# A Game Theoretic Approach for Plug-in Hybrid Electrical Vehicle Load Management in the Smart Grid

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Abstract — Plug in hybrid electric vehicles (PHEVs) have been recently introduced as environmentally friendly and fuel economic vehicles. It is believed that they will be widely adopted in the coming years, which puts the resilience of the electric grid in question. The large amount of electricity drawn by simultaneously charging multiple PHEVs can cause electric overloads which will either destroy grid components or shorten the equipment's life expectancy. In this paper, we propose a smart PHEVs charging algorithm to efficiently manage random and uncontrolled PHEV charging. The algorithm is based on a modified regret matching procedure that leads to a set of correlated equilibrium. A game theoretic approach is used to formulate the PHEV charging problem into a game where the players are the PHEVs and the strategies are their charging schedules. The proposed algorithm preserves users' privacy in terms of charging schedules and does not incur much overhead on the system. Simulation results show that proposed algorithm achieves high saving in terms of the total energy costs, individual costs, and flattens the overall charging load. The algorithm is also scalable and converges in an acceptable time.

Keywords – load; energy scheduling; smart grid; payoff; PHEV; game theory.

## I. INTRODUCTION

THE electric vehicle technology is becoming more popular across the world as concerns over energy conservation and the environment arise. In fact, according to [1] the overall fuel conversion efficiency of electric vehicles is 59%-62% compared to 17%-21% of fuelbased vehicles. An electric vehicle is any vehicle that is based on electrical propulsion. For instance, Plug-in Hybrid Electrical Vehicles (PHEVs) rely on both a combustion engine and a battery to operate. Battery electrical vehicle is another kind that functions solely using rechargeable electrical battery. A study [2] predicts that by 2015, the number of PHEVs on the road will reach 1.7 million, and that the United States will have one million charging stations. widespread use of EVs would put a tremendous new load on the current electricity grid, causing power quality degradation, consumer equipment damage, or even utility equipment failure resulting in blackouts. To overcome these challenges, smart grids have emerged as the next generation electrical power grid systems. A smart grid is an intelligent electricity

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network based on advanced new technologies such as communication, control, and market design. Its main goals are to enhance the efficiency and the reliability of the power grid, improve the quality of power supply, and save energy [3]. Currently, smart grids are subject to development in order to address the increasing energy demand [4], stemming from residential loads especially the uncontrolled and random charging patterns of PHEVs.

According to the FERC staff report [5], residential load management strategies can be divided into two categories. Time-based programs aim to encourage users to manage their loads individually, such as critical peak pricing (CPP) that drives users towards reducing their peak hour's consumptions. A common problem of time-based solutions is that they tend to shift a considerable amount of load from a typical peak hour to a typical non-peak hour, without any coordination this would create a rebound peak. Incentive-based, on the other hand, requires the utility to send some kind of load control messages to the customers.

In this paper, we propose a smart PHEV charging algorithm that significantly balances the load at the level of the grid hence reducing the total energy cost. This incentivebased algorithm takes into consideration the aggregated residential PHEV load to solve the energy consumption problem. The total load at each unit of time is used as a metric to satisfy the desired energy saving. We consider a common energy source shared by multiple customers, such as the case of an electricity generator supplying a residential neighborhood. We assume that smart PHEVs are connected to the utility company through a communication network either directly or via a local controller. We model the smart charging problem as a game, where each player executes a modified regret matching procedure to select the best charging time that would reduce the overall load and energy cost. This provides players with the incentives to collaborate. The proposed algorithm is highly distributed, semi-decentralized, and incurs very limited overhead on the network. Moreover, it converges in acceptable times, provides high savings in terms of energy cost, and balances the overall PHEVs demands, which automatically removes the peak load.

The rest of the paper is organized as follows. In Section II, we discuss related work. We then present the proposed scheme in Section III. Performance evaluation results are shown in section IV while Section V concludes the paper.

### II. RELATED WORKS

Several papers have been recently published addressing the consequences of uncontrolled PHEVs charging on the electrical grid. In fact, [6] and [7] show that as the number of PHEVs increases in the distribution system, the transformers, which are originally designed to handle overloading only for short periods of time, may break and cause grid failure.

As a result, the past few years have witnessed major interest from researchers on the issue of smart grid demand response using load scheduling.

For instance, the authors in [8] compare two control strategies, both of which are based on optimization. The local strategy schedules PHEV charging according to the loads of other appliances in a house, while the global one schedules PHEV charging taking into consideration global load information in a neighborhood.

