2015 European Control Conference (ECC)
July 15-17, 2015. Linz, Austria

Online Adaptive Approach for a Game-theoretic Strategy for Complete Vehicle Energy Management

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Abstract— **This paper introduces an adaptive approach for a game-theoretic strategy on Complete Vehicle Energy Management. The proposed method enhances the game-theoretic approach such that the strategy is able to adapt to real driving behavior. The classical game-theoretic approach relies on one probability distribution function whereas the proposed approach is made adaptive by using dedicated probability distribution functions for different drive patterns. Owing to the adaptability of the proposed approach, the strategy is further refined by proposing dedicated objective functions for the driver player and for the auxiliary player. Next, an algorithm is developed to classify the measured driving history into one of the pre-defined drive pattern and employ the corresponding game-theoretic strategy. Multiple strategies are simulated with a model of a parallel hybrid heavy-duty truck with a battery and electric auxiliaries. The fuel reduction results are compared and the adaptive game-theoretic approach shows an improved and a more robust performance over different drive-cycles compared to the non-adaptive one.**

I. Introduction

Hybrid Electric Vehicles (HEVs) have attracted attention in the automotive industry for their fuel saving potential. To exploit this potential, careful design is required involving aspects such as the choice of energy sources, drivetrain topology, component size and control strategy [4]. Because HEVs are equipped with at least two energy sources, i.e., the fuel tank and the battery, the control strategy design focuses on developing an energy management strategy which decides on how to optimally distribute the power request for vehicle propulsion among the available energy sources. See e.g., [13], [12], [14], [11] and the references therein for existing energy management strategies for HEVs.

The concept of Complete Vehicle Energy Management (CVEM) is discussed in [8] where all subsystems in the vehicle and auxiliaries with energy buffers are taken into account in the energy management algorithm. In [3], a single-leader multi-follower game-theoretic framework is proposed where the driver is considered as the leader and each auxiliary is considered as a follower. A common cost function is formulated between the leader and the follower where the fuel mass flow is weighted together with a probability distribution function, G, that characterizes the likelihood of the driving behaviour in terms of the requested torque and speed over a drive-cycle. Up to now, using one G function in the game-theoretic setting has been explored in [7], [6], [5] and [3]. On the other hand, it should be noted that G functions may vary significantly for different drive-cycles and drivers. Consequently, the use of only one G function in the optimization criterion for an energy management system may not produce robust results when varying over different drive-cycles and different drivers. In this paper, the possibility of adapting the game-theoretic approach to improve the robustness over different drive-cycles is explored. The main contributions of this paper are as follows.

1. **Adaptability** The optimal strategies in the energy management system are computed on the basis of a cost function that changes with different driving behaviour. As such, the energy management strategy is made adaptive to the driver and the drive-cycle.

2. **Robust performance** If the optimization criterion adapts itself to the real driving behaviour, the uncertainty in the drive pattern represented by G can be reduced. This leads to potential performance improvement in terms of extra fuel savings.

We will show the merits of such an approach in this paper. The remainder of this paper is structured as follows: Section II presents the modelling of the components in the simulation model. Section III formulates the optimization problem considered in this paper and provides an outline of the proposed adaptive approach. Section IV and Section V describe the design of the game-theoretic strategy and the online classification technique in detail, respectively. A case study is presented in Section VI and the conclusions are given in Section VII.

II. Component Modelling

The type of vehicle we consider in this paper is a parallel hybrid heavy-duty truck. Its power net consists of a high voltage battery and additional auxiliaries with electric power demand P_{elec} (Figure 1). In this paper, we focus on the control of the high voltage battery for energy management. The drive train block contains all the drive line components from the transmission to the wheels. The goal of the energy management strategy is to control the battery power P_B such that the fuel consumption is minimized. It is assumed that the drivability is not influenced when different energy management strategies are applied. This implies that the vehicle speed, the gear ratio, and the drive train torque remain fixed. The models describing the components of the vehicle under consideration are given below.

This work has been carried out with financial support from the FP7 of the European Commission under the grant CONVENIENT (312314).

