Design of Power Controller for Hybrid Vehicle

Hubert H. Chin and Ayat A. Jafari
Electrical Engineering and Computer Science Department
New York Institute of Technology
Old Westbury, New York 11568

Abstract- In this paper a new solution for the design of a control system of a hybrid vehicle is presented using game model approaches. Hybrid vehicles combined the benefits of a gasoline engine and an electric motor, which can be utilized in parallel through a mechanical transmission. Players are considered to be the electric motor and the gasoline engine. Payoff matrices of the bimatrix game are calculated by a power controller of the hybrid vehicle. The Lemke-Howson algorithm used to compute a Nash equilibrium point which is a pair of strategies for both players. The solution concept is to integrate torque for satisfying the driver pedal motion. The ratio of the power contribution between the gasoline engine and electric motor is the key point for efficient driving, whilst satisfying power demands. The main contribution of this paper is the development of a power controller which improves fuel economy and reduces charging cycles thus minimizing emissions.

Keywords- Hybrid Vehicle, Power Controller, Microcontrollers, Bimatrix Game, Nash Equilibrium.

I. INTRODUCTION

Environmental concerns and foreign oil dependence have motivated legislative action by governments around the world to reduce tailpipe emissions. Modern automobile designs must satisfy challenging clean-energy technology requirements. With increasing concern over the global commitments, gasoline engines must improve fuel economy and reduce engine emissions: Carbon Dioxide (CO2), Carbon Monoxide (CO), Hydrocarbon (HC), Nitrogen Oxide (NOX), and Particulate Matter (PM). While customers demand performance and efficiency, all of these objectives must be delivered at low cost and high reliability. To meet these challenges engineers are looking to modern controls, innovative drivetrains, and advanced computer technology.

In 1905, H. Piper filed a patent for a petrol-electric hybrid vehicle. His idea was to use an electric motor to assist an internal-combustion engine, mainly to augment the engine to let the vehicle accelerate to 40 kilometers (25 miles) per hour in 10 seconds, instead of the usual 30. By the time the patent was issued, three and a half years later, engines had become powerful enough to achieve his idea. At that time cheap petrol and advances in conventional engine technology, gradually killed off the hybrid vehicle. However its technology did not disappear. Its development was continuing in the research arena.

Now major automotive manufacturers are interested in the development of hybrid vehicles. To date, the Toyota Prius and Honda Insight are equipped with an internal combustion engine (gasoline) in combination with an electric motor (nickel-metal-battery) for hybrids which have attracted the most attention. Meanwhile, DaimlerChrysler, Ford and General Motors intend to introduce hybrid light trucks in the USA. These five companies, as well as Nissan, Renault and Volkswagen, are also demonstrating prototype hybrids.

A hybrid vehicle can run on purely the engine, purely batteries, or a combination of both. Toyota Prius, Ford Escape, and Mercury Mariner are examples of this setup, as these cars can be moved forward on battery power alone. They need a large capacity battery pack for battery-only operation. These vehicles have options to put a sharing power path that allows more flexibility in the drivetrain by interconverting mechanical and electrical powers, at some cost in complexity. To balance the forces from each portion, the vehicles use a differential-style linkage between the engine and motor connected to the head end of the transmission.

Hybrid vehicles offer a virtually unlimited number of system configurations characterized by their power sharing patterns and control strategies. The overall efficiency of a hybrid vehicle is determined by its configuration of the components and the utilization of power control strategies. For example, the operation of a vehicle with an electric motor on a highway will result in a much higher energy use and lower efficiency than for a vehicle with a gasoline engine. On the other hand, a vehicle in a low speed conditions will operate more efficiently on an electric motor than a gasoline engine. Due to the wide range of traffic conditions, the objective of hybrid vehicles cannot be achieved without an optimal controller for controlling operation under the driver's intentions. The power controller is the element which ensures that the vehicle's power resources are shared in a dynamic real-time manner.

Powell and Pilutti [1] have used a combination of several controllers, one for every section of the vehicle, due to the highly nonlinear system. The fuel consumption was relatively high. Sacks and Cox [2] have proposed neuro-adaptive controllers; the major advantages are robustness to different driving and road conditions. Lee et al. [3] have used fuzzy systems, a fuzzy predictive controller with nine rules for converting the driver's commands to appropriate torques and another fuzzy controller with 25 rules. Ippolito et al. [4] have used fuzzy c-means, along with genetic algorithms, for power-flow management in different driving cycles of hybrid vehicles. In their method, there is a need for some off-line training for the controller, but they have achieved relatively low fuel consumption and smooth simulation results.

