Optimal Management and Integration of Electric Vehicles to the grid: Dynamic Programming and Game Theory Approach

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***Abstract*—This paper provides a Dynamic Programming (DP) approach to optimally manage the charging schedule of electrical vehicles (EV) based on a decentralized global optimization frame given by the use of a Game Theory approach. The paper provides a detailed explanation of a forward induction DP algorithm and shows its adaptation to the problem of optimal charging of one EV with the corresponding constraints of limiting power consumption, minimal and maximal states of charge, desired states of charge, etc. The extension to multiple EVs is provided by the adaptation of a N-person non-cooperative game approach. In this game, the payoff of each player is based on a utility function that aims to reduce the distance between the total load and the average load, achieving load curve flattening.**

***Index Terms*—Plug-In Hybrid Electric Vehicles, Smart charging, Vehicle-to-Grid, Dynamic Programming, Peak shaving, Valley filling, Game theory, N-person non-cooperative game, Interfacing and Control of Energy Storage Technology, Optimization.**

I. Introduction

Hybrid Vehicles (HV), Plug-In Hybrid Electric Vehicles (PHEV), and Electric Vehicles (EV) are some of the most outstanding solutions to the environmental and economic issues caused by the high levels of fossil fuel consumption of Conventional Vehicles (CV). The advantages of CVs compared to EVs can be sumarized in the autonomy. The fact that current battery technologies do not support great storage capacities or a large number of charging-discharging cycles, or the fact that fast charging devices dramatically reduce the state of health of batteries [1], [2]. Similar to CVs, HVs have a larger energy storage capacity that is applied to reduce fuel consumption during peak accelerations. On the other hand, with larger energy storage capacities that HVs, PHEVs combine the functionallity of CVs, HVs and EVs. PHEVs can work on all electric mode with certain autonomy, they can work as CVs and also, they can reduce consumption during strong accelerations, as HVs do [1], [3], [4].

EVs and PHEVs have to be connected to an energy source in order to charge their batteries. Ignoring the existence of dedicated infrastructures, EVs and PHEVs represent an important quantity of new load to residential or distribution grids [3], [5], [6]. These large amounts of new load represent multiple impacts, locally and globally, such as voltage levels, unbalanced loads, harmonics, stability issues, etc. [7], [8]. However, EVs and PHEVs, through their energy storage capacity, also represent valuable opportunities to the solution of power quality issues on the distribution grids [5], [7], [9]. With optimally managed bidirectional chargers and improved energy storage systems, EVs and PHEVs become mobile energy storage systems and controllable loads [9].

Optimal management strategies for energy consumption become crucial for the integration of EVs and PHEVs to distribution grids [3], [5], [7], [9]–[13]. Authors of [14], [15] propose centralized methods for charging EVs, one is formulated with Linear Programming (LP), a second one is formulated with Mixed Linear Integer Programming (MILP) knowing some information of demand, number of PHEVs and prices. A third method, includes unexpected leaving and arriving behaviour and is solved by employing heuristic algorithms. In [7], [16], authors propose optimization approaches taking into account the presence of charging infrastructures on buildings. Authors of [7] provide a comparison between a centralized approach and a decentralized approach following a non-cooperative game theory scheme. Other schemes following similar game theory approaches are [5], [9], [17]. An approach where EV charging profiles are optimally distributed over a range of time in order to minimize the variance of the demand over time is presented in [18]. Authors of [19] present an optimization method to define charging schedules for one electric vehicle taking into account stochastic behaviour of arrival and departure times. In [20], a dynamic programming approach is proposed for battery charging and power management when driving for one PHEV, aiming to reduce CO_2 emissions. Authors of [21] compare multiple approaches to solve the optimal centralized coordination of charging schedules of multiple EVs and PHEVs. Backward induction DP and Quadratic Programming (QP) techniques are compared as solvers for the centralized optimization approach.

This paper provides a Dynamic Programming (DP) approach to optimally manage the charging schedule of electrical vehicles (EV) based on a decentralized global optimization frame given by the use of a Game Theory approach. The paper provides a detailed explanation of a forward induction DP algorithm and shows its adaptation to the problem of optimal charging of one EV with the corresponding constraints of limiting power consumption, minimal and maximal states of charge, desired states of charge, etc. Given the fact that the state and control action spaces are finite, the optimal policies can be tractably computed with the induction algorithm [22]. The extension to multiple EVs is provided by the adaptation of a N-person non-cooperative game approach. In this game, the payoff of each player is based on a utility function that aims to reduce the distance between the total load and the average load, achieving load curve flattening.

The paper is organized as follows. Section II describes general approaches followed to solve the charging problem of multiple EVs with centralized schemes. Section III introduces and describes the adaptation of the DP algorithm to the charging problem of one EV. Section IV describes the N-person non-cooperative game approach followed to manage the problem of charging multiple EVs along with the DP algorithm in a decentralized scheme. Section V presents some of the simulation results. Finally, Section V concludes the paper.

