

Energy Efficient Navigation Management for Hybrid Electric Vehicles on Highways

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ABSTRACT

Plug-in Hybrid Electric Vehicles (PHEVs) are gaining popularity due to their economical efficiency as well as their contribution to environmental preservation. PHEVs allow the driver to use exclusively electric power for 30–50 miles of driving, and switch to gasoline for longer trips. The more gasoline a vehicle uses, the higher cost is required for the trip. However, a PHEV cannot go long with its stored electricity without being recharged. Thus, it needs frequent recharging as compared to traditional engine vehicles powered by gasoline. Moreover, the battery recharging time is usually long, which leads to longer delays on a trip. Therefore, for the deployment of the PHEV technology it is necessary to provide a flexible navigation management scheme considering an efficient recharging scheduling, which allows choosing an optimal route based on the fuel-cost and time-to-destination constraints. In this paper, we show that this PHEV navigation management problem is *NP-Complete* and present a formal model to solve the problem using *Satisfiability Modulo Theories* (SMT) that provides a vehicle driver a routing plan, as well as the potential charging points that satisfy the requirements (e.g., the maximum fuel cost and the maximum waiting time). We also present a price-based navigation control technique to achieve better load balance for the system. Our evaluations show that the formalization can be efficiently solved even with large sizes of highway topologies and large number of charging stations.

Categories and Subject Descriptors

H.4 [Information Systems Applications]: Miscellaneous;
F.4.3 [Mathematical Logic and Formal Languages]:
Formal Languages—*Decision Problems*

General Terms

Management

Keywords

Plug-in Hybrid Vehicle, Navigation Plan, Formal Model

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1. INTRODUCTION

A Plug-in Hybrid Electric Vehicle (PHEV) is typically equipped with a rechargeable battery and can use the electric power stored in the battery as energy, as an alternative to traditional gasoline. This battery can be fully recharged by connecting to the power grid with an extension cord. In these days, PHEVs have larger batteries that allow the drivers to use electric power exclusively for 30–50 miles of driving before switching to gasoline for longer trips [1]. The cost for electricity to power PHEVs has been estimated at less than one fourth of the cost of gasoline [2].

An electric vehicle (EV) cannot travel long with stored electricity without being recharged. In case of longer trips, an EV needs frequent recharging. Recharging takes more time than refueling. The long recharging time leads to a longer delay in the trip. However, unlike EV, a PHEV can use gasoline when the battery power is finished instead of being constantly recharged. Hence, it is necessary to select the route that allows for an optimal recharging schedule plan for a PHEV in order to satisfy different constraints. Vehicle drivers usually have constraints on time, as well as on the fuel cost for a trip. They may have additional requirements like passing through some specific points (intermediate points of interest).

In this paper, we model PHEV navigation management as a constraint-satisfaction problem and use *Satisfiability Modulo Theories* (SMT) [3] to obtain satisfiable solutions, which include the routing plan, as well as the charging points satisfying the fuel-cost, the driving-time, and the intermediate points of interest requirements. We assume that a service provider can execute our model to compute a navigation plan for a PHEV based on the PHEV's requirements and the information about the highway system, the charging stations, the charging prices, etc. Therefore, a PHEV can obtain a navigation plan for the service provider at the beginning of its trip, which can be updated on-demand according to the latest information on the charging stations and traffic. We also present a navigation control technique to adjust the charging prices of the stations at each time slot with the motivation of indirectly distributing the vehicles in the roads and the charging stations. In summary, our main contributions are as follows:

- We define the *PHEV Navigation Management Problem* and show that the problem is *NP-complete* using a *graph theoretical* approach.
- We formally model the navigation management problem using SMT. We demonstrate the model using an

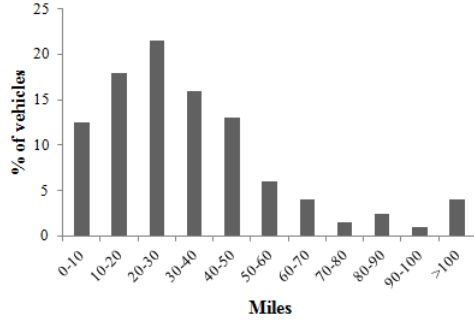


Figure 1: Percentages of vehicles with respect to daily miles driven.

example. We evaluate the efficiency of our proposed model by executing simulation experiments. Our model takes less than a minute for a highway system consisting of 500 exits and a similar number of charging stations.

- We present a price-based navigation control technique to achieve better load balancing for the system.
- In addition, this work contributes to the green management, as it inherently minimizes the usage of gasoline by efficiently managing the use of the recharging technology.

This paper is organized as follows: Section 2 presents the state of the art of PHEVs, the PHEV navigation management problem. Section 3 presents the formal modeling of the navigation problem along with an illustrative example. Section 4 presents the navigation control technique and its modeling. Section 5 presents evaluation results. Section 6 presents the related works. Section 7 concludes the paper.

2. BACKGROUND AND MOTIVATION

In this section, we first describe the state of the art of PHEVs. Then, we define the navigation management problem for a PHEV and discuss its complexity. Lastly, we discuss the motivation of using SMT for solving our model.

2.1 PHEV: State of the Art

Electric vehicles are gaining popularity in the market. It is expected that more than 10 million electric vehicles (pure and hybrid electric vehicles) will be in the United States (U.S.) by 2020 [4]. There are different models of PHEVs existing in the market [1, 2]. These cars usually can operate in the range of 30–50 miles after a full charge as a pure EV. Due to the recent development in PHEV technology, few well-advanced PHEVs are released to the market. For example, the Hyundai BlueOn [5] can go more than 80 miles per charge and can be recharged to 80% of the capacity in 25 minutes. Though most of the cars run trips within short distances, over 15% of the cars in the U.S. travel in the range of 50–100 miles, while around 3–4% of the cars run for trips more than 100 miles (see Figure 1) [6]. PHEVs are used for not only short-distance trips, but also long-distance journeys.