The goal of this paper is to develop the analytic solution for the fabrication of a discrete power controller. We have selected the MSP430 series microcontrollers [5] for the computation of our solution. It is a fairly standard 16-bit microprocessor and has many more configuration registers. Various combinations of the input parameters and respective performance functions are stored in the controller memory and the solutions of the game interpolates between the optimal strategies using games depending on previous selections.

A 500-millisecond period must be executed to find strategies of both engine and motor which would allow the vehicle to respond to the road conditions and driver's demands. That means that the output of the power controller must be varied to correspond with the general driving pattern. A typical road load profile consists of a number of cycles that include an initial acceleration phase, cruising phase including one or several sections at approximately constant speeds, separated by short periods of acceleration or deceleration, and the final phase of deceleration to stop. The power control on each of such cycles ideally would be such that the battery state of charge at the end of the cycle would be equal to that at the beginning of the cycle.

This paper describes a power controller that combines both power sources thus enabling the power control techniques. It is organized as follows: In Section 2, the configuration of a hybrid vehicle is described, followed by the hybrid modeling of bimatrix games in Section 3. The development of power controller is introduced in Section 4. Conclusions are presented in Section 5.

II. HYBRID VEHICLE CONFIGURATION

The focus of the hybrid vehicle development shifted towards parallel hybrid vehicle systems and more advanced power control strategies. The parallel configuration can be categorized by the way the two sources of power have a mechanical parallel connection to a mechanical transmission. The transmission provides the means for disengaging the drive and for changing gear. The disengaging of the drive is easily provided by simply turning off the motor. Gear changing is accomplished by using 'field weakening', which gives the electrical motor an approximately constant power versus speed characteristic. The speeds at this axis must be identical and the supplied torques adds together. When only one of the two sources is being used, the other must either also rotate in an idling manner or be connected by a one-way clutch or freewheel.

In the normal condition, it is usual to join the two sources through a differential gear. Thus the torques supplied must be the same and the speeds add up, the exact ratio depending on the differential characteristics. When only one of the two sources is being used, the other must still supply a large part of the torque or be fitted with a reverse one-way clutch or automatic clamp. In some cases, the combustion engine is the dominant portion (the electric motor turns on only when a boost is needed) and vice versa. Others can run with just the electric system operating.

Most designs combine a capacitor and a battery into one motor power, often located between the combustion engine and the transmission, replacing both the conventional starter motor and the alternator. To store power, a hybrid uses a large battery pack with a higher voltage than the normal automotive 12 volts. Accessories such as power steering and air conditioning are powered by electric motors instead of being attached to the combustion engine. This allows efficiency gains as the accessories can run at a constant speed, regardless of how fast the combustion engine is running. The use of a capacitor bank significantly reduces the power and energy cycling requirements of the battery system, thus increasing battery life. As an alternatively, a smaller battery bank can be used to reduce the vehicle weight. The peak torque, peak power and maximum speed specifications for the battery motor are 25 Nm, 10 kW, and 6000 revolutions per minute (rpm) and its function is to provide larger energy capacity at lower power levels.

A. Gasoline Engine Controller

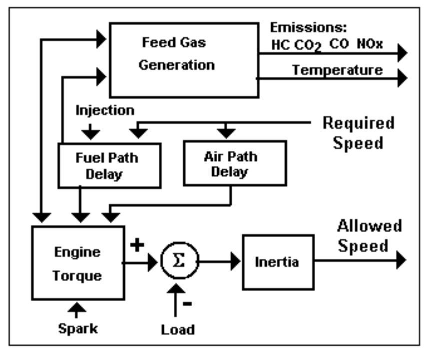

Figure 1: Gasoline Engine Controller

The first generation of gasoline engines regulated by controllers introduced major improvements in fuel economy and emission reductions over their purely mechanically controlled predecessors. In the conventional fuel injection engine, fuel is metered to form a homogeneous and generally stoichiometric mixture. The measurements are based on inlet air flow or intake manifold pressure, and injected into the intake port of each cylinder upstream of the intake valve. Emission control relies primarily on a three way catalyst system to convert the HC, CO2, CO and NOX emissions in the exhaust [6].