II. Optimal Charging Problem

The problem of charging electric vehicles subject to a certain number of constraints can be addressed as follows,

$$\min_{\mathbf{p}_i\in\mathbf{P}_i, i=\{1,\cdots,N\}} cost(\mathbf{p}_1,\mathbf{p}_2,\cdots,\mathbf{p}_i,\cdots,\mathbf{p}_N) \tag{1}$$

where $cost(\cdot)$ is a function having as parameters the power consumption schedules $\mathbf{p}_i$ of the N electric vehicles to be charged. The power quantities at each instant of a discretized charging period are the elements of the vector $\mathbf{p}_i$.

$$\mathbf{p}_i = \left[p_i^1, p_i^2, \cdots, p_i^t, \cdots, p_i^T\right]$$

The set $\mathbf{P}_i$ of all possible power consumption schedules for the vehicle i can be defined by the constraints that are described as follows. The *power consumption/injection* limits depend on the type of charger. These limits can be expressed by the following inequalities:

$$-p_{\text{max}} \le p_i^t \le p_{\text{max}}, \forall t \in \{1,\cdots,T\}$$

The *state of charge* denoted by soc_i must always be bounded by a certain upper limit (soc_{max}) and a lower limit (soc_{min}) both defined to decrease the impact of the charging and discharging cycles on the battery's health.

$$soc_{\text{min}} \le soc_i^0 + \Delta t \sum_{t=1}^{\tau} p_i^t \le soc_{\text{max}}, \forall \tau \in \{1,\cdots,T-1\}$$

The final or *desired state of charge* soc_{des} can be defined as an equality constraint as follows,

$$soc_i^0 + \Delta t \sum_{t=1}^{T} p_i^t = soc_{\text{des}}$$

or as an interval of desirable states of charge (more relaxed constraint) as follows,

$$soc_{\text{des}} \le soc_i^0 + \Delta t \sum_{t=1}^{T} p_i^t \le soc_{\text{max}}$$

Additionally, other types of constraints can be included. For example, with an approximated model of the grid, the voltage level at every node of the grid can be included as a constraint in order to provide a voltage support service with the storage capacity of the batteries [21], [23]. The inherent centralization of this type of approach demands great computation capacity, collecting and handling great quantities of information.

The following sections provide the details of a forward induction dynamic programming algorithm adapted to solve the charging schedule problem of one EV. Then, a non-cooperative N-person game is presented in order to link the DP solvers of multiple EVs and optimally manage the charging of multiple EVs in a decentralized scheme.

III. Dynamic Programming Algorithm

The dynamic programming technique is usually employed to solve certain types of problems where decisions have to be taken in stages. In this type of situations, the objective is to take a sequence of decisions in order to minimize an undesirable outcome or maximize a profit. Usually, this type of problems can be represented as discrete-time dynamic systems where the outcome is an additive function over time (the discrete stages). The sequence of decisions are taken in order to influence the state variables that describe the system [24]–[28]. The system can be described as follows,

$$x_{k+1} = f_k(x_k, u_k, w_k), \ k = 0, 1, \cdots, T-1$$

with k is the discrete time index, f_k is the function that describes the system, x_k is system's state at time instant k, u_k is the control decision taken in k to steer the system from x_k to x_{k+1}, and w_k is a random variable that depends on the system's type. The expected value, with respect to the joint distribution of the random variables, of the total outcome function is expressed as,

$$\mathbb{E}\left\{\sum_{k=0}^{T-1} g(x_k, u_k, w_k) + g_T(x_T)\right\}$$

with $g(x_k, u_k, w_k)$ is the cost at time k, which depends on the of state, the control, and the random variables involved. Let us define *admissible control laws* as a sequences $\mu = \{\sigma_0, \sigma_1, \cdots, \sigma_{T-1}\}$ of functions $\sigma_k(x_k) = u_k$ mapping states x_k into controls u_k, such that $u_k \in U_k(x_k) \subset \mathbf{U}_k$. The set $\mathbf{U}_k$ is the set of all possible control decisions at time k and $U_k(x_k)$ is a non-empty subset of $\mathbf{U}_k$ that depends on the current state $x_k \in \mathbf{X}_k$. The set $\mathbf{X}_k$ contains all possible states x_k at time k. Given an initial state x_0 and an admissible control law $\mu = \{\sigma_0, \sigma_1, \cdots, \sigma_{N-1}\}$, the total expected cost associated to this control law, starting at state x_0, is defined as,

$$J_\mu(x_0) = \mathbb{E}\left\{\sum_{k=0}^{T-1} g(x_k, \sigma_k(x_k), w_k) + g_T(x_T)\right\}.$$

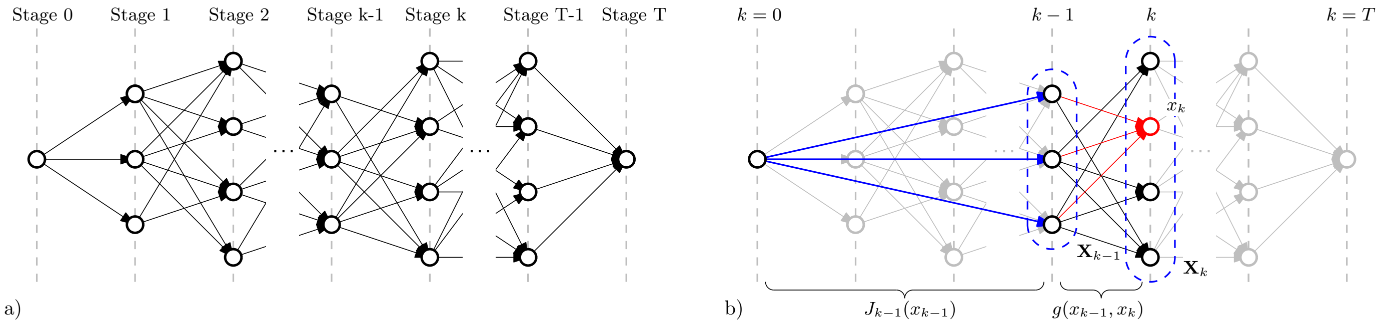

Fig. 1. Multi-stage shortest path problem: a) Multi-stage problem example, (b) Forward Dynamic Programming approach.