Since PHEVs cannot run for long distances with the electric power only, the battery needs to be recharged to use electric power instead of gasoline for further operation in the long trips. Again, PHEVs need long recharging times,

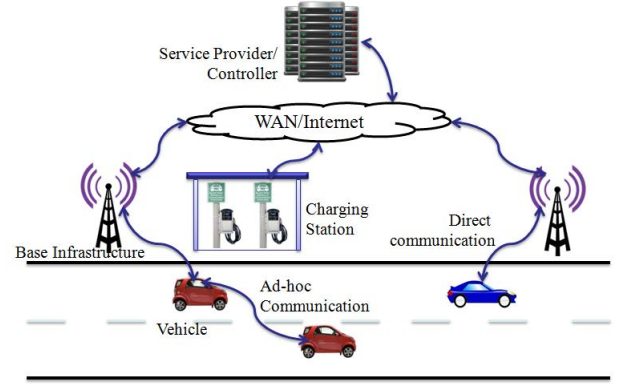


Figure 2: The communication model for the navigation management system.

which leads to long waiting at charging stations. Thus, for travel efficiency and comfortable driving, it is necessary to optimally plan a trip, i.e., the routing path and the electricity recharging schedule. There are few existing works related to the PHEV navigation management problem. For example, a fuel-efficient navigation mechanism is developed in [7], while an optimal recharging scheduling for EVs is proposed in [8]. However, neither of these models uses alternative fuels (electricity and gasoline) by PHEVs satisfying the constraints for cost-effectiveness and time-efficiency.

In this paper, we assume a centralized system that provides a navigation plan to a PHEV for a highway trip satisfying the PHEV's (i.e., its driver's) requirements. The service provider will execute our navigation management model for synthesizing the navigation plan. However, a PHEV can also run the model by itself, if it is equipped with a processor for executing SMT-based solver. For a navigation planning, along with the PHEV's requirements, our model needs different static and dynamic information about the highway system and the traffic. We assume that all the charging stations and the PHEVs are connected to the service provider through a wired or wireless, infrastructure-based or infrastructure-less communication model, e.g., as shown in Figure 2. The stations and PHEVs provide necessary local information to the service provider. The provider updates the dynamic part of the information system frequently with the collected data. This dynamic information includes the current (real) status and future (predicted) status of the charging stations and the traffic on the highways (especially, the number of waiting vehicles in the stations and the average traffic speeds on the roads). The information specific to a future time is predicted from the past records and current status. Since traffic can be very randomly distributed, spatially and temporally, the exact knowledge about the future may be different from the forecasted information. However, a PHEV can frequently update the navigation plan from the service provider based on the latest information and the same or modified requirements.

2.2 PHEV Navigation Management Problem

The goal of the navigation management problem is to find an efficient navigation plan on a highway system under a number of constraints. Along with the basic objective of reaching the destination, on the way towards the destination, a vehicle may require to pass through one or more intermediate points of interest, i.e., the *via-points*. There

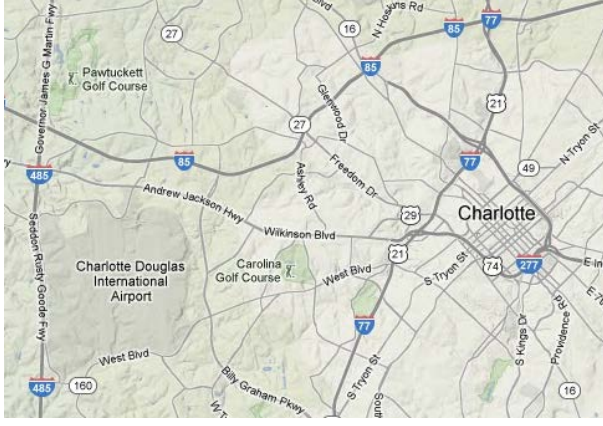


Figure 3: An excerpt of the highway system in United States. The thick lines represent the highways.

are usually different constraints in terms of time and cost. There can be a time-to-reach-destination (or simply time) constraint, which means the vehicle should reach the destination on or before a specified time. There may also be time constraints to reach the via-points. The fuel-cost (or simply cost) incurred in a trip depends on the type and amount of the consumed fuel. A hybrid vehicle can use either electricity or gasoline as fuel. We assume that the vehicle does not use both kinds of fuel simultaneously, but uses them sequentially. Hence, the cost is the summation of the price of electricity (the battery recharging cost) and the price of the amount of gasoline used throughout the trip. The constraint on fuel-cost specifies that the cost cannot exceed a given value. We assume that a PHEV can recharge its battery at the charging stations only. Charging prices and waiting queues are usually different at different stations. Due to the time-varying price model of power-grids, a particular station may have different prices at different time slots. The objective of the PHEV navigation management problem is to find a driving routing plan from the starting point to the destination that potentially enables the vehicle to reach the destination, including the intermediate points of interest, within associated time constraints, as well as potentially takes fuel-cost within the cost limit. In the following, we also formally define the problem and show that the problem is *NP-Complete*.

DEFINITION 1 (PHEV NAVIGATION MANAGEMENT PROBLEM).

Suppose we have a directed graph $G = (V, E)$. Every edge $e \in E$ has an associated cost $w(e)$ (corresponds to the time to travel from the start point to the end point of the edge). The set $V_1 \subseteq V$ is the set of charge stations, and every charge station $v \in V_1$ has an associated cost $w(v)$ (corresponds to the recharging cost in the station). For any path P from source S to destination D ($S, D \in V$), the time cost T_P is the summation of costs of all edges in P , and the recharging cost T_H is the summation of costs of all selected charging stations in P . Given two integer values C_t, C_h , the PHEV Navigation Management Problem is defined in a simple form as if a path P is possible from S to D such that $T_P \leq C_t$ and $T_H \leq C_h$.

THEOREM 1. *The PHEV Navigation Management problem is NP-complete.*

PROOF. We can reduce the two-objective shortest path problem to the PHEV navigation management problem. Suppose we have an instance of two-objective shortest path problem with a directed graph $G_1 = (V_1, E_1)$, where every edge $e \in E_1$ has two types of associated costs $c_1(e)$ and $c_2(e)$. The two-objective shortest path problem asks if there exists a path P from source S to D ($S, D \in V$) and the summation of cost type 1 of all links in the path is less than C_1 and the summation of cost type 2 of all links in the path is less than C_2 (C_1 and C_2 are integer numbers given as input). We can construct a new directed graph G based on G_1 . G has all the nodes of G_1 . In addition, G has a new station node v' for every link v_1v_2 of G_1 . We add a link from v_1 to v' and another link from v' to v_2 in G . We define the cost of link $v'v_2$ in G to be $c_1(v_1v_2)$ in G_1 , the cost of link v_1v' in G to be 0, and the cost of v' in G to be $c_2(v_1v_2)$ in G_1 . Now we can see that the two types of cost of a path in G_1 is exactly the time cost and recharging cost of the corresponding path in G . Since the two-objective shortest path problem is NP-complete [9], the navigation management problem is also NP-complete. \square

2.3 Efficiency of Using SMT

In this work, we formalize our proposed navigation management models using SMT. SMT is a powerful logic to solve constraint satisfaction problems that arise in many diverse areas including software and hardware verification, test-case generation, scheduling, planning, etc. [3]. SMT is the problem of determining whether a formula is satisfiable or not. For example, the SMT instance with the following two constraints is satisfiable with the assignments of $x = 1$ and $y = 0$:

$$(x + y < 2) \vee (x - 2y > 0)$$

$$x \leq 1$$

An SMT instance is a formula in first-order logic, where some functions and predicate symbols have additional interpretations. In SMT, complex Boolean logics are replaced by first order logics (predicates or functions) using a variety of underlying theories, e.g., the theory of equality, linear arithmetic, difference logic, etc. Modern SMT solvers can check formulas with hundreds of thousands of variables, and millions of clauses [10]. Our evaluations show that the formalization can be solved for problems with thousands of exits and charging stations.