Figure 1 illustrates that high simultaneous conversion efficiencies for the three species occur only in a narrow band around stoichiometry, emphasizing the criticality of minimizing tailpipe emissions. An overview of the challenges related to emissions control in the design and development of drivetrain control systems for modern passenger vehicles may be found in [7].

Considerable effort as well is made to minimize engine emissions to reduce the amount of costly precious metal required in the three-way catalytic converter. Typically, NOX reduction is accomplished by reducing combustion temperature through exhaust gas recirculation. It can be introduced externally via a valve that connects the intake and exhaust manifolds or internally via variable camshaft timing control. The control can improve fuel economy in addition to reducing emissions. The challenges arise from dynamic interactions in the engines breathing process.

B. Electric Motor Controller

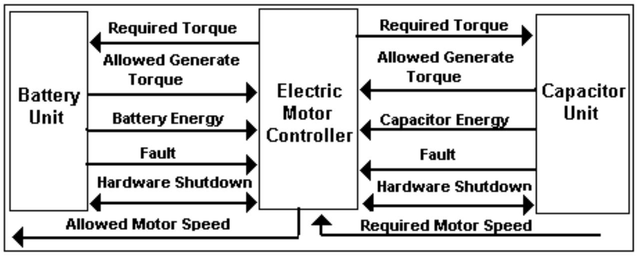

Figure 2: Electri Motor Controller

The electric motor controller in figure 2 is linked with batteries and capacitors where the batteries have a high energy-storage capacity and a reasonable power-handling capability. Capacitors offer low energy but high power. The combination of batteries and capacitors gives better performance than either would on their own. If batteries alone were used, then their weight would have to be high to satisfy the power requirement. If capacitors alone were used, then capacity would be a problem.

Capacitors have significantly better lifetimes and higher energy efficiencies than batteries. The peak torque, peak power and maximum speed specifications for the capacitor motor are 80 Nm, 32 kW, and 6000 rpm respectively. The specifications for battery motor are 25 Nm, 10 kW, and 6000 rpm respectively, but the battery motor provides larger energy capacity at lower power levels. The use of a capacitor bank significantly reduces the power and energy cycling requirements of the battery system, thus increasing battery life. The main function of the capacitor motor is to provide or accept short bursts of torque for accelerating, decelerating, or hill climbing.

The controller uses capacitors to extend battery life in such a dual energy-storage system to improve motor efficiency. The controller has two categories of battery units monitoring. The first category monitors the measured parameters as they approach their limits. The measured parameters are cell voltages and current, DC link voltage, power, speed change over time, phase current, and several temperatures. If the maximum allowed motoring torque were too rapidly reduced the resulting sudden unexpected power decreases would be irritating to the vehicle operator and, in the extreme case, could cause a road accident. This torque monitoring prevents the operator from noticing the power controller's impact on car's drivability. The second category monitors the measured parameters as they exceed their limits or as digital type faults occur in the controller. In the latter event, motor torque is zeroed and a fault is indicated to the controller. If the measured parameter continues increasing to a predetermined value outside the limit, a complete shutdown of the motor is initiated. The motor controller ensures that the maximum motor current is not exceeded and that the electric motor is not working when it is not needed.

Our design uses two electrical energy sources which are bi-directionally switched between the batteries and capacitors. The advantage is that this approach increases system redundancy. The disadvantage is that the motor transferring power between the battery and capacitors will be relatively low. The impact on efficiency is reduced if the controller performs most of the power transfer between the batteries and capacitors at high vehicle speed.

C. Parallel Configuration and Power Controller

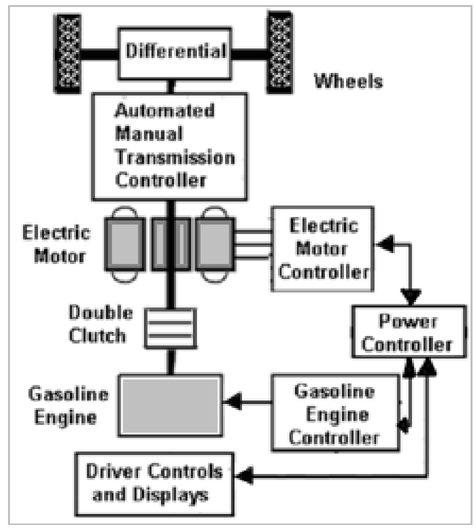

Figure 3: Hybrid Configuration

Figure 3 presents the block diagram of an electrical motor and a gasoline engine that are combined together to drive the vehicle [8]. The power controller designed using the game method integrates two power sources to provide the torque. The requested torque from the driver is calculated by optimum strategies output torque for the electrical motor controller and gasoline engine controller at any time. The electrical motor controller functions as the power contributing element when torque is needed for driving the vehicle. It also works as a generator when the state of charge of batteries is low and there is need to charge the batteries. The gasoline engine controller controls the engine by changing the throttle angle in order to produce the required torque.