The authors in [9] propose an optimization-based solution to minimize both the operational and the PHEVs emissions costs by dispensing the peak-hours loads into off-peak hours.

In [10] the authors focus on optimization techniques to flatten user demands individually by controlling home appliances and thermal devices. In the proposed scheme, each user independently predicts the price of the next day and optimizes its own consumption. The lack of coordination and the individual price reaction cause rebound peaks to be created in new specific periods instead of flattening the load.

In [11], two solutions for PHEV charging have been proposed: an optimal quadratic programming scheduler and a multi-agent system. While the scheduler gives the optimal solution by flattening peak loads and charging PHEVs, it suffers from scalability issues. The multi-agent system solves the scalability problem and is able to adapt to unpredictable information, however it adopts a fixed pricing scheme and fails to prevent cheating.

In [12], a game based on an optimization technique is proposed to balance the overall load and reducing the peak-to-average energy consumption ratio in a given neighborhood. However, the users privacy is not preserved as each user has to broadcast his energy consumption schedules to other users.

In [13], the authors formulate the scheduling problem as an optimization problem taking into consideration user preferences as the concept of discomfort level. The main goal is to balance the load and minimize user inconvenience caused by shifting users' demand schedules.

In [14], the authors tackle the problem of overloading at the level of the distribution transformer due to EVs penetration. The proposed solution is based on a load shaping tool that introduces load control strategies inside the household by taking into account preset user preferences and priorities. However, this scheme is only applied when overloading happens and hence does not flatten the overall load especially during the peak hours.

In [15], a distributed framework for demand response and user adaptation in the smart grid is proposed using the concept of congestion pricing. The adaptation mechanism is based on

the current price and a willingness to pay parameter that changes the behavior of the system and controls user's demands. However, the main disadvantage of this paper is the fact that the willingness to pay parameter is introduced as a way to impact the price. As in [10], [16] also relies on price prediction to propose two optimal PHEV charging algorithms. Both techniques are based on dynamic programming methods to achieve optimal economic solutions for the user.

Unlike previous works that propose optimal centralized solutions, our algorithm is highly distributed and semi-decentralized. It scales to a large number of PHEVs and converges in acceptable times. In addition, the game theoretic approach used employs two way communications between the users and the electric utility without incurring much overhead on the network. The algorithm allows the users to leave and join the game at will and dynamically adapts to these changes. In the absence of coordination or cooperation between users, the proposed algorithm achieves the best result possible. Overall, the local optimal performances of all users lead to a significant system performance.

# III. MODIFIED REGRET MATCHING BASED SCHEDULING

# A. PHEV Scheduling Game

Rather than choosing a centralized optimization solution that suffers from scalability issues, the technique adopted in this paper is based on game theory in order to decentralize the scheduling algorithm. In this game, the players will execute a reinforcement procedure based on regret matching [17]. Residential users are modeled as players who are willing to charge their PHEVs. The time needed to charge a PHEV differs from one player to another depending on the battery capacity and its remaining charge. We assume that the time between the arrival and departure of a PHEV is divided into equal time slots. Thus, each player first calculates the total number of time slots needed to charge his PHEV. Since the user only cares about having his car ready at his departure time on next morning, our procedure takes advantage of selecting non-consecutive charging time slots to shift the load and reduce the total energy cost. This creates a number of charging schedules, which constitute the set of actions (or strategies) for each user. A payoff function is associated with each action a player makes. The ultimate goal of each player is to maximize its payoff. Therefore, the payoff function is inversely proportional to the price the user pays for his total PHEV energy consumption. We assume that the energy price depends on two factors: the energy cost set by the electric utility, and the total number of users charging during in a given time slot [12].

The players are denoted by a set  $N = \{1,2,3,...,i\}$ , where n is the total number of players present in the system. Each player  $n \in N$  selects the best charging schedule from his set of schedules to maximize his payoff. We define a charging schedule as:

$$a^n = [a_1^n, a_2^n, \dots, a_h^n].$$
 (1)

where

 $a_h^n = \begin{cases} 1 & \text{if player n is charging his PHEV in time slot h} \\ 0 & \text{otherwise} \end{cases}$ 

**Algorithm 1:** PHEV Charging Algorithm Based on Modified Regret Matching for Player  $n \in N$ 

1: randomly select an initial charging schedule  $s_1^n$ set  $p_1^n(j) = \frac{\delta}{m^n}$  for every  $j \in S^n$ 2: for each stage  $t = 1, 2, \dots$  do 3: 4: send  $s_t^n$  to the grid utility if  $\sum_{n=1}^{N} a_h^n$  and  $C_h$  are received from the grid utility 5: 6: use equation (2) to compute  $u_t^n(s_t^n, s_t^{-n})$ 7: use equations (3) and (4) to compute  $Q_t^n(j, k)$ use equations (5) and (6) to compute  $p_{t+1}^n$ 8: use play probabilities  $p_{t+1}^n$  to select a charging 9: schedule for the next stage 10: end if 11: end for

and

$$\sum_{h=0}^{H} a_h^n = T$$

where T is the total number of time slots needed to charge the battery and H is the total number of time slots between the arrival and departure times.