3. PHEV NAVIGATION MANAGEMENT MODEL

In this section, we formalize the PHEV navigation management problem as a satisfaction of a number of constraints.

3.1 Highway System

A highway road usually consists of a good number of entrance and exit points. Figure 3 shows a map of the highways around the city of Charlotte (courtesy of Google Maps [11]). Since an exit is co-located with an entrance point, we consider both as a single point and use the word *exit point* (or simply *point*) to denote both of them. A *link road* may connect an exit of a highway to an entrance of another highway. We assume that all the roads are bidirectional. We do not consider any traffic jam or signaling system in the highways, rather the effects of their existence are realized in the average vehicle speeds of the roads.

The charging stations are deployed at different exits. If there is a charging station at an exit, a PHEV can charge its battery there, and then continue moving to its destination. The transport network (i.e., the highway system) and the placements of the stations are known. It is worth mentioning that we do not consider gas stations in our model assuming that a PHEV can store/carry enough gasoline that is required for the whole trip. This assumption is fair as usually a vehicle can drive 500 miles easily with a full-tank of gasoline on highways. Moreover, the inclusion of gas stations in the model would be similar to that of charging stations and there would be no additional complexity in the modeling. Charging stations may have different prices for charging. Due to the time-varying pricing model, different times of the day can also have different prices. We assume that the time in a day is divided into 24 slots on an hourly basis. The charging price remains the same within a slot, while it may vary between the slots. Usually recharging a PHEV takes a substantial amount of time. So, there can be some vehicles waiting to recharge in a station. The queue size of the waiting cars can be different at different times of the day. For example, the number of waiting vehicles at the end of the day is usually larger than the number at midnight.

3.2 System Model

In this subsection, we present the modeling of highways, stations, PHEVs, and user requirements.

Highway. We define the highway system as a collection of highways, the exit points, and the link roads (simply roads) between the highways. With no loss of reality, we define a highway in terms of exit points and link roads, also. That is, a highway is modeled as a sequence of exit points and the roads between them. A point is denoted by a pair of parameters $\{h, l\}$: the highway h and the location on the highway l . $L_{h,l}^{\hat{h},\hat{l}}$ represents the road between $\{\hat{h}, \hat{l}\}$ to $\{h, l\}$. $D_{h,l}^{\hat{h},\hat{l}}$ is the distance (miles) between $\{\hat{h}, \hat{l}\}$ to $\{h, l\}$. The average speed at which a vehicle moves in the road is $S_{h,l}^{\hat{h},\hat{l}}$ (miles/minute). The average speed comes from the speed limit of the road and/or the traffic on the road.

Station. $S_{h,l}$ denotes that a charging-station exists at point $\{h, l\}$. It has different properties. The price of charging (per kWh) at a time slot t is denoted by $Ps_{h,l,t}$ (dollars). The expected queue length of the waiting vehicles at the time slot t is $Qs_{h,l,t}$. Recharging the battery of a vehicle takes a time $Ts_{h,l}$ (minutes) in an average. We assume that if a battery is recharged, it is recharged until it has reached its full capacity. We do not consider the number of charging outlets, since this can easily be reflected in the queue size.

Vehicle. A plug-in hybrid vehicle V has different properties: the current location Sv (an entrance point), the destination Dv (an exit point), the stored electric charge Ev (kWh), the battery capacity Cv (kWh), the electric energy consumption rate Re (miles/kWh), and the gasoline consumption rate Rg (miles/gallon). We assume that any vehicle drives on a road at the speed limit (i.e., $S_{h,l}^{\hat{h},\hat{l}}$) of the road. The price of gasoline Pg is assumed as constant in the system.

User Requirements. We consider that the vehicle is required to satisfy two major constraints. The first constraint is on the cost Cp (dollars); that is, the price of the fuel consumed by the vehicle should be within a cost bound. The second

constraint is on the traveling time Ct_{Dv} (minutes); that is, the vehicle has to reach the destination Dv within a time limit. There can be more constraints. The vehicle might need to go via some intermediate points ($\{h, l\}$ s). These via-points are denoted as a set I . Moreover, a vehicle might be required to reach a via-point within a time constraint ($Ct_{h,l}$).

3.3 Parameters for Modeling Vehicle Routing

We use the following parameters in order to model the routing of a vehicle from a source to a destination:

- *Visited point.* $X_{h,l}$ denotes that the vehicle reaches the point $\{h, l\}$; that is, it moves to location l on highway h . It is a boolean value. If the vehicle reaches the destination, the goal is achieved (i.e., no more movement is required). $X_{h,l}^{\hat{h},\hat{l}}$ denotes that the vehicle moves from $\{\hat{h}, \hat{l}\}$ to $\{h, l\}$.
- *Station selected for recharging.* $S_{h,l}$ denotes whether the vehicle recharges at the station located at the point $\{h, l\}$.
- *Stored electricity.* $C_{h,l}$ represents the stored electricity (electric charge) of the vehicle at the point $\{h, l\}$.
- *Effective stored electricity.* $\hat{C}_{h,l}$ is the effective electricity, at the point $\{h, l\}$. Obviously, $\hat{C}_{h,l} \geq C_{h,l}$. If the vehicle is recharged from the charging station at $\{h, l\}$, it is greater than $C_{h,l}$.
- *Fuel-cost.* $P_{h,l}$ is the cost that the vehicle has spent to reach the point $\{h, l\}$, starting from the source.
- *Time-to-reach.* $T_{h,l}$ is the time at which the vehicle has reached the point $\{h, l\}$.
- *Slot of a time.* $T_{h,l}^s$ represents the time slot in which the time $T_{h,l}$ falls.

3.4 Vehicle Routing Model

The routing of a vehicle from a source to a destination is a sequence of moves from one point to another point starting from the source until the destination. The cost and time incurred due to this sequence of moves must be within the cost and time constraints.