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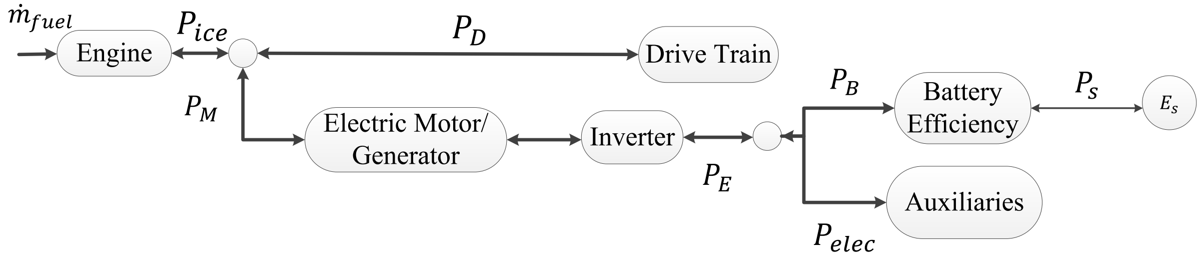

Fig. 1. Power flows within the vehicle simulation model

A. Engine

The internal combustion engine is represented by its fuel consumption as function of requested power P_{ice} and its rotational speed ω. It is given by the following quadratic function which is obtained by approximating a mean-value static fuel consumption map,

$$\dot{m}_{fuel} = \max(a(\omega)P_{ice}^2 + b(\omega)P_{ice} + c(\omega), 0), \quad (1)$$

with $\dot{m}_{fuel}$ [g/s] being the fuel mass rate. $a(\omega)$, $b(\omega)$ and $c(\omega)$ are real-valued coefficients that depend on the engine rotational speed ω. From the power balance indicated in Figure 1, we have $P_{ice} = P_D + P_M$. Here, P_D and P_M represent the requested driving power and the electric motor/generator power on the mechanical side, respectively. For simplicity, we assume an ideal inverter and an ideal electric motor/generator in the sense that $P_M = P_E$ where P_E denotes the electric power. Using this assumption and again the power balances in Figure 1, we have

$$P_{ice} = P_D + P_E = P_D + P_B + P_{elec}. \quad (2)$$

B. Battery

The battery energy E_s is modelled as an integration of the net power P_s stored in or retrieved from the battery. For a given sample time h, the discrete time model for E_s is given below

$$E_s(k+1) = E_s(k) + h \cdot P_s(k), \quad (3)$$

with $k \in \mathbb{N}$ denoting the discrete sample step. Using the approach described in [10], the power loss of the battery is taken into account. Hence, the battery output power P_B is given by

$$P_B = \gamma_{bat} P_s^2 + P_s, \quad (4)$$

where a positive constant γ_{bat} represents the quadratic loss during charging and discharging.

III. Problem Formulation and Proposed Solution

A. Problem Formulation

The objective of energy management is to minimize the overall fuel usage over a certain drive-cycle with length $t_f = k_f h$ for some $k_f \in \mathbb{N}$, while satisfying several constraints. This can be formulated by the following optimization problem

$$\min_{P_s} \sum_{k=0}^{k_f-1} \dot{m}_{fuel}(k) \cdot h \quad (5)$$

subject to (2) - (4) and

$$P_{B,min} \le P_B(k) \le P_{B,max}, \quad (6)$$
$$E_{min} \le E_s(k) \le E_{max}, \quad (7)$$
$$E_s(k_f) = E_s(0), \quad (8)$$

Note that (8) dictates that the battery is charge-sustaining such that the energy stored in the battery at the end of the drive-cycle is the same as its initial value. Constraint (6) imposes power limits on the battery and constraint (7) restrains the operating range of the battery.

With the exact information of P_D, (5) can be solved using dynamic programming (DP) [2]. However, the control strategy obtained in this way turns out to be non-causal as it depends on the future information of P_D. In order to have a strategy that uses only the current measurement of P_D rather than the future values, an intermediate cost function which involves the driver's power request and the auxiliaries' power requests, is commonly used (see e.g., [6], [5], [9], [7] and [3]). The authors in [5] consider the application where only the battery is present in the powertrain. Given the fact that only the current measurement of the powertrain operating condition is available, the authors in [5] re-formulate (5) as the following game problem within the prediction time horizon $t_p = k_p h$ for some $k_p \in \mathbb{N}$:

$$\max_{w(0)} \min_{u(0)} \cdots \max_{w(k_p-1)} \min_{u(k_p-1)} \sum_{k=0}^{k_p-1} L(x(k), u(k), w(k)) \quad (9)$$

where the stage cost is

$$L(x, u, w) = \text{Fuel}(u, w) + \mu(x_{ref} - x)^2 + \beta G(w), \quad (10)$$

where $x(k)$ and x_{ref} are the state and the reference state value for the battery, respectively, $\mu \ge 0$ and $\beta \ge 0$ are weighting parameters, $u(k)$ is the powertrain control variable, $w(k)$ is the powertrain operating condition selected by the driver and Fuel is the function that represents fuel rate. The function $G(w)$, is a static probability distribution function which describes the probability of a certain power-train operating condition to occur. The probability of each operating point is proportional to the time spent at this point over a typical drive-cycle. However, there are a few major shortcomings with this approach:

- **Non-robustness against Different Driving Behaviours** Because (9) suggests that the driver is maximizing the cumulative effect of (10), the term $G(w)$ encourages w to be selected as powertrain operating conditions that have large probability density over a specific drive-cycle. However, different drive-cycles and different drivers can lead to different driving behaviour and thus generate different G functions. Whether one G is representative enough to cover all possible real world driving scenarios remains an open question. As a result, using only one G might produce a poor description of the real driving behaviour.
- **Tuning of** β
It is seen in (10) that G is weighted by a factor β against the Fuel and the state deviation. By choosing different values for β, the weight of G becomes more (less) dominant. That is, a different level of confidence is placed on how much the real-world drive-cycle mimics the drive pattern captured by G. The extreme case is to choose β as zero which implies no confidence in G. As a result, a worst-case w, maximizing fuel consumption, is assumed. This assumption might lead to conservative result in terms of fuel economy. In general, the tuning of β is difficult as it is uncertain whether G is representative for the real driving behaviour. Differences in real driving behaviour can imply different preferred values for β.

B. Proposed Solution - an Adaptive Approach

In this paper, we present an adaptive game-theoretic approach to tackle the challenges formulated in the previous section. The basic structure of such an adaptive approach can be summarized in the following steps:

1.: We select N_w representative drive-cycles and construct a corresponding probability distribution function $G_j(w)$, $j = 1, 2, ..., N_w$ for each of them.

2.: Next, we design the game-theoretic strategies offline where full confidence is placed on $G_j(w)$ by carefully decomposing the cost function J_i defined for the ith auxiliary. The role of β in (10) is eliminated in this step. (More details are provided in Section IV).

3.: We develop an online algorithm that periodically classifies the recent driving history into one of N_w drive patterns represented by G_j. Consequently, the game-theoretic strategies corresponding to the identified drive pattern is adopted until the next time instance of the classification (More details are provided in Section V).

IV. Design of Stackelberg Strategy with Adaptive Cost Function

A. Decomposing the Cost Function

The authors in [5] propose a single-leader-single-follower game-theoretic framework to characterize the optimization problem (9) where the driver is the leader and the complete powertrain is the follower. In order to consider all the auxiliaries for CVEM, we adopt the single-leader multi-follower game-theoretic framework presented in [3] where the driver is selected as the leader and all auxiliaries are selected as individual followers. In the remainder of this paper, let $w_k = w(k) = (\tau(k), \omega(k))$ denote the requested wheel torque and rotational speed at the input side of the gearbox at time instance $t_k = kh$. Assume that for any $k \geq 0$, $w(k) \in W$ where

$$W = \{(\tau, \omega) \mid \tau_{min} \leq \tau \leq \tau_{max}, \omega_{min} \leq \omega \leq \omega_{max}\}.$$

Subsequently, let $u_{i,k} = u_i(k)$ denote the power request $P_i(k)$ for the ith follower, $i = 1, \ldots, N$ where N is the number of followers. We assume for all $k \geq 0$, $u_i(k) \in U_i$ where $U_i = \{P_i \mid P_{i,min} \leq P_i \leq P_{i,max}\}$. Assume that the ith follower has been modelled by a discrete time state space model

$$x_i(k+1) = g_i(x_i(k), u_i(k)).$$

In the ideal situation where energy conversions take place without losses, the engine power P_{ice} satisfies $P_{ice} = u_i + u_{-i} + \tau\omega$, for all $i = 1, \ldots, N$ with u_{-i} denoting the total power requests from other followers $\sum_{r \neq i} u_r$. Furthermore, by using (1), $\dot{m}_{fuel}$ can be expressed in terms of u_i, u_{-i} and w. Hence, we can define the fuel rate function $\text{Fuel}_i(w, u_i, u_{-i})$ such that for all $i = 1, \ldots, N$,

$$\dot{m}_{fuel} = \text{Fuel}_i(w, u_i, u_{-i}).$$

The authors in [3] formulate a cost function J_i for the ith follower similar to (9) where fuel consumption and the function G are weighted together in the incremental cost:

$$J_i = \Sigma_{\hat{k}=0}^{k_p-1}(h \cdot (\text{Fuel}_i(\hat{k}) + \beta G(w_{\hat{k}}))) + \mu_i(x_{i,k_p} - x_{i,0})^2, \tag{11}$$

with $\hat{k} \in \mathbb{N}$ being the discrete prediction step. x_{i,k_p} and $x_{i,0}$ are the end value and initial value for x_i within the prediction horizon. Here, we let $\text{Fuel}_i(\hat{k})$ denote $\text{Fuel}_i(w_{\hat{k}}, u_{i,\hat{k}}, u_{-i,\hat{k}})$ for the simplification of notation. Concerning the non-robustness problem described in Section III, we seek an adaptive cost function that incorporates different G functions. Hence, we replace G in (11) by G_j and define

$$J_{ij} = \Sigma_{\hat{k}=0}^{k_p-1}(h \cdot (\text{Fuel}_i(\hat{k}) + \beta_j G_j(w_{\hat{k}}))) + \mu_{ij}(x_{i,k_p} - x_{i,0})^2. \tag{12}$$

As mentioned in III, by choosing different values for β_j, we place a different level of confidence on how much the real-world driving mimics the drive patterns represented by G_j. As we propose now an approach that adapts itself to the real driving behaviour, we remove the uncertainty in G_j by decomposing (12) into J_j and $J_{aux,ij}$ and assign them to the leader and the follower, respectively:

$$\begin{aligned} J_j(w) &= \Sigma_{\hat{k}=0}^{k_p-1} L_j(\hat{k}), & (13)\\ J_{aux,ij}(w, x_i, u_i, u_{-i}) &= \Sigma_{\hat{k}=0}^{k_p-1} L_{aux,i}(\hat{k}) \\ &\quad + \mu_{ij}(x_{i,k_p} - x_{i,0})^2, & (14) \end{aligned}$$

where we use the abbreviated notation $L_j(\hat{k})$ for the stage cost of the leader $L_j(w_{\hat{k}})$ and the abbreviated notation $L_{aux,i}(\hat{k})$ for the stage cost of the follower $L_{aux,i}(w_{\hat{k}}, u_{i,\hat{k}}, u_{-i,\hat{k}})$. Moreover, L_j and $L_{aux,i}$ are given as

$$L_j = h \cdot G_j(w), \tag{15}$$

$$L_{aux,i} = h \cdot \text{Fuel}_i(w, u_i, u_{-i}). \tag{16}$$

Clearly, if $\beta = \beta_j$ and $\mu_i = \mu_{ij}$, we return to (11):

$$J_{ij} = \beta J_j + J_{aux,ij}. \tag{17}$$

If (14) is to be minimized, we recognize a terminal cost penalizing the squared difference between x_{i,k_p} and $x_{i,0}$. Because the prediction horizon is normally much shorter than the real drive-cycle, this terminal cost discourages any deviation of x_i which introduces conservativeness of the operating range for x_i. To remove this conservativeness, μ_{ij} can be selected as zero. However, such selection for μ_{ij} implies no penalty on the state value (for the battery, this will probably lead to complete depletion of the battery at the end of the drive-cycle). To reduce the conservativeness and still keep a penalty on the state variation, we remove the exponent in the terminal cost so that it becomes $\mu_{ij}(x_{i,0} - x_{i,k_p})$. As a result, (14) becomes

$$J_{aux,ij} = \Sigma_{\hat{k}=0}^{k_p-1} L_{aux,i}(\hat{k}) + \mu_{ij}(x_{i,0} - x_{i,k_p}). \tag{18}$$

Such terminal cost penalizes the increase (or decreases) of state variation from $x_{i,0}$ and encourages the decrease (or increase) of state variation from $x_{i,0}$ depending on the sign of μ_{ij}. In this way, the aggregated cost $\Sigma_{\hat{k}=0}^{k_p-1} L_{aux,i}(\hat{k})$ and the terminal cost $\mu_{ij}(x_{i,0} - x_{i,k_p})$ are counter acting each other in the sense that their preferences for operating x_i are contradicting. For example, considering the high-voltage battery, if we minimize (18) and select a negative μ_{ij}, the aggregated cost discourages x_{i,k_p} to go above $x_{i,0}$ and encourages x_{i,k_p} to go below $x_{i,0}$ for fuel economy while the terminal cost encourages x_{i,k_p} to go above $x_{i,0}$ and discourages x_{i,k_p} to go below $x_{i,0}$ for charge sustaining. Such a balancing effect is not achievable with a squared or absolute valued terminal cost since it penalizes the operation in both directions, which motives the selection of a linear terminal cost as shown in (18).