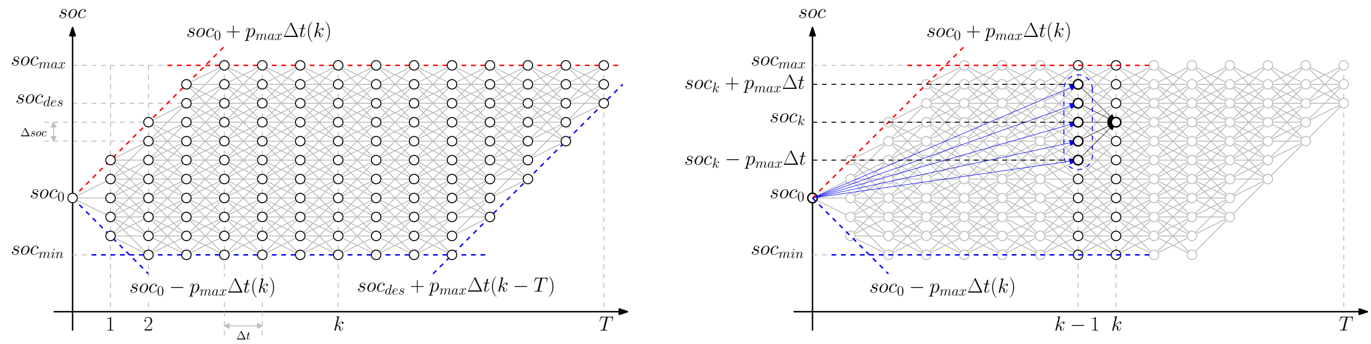

Fig. 2. Adaptation of the problem of one EV charging schedule to the Forward Dynamic Programming Algorithm.

It can be noticed that the expected cost of the admissible control law depends on the initial state x_0. Now, knowing the set $\mathbf{N}$ of admissible control laws, an *optimal control law* μ^*, starting from x_0, can be found as,

$$J_{\mu^*}(x_0) = J^*(x_0) = \min_{\mu \in \mathbf{N}} J_\mu(x_0). \tag{2}$$

The optimal control law is the optimal sequence of decisions. The function $J^*(x_0)$ is also called *optimal cost function*.

A. Finite-state Problem and the DP Algorithm

Let us consider the case where the state variable at each time k is discretized with a defined step size. Then, the problem looks like a a multi-stage shortest path problem as the one shown on Fig. 1(a). Neglecting the presence of random noise, let us assume that applying feasible control actions u_{k-1}, the cost at time $k-1$ depends only on the transition between states x_{k-1} and x_k, and it is given by a function $g(x_{k-1}, x_k)$. If a control action being able to bring the system from state x_{k-1} to x_k is unfeasible, it is represented by a cost function $g(x_{k-1}, x_k) = \infty$. On Fig. 1(a), if a feasible control exists, the transition is represented by an arc and the cost of transition is given by $g(x_{k-1}, x_k)$. With this assumptions, the Dynamic Programming Algorithm is presented as the following pseudo-code [24].

The idea is to solve the optimization problem splitting the optimal cost function (2) in temporal sub-functions of optimal cost, based on the Bellman's *principle of optimality* [26]. The algorithm finds the optimal trajectory of the state variable and the optimal control sequence in an iterative scheme over the time. The approach followed in this paper is the forward induction which is illustrated in Fig. 1(b). The algorithm proceeds forward in time (with the first loop) from $k = 2$ until the final state in $k = T$. For each state in $\mathbf{X}_k$, the algorithm finds the path of optimal cost starting from the root state at stage 0 and arriving to the evaluated state in $\mathbf{X}_k$. Thus, the *optimal cost function* is constructed.

Algorithm 1 Forward Dynamic Programming Algorithm

for all $x_1 \in \mathbf{X}_1$ **do**
    $J_1(x_1) = g(x_0, x_1)$
end for
for $k = \{2, 3, \cdots, T\}$ **do**
    for all $x_k \in \mathbf{X}_k$ **do**
        $J_k(x_k) =$
        $$\min_{x_{k-1} \in \mathbf{X}_{k-1}} [g(x_{k-1}, x_k) + J_{k-1}(x_{k-1})]$$
    end for
end for