The number of available charging schedules for each player represent his actions, denoted by the set  $S^n = \{1,2,3,\ldots,s\}$ , where s is the total number of available PHEV charging schedules.

The game is played in a repeated fashion where, at each stage t, player n selects a charging schedule from the available set  $S^n$ , denoted as  $s^n_t$ . The payoff of player n at stage t is denoted by  $U^n_t = u^n(s_t)$ . One of the advantages of this game is the fact that each user n only observes his own payoffs and actions and does not need to know anything about other players. For instance, the actions taken by all the other players and their payoffs are kept private and not exchanged between players. As a result, the proposed game ensures the privacy of its users.

We define the utility function of each user  $n \in N$  at stage t as follows:

$$U_t^n(s_t^n, s_t^{-n}) = -\frac{\sum_{h=0}^H \left[ a_h^n * C_h * \sum_{n=1}^N a_h^n \right]}{N * \sum_{h=0}^H \left[ a^n * C_h \right]}.$$
 (2)

Where  $s_t^n$  and  $s_t^{-n}$  represent the actions chosen by player n and the actions selected by all the other players except n at stage t.  $C_h$  represents the cost of energy in time slot h used for illustration purposes and can be removed since the utility is a function of load.  $\sum_{n=1}^{N} a_h^n$  represents the total number of users charging their PHEVs at time slot h. These two variables are provided by the grid utility to each user in order to calculate its payoff. The term  $C_h * \sum_{n=1}^{N} a_h^n$  represents the artificial total cost of energy at slot h which depends on the number of users connected on that same slot. As more users are connected to the grid during a specific timeslot, the price increases.  $\sum_{h=0}^{H} a_n^n$  represents the number of time slots needed to charge user's n PHEV. Thus, the nominator in (2) corresponds to the total price the player is going to pay when selecting action  $s_t^n$ .

The best case scenario would be that a player n is the only one charging his PHEV during specific time slots. This will induce the minimum price. On the other hand, the worst case consists of a scenario where all players in the system charge their PHEVs during the same time slots chosen by player n. In this case, the maximum price would be  $N*\sum_{h=0}^{H}a^n*C_h$  and  $U_t^n(s_t^n,s_t^{-n})=-1$ . As the user deviates from overloaded time slots, the nominator in (2) goes to 0, making  $U_t^n(s_t^n,s_t^{-n})\to 0$ , increasing the payoff.

In order to construct the payoff function, each player needs the aggregated load at each time slot  $h\left(\sum_{n=1}^{N}a_{h}^{n}\right)$  as well as the time slot  $\cot C_{h}$ . These values are communicated to each player by the utility grid. In fact, at the start of each stage, players send their chosen actions (charging schedules) to the utility grid, which calculates the aggregated load for each time slot and send it back to all the players in addition to  $C_{h}$ .

Therefore, a limited number of messages are exchanged in the system. A total of N messages are sent to the grid utility, which in its turn replies by N messages at each stage. Moreover, the proposed algorithm is not affected by players joining and leaving the game. In fact, as a player joins or leaves the system, he only needs to notify the grid utility by sending a control message. The utility grid then triggers all players to start a new game.

# B. Modified Regret Matching Based Algorithm

The proposed game approach relies a modified regret matching procedure [17] performed by each player. The average regret of player n from not having used action k instead of j at stage t is defined as:

$$Q_t^n(j,k) = \max\{C_t^n(j,k), 0\}.$$
 (3)

Where

$$C_t^n(j,k) = \frac{1}{t} \sum_{\tau \le t : s_\tau^n = k} \frac{p_\tau^n(j)}{p_\tau^n(k)} U_\tau^n - \frac{1}{t} \sum_{\tau \le t : s_\tau^n = j} U_\tau^n.$$
 (4)

Equation (3) represents an estimated value of the modified regret of player n at stage t for not having selected charging schedule k every time charging schedule j was selected in the past. It is a measure of the difference in the payoff over the stages when charging schedule k was selected and the stages when charging schedule j was selected.  $p_{\tau}^n$  stands for the play probabilities at stage  $\tau$ , hence  $p_{\tau}^n(j)$  is the probability that n choses charging schedule j at stage  $\tau$ .