Note that although we focus our discussion on fuel economy in this paper, this method is capable of including other design specifications such as battery degradation, emission management, etc. by adding extra terms to the cost function (18), similar to what is done in [7].

B. Formulating the Sequential Game Problem

Now both J_j and $J_{aux,ij}$ have been defined, we formulate a dynamic game over the steps $\hat{k} = 0, 1, ..., k_p - 1$ between the leader and the follower. The leader focuses on maximizing J_j and the follower focuses on minimizing $J_{aux,ij}$. At stage $\hat{k}$, the leader has to announce his action first without knowing the decision from the follower. Therefore, he has to assume the worst-case response from the follower. Consequently, we define the cost-to-go function $V_{j,\hat{k}}$ associated to (13) with the following recursive law

$$V_{j,\hat{k}} = \max_{w \in W} \min_{u_i \in \hat{U}_{ij,\hat{k}}} \{L_j(w) + V_{j,\hat{k}+1}\}, \tag{19}$$

where $\hat{U}_{ij,\hat{k}}$ is the set of optimal response of the follower at stage $\hat{k}$. In this setting, the leader selects the strategy $w^*_{ij,\hat{k}}$ that satisfies

$$w^*_{ij,\hat{k}} = \arg\max_{w \in W} \min_{u_i \in \hat{U}_{ij,\hat{k}}} \{L_j(w) + V_{j,\hat{k}+1}\}. \tag{20}$$

(20) is the optimal strategy for the leader in a sequential game where decisions are made sequentially between the leader and the follower. Such a strategy is referred to as the Stackelberg strategy of the leader. As it is seen from (20), $V_{j,\hat{k}+1}$ and L_j do not depend on u_i and hence (20) reduces to

$$w^*_{j,\hat{k}} = \arg\max_{w \in W} \{L_j(w) + V_{j,\hat{k}+1}\}. \tag{21}$$

Furthermore, because $V_{j,\hat{k}+1}$ also does not depend on w, (21) boils down to

$$w^*_j = \arg\max_{w \in W} L_j(w). \tag{22}$$

After the leader has announced his strategy at stage $\hat{k}$, the follower rationally reacts to any possible action $w_{\hat{k}} \in W$ from the leader. Define the cost-to-go function $V_{ij,\hat{k}}$ associated to (18) with the recursive law given below

$$V_{ij,\hat{k}} = \min_{u_i \in U_i} \{L_{aux,i}(w_{\hat{k}}, u_i, u_{-i,\hat{k}}) + V_{ij,\hat{k}+1}(w^*_j, g_i(x_i, u_i), u^{-i,\hat{k}+1})\}, \tag{23}$$

where $u^{-i,\hat{k}+1}$ is the aggregated decisions of other followers from stage $\hat{k}+1$ till $k_p - 1$, i.e., $u^{-i,\hat{k}+1} = \{u_{-i,l}\}_{l=\hat{k}+1}^{k_p-1}$. The follower's optimal response (Stackelberg) strategy $\hat{u}_{ij,\hat{k}}$ is then the minimizer of (23). If the leader selects his Stackelberg strategy (22) at stage $\hat{k}$, the strategy pair $(w^*_j, \hat{u}_{ij,\hat{k}}(w^*_j))$ constitutes a Stackelberg equilibrium pair [1]. Next, we employ the receding horizon principle and assume $u_{-i,0} = u_{-i,1} = \cdots = u_{-i,k_p-1}$. As a result, the time invariant game-theoretic strategy taken by the follower is the Stackelberg strategy at $\hat{k} = 0$ as given below

$$\hat{u}_{ij} = \arg\min_{u_i \in U_i} \{L_{aux,i}(w_0, u_i, u_{-i,0}) + V_{ij,1}(w^*_j, g_i(x_{i,0}, u_i), u_{-i,0})\}. \tag{24}$$

The game-theoretic strategies of the follower corresponding to each G_j are computed offline and stored in look-up-tables for gridded values of $x_{i,0}$, w_0 and $u_{-i,0}$.