B. The EV battery charging problem adapted to the DP Algorithm

For the battery charging problem of an EV, one possible adaptation is described here. The system (neglecting the random noise) can be described by,

$$soc_{k+1} = soc_k + \Delta t p_k$$

where the state and the control variables, at time k, are the state of charge soc_k and the power consumed by the charger p_k respectively. The set $\mathbf{U}_k$ of possible control decisions is defined by the following inequalities,

$$-p_{\max} \leq p_k \leq p_{\max}, \forall k \in \{0, \cdots, T-1\}$$

The set $\mathbf{X}_k$ is defined depending on the limits that should be imposed to the state of charge at each time k. A priori, the boundaries are the defined by the minimal $soc_{\min}$ and maximal $soc_{\max}$ states of charge allowed. However, some other boundaries are fixed depending on the initial state of charge soc_0 and the minimal desired state of charge soc_{des} at the end of the charging period. The precise boundaries can be observed in Fig. 2(a). The boundaries are defined by the following inequalities,

$$\underline{soc}_k \leq soc_k \leq \overline{soc}_k$$

where $\underline{soc}_k$ and $\overline{soc}_k$ are functions depending on k as follows,

$$\underline{soc}_k = \max \left\{ \begin{array}{c} soc_{\min}, \\ soc_0 - p_{\max}\Delta t(k), \\ soc_{\text{des}} + p_{\max}\Delta t\,(k-T) \end{array} \right\}$$

$$\overline{soc}_k = \min \left\{ \begin{array}{c} soc_{\max}, \\ soc_0 + p_{\max}\Delta t(k) \end{array} \right\}.$$

As it can be noticed on Fig. 2(a), $\underline{soc}_k$ and $\overline{soc}_k$ depend not only on the upper and lower limits allowed for the state of charge, but also on the maximum allowed rate of charge and discharge (the maximum input/output power of the charger p_{max}). On the other hand, the subset $U_k(soc_k) \subset \mathbf{U}_k$ depends on the state variable soc_k in the sense that at certain states of charge, certain rates of charge/discharge p_k are forbidden. For example, if the state of charge at time k is close enough to the maximal state of charge $soc_{\max}$, then it is not possible to charge the battery at rates p_k producing future states of charge $soc_{k+1} > soc_{\max}$. In this sense, and for the purpose of the forward DP algorithm adaptation, let us define the subset U_{k-1} with respect to the state of arrival soc_k. Knowing the arrival state soc_k, the possible root states in $k-1$ are defined by the following inequalities,

$$\underline{soc}_{k-1} \leq soc_{k-1} \leq \overline{soc}_{k-1}$$

where $\underline{soc}_{k-1}$ and $\overline{soc}_{k-1}$ are functions depending on soc_k as follows,

$$\underline{soc}_{k-1} = \max \left\{ \begin{array}{c} soc_{\min}, \\ soc_0 - p_{\max}\Delta t(k-1), \\ soc_k - p_{\max}\Delta t \end{array} \right\}$$

$$\overline{soc}_{k-1} = \min \left\{ \begin{array}{c} soc_{\max}, \\ soc_0 + p_{\max}\Delta t(k-1), \\ soc_k + p_{\max}\Delta t \end{array} \right\}.$$

These limits are better illustrated on Fig. 2(b). Based on these inequalities, the subset $U_{k-1}(soc_k)$ is defined by,

$$\frac{soc_k - \overline{soc}_{k-1}}{\Delta t} \leq p_{k-1} \leq \frac{soc_k - \underline{soc}_{k-1}}{\Delta t}$$

Including more that one EV in the DP algorithm will exponentially increase the quantity of states at each step of time. This will increase memory requirements and computation time, and the issues associated to centralized algorithms of section II. Thus, in order to provide a link between multiple EVs and optimally manage the charging schedules, a Game Theory approach is followed. This approach is explained in the following section.

IV. Management of Multiple EVs: DP & Game Theory

In order to provide a decentralized approach to manage the charging operation of multiple EVs, a N-person non-cooperative game approach is followed. Each EV is considered as a player and the strategies for player i are defined as the possible charging profiles $\mathbf{p}^i = [p_1^i, p_2^i, \cdots, p_k^i, \cdots, p_T^i]$. On the other hand, for player i, the strategies from other $N-1$ players are grouped in the following expression,

$$\mathbf{p}^{-i} = [\mathbf{p}^1, \mathbf{p}^2, \cdots, \mathbf{p}^{i-1}, \mathbf{p}^{i+1}, \cdots, \mathbf{p}^N]$$

The payoff function for player i, depends on its strategy and the strategies followed by the other players. Here, it is defined as the negative of the distance between the total load at each step of time and the average load, as follows,

$$G_i(\mathbf{p}^i, \mathbf{p}^{-i}) = -\sum_{k=1}^{T} \left(\left(p_k^i + \sum_{j=1, j\neq i}^{N} p_k^j + l_k \right) - L_{mean} \right)^2$$

where, l_k is the load of the grid (without EVs) and L_{mean} is the average load during the whole charging period, including the grid load and the EVs' load. L_{mean} is defined by,

$$L_{mean} = \frac{\frac{1}{\Delta t}\sum_{i=1}^{N}\left(soc_{des}^i - soc_0^i\right) + \sum_{k=1}^{T} l_k}{T}.$$

In order to simplify the payoff expression, let us regroup some terms in $L_k^{-i} = -\sum_{j=1, j\neq i}^{N} p_k^j - l_k + L_{mean}$. Then, the payoff for player i is given by,

$$G_i(\mathbf{p}^i, \mathbf{p}^{-i}) = -\sum_{k=1}^{T} \left(p_k^i - L_k^{-i}\right)^2. \quad (3)$$