3.4.1 Modeling Vehicle Movement

A vehicle can move to the point $\{h, l\}$ from the point $\{\hat{h}, \hat{l}\}$, if the vehicle is in $\{\hat{h}, \hat{l}\}$ (i.e., it is already reached) and there is a road from $\{\hat{h}, \hat{l}\}$ to $\{h, l\}$. The vehicle can move to $\{h, l\}$ from any of the neighboring points (i.e., the points connected to $\{h, l\}$).

$$X_{h,l}^{\hat{h},\hat{l}} \rightarrow X_{\hat{h},\hat{l}} \wedge L_{h,l}^{\hat{h},\hat{l}} \quad (1)$$

$$X_{h,l} \rightarrow \bigvee_{\hat{h},\hat{l}} X_{h,l}^{\hat{h},\hat{l}} \quad (2)$$

There is no need to move from the destination or to the source. In order to reduce the search space, we consider these issues as constraints, which are encoded as follows:

$$\forall_{h,l}(\{\hat{h}, \hat{l}\} = Dv) \rightarrow \neg X_{h,l}^{\hat{h},\hat{l}} \quad (3)$$

$$\forall_{\hat{h}, \hat{l}} (\{h, l\} = Sv) \rightarrow \neg X_{h, l}^{\hat{h}, \hat{l}} \quad (4)$$

In our model, we assume that no point is revisited. Due to this assumption, it is obvious that, if there are any intermediate destinations (i.e., via-points), they have to be on the way towards the final destination. If there is a requirement to backtrack a path to reach the final (or an intermediate) destination, which could be mandatory due to system based on constraints, then the model will fail to find a routing plan. The following constraints ensure that no point is revisited:

$$X_{h, l}^{\hat{h}, \hat{l}} \rightarrow \bigwedge_{\hat{h}', \hat{l}' \neq \hat{h}, \hat{l}} \neg X_{h, l}^{\hat{h}', \hat{l}'} \quad (5)$$

$$X_{h, l}^{\hat{h}, \hat{l}} \rightarrow \bigwedge_{h', l' \neq h, l} \neg X_{h', l'}^{\hat{h}, \hat{l}} \quad (6)$$

3.4.2 Modeling Battery Recharging

The battery of the vehicle can be recharged at a charging station provided that the vehicle reaches the corresponding exit point of the station, as shown in (7). If the battery is recharged, the effective charge of the battery increases. If it is not recharged, then the effective charge remains the same. We assume that when a battery is recharged, it is always recharged fully up to its capacity.

$$Sc_{h, l} \rightarrow X_{h, l} \wedge S_{h, l} \quad (7)$$

$$Sc_{h, l} \rightarrow (\hat{C}_{h, l} = Cv) \quad (8)$$

$$\neg Sc_{h, l} \rightarrow (\hat{C}_{h, l} = C_{h, l}) \quad (9)$$

The formalization of the stored electricity of the vehicle when it reaches a point is shown in (10). The stored electricity at the point $\{h, l\}$ depends on the stored electricity at the last visited point $\{\hat{h}, \hat{l}\}$ and the charge required to cross the distance between these points.

$$X_{h, l}^{\hat{h}, \hat{l}} \rightarrow (B \rightarrow ((C_{h, l} = \hat{C}_{\hat{h}, \hat{l}} - C) \wedge (\hat{D}_{h, l} = 0))) \wedge (\neg B \rightarrow ((C_{h, l} = 0) \wedge (\hat{D}_{h, l} = D_{\hat{h}, \hat{l}}^{\hat{h}, \hat{l}} - D))) \quad (10)$$

In (10), C and D represent $(D_{h, l}^{\hat{h}, \hat{l}}/Re)$ and $(\hat{C}_{\hat{h}, \hat{l}} \times Re)$ respectively, and B denotes the Boolean result of $(\hat{C}_{\hat{h}, \hat{l}} \geq C)$.

3.4.3 Modeling Time and Cost

The computation of the cost of traveling from a point to another point depends on the energy (fuel) type that has been used by the vehicle. We assume that the vehicle's primary energy source is the battery. If the battery is out of charge, then gasoline is used as the energy source. Hence, in a trip, the vehicle can use any one of them or both based on the availability of the battery charge. The cost, i.e., the price of consumed fuel, $P_{h, l}$ to reach the point $\{h, l\}$ is modeled in (11). In the equation, \hat{t} stands for $T_{h, l}^{\hat{h}, \hat{l}}$, and $PG_{h, l}$ represents $((D_{h, l}^{\hat{h}, \hat{l}} - \hat{D}_{h, l})/Rg) \times Pg$, which denotes the price of the amount of gasoline that is consumed to reach $\{h, l\}$.

$$X_{h, l}^{\hat{h}, \hat{l}} \rightarrow (Sc_{\hat{h}, \hat{l}} \rightarrow (P_{h, l} = (P_{\hat{h}, \hat{l}} + PG_{h, l} + Ps_{\hat{h}, \hat{l}, \hat{t}} \times C_{h, l}))) \wedge (\neg Sc_{\hat{h}, \hat{l}} \rightarrow (P_{h, l} = (P_{\hat{h}, \hat{l}} + PG_{h, l}))) \quad (11)$$

The time required for traveling does not depend on the energy source (fuel) type (refer to Section 3.2). It depends on

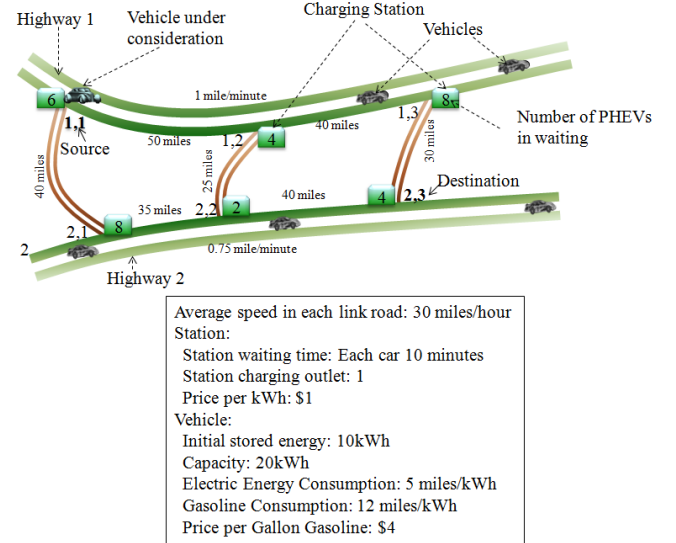


Figure 4: An example of the system with the values of different parameters.