V. Online Classification of the Driving History

A. Method Overview

After we have designed the game-theoretic strategy for different drive patterns offline, an online algorithm is needed to classify the driving history into one of N_w drive patterns represented by G_j. This online classification procedure is illustrated in Figure 2. The function $G_{ol}(w, t_k)$ is constructed online where $t_k = kh$ is the sampled discrete time. $G_{ol}(w, t_k)$ takes the recorded driving history in a sliding

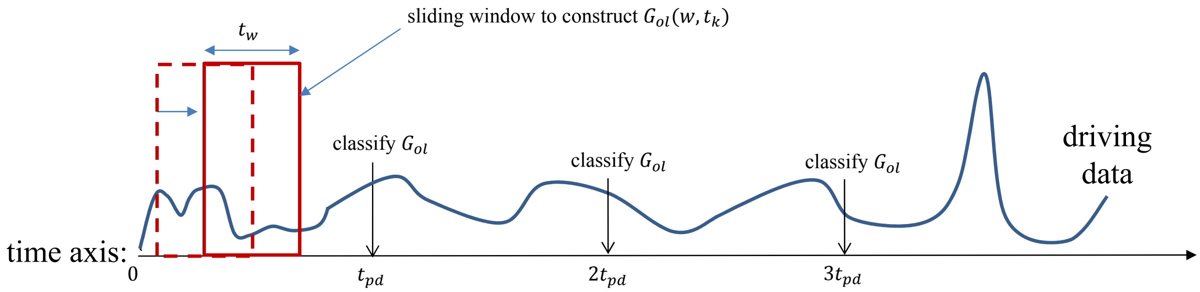

Fig. 2. Online classification of the driving history

window with length t_w and evolves with time in a receding horizon framework. After every time interval t_{pd}, an algorithm classifies G_{ol} into one of N_w classes of drive patterns represented by G_j. Once G_{ol} is classified, the corresponding game-theoretic strategy is employed within the next time interval t_{pd}. This procedure is repeated throughout the drive-cycle. The time period t_{pd} and t_w are parameters and in general should be in a scale of minutes since a sufficient amount of data is needed to construct a meaningful G_{ol} and fast switching between strategies is also not desired. Besides, such a choice also gives the ECU (Electronic Control Unit) in the vehicle sufficient time to run the online classification algorithm which will be discussed next.

B. Classification Algorithm

The purpose of the classification algorithm is to classify function $G_{ol}(w, t_k)$ into one of G_j functions. It is seen in Section IV that the leader focuses on maximizing (13) where $w = (\tau, \omega) \in W$. Furthermore, we assume the vehicle's inertia to be infinitely large, i.e., ω does not change within the prediction horizon $t_p = k_p h$. As a result, the decision space for the driver reduces to one dimension as $W = \{(\tau, \omega) \mid \tau_{min} \leq \tau \leq \tau_{max}, \omega = \omega(k)\}$. This means the leader is maximizing (13) by selecting τ. Then it is logical that the torque requests $\bar{\tau}_j(\omega)$ corresponding to the maximum values of G_j for each ω, i.e., $\bar{\tau}_j(\omega) = \arg\max_\tau G_j((\tau, \omega))$, are considered as important characteristics of G_j.

Following this logic, the classification algorithm focuses on fitting the characteristics of the online constructed G_{ol} to those of G_j by solving the minimization problem (25)

$$\min_{j \in \{1,2,\ldots,N_w\}} \Sigma_\omega \left(\frac{\bar{\tau}_j(\omega) - \bar{\tau}_{ol}(\omega)}{\tau_{max}} \right)^2 \cdot G_{ol}((\bar{\tau}_{ol}(\omega), \omega), t_k), \tag{25}$$

where $\bar{\tau}_j(\omega) = \arg\max_\tau G_j((\tau, \omega))$ and $\bar{\tau}_{ol}(\omega) = \arg\max_\tau G_{ol}((\tau, \omega), t_k)$. The algorithm penalizes the difference between torque values corresponding to the maximum values of G_j and G_{ol} at each ω. This error is also weighted by $G_{ol}((\bar{\tau}_{ol}(\omega), \omega), t_k)$ so that the errors at operating points that occurred more frequently in the driving history receive higher penalty.

VI. Case Study

In this case study, we would like to evaluate the robustness of the adaptive game-theoretic approach together with the non-adaptive one over different drive-cycles. The results will be compared in the simulation model from Section II.

A. Typical Drive-cycles for Constructing G_j

In total, we utilize 9 typical drive-cycles which cover different types of areas, e.g., city flat, secondary flat, etc. These drive-cycles contain information about τ and ω which are necessary for the construction of G_j. Table I provides the information of the average values throughout the drive-cycles.