It is important to note that this utility function is strictly concave. Now, given fixed strategies from the other players $\mathbf{p}^{-i}$, the *best response strategy* $\mathbf{p}^{i*}$ for player i is defined by,

$$\mathbf{p}^{i*} \in \arg\max_{\mathbf{p}^i \in \mathbf{P}^i} G_i(\mathbf{p}^i, \mathbf{p}^{-i}).$$

The set $\mathbf{P}^i$ is defined by the constraints detailed in section II. Given the fact that these constraints are linear, the set $\mathbf{P}^i$ is convex, closed and bounded. The *Nash equilibrium* is defined as a state (or vector of strategies $\mathbf{p}^* = [\mathbf{p}^{i*}, \mathbf{p}^{-i*}]$) in which no player con improve its payoff by unilaterally deviating from its equilibrium strategy $\mathbf{p}^{i*}$. In the Nash equilibrium state, if one player deviates its strategy from its equilibrium strategy (while other player maintain their equilibrium strategies), then its payoff can only be reduced [7], [29], [30].

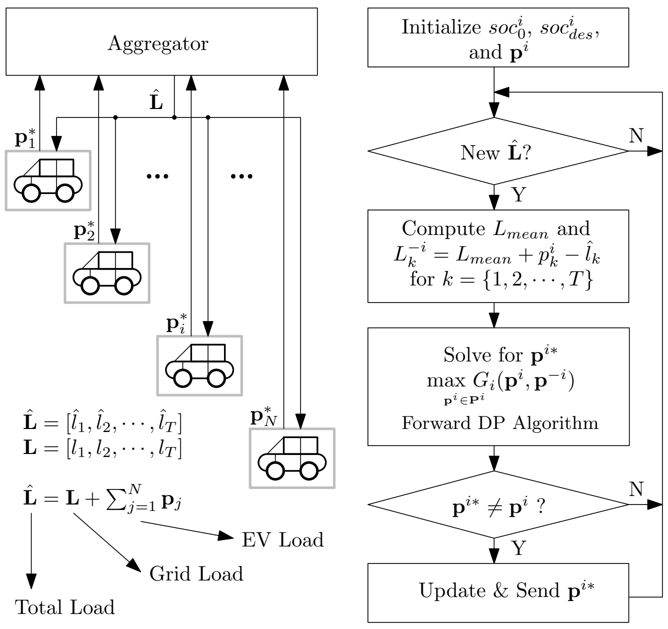

Fig. 3. Algorithm of the EVs N-person game.

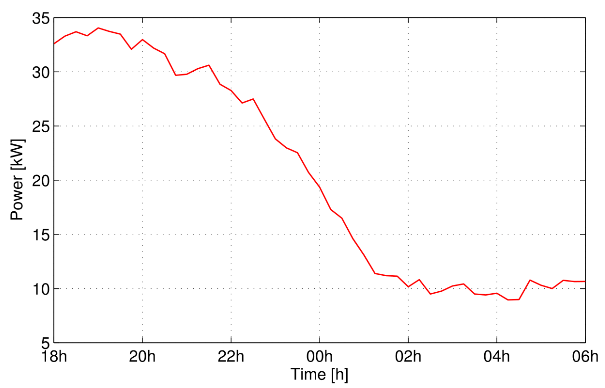

Fig. 4. Load profile of the grid without EVs.

Given the strict concavity of utility functions for each player and the fact that sets $\mathbf{P}^i$ are convex, closed and bounded, the game is a strictly concave N-person game where the Nash equilibrium exists and it is unique [7], [17], [31].

The approach followed to find the Nash equilibrium is an algorithm similar to the one proposed by [7]. Each player receives an updated total load profile $\hat{\mathbf{L}}$ (including all other players' strategies and grid load) from a centralized agent or aggregator. Based on the total load, the player solves the local optimization problem (with the Forward DP algorithm) to find the best response strategy $\mathbf{p}^{i*}$ and if it is different from the previous strategy $\mathbf{p}^i$, the player sends its updated best response strategy to the aggregator. The algorithm is summarized in Fig. 3. As it is proved by authors of [7], this best response algoritm converges to the nash equilibrium if players choose their best responses (maximize their payoff) in a sequential and asynchronous fashion.

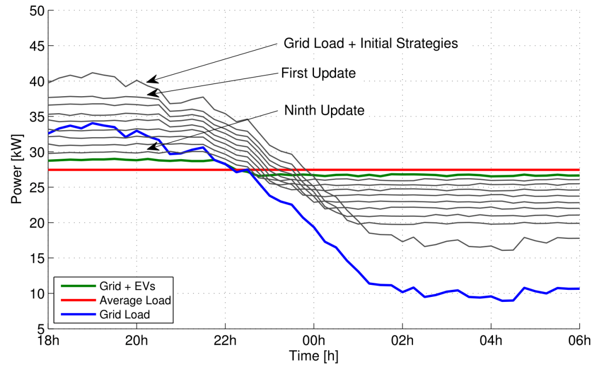

Fig. 5. Evolution of the game with 10 EVs

The role of the aggregator in the algorithm is the compilation of information from players and the broadcasting of updated information. Players interact with other player only through the aggregator. Additionally, the algorithm bases the optimal charging schedules on the computation of a short-term load forecast $\mathbf{L}$ of the transformer load with time steps of 15 minutes. This kind of forecast is possible with appropriate historical information [32], [33]. This is the second task of the aggregator.