the average speed of the road. Equation (12) shows the modeling of time computation. Here, T represents $(D_{h, l}^{\hat{h}, \hat{l}}/S_{h, l})$.

$$X_{h, l}^{\hat{h}, \hat{l}} \rightarrow (\neg Sc_{\hat{h}, \hat{l}} \rightarrow ((T_{h, l} = (T_{\hat{h}, \hat{l}} + T)))) \wedge (Sc_{\hat{h}, \hat{l}} \rightarrow (T_{h, l} = (T_{\hat{h}, \hat{l}} + T + Qs_{\hat{h}, \hat{l}, \hat{t}} \times Ts_{\hat{h}, \hat{l}}))) \quad (12)$$

3.4.4 Modeling User Requirements

The main objective of the model is to find a route from the source to the destination. There are three more user requirements on the navigation plan: limited cost (Cp), time boundary (Ct), and the intermediate points of interest (I) with/without associated time boundaries ($Ct_{h, l}$ for $\{h, l\} \in I$). The following equations represent these constraints:

$$X_{Sv} \wedge X_{Dv} \bigwedge_{\{h, l\} \in I} X_{h, l} \quad (13)$$

$$P_{Dv} \leq Cp \quad (14)$$

$$T_{Dv} \leq Ct \quad (15)$$

$$\forall_{\{h, l\} \in I} T_{h, l} \leq Ct_{h, l} \quad (16)$$

It is easy to understand that, in the navigation plan, a point cannot be reached by the vehicle taking more cost or time than the overall cost or time boundaries (i.e., Cp or Ct), respectively. We incorporate these constraints as follows:

$$X_{h, l} \rightarrow P_{h, l} \leq Cp \quad (17)$$

$$X_{h, l} \rightarrow T_{h, l} \leq Ct \quad (18)$$

We need to consider the constraints that initialize the model. These constraints are about the stored electric capacity, the time and the cost at the source. The starting time of travel (i.e., time at source) is initialized with the current time. Equation (19) models the initialization constraints.

$$(C_{Sv} = Ev) \wedge (T_{Sv} = T_{current}) \wedge (P_{Sv} = 0) \quad (19)$$

Table 1: Input to the Example

```
# Numbers of highways and numbers of exits of each highway.
2 3 3

# Seven roads: ((highway, exit) (highway, exit)) pair,
# with length and average speed (speed limit).
7
1 1 1 2 50 3
1 2 1 3 40 3
1 1 2 1 40 1
1 2 2 2 25 1
1 3 2 3 30 1
2 1 2 2 35 3
2 2 2 3 40 3

# Five charging stations (position (highway, exit), price/kWh,
# queue size, waiting time).
5
1 1 1 6 10
1 2 1 4 10
1 3 1 8 10
2 1 1 2 10
2 3 1 4 10

# Nine properties of the vehicle: (position (highway, exit),
# destination (highway, exit), stored electricity, battery capacity,
# miles/kWh, miles/gallon, gasoline price/gallon).
1 1 2 3 10 20 5 12 4

# Starting time, two constraints (time, cost).
50 140 20
```

Table 2: Z3 Output from the Example

```
(define X11 true)
(define X23 true)
(define Cs11 false)
(define Cs12 true)
(define X12 true)
(define Cs13 false)
(define X13 true)
(define Cs21 false)
(define X21 false)
(define Cs22 false)
(define X22 false)
.....
```

3.5 Implementation: SMT Encoding

We implement our model by encoding the system configuration and the constraints into SMT using *Z3* SMT solver [12]. We use the *Z3 .Net API* for encoding the formalization of the model that we have already described in this section. We use three types of terms: *boolean*, *integer*, and *real*. Though many parameters (e.g., distance, speed, queue size, etc) in the model are usually integer values, we encode them as real terms, since we need to do some division operations in computing the time and the cost incurred for the vehicle routing. The system configurations and the constraints are given in a text file (*input* file). Executing the model (in *Z3*), we obtain the verification result as either satisfiable (*sat*) or unsatisfiable (*unsat*). If the result is *unsat*, it means that the problem has no route from the source to the destination satisfying the constraints. In the case of *sat*, we get the navigation plan from the assignments of the variables, $X_{h,l}$ and $Cs_{h,l}$. $X_{h,l}$ shows the route, while $Cs_{h,l}$ represents the station that is selected for recharging. The results from our model is also printed in a text file (*output* file).

3.5.1 Example

Figure 4 presents a small example problem. The corre-

Algorithm 1 Optimal Configuration Determination

```
 $Cp_{min}$  &  $Cp_{max}$  are the minimum and maximum values of  $Cp$ .
 $K_{max}$  is the maximum number of executions of the loop-body.
if Solver returns SAT then
  Get Model,  $M$ .
  Update  $Cp_{max}$  according to the value of  $P_{Dv}$  in  $M$ .
   $K = 0$ .
  repeat
     $Cp = (Cp_{min} + Cp_{max})/2$ .
    Update the constraints (14) & (17) associated to  $Cp$ .
    if Solver returns SAT then
      Get Model,  $M$ .
      Update  $Cp_{max}$  according to the value of  $P_{Dv}$  in  $M$ .
    else
       $Cp_{min} = Cp$ .
    end if
     $K = K + 1$ .
  until ( $Cp_{max} - Cp_{min} \approx 0$ ) or ( $K = K_{max}$ ).
end if
```

sponding input file is shown in Table 1. We want to find a possible route that satisfies the given constraints. In order to keep the example simple, we consider the same charging price and queue size for a particular station irrespective of the time of a day. The execution of the model corresponding to the example gives a *sat* result. The important part of the solution (i.e., the assignments to different variables of the model) is shown in Table 2. From the assignments, we find that one of the possible routes for the vehicle is $[\{1, 1\}, \{2, 1\}, \{2, 2\}, \{2, 3\}]$, and the battery should be recharged at the station of the exit $\{2, 1\}$. The fuel cost through this route is \$18, while the time cost is 85 minutes, both of which satisfy the constraints.

3.6 Optimal Navigation Plan Determination

The verification result comprehensively represents a consistent PHEV navigation plan for the network, satisfying the user requirements. Usually, there are more than one model that satisfy the constraints. These models also take different time and cost, though all of them take time and cost less than or equal to the time constraint (Ct) and the cost constraint (Cp). Observing these models, one can choose the most cost (or time) efficient route among all alternative satisfiable models for the same set of constraints. We propose Algorithm 1 that considers two values: Cp_{max} and Cp_{min} , and finds the optimal navigation plan based on the cost. Typically, Cp_{max} is the user's given constraint Cp , while Cp_{min} is zero.