TABLE I

TYPICAL DRIVE-CYCLES OVERVIEW

cycle type	average τ [Nm]	average ω [rad/sec]
$j = 1$: city flat	464.2	116.4
$j = 2$: secondary flat	884.0	115.0
$j = 3$: highway flat	942.8	130.1
$j = 4$: city hilly	620.7	131.2
$j = 5$: secondary hilly	841.2	131.3
$j = 6$: highway hilly	929.6	136.5
$j = 7$: city mountainous	591.0	131.2
$j = 8$: secondary mountainous	832.0	147.5
$j = 9$: highway mountainous	984.8	143.7

B. Drive-cycles for Evaluating the Strategies

We take in total four drive-cycles for evaluating the strategies. They consist of three drive-cycles for long haul application and one drive-cycle for the application of urban delivery. The descriptions of these cycles can be found in Table II.

TABLE II

DRIVE-CYCLES FOR EVALUATING THE STRATEGIES

cycle name	cycle description
Ardennen	a route through Belgium
Ancona Giessen	a route pan Europe
ACEA_LH	long haul cycle proposed by ACEA
ACEA_UD	urban delivery cycle proposed by ACEA

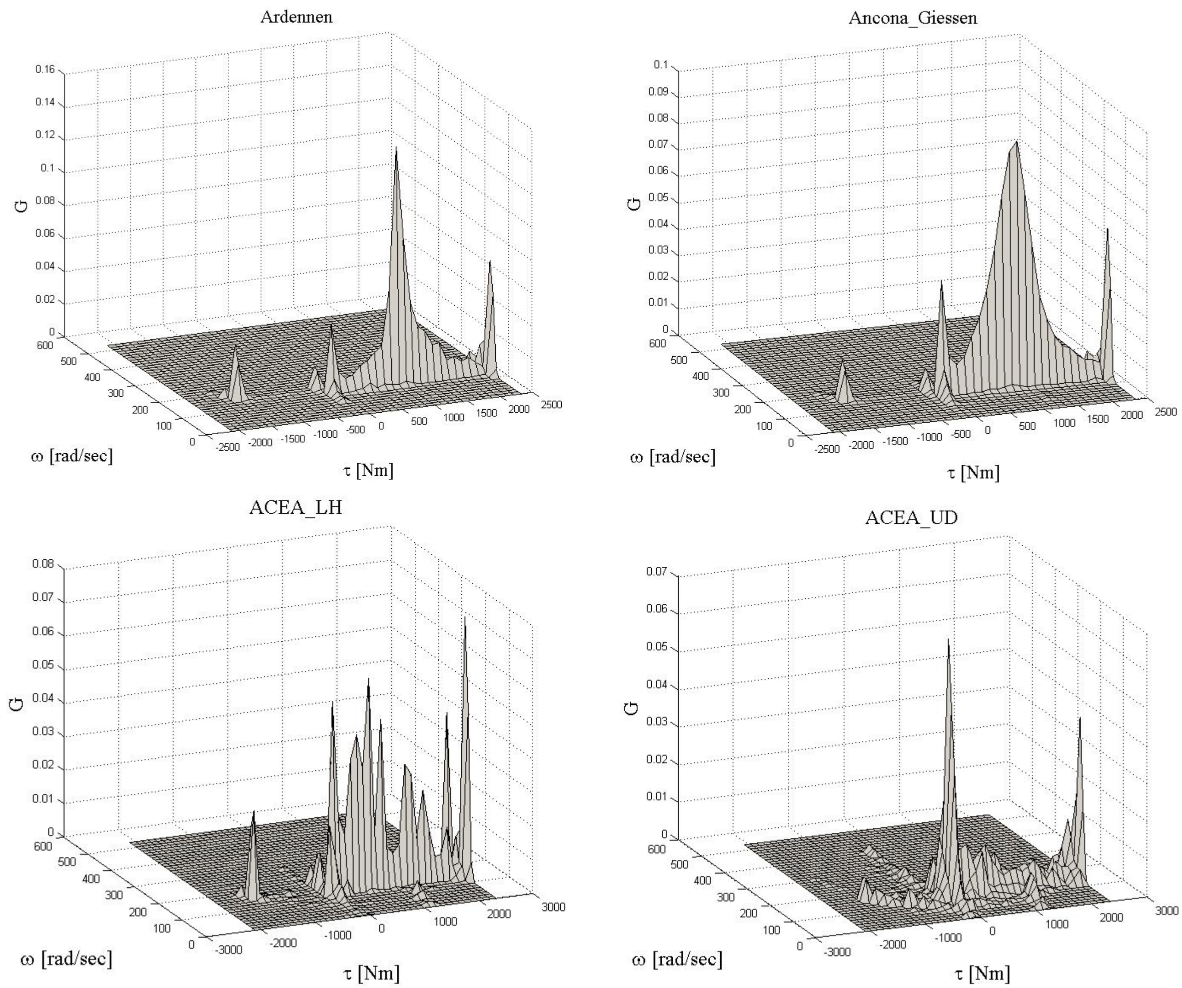

Fig. 3. G functions

The G function for each of these drive-cyles is shown in Figure 3. It is seen that the G function of Ardennen and Ancona Giessen looks similar: for these long-haul cycles, the majority of the operating points are concentrated around cruising on the highway which results in high peaks in the G function at $\tau = 900$ Nm, $\omega = 132$ rad/sec. Some similarities can also be observed in the G function of ACEA_LH. But the G function of ACEA_UD, on the contrary, shows few similarities.

C. Strategies for Comparison

In this case study, we give a constant power profile for P_{elec} and design the following strategies for comparison:

1. **Baseline**: non-hybrid mode where the battery is not utilized, i.e., the battery is never charged or discharged.

2. **GT**: the game-theoretic approach presented in [3], where one G function is considered, is used to determine the battery power P_s. Then, P_B can be found by using (4). The long haul drive-cycle Ancona Giessen is selected to design the G function as three out of four drive-cycles that we consider are for long haul application.

3. **Adaptive GT**: the proposed adaptive game-theoretic approach from Section III-B is adopted to determine the battery power P_s and then P_B is found by using (4). The drive-cycles from Table I are used to generate G_j and construct the Stackelberg strategy (24) as discussed in Section IV. Then, the classification technique discussed in Section V selects the corresponding Stackelberg strategies online.

Because the lumped additional auxiliaries always request constant power in this case study, the equilibrium strategies can be found by the second level simultaneous game described in [3] without iteration. For the classification algorithm, both t_{pd} and t_w are chosen to be 10 minutes.

4. **Dynamic Programming (DP)**: optimal solution for the battery power P_B is obtained using dynamic programming.