The transition cost function of the DP algorithm is defined based on the payoff function of the N-person game as follows,

$$g_k^i = -(p_k^i - L_k^{-i})^2 = -\left(\frac{soc_k^i - soc_{k-1}^i}{\Delta t} - L_k^{-i} \right)^2 . \quad (4)$$

If players update their best strategy responses in an asynchronous fashion, their payoffs will either increase or remain the same. Since the distance to the average load is bounded below (in the best case it could be zero), then the algorithm will converge to the desired point or the Nash equilibrium [7].

V. Results

In order to test the charging management approach, a test is proposed with 10 EVs with the following characteristics. The capacity of the batteries is 20kWh, and the allowed interval of charge is 30% to 80% (6kWh to 16kWh). The state of charge is discretized with a step $\Delta soc = 80$Wh. Thus, the whole range is divided in 126 steps from 6kWh to 16kWh both included.

The range of power of the charger is -3.2kW to $+3.2$kW ($p_{max} = 3.2$kW). With $\Delta soc = 80$kWh and a time step $\Delta t = 0.25$h, the corresponding step size of input/output power is $\Delta p = 320$W. Thus, the range of power is divided in 21 steps including 0W.

The period of charge is chosen between 18h in the evening and 06h in the morning. During this period of time, the load of the grid without EVs is shown on Fig 4.

The initial state of charge of each EV is chosen randomly with a uniform distribution between 30% and 40%. The initial states of charge are 37.6%, 36.8%, 33.6%, 40.0%, 31.2%, 38.8%, 34.4%, 36.4%, 37.6%, and 37.2%.

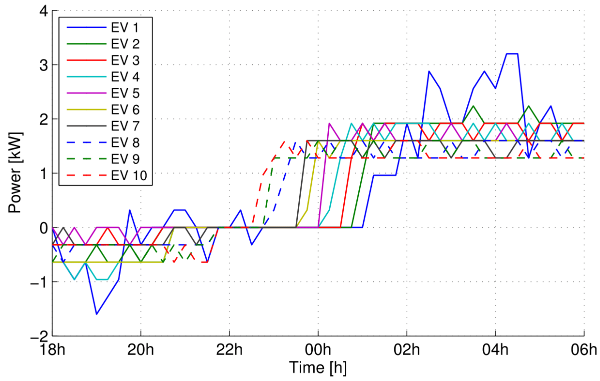

Fig. 6. Optimal power consumption profiles, final strategies in the Nash equilibrium

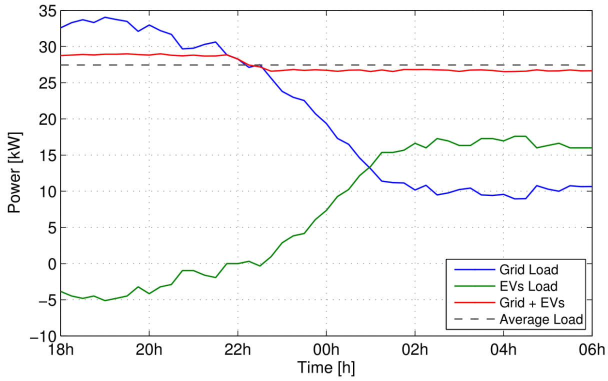

Fig. 8. Peak reduction and valley filling resulting from the management of the charging schedules.

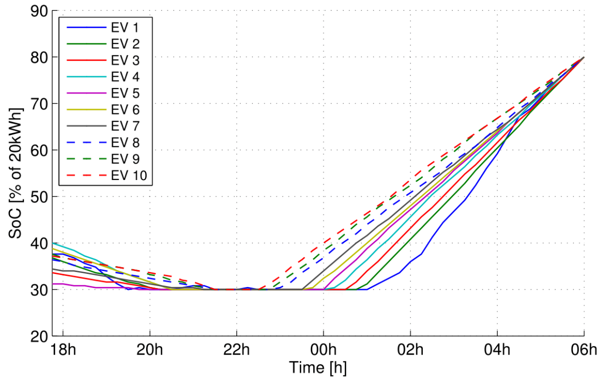

Fig. 7. Optimal charging schedules, states of charges for each player.

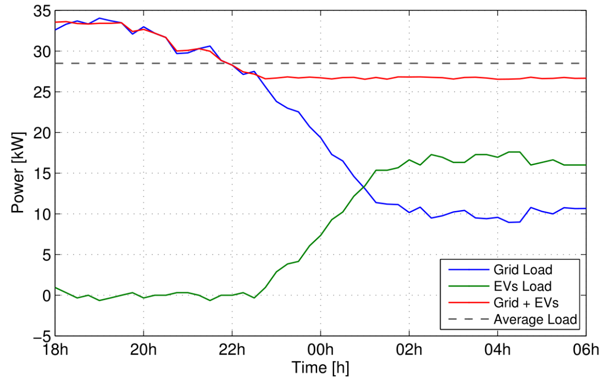

Fig. 9. Management of the charging profiles for initially discharged EVs.