The operating style of each source (i.e. engine or motor) and the amount of their contribution in producing the required torque at any time is determined by a power controller. Moreover, this controller should minimize the fuel consumption. However, minimizing the fuel consumption could lead to considerable torque reduction, which may not yield very pleasant driving conditions. Therefore, in addition to fuel minimization, it is sometimes necessary to maximize the torque as well. That is, although the game model has been designed for fuel economy and reducing air pollution, they must meet some minimum requirements on drivability. The hybrid vehicle provides four modes of operation which are selected by the driver. The contributions of the gasoline engine and the electric motor in different driving conditions are defined as follows:

1. Engine mode: The motor is off, and the gasoline engine provides the entire required torque.
2. Motor mode: The engine is off, and the battery provides electrical energy to power the motor (or the reverse when regenerative braking is engaged). Used for idling as well when the battery state-of-charge (SOC) is high.
3. Mixed mode: The vehicle is cruising and the power is shared between the gasoline and electric motor. In general, when the required torque is more than the maximum torque of gasoline engine, the electric motor compensates for the extra torque. If the battery state-of-charge is low, part of the power from the generator is directed towards charging the battery.
4. Battery charge mode: Also used for idling, except that in this case the battery state-of-charge is low and requires charging which is provided by the engine.

III. GAME MODEL FORMULATION

This paper concerns the game theoretical approach to designing power controllers for hybrid vehicles. The contribution of torque is formulated by a gasoline engine and an electric motor in which they are playing as two non-cooperative players to find their stable set of Nash equilibria [9] of a bimatrix game. Bimatrix games are among the most basic models in game theory, and solutions of the game are pair of optimal strategies for sharing torque between two power sources. The payoff matrices are predefined and determined by referee's choices. The strategic actions are specified by two players, a finite set of "pure or mixed" strategies for each player and a payoff for each player for each strategy profile (which is a tuple of strategies, one for each player).

The game is played by each player independently and simultaneously choosing one strategy, whereupon the players receive their respective payoffs. A player is allowed to randomize according to a probability distribution on his pure strategy set, which defines a mixed strategy for that player. Players are then interested in maximizing their expected payoffs. The game will play repeatedly with in a finite time (500 milliseconds).

The game is specified by two M×N payoff matrices A and B in $R^{M\times N}$, where the m rows are the pure strategies i of player 1 and the n columns the pure strategies j of player 2, with resulting matrix entries a_{ij} and b_{ij} as payoffs to player 1 and 2, respectively. The game here is of the non-cooperative bimatrix type. M and N denote the sets of strategies of Player 1 and Player 2. When player 1 takes the strategy $i \in M$ and player the strategy $j \in N$, the pair (a_{ij}, b_{ij}) is payoffs for the players 1 and 2 respectively according to matrices A and B. The payoffs can be considered as a Nash equilibrium, if it satisfies the following conditions.

For fixed j, $a_{ij} \geq a_{kj}$ for $\forall\, k \in M$;
For fixed i, $b_{ij} \geq b_{ih}$ for $\forall\, h \in N$.

Nash equilibrium is a steady state in an environment in which players act repeatedly and ignore any strategic link that may exist between successive interactions. In this sense, a mixed strategy represents information that players have about past interactions. We define the mixed strategies as follows.

A mixed strategy for player 1 is a probabilistic decision vector $X = (x_1, x_2, \ldots x_M) \in R^M$ such that $x_i \geq 0$ and Sum of x_i = 1. We define the same for player 2. X and Y denote the set of mixed strategies for player 1 and 2. For vector X, X^T presents the transpose of X. A pure strategy is a mixed strategy that has only one nonzero component.