Table III summarizes the parameters used in the simulations. In all simulations, the end value of E_s gets close to the initial value such that no correction is needed for the fuel consumption result.

TABLE III

SIMULATION PARAMETERS

Name	Value	Name	Value
$P_{B,min}$ $[kW]$	-110	$P_{B,max}$ $[kW]$	110
E_{min}/E^{*}_{bat}	0.3	E_{max}/E^{*}_{bat}	0.7
γ_{bat} $[-]$	1.12×10^{-6}		

*E_{bat} denotes the maximum energy that can be stored in the battery.

D. Simulation Results

The fuel reduction of different strategies are shown in Table IV. It is observed that adaptive GT achieves better fuel economy in all drive-cycles compared to GT. Besides, the adaptive GT is reducing the gap between the performance of GT and DP. When we compare the fuel reduction for different cycles, adaptive GT outperforms GT up to 0.92 %. Adaptive GT shows significantly more benefits on the urban delivery cycle compared to the other long haul cycles. Another interesting observation is that the performance of GT on a cycle gets closer to adaptive GT if the G function of the cycle concerned looks more similar to the G function of Ancona Giessen which we used to design the strategy GT. It is concluded that the GT approach designed with a G function concerning a certain type of vehicle application shows reasonable performance for such application where the driving behaviour is correctly captured. But it will lose performance if a different type of application is considered. On the contrary, the adaptive GT approach eliminates the dependency of the strategy on types of application. Instead, it recognizes the drive pattern and employs the corresponding strategy. As a result, the adaptive GT strategy shows a robust performance on different drive-cycles.

TABLE IV

FUEL REDUCTION OVERVIEW

	Ardennen	Ancona Giessen	ACEA_UD	ACEA_LH
GT [%]	3.44	2.14	6.27	3.79
Adaptive GT [%]	3.61	2.23	7.19	4.06
DP [%]	4.06	2.55	7.52	4.60

VII. CONCLUSIONS

This paper proposes an adaptive approach for a game-theoretic strategy in CVEM. This approach first utilizes several typical drive-cycles to generate and store the corresponding game-theoretic strategies of auxiliaries in look-up-tables offline. Next, an algorithm classifies the recent driving history into one of the typical drive patterns periodically. As a result, the game-theoretic strategies corresponding to the identified drive pattern are adopted until the next time instance of classification. In the case study, multiple strategies are simulated on a model of a parallel hybrid heavy-duty truck with a battery and additional auxiliaries. The adaptive game-theoretic approach shows a more robust performance over different drive-cycles compared to the game-theoretic approach presented in [3] in terms of fuel economy.

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