Given the initial state of charge, the initial strategy for player i is defined with the policy of charging uniformly during the whole period of charge (the same rate of power consumption p^i_{init}). The rate is defined as,

$$p^i_{init} = \frac{soc^i_{des} - soc^i_0}{T \Delta t}$$

where T is the total number of steps in the charging period. In this case, $T = 49$.

Given the initial strategies, the initial load curve including grid and EVs is shown on Fig. 5. As it can be seen, this initial curve follows the same shape of the grid load with a higher average corresponding to the EVs load. The subsequent updates are also shown. It is important to note that once the last EV updates its strategy, the final load curve does not change much since the Nash equilibrium is almost reached. Also, it is important to notice how the total load curve becomes flatter with the progress of the game. This occurs because the utility of each player is higher if the total load at each step of the changing period is closer to the average load.

The optimal strategies or the final power consumption schedules are shown on Fig. 6. On the other hand, the profiles of state of charge are shown on Fig. 7. It is important to notice how each player respects the constraints of minimum state of charge (30%), final desired state of charge (80%), and power limits (± 3.2kW).

Fig. 8 shows a comparison between the grid load, the optimal total load (grid + EVs), the average load, and the load of EVs. It is important to notice that the consumption of EVs is mostly moved to off-peak hours (valley filling). During peak demand hours, EVs tend to discharge their batteries (if energy is available) to provide peak reduction. If the initial states of charge are minimal (30%), batteries can not be discharged and EVs cannot provide peak reduction. This case can be observed in Fig. 9. However, EVs are still charged mostly during off-peak hours.

VI. Conclusion

This paper provides a DP - Game theory approach to compute optimal charging schedules of multiple PHEVs in a decentralized scheme. Additionally, the proposed strategy employs the energy storage capacity of the PHEVs in order to provide a service of load flattening. The proposed scheme is evaluated under multiple cases in order to test its capabilities. Then, the results are analyzed in detail. For perspectives, multiple alternative tests can be considered in the future: the inclusion of a differentiation between energy selling prices and buying prices for different utility functions; the analysis under stochastic conditions for the prices or loading profiles; the optimal scheduling of other controllable house loads, etc.

VII. Acknowledgment

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References

[1] H. Turker, S. Bacha, and D. Chatroux, "Impact of plug-in hybrid electric vehicles (phevs) on the french electric grid," in *2010 IEEE PES Innov. Smart Grid Tech. Conf. Europe (ISGT Europe)*, Oct 2010, pp. 1–8.

[2] J. Moreno, M. Ortuzar, and J. Dixon, "Energy-management system for a hybrid electric vehicle, using ultracapacitors and neural networks," *IEEE Trans. Ind. Elec.*, vol. 53, no. 2, pp. 614–623, April 2006.

[3] H. Turker, S. Bacha, D. Chatroux, and A. Hably, "Low-voltage transformer loss-of-life assessments for a high penetration of plug-in hybrid electric vehicles (phevs)," *IEEE Trans. Power Del.*, vol. 27, no. 3, pp. 1323–1331, July 2012.

[4] F. Mapelli, D. Tarsitano, and M. Mauri, "Plug-in hybrid electric vehicle: Modeling, prototype realization, and inverter losses reduction analysis," *IEEE Trans. Ind. Elec.*, vol. 57, no. 2, pp. 598–607, Feb 2010.

[5] B.-G. Kim, S. Ren, M. van der Schaar, and J.-W. Lee, "Bidirectional energy trading and residential load scheduling with electric vehicles in the smart grid," *IEEE Journal Selected Areas in Comm.*, vol. 31, no. 7, pp. 1219–1234, July 2013.

[6] K. Kumar, B. Sivaneasan, P. So, and D. Wang, "Methodology for optimizing the number of electric vehicles deployed under a smart grid," in *IECON 2013 - 39th Annual Conf.IEEE Ind. Elec. Soc.*, Nov 2013, pp. 4647–4652.

[7] H. K. Nguyen and J. B. Song, "Optimal charging and discharging for multiple phevs with demand side management in vehicle-to-building," *Journal Communications and Networks*, vol. 14, no. 6, pp. 662–671, Dec 2012.

[8] W. Su, J. Wang, K. Zhang, and M.-Y. Chow, "Framework for investigating the impact of phev charging on power distribution system and transportation network," in *IECON 2012 - 38th Annual Conf. IEEE Ind. Elec. Soc.*, Oct 2012, pp. 4735–4740.

[9] R. Couillet, S. Perlaza, H. Tembine, and M. Debbah, "Electrical vehicles in the smart grid: A mean field game analysis," *IEEE Journal Selected Areas in Comm.*, vol. 30, no. 6, pp. 1086–1096, July 2012.

[10] H. Turker, S. Bacha, and A. Hably, "Rule-based charging of plug-in electric vehicles (pevs): Impacts on the aging rate of low-voltage transformers," *IEEE Trans. Power Del.*, vol. PP, no. 99, pp. 1–1, 2014.

[11] H. Turker, M. Hauck, A. Hably, and S. Bacha, "A tool of vehicle-to-grid (v2g) concept for voltage plan control of residential electric grid areas with plug-in hybrid electric vehicles (phevs)," in *IECON 2012 - 38th Annual Conf. IEEE Ind. Elec. Soc.*, Oct 2012, pp. 2883–2888.