Each player n selects an action j at stage t, the probability (5) of choosing another action k at stage t+1 is approximately proportional to the average regret from j to k; the same action j is selected again using the remaining probability (6). Actions with higher regrets at the current stage will be selected with a higher probability in the next stage. Over time, the regrets will be reduced as well as the average regret of any two actions for each player.

$$p_{t+1}^{n}(k) = \left(1 - \frac{\delta}{t^{\gamma}}\right) \min\left\{\frac{1}{\mu} Q_{t}^{n}(j, k), \frac{1}{m^{n} - 1}\right\} + \frac{\delta}{t^{\gamma} m^{n}}.$$
 (5)

$$p_{t+1}^n(j) = 1 - \sum_{k \in S^n: k \neq j} p_{t+1}^n(k).$$
 (6)

where  $j=s_t^n$  is the action of player n in stage t. At t=1,  $p_1^n$  are chosen randomly such that  $p_1^n(j) \geq \delta/m^n$  where  $m^n$  is the number of actions of player n. According to [17],  $0 < \delta < 1$ ,  $0 < \gamma < 0.25$  and  $\mu > 2M^n(m^n-1)$  for all n, where  $M^n$  is an upper bound on  $|u^n(s)|$  for all  $s \in S$  and  $m^n$  is the number of actions of n. Play probabilities (5) and (6) depend only on  $U_1^n, U_2^n, \dots U_t^n$ . This implies that every player n does not need to exchange any information with other players. As a result, the algorithm generates a very limited number of message overhead.

When the game converges, the authors in [17] prove that the players do not regret their dynamic choice of charging schedules based on Algorithm 1. In fact, each player achieves a reasonable payoff against variations of other players' choices, minimizing its consumption price. This leads to a significant system performance, where the overall total energy cost is reduced and the total load is well distributed over all time slots. The proposed algorithm might not achieve the optimal solution for each user, but it attains sub-optimal result in the absence of coordination among players.

### IV. PERFORMANCE EVALUATION

### A. Simulation Setting

We used OMNET++ discrete event network simulator in order to evaluate the performance of the proposed scheduling algorithm. In our simulations, we consider the following simulation settings for regret calculation:  $\delta = 0.01$ ,  $\gamma = 0.24$ and  $\mu = 2 * 0.1(m^n - 1)$ . According to [18], the capacity of PHEV batteries varies from 16 kWh to 53 kWh depending on the model. A total of 3 to 4 hours is therefore needed for a battery to be fully charged with plugs of 6.6kW or 16kW capacities. In our simulations, we adopt the Chevy Volt model as in [14], where normal charge requires a charging circuit of (120V/12A), charge Power is 1.44kW, and charge duration is 5.6 hour. We assume that users do not wait for their battery to be completely depleted in order to charge it. We assume that all users have at least 1.6 hours charging capacity left in their batteries and hence need a maximum of 4 hours charging time. Most of the time, users charge their PHEVs overnight, more specifically after they get back from work around 7p.m. We assume that they need their PHEVs to be ready at 7a.m. before going back to work. As a result, we set the total number of charging slots to 12 hours. In our simulations, we consider a scenario where users plug their PHEVs within the interval [7p.m., 1a.m.] to model a more realistic scenario where users perform some after work activities (such as shopping or staying out late for dinner). We use the following quadratic cost function as in [12]:  $Cost_h(L_h) = \alpha_h L_h^2 + \beta_h L_h + \theta_h$ where  $L_h = \sum_{n=1}^{N} a_h^n$  represents the total load per hour. We consider  $\beta_h = 0$ ,  $\theta_h = 0$  and  $\alpha_h = 0.2$  cents for simplicity purposes. The adopted cost function is used only for giving artificial cost rates and depends on the load.

