$

$$\bar{S}_{100} \approx N(75, 0.25)$$