[12] A. Ovalle, A. Hably, S. Bacha, and M. Ahmed, "Voltage support by optimal integration of plug-in hybrid electric vehicles to a residential grid," in *IECON 2014 - 40th Annual Conference on IEEE Industrial Electronics Society*, Nov 2014.

[13] V. L. Nguyen, T. T. Quoc, S. Bacha, and N. A. Luu, "Charging strategies to minimize the energy cost for an electric vehicle fleet," in *IEEE PES ISGT Istambul*, Oct 2014.

[14] C. Jin, J. Tang, and P. Ghosh, "Optimizing electric vehicle charging: A customer's perspective," *IEEE Trans. Vehicular Tech.*, vol. 62, no. 7, pp. 2919–2927, Sept 2013.

[15] ——, "Optimizing electric vehicle charging with energy storage in the electricity market," *IEEE Transactions on Smart Grid*, vol. 4, no. 1, pp. 311–320, March 2013.

[16] C. Pang, P. Dutta, and M. Kezunovic, "Bevs/phevs as dispersed energy storage for v2b uses in the smart grid," *IEEE Trans. Smart Grid*, vol. 3, no. 1, pp. 473–482, March 2012.

[17] C. Wu, H. Mohsenian-Rad, J. Huang, and A. Wang, "Demand side management for wind power integration in microgrid using dynamic potential game theory," in *2011 IEEE GLOBECOM Workshops (GC Wkshps)*, Dec 2011, pp. 1199–1204.

[18] P. Zhang, K. Qian, C. Zhou, B. Stewart, and D. Hepburn, "A methodology for optimization of power systems demand due to electric vehicle charging load," *IEEE Trans. Power Syst.*, vol. 27, no. 3, pp. 1628–1636, Aug 2012.

[19] A. Aabrandt, P. Andersen, A. Pedersen, S. You, B. Poulsen, N. O'Connell, and J. Ostergaard, "Prediction and optimization methods for electric vehicle charging schedules in the edison project," in *2012 IEEE PES Innov. Smart Grid Tech. (ISGT)*, Jan 2012, pp. 1–7.

[20] R. Patil, J. Kelly, Z. Filipi, and H. Fathy, "A framework for the integrated optimization of charging and power management in plug-in hybrid electric vehicles," *IEEE Trans. Vehicular Tech.*, vol. 62, no. 6, pp. 2402–2412, July 2013.

[21] K. Clement-Nyns, E. Haesen, and J. Driesen, "The impact of charging plug-in hybrid electric vehicles on a residential distribution grid," *IEEE Trans. Power Syst.*, vol. 25, no. 1, pp. 371–380, Feb 2010.

[22] Y. Wang, B. O'Donoghue, and S. Boyd, "Approximate dynamic programming via iterated bellman inequalities," *International Journal of Robust and Nonlinear Control*, 2014.

[23] K. Clement-Nyns, E. Haesen, and J. Driesen, "The impact of vehicle-to-grid on the distribution grid," *Electric Power Systems Research*, vol. 81, no. 1, pp. 185 – 192, 2011.

[24] Y. Riffonneau, S. Bacha, F. Barruel, and S. Ploix, "Optimal power flow management for grid connected pv systems with batteries," *IEEE Transactions on Sustainable Energy*, vol. 2, no. 3, pp. 309–320, July 2011.

[25] M. Lara and L. Doyen, *Sustainable Management of Natural Resources: Mathematical Models and Methods*, ser. Environmental Science and Engineering. Springer, 2008.

[26] D. Bertsekas, *Dynamic Programming and Optimal Control: Approximate dynamic programming*, ser. Athena Scientific optimization and computation series. Athena Scientific, 2012.

[27] E. Bilbao, P. Barrade, I. Etxeberria-Otadui, A. Rufer, S. Luri, and I. Gil, "Optimal energy management strategy of an improved elevator with energy storage capacity based on dynamic programming," *IEEE Transactions on Industry Applications*, vol. 50, no. 2, pp. 1233–1244, March 2014.

[28] P. Haessig, T. Kovaltchouk, B. Multon, H. B. Ahmed, and S. Lascaud, "Computing an optimal control policy for an energy storage," *CoRR*, vol. abs/1404.6389, 2014. [Online]. Available: http://arxiv.org/abs/1404.6389

[29] J. Webb, *Game Theory: Decisions, Interaction and Evolution*, ser. Springer Undergraduate Mathematics Series. Springer, 2007.

[30] J. Hofbauer and K. Sigmund, *Evolutionary Games and Population Dynamics*. Cambridge University Press, 1998.

[31] J. B. Rosen, "Existence and uniqueness of equilibrium points for concave N-Person games," *Econometrica*, vol. 33, no. 3, pp. 520–534, 1965.

[32] E. Paparoditis and T. Sapatinas, "Short-term load forecasting: The similar shape functional time-series predictor," *Power Systems, IEEE Transactions on*, vol. 28, no. 4, pp. 3818–3825, Nov 2013.

[33] K. Basu, L. Hawarah, N. Arghira, H. Joumaa, and S. Ploix, "A prediction system for home appliance usage," *Energy and Buildings*, vol. 67, no. 0, pp. 668 – 679, 2013.