The algorithm utilizes a binary search method to find the optimal value. Algorithm 1 usually takes a longer time compared to the time for finding a satisfiable model only, since the algorithm requires several invocations of the model synthesis. The complexity of the algorithm is $O(T_{verify} \times \log_2 D)$, where T_{verify} is the synthesis time and D is the difference between $Th_{A,max}$ and $Th_{A,min}$. Since T_{verify} is very high in unsatisfiable cases as well as in tight constraint-based cases (see Section 5 for details), the time for finding the optimal would be very high for a large number of vehicles. However, if the navigation management service is provided online basis, where the SMT model is executed by an SMT solver running in a centralized server, the navigation management service provider can utilize powerful machines to compute this optimization. Even, the algorithm lets the provider can control the number of steps (K_{max}) to reduce the optimization time. In this case, a vehicle may receive a

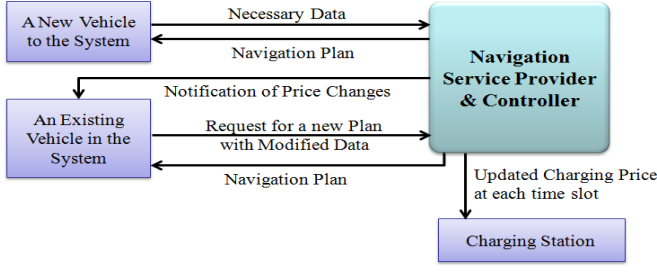


Figure 5: The information flow in the control system

semi-optimal navigation plan.

4. PRICE-BASED NAVIGATION CONTROL

The model we have discussed in Section 3 finds a satisfiable navigation plan for a PHEV under a number of given constraints. The main constraints are time and cost. The traveling cost depends not only on the electricity price but also on the availability of the charging stations, the queue lengths of waiting vehicles in the stations, and the time constraint to reach the destination. If many vehicles choose the same routes or route segments (i.e., roads), the queue sizes of the charging stations on these routes have much higher possibility of being longer compared to that of other stations. Moreover, a large number of vehicles may create traffic jams on these roads. Hence, it is important to control the traffic flow for distributing the load among the roads in a balanced way, so that the roads have no or less traffic jams and the charging stations have smaller waiting queues. In this section, we present a simple load control technique, which motivates the navigation management model to choose the roads with lower loads. The service provider will play the controller role. However, the success of the model depends on the even distribution of the charging stations throughout the system.

4.1 Basic Idea

The basic idea we follow in our control technique is to adjust the charging prices of the stations at each time slot to indirectly distribute the vehicles in the roads and the charging stations. The control technique is shown in Figure 5. When a vehicle enters the system, it establishes communication with the service provider/controller. The vehicle sends necessary information to the controller. The controller computes a satisfiable navigation plan based on the model discussed in Section 3.2 and sends it back to the vehicle. Hence, the controller has the navigation plans of all the existing vehicles in the system. From these plans, the controller can know the traffic loads of each road segments and adjust the charging prices at each time slot. The price adjustment technique considers that the existing routing paths are expected to be unchanged. Hence, the technique considers the cost requirement of each vehicle as a constraint, so that the navigation plan for the vehicle will remain valid even after any price adjustment. The controller can notify each existing vehicle in the system if there is a price adjustment. Then a vehicle may request the controller for a new navigation plan.

In order to adjust the charging prices to achieve better load balance, the controller gives a weight for each point $\{h, l\}$ associated to a charging station during a time slot t based on the number of vehicles that have the point on its

routing path during t — the more busy the point is during t , the higher weight given to it at t . The control chooses the prices according to these weights— higher weights get higher prices, while lower weights have lower prices. The prices during a time slot t should be within some minimum $(P_{min,t})$ and maximum $(P_{max,t})$ bounds. Moreover, the average of the charging prices of all the charging stations during a time slot t is constrained to be equal to $P_{avg,t}$, where $P_{avg,t} = (P_{min,t} + P_{max,t})/2$.

4.2 Parameters for Navigation Control

We define \mathbb{S} as the set of the charging stations, i.e., the set of the points where the stations are located. We also define \mathbb{V} as the set of the vehicles currently existing in the system. At the beginning of the time slot \bar{t} , the controller updates the charging prices for the stations for a number of future time slots (i.e., time $t \geq \bar{t}$) by modeling a constraint satisfaction problem. We use the following parameters in the model:

- *Charging Cost.* PS_v denotes the total cost that the vehicle $v \in \mathbb{V}$ spent for charging the battery during the trip. On the way from the source to the destination the vehicle may need to charge its battery several times, here, m times. We define PS_v as follows:

$$PS_v = PS_{h_1, l_1, t_1} \times C_{h_1, l_1} + \dots + PS_{h_m, l_m, t_m} \times C_{h_m, l_m}$$

Here, PS_{h_i, l_i, t_i} ($1 \leq i \leq m$) denotes the charging price of the charging station located on the point $\{h_i, l_i\} \in \mathbb{S}$, where the vehicle charges its battery during the time slot t_i . In the case when the time slot $t_i < \bar{t}$, the vehicle has already charged its battery, while in the case when the times slot $t_i \geq \bar{t}$, the vehicle will charge its battery. Hence, the updated prices have an impact on PS_v for the time slots equal or higher than \bar{t} .

- *Gasoline Cost.* PG_v denotes the total cost that the vehicle spent for gasoline during the trip. PG_v is computed as

$$PG_v = (PG_{h_1, l_1})_v + \dots + (PG_{h_n, l_n})_v$$

Here, $\{h_i, l_i\}$ is the i 'th point ($1 \leq i \leq n$) on the routing path from the source to the destination and PG_{h_1, l_1} is already defined in Section 3.4.3. Since the gasoline price is kept unchanged, PG_v remains the same, irrespective of the change in the charging prices.

- *Weight.* $W_{h, l, t}$ denotes the weight of the vehicles passing through the point $h, l \in \mathbb{S}$ during the slot t ($t \geq \bar{t}$), i.e., the ratio of the number of vehicles passing (actually expected to pass according to the vehicles' navigation plans) through the point $\{h, l\}$ over the total number of vehicles passing all the points in the set \mathbb{S} during t .

$$W_{h, l, t} = \frac{Nv_{h, l, t}}{\sum_{\{h', l'\} \in \mathbb{S}} Nv_{h', l', t}}$$

Here, $Nv_{h, l, t}$ represents the total number of vehicles passing through the point $\{h, l\}$, which is computed from $X_{h, l}^{\hat{h}, \hat{l}}$ for any $\{\hat{h}, \hat{l}\}$ and $T_{h, l}^s$ associated to all vehicles.