A strategy for players is the torque which will be contributed to the hybrid vehicle. The measurement of pure strategy is a scale from 0 to 6000 rpm. Where torque = 5252 x horsepower / speed rpm. Both players have 60 pure strategies. For example, if 80% by player 1 involved choosing strategy i of 4000 rpm and 20% involved choosing k of 5000 rpm, then these i and k strategies form the payoff matrix can form 4200 rpm about the behavior of player 1. In equilibrium, the strategies will remain constant over 500 milliseconds.

The size of each payoff matrix is 60 by 60. The payoff matrices are based on goals of maintaining the battery state-of-charge (SOC), optimizing fuel economy and emissions. Moreover, it also satisfies the dynamic state of drivability requirements, such as driving distance, driving conditions, environment conditions, driver's power demands and vehicle performance conditions. Calculations of all contributed categories allow prioritization of the relative importance of each of these factors. This prioritization is a weighting factor of each category. However, the values of each payoff matrix cannot directly compare energy used (in Joules) and emissions output (in grams). In order to combine and weight the categories, entries of matrices have to convert values to similar non-dimensional scales.

The payoff matrix is presented by $[a_{ij}]_{mxn}$. The Payoff of each entry $a_{ij} = w_1$ (fuel-consumption) $+ w_2$ PM $+ w_3$ HC $+ w_4$ CO $+ w_5$ NOx $+ w_6$ (Engine-temperature) $+ w_7$ (SOC- deviation) $+ w_8$ (extra-weight) $+ \ldots + w_S$ (deriver's demands). Where, w_t is the weighting factor for each factor, t = 1,2, ...s; i = 1,2, … m; and j = 1,2, …n.

The weighting factors are illustrated as follows:

(w_1): Fuel consumption weighting factor is 4 of its scale. This means the maximum fuel usage would use 4 times more fuel than required to reach the destination. (Operating where the normalized weighting factor is 1)

(w_2): PM weighting factor is 5 of its scale value.

(w_3): HC weighting factor is 0.5 of its scale value.

(w_4): CO weighting factor is 2 of its scale value.

(w_5): NOx weighting factor is 35 of its scale value.

(w_7): SOC deviation weighting factor is 15 of its scale which is measured by a quadratic distance between the current SOC value and the SOC reference point. (etc.)

Lemke-Howson [10] found an elegant pivot algorithm to compute Nash equilibrium payoffs by:

(a) guessing the support of the equilibrium, i.e., the set of pure strategies each player uses with positive probability (which can always be done by systematically checking all possibilities, if necessary);

(b) finding a pair of mixed strategies with this support, such that all the pure strategies designated to either player give that player the same payoff against the other player's mixed strategy;

(c) computing the corresponding payoffs; and, finally; and

(d) checking that none of the other pure strategies of either player gives that player a higher payoff.

IV. POWER CONTROLLER DESIGN

The power controller of the gasoline engine and the electric motor in different driving conditions is defined as follows:

The first stage is to estimate the approximate level and duration of the torque for various driving conditions.

The second stage is to select payoff matrices which were made between the motor and the engine.

The third stage is to solve the bimatrix game for finding a Nash equilibrium and determines the contributed power and energy by means of detailed modeling of typical and extreme drive cycles in conjunction with the solution of bimatrix game.

The final stage is to obtain a pair of pure or mixed strategies. To some extent, the optimal strategies affect the power sharing. The values are sent to the motor and gasoline controllers.

The solutions, in terms of sharing power and energy storage, were determined using bimatix game which involved a four-stage process.

The design involves two phases. The overall goal is to produce a controller such that the closed loop system exhibits some desired emergent behavior. The emergent behavior is usually described linguistically (increased safety, improved torque efficiency, reduced environmental impact, etc.).

In the first phase of the design process (top-down) performance specification are established to quantify the desired emergent behavior. The performance specifications then get parsed to a preliminary high level (power) controller design. This process imposes requirements on the low level (Engine and Motor) controller. We use the Lemke-Howson algorithm to determine under what conditions the requirements can be met and produce a controller to meet them whenever possible. The Lemke-Howson algorithm makes use of ideas from game theory. The performance is treated as a game between two players: the actions of the player itself (gasoline engine) and the actions of the second player (electric motor).

In the second phase of the design process (bottom-up) the high level controller is modified to take into account these restrictions. The implication is that, if the process is completed successfully, the closed loop hybrid vehicle is guaranteed to satisfy the performance specifications without the need for further verification. To conclude the bottom-up phase, the election of the resulting hybrid controller on the physical process needs to be abstracted so that the result in emergent behavior can be evaluated. The bottom-up phase of the design process can also be thought of as a verification technique. Optimal control is used to determine the worst possible evolution of each player with respect to some desired property.