# B. Simulation Results

The first scenario simulated consists of 50 PHEVS. Fig. 1 and Fig. 2 illustrate the energy consumption and the cost of

users, respectively, for the same scenario with and without using the regret based scheduling algorithm. In the normal charging procedure, users start charging their PHEVs randomly between 7p.m. and 1a.m. The total charging time is randomly selected from the range [1, 4] hours. As a result, we can see that the demands are higher within the first 6 hours of the night. This explains the peak seen in the period 7p.m. to 1a.m. in both figures. Using the proposed regret based algorithm, we notice that the loads are more evenly distributed over the different hours of the night. A total saving of 49% is achieved with the use of Algorithm 1. The highest energy cost per hour is reduced from 2.5\$ (7p.m. to 8p.m.) using normal charging to 0.5\$ (2a.m. to 3a.m.) using the proposed algorithm. Although the total energy consumed remains the same in both cases; the main difference lies in the fact that using Algorithm 1 results in distributing energy consumption efficiently for the whole neighborhood, which in turn reduces individual user's costs. This is illustrated in Fig. 3 which shows the total charging cost for individual users. For illustration purposes, only 10 PHEVs from the described above scenario are shown in Fig. 3.

To study the effects of the number of PHEVs on the performance of the algorithm, we performed an extra set of simulations with PHEVs ranging from 10 to 100, modeling different neighborhood densities. Each simulation was run for 200s and repeated 30 times. Every point in the graphs represents the average result of the 30 independent runs, the 98% confidence interval is also provided.

Fig. 4 shows the savings as a function of the total number of PHEVs in the neighborhood. The saving is calculated as the percentage difference between the total cost of all the users with and without using the algorithm. For instance, users achieve savings of almost 58% as is the case for 70, 80 and 90 in Fig 4. We also see that in the worst case, users achieve a saving of 40% in their total cost for 10 users. The lower saving can be explained by the fact that the demands of 10 users are lower than those of more users. This produces less condensed time slots during the charging interval (7p.m. to 1a.m.) using the normal charging method, leading to smaller savings using the proposed algorithm.

Fig. 5 shows the equilibrium time as a function of the total number of PHEVs in the neighborhood. The equilibrium time

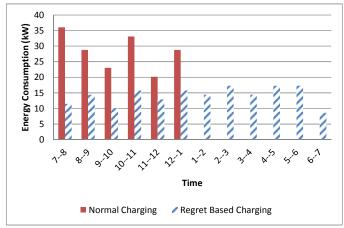


Fig. 1. Energy consumed with and without the regret based charging

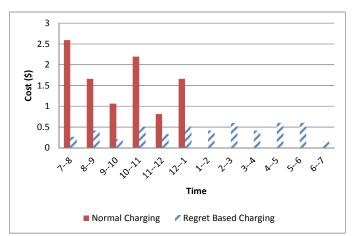


Fig. 2. Total cost with and without the regret based charging

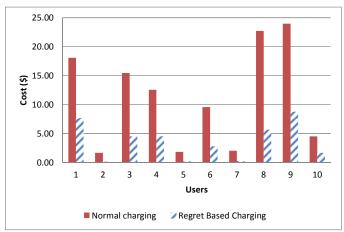


Fig. 3. Total charging cost for each user with and without the regret based charging

is the number of rounds required for the algorithm to converge. Convergence is achieved when users stick to the same charging schedule and decide not to change it in the following rounds. We see that it takes the algorithm at worst 28 rounds to achieve equilibrium, while it takes only 25 rounds in the best case. We also notice that the convergence of the algorithm does not depend on the number of users, which shows that the algorithm is scalable.

# V. CONCLUSION

In this paper, we proposed a smart PHEV charging algorithm in order to manage the tremendous new load put on the electricity grid by the penetration of EVs. The adopted game theoretic approach, used to formulate the energy consumption problem, is preserving the user's privacy and incurring very limited overhead on the network. Moreover, the highly distributed proposed algorithm is minimizing both the total energy cost and the individual users' costs. Simulation results also show that the algorithm converges in acceptable time and is scalable.

As future work, we plan to include the user preferences in proposed algorithm by providing a way for users to prioritize their comfort over their saving. We will also run more simulation scenarios to study the behaviour and effectiveness of the proposed algorithm in different settings. Finally, we plan to extend the algorithm to support other appliances in

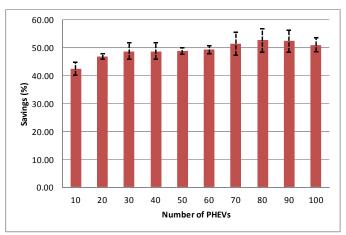


Fig. 4. Savings as a function of the number of PHEVs

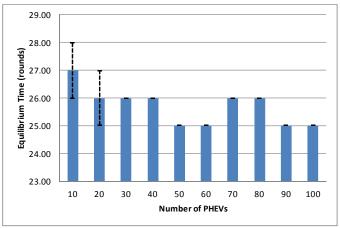


Fig. 5. Equilibrium time as a function of the number of PHEVs

addition to PHEVs and compare the results with the optimal solution.

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