4.3 Control Model

The gasoline cost PG_v for a vehicle and the weight $W_{h,l,t}$ for each gasoline station during a time slot are taken as computed value which are constant in the model. We synthesize the charging prices for each station at different upcoming time slots, i.e., for each time slot $t \geq \bar{t}$; while the charging prices during previous time slots are invariable.

The charging price of a gas station during a slot t is proportional to the weights of the vehicles passing through the point associated to the station. This is formalized by the following relation:

$$W_{h,l,t} > W_{h',l',t} \rightarrow P_{s_{h,l,t}} > P_{s_{h',l',t}} \quad (20)$$

The prices cannot be less than $P_{min,t}$ and more than $P_{max,t}$ during the time slot t . If the weight is zero, i.e., no vehicles passing through the point corresponding to the charging station, the price is taken as the minimum. We consider the similar constraint for maximum weight (i.e., 1). These constraints are formalized as follows:

$$(P_{s_{h,l,t}} \geq P_{min}) \wedge (P_{s_{h,l,t}} \leq P_{max,t}) \quad (21)$$

$$(W_{h,l,t} = 0) \rightarrow (P_{s_{h,l,t}} = P_{min,t}) \quad (22)$$

It is also ensured that the average charging prices during a time slot is equal to the regular, i.e., average price.

$$\frac{\sum_{h,l,S_{h,l}} P_{s_{h,l,t}}}{\sum_{\{h,l\} \in \mathbb{S}} 1} = P_{avg,t} \quad (23)$$

The crucial constraint is that the vehicles already in the system have to be able to reach their destination within the cost limit. Since the prices may change (for the upcoming time slots), we need to verify whether the cost for a vehicle v remains within the limit Cp_v .

$$PG_v + PS_v \leq Cp_v \quad (24)$$

4.4 Implementation

The implementation of our price-based control model is similar to that of our PHEV navigation management mode. However, the most important point of our control model is that often the synthesis of the model returns with an *unsat* result. This is due to the constraints (20) and (24). Since (24) must hold, we encode (20) as a *soft clause* (also known as *assumption*) [12] for each pair of charging stations. We synthesize the charging prices by satisfying the maximum number of assumptions. Due to the space limitation, we defer describing this soft-clause implementation procedure.

4.5 Example

In Figure 6, we show the change of the charging prices with the time slots. We consider the same highway system as shown in Figure 4. In this example, we take 100 vehicles, which are arbitrarily distributed during 24 time slots. For each vehicle, the source and destination, and the starting time of the trip are chosen randomly following uniform distribution. The maximum and minimum charging prices are taken as \$1.5 and \$0.5, respectively. The initial price for each station is taken randomly. The cost and time constraints are taken from the ranges of 10-50 dollars and 100-200 minutes, respectively. Figure 6 shows the charging prices for two stations: the charging stations located at $\{1,2\}$ and $\{2,2\}$. We

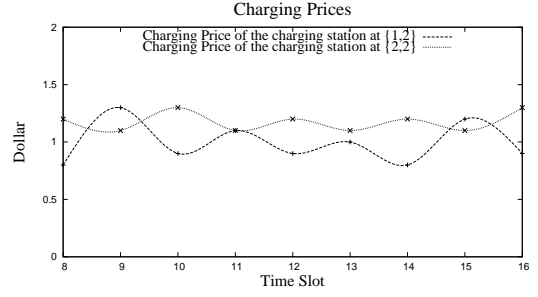


Figure 6: Change of charging prices with the time.

see that the prices are changing with the time. It is interesting to observe that the charging price of the station at $\{1,2\}$ is mostly less than the charging price of the station at $\{2,2\}$, except few cases. The reason behind this behavior is that the connecting roads to the point $\{2,2\}$ are shorter compared to the connecting roads to the point $\{1,2\}$. That is why the navigation management plans contain the point $\{2,2\}$ more than the point $\{1,2\}$, especially due to the longest road segment between $\{1,1\}$ and $\{1,2\}$. Hence, to divert the vehicles to the point $\{1,2\}$, most of the times the charging price of the station at $\{1,2\}$ is kept low.

5. EVALUATION

We evaluated the scalability of our proposed navigation management model and control model. Due to the unavailability of the real-life data, we analyzed the models using different synthetic data. We mainly present the evaluation results for our navigation management model, since we observed similar behaviors for both of the models.

5.1 Methodology

We evaluated the scalability of our proposed models by analyzing the *time* and *space* required for synthesizing the outputs. The problem size depends mainly on the number of exits. We consider the problem size as the total number of link roads, which is proportional to the total number of exits (i.e. the multiplication of the number of highways and the number of exits per highway). We assume that the number of stations are proportional to the number of exits. The length of a road is taken arbitrarily from the range of 2–20 miles. The number of highways is taken between 2 to 10, and the number of exits of a highway is chosen up to 1000. The highways are considered parallel with each other and an exit of a highway can be connected to an exit of a neighboring highway (as shown in Figure 4). The average vehicle speed on a road is randomly chosen between 0.5 to 1.5 miles/minute. For the gasoline and electricity prices, the ranges are taken as \$3–\$5/gallon and \$1–\$3/full-charge, respectively. The capacity of a battery is taken randomly from the range of 10–40 KWh. We randomly choose the mileage achieved by a vehicle from the range of 12–30 miles/gallon of gasoline and the range of 2–4 miles/kWh. We encoded our model using Z3 .NET API and ran the verification of the model in an Intel Core i3 Processor with 4 GB memory under Windows 7 OS.

5.2 Experimental Results with Discussion

Impact of the Problem Size: Figure 7(a) shows the model synthesis or verification time with respect to the problem size. In this analysis, we considered that three out of

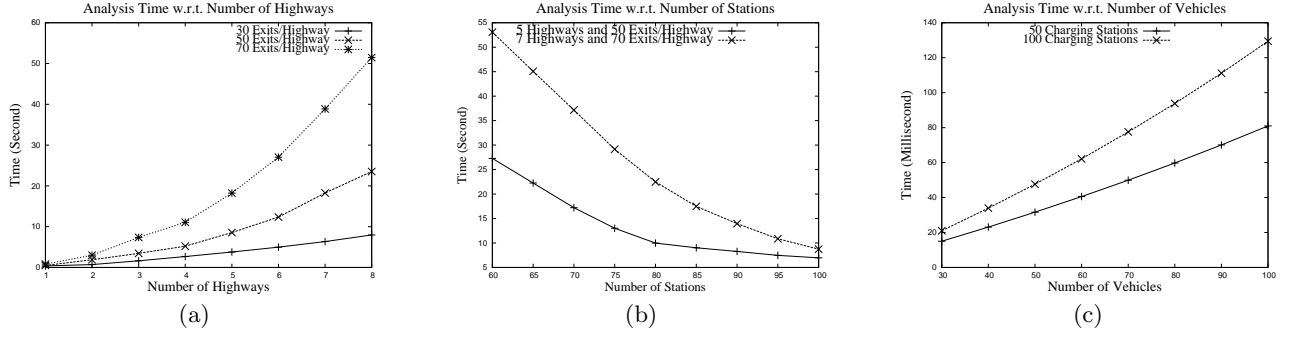


Figure 7: (a) The navigation management synthesis time w.r.t. the number of highways, (b) the impact of the number of stations on the navigation management synthesis time, and (c) the impact of the number of vehicles on the control model synthesis time.