If the performance specifications are satisfied in this case, then they are guaranteed to be satisfied for all other evolutions as well. If the performance specifications are not satisfied, the verification process reveals ways in which the design can be modified in terms of restrictions on the actions of other players and on the switching patterns of the hybrid control scheme. If these restrictions are implemented in the discrete design the modified controller is guaranteed to satisfy the performance specifications.

On final consideration, there seem to be two main problems in the design of hybrid vehicles. The first problem is how to quantify emergent behavior requirements, i.e. how to come up with appropriate quantitative performance specifications. The second problem is how to design a power controller to meet these performance specifications. In terms of game theoretic approaches, the first problem manifests itself in the modeling of the network and link layers and the second problem manifests itself in the design of the coordination and regulation layer controllers. The top-down and bottom-up phases discussed above contain a little of both problems: the first part of the top-down phase and the last part of the bottom-up phase deal with the first problem while the rest of the design process deals with the second.

For implementation, a Toyota Prius with parallel configuration is the baseline vehicle. The basic control strategy of the Toyota Prius is an electric assist where the motor adds additional power when needed; the battery is mainly a peak power device [11]. The Prius uses the motor exclusively on takeoff and at low speeds. We have to convert some components and computer equipment to satisfy our requirements.

The battery packs used the Lithium-ion cells with 6 Ah capacities and a nominal output voltage of 3.6V. In order for a Prius to drive preferentially on electric power alone, it would need a 70kW or so battery pack, since the Prius car has approximately 70 kW of total usable power with the engine off. The C-rating for an energy-type battery pack is typically no higher than 5 or so, meaning that it can provide power at best approximately 5 times its 1 hour rated capacity. So a 70 kW battery is going to have about 14 kWHr of energy on board. Approximately 60% is available for hybrid vehicle discharge within safe and durable limits, so we have a usable 8.4 kWHr of battery energy.

Other components include controllers that use microprocessors which can process the inputs from sensors in real time and an electronic control unit that contains the hardware and software (firmware). The hardware consists of electronic components on a printed circuit board (PCB) and a ceramic substrate or a thin laminate substrate. The main component on this circuit board is a microcontroller chip. The software is stored in the microcontroller or other chips on the PCB, typically in EPROMs or flash memory so the CPU can be re-programmed by uploading updated code or replacing chips. The main tasks of microprocessors are listed as follows:

- The microcontroller must monitor the engine rpm (tap off the spark plug coil) and control the engine throttle. It will return the engine throttle from idle to its maximum.
- The microcontroller must monitor the battery voltage (voltage divider) and control the electric motor (PWM RC servo). The servo will let the battery voltage reach its maximum.
- The microcontroller will display key diagnostics (engine temperature, rpm, car max speed) on an alpha-numeric LCD (liquid crystal display) display.

V. CONCLUSION

In this paper, an optimal method based on bimatrix game model for controlling parallel hybrid vehicles was presented. In parallel hybrid vehicles the required torque for driving and operating the onboard accessories is generated by a combination of internal-combustion engine and an electric motor. By considering various possible driving conditions, the problem was cast as an optimal control problem for a hybrid vehicle. The controller divides power sharing between the internal combustion engine and the electric motor which is the key point for efficient driving. The controller will be designed based on the desired torque for driving and the state of charge of batteries. The output of the controller adjusts the throttle in the combustion engine.

It was found that improvements in fuel economy were derived mainly from optimizing the power contribution policy and discharging/charging schedule, and relieving the engine load through more efficient motor/battery operations. In the near future testing, bimatrix games will be played at every 500 milliseconds intervals under the following conditions:

- On a conservative basis, hybrid vehicle requires 0.25 kWh per mile driven;
- With average 9 kWh of daily production, vehicle can drive 50 miles;
- A typical testing range (according to 2001 DOT survey) is 24 miles; and
- The area of one parking space can provide enough energy for a hybrid vehicle to drive the typical testing range.

We expect the optimal results as follows:

- Carbon Dioxide emissions reduced by 60 to 70%;
- Gasoline consumption decreased by 60%;
- Cost per mile driven lowered by 30%; and
- Mixed mode operation => ~80 mpg (up to 25 miles)

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