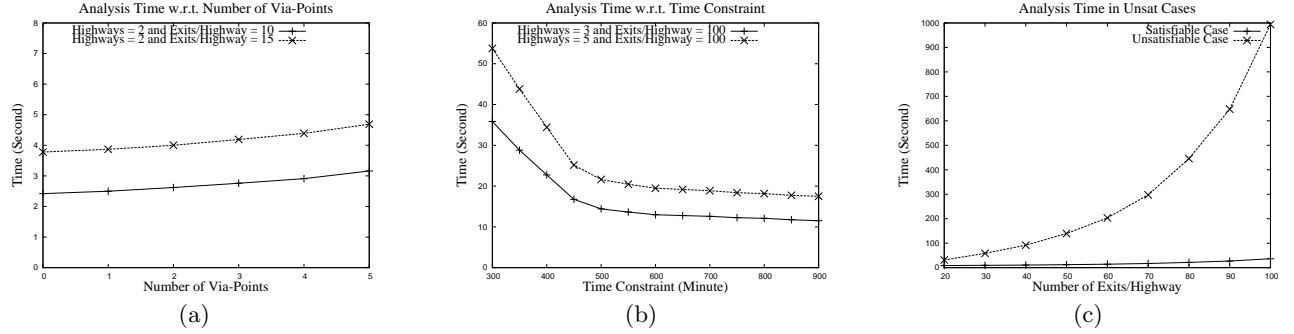


Figure 8: (a) The synthesis time w.r.t. the number of intermediate points of interests, (b) the impact of time constraint on the synthesis time, and (c) the model verification time in case of unsatisfiable problems.

four exit points have charging stations. We observed that the analysis time is linearly dependent on the problem size. We varied the problem size with respect to the number of highways (Figure 7(a)). The cost incurred in a trip depends on the distance driven by the vehicle and the type of fuel that is used. The more electricity (battery) is used as fuel compared to gasoline, the less the cost is. A battery requires frequent charging; hence, the availability of the charging stations are important. The higher the number of charging stations, the more options available to choose stations for charging. Figure 7(b) justifies this argument. It shows that, if the ratio (in percentage) increases (from 60% to 100%), the analysis time decreases. However, recharging takes a significant amount of time, which increases the amount of time spent for traveling. As a result, very often it is not possible to choose many stations for recharging, as the time spent for traveling has to satisfy the time constraint.

We also evaluated the scalability of our control model, and the results are presented in Figure 7(c), which shows the synthesis time with respect to the number of vehicles. In these experiments we considered 5 highways and 50 exits per highway in two different numbers of charging stations (50 and 100). We observed that the synthesis time increases almost linearly with the increase of the number of vehicles.

Impact of the Points of Interest: We observed that the number of intermediate points of interest (i.e., via-points) has impact on the synthesis time. The results are shown in Figure 8(a) under two different problem sizes. The figure shows that time increases with the increase in the number of via-points on the route. Since the number of points of interest (i.e., intermediate destinations) increases and each point is associated with a time constraint to be reached, a

higher search time is required to find a solution. As a result, the analysis time increases.

Impact of the Constraint Tightness: We analyzed the impact of the tight/relaxed constraints on the model synthesis time. Tightening (relaxing) a time/cost constraint means decreasing (increasing) the time/cost constraint value. The analysis result is shown in Figure 8(b) by varying the time constraint value. In this analysis, we considered a fixed number of exit points (i.e. the number of highways is 5 and the number of exits in each highway is 50). We observed that the execution time increases with the reduction of the time constraint value. This is due to the reason that tightening a constraint reduces the number of possible solutions to the model; as a result, more search is usually required to find a solution. We also observed the same behavior in the case of the cost constraint.

Performance in the Unsatisfied Cases: In the cases of very tight constraints (e.g., very low values for the time/cost constraints), there may not be any satisfiable solution. In such cases, the SMT solver takes significantly longer time to give the unsatisfiable results compared to the required time in satisfiable cases. Figure 8(c) shows such a comparison between the satisfiable and unsatisfiable cases by varying the number of exits. The reason behind this behavior is that the SMT solver requires verifying all possible ways to conclude that there is no solution based on the given constraints.

Space Complexity: The space (memory) requirement by the SMT solver [12] for our model is evaluated by changing the number of exits. The evaluation is done considering the memory required for encoding the problem. The required memory for a model synthesis is the sum of the memory for modeling the system properties and the memory

Table 3: The Memory Space Requirement

Exits	Memory (MB)	Time (Second)
500	69.07	40.54
1000	138.14	205.62
2000	276.28	759.82
3000	414.43	1541.55
4000	552.57	3721.00
5000	703.84	8629.00
5500	751.95	12591.03
6000	<i>Out of Memory Exception</i>	

for modeling the constraints. The analysis result is shown in Table 3. We observed that the memory requirement increases linearly with the increase in the number of exits. An increase in the model size depends on the problem size (e.g., the number of exit points). In Table 3, we can see that if the number of exit points increases more than 5500, the Z3 SMT solver [12] failed to give an answer. It gave an exception (*out-of-memory-exception*). This is due to the 2GB memory limit of a Dot NET program. However, one can bypass this limitation by using a self or commercially developed SMT solver, which is customized for increasing the memory limit.

6. RELATED WORK

Raghu K. Ganti et al. develop a navigation service, named GreenGPS [7], for traditional vehicles. GreenGPS provides the most *fuel-efficient route* between two points, which may be different from the shortest and fastest routes provided by the traditional navigation tools like Google Maps [11]. GreenGPS collects the necessary data, which is continuously updated by the participating vehicles and answers queries on the most fuel-efficient route. GreenGPS cannot work for PHEVs, as it does not consider the recharging/refueling of cars. It considers mainly the traffic congestion and the stop lights in order to find out the most fuel-efficient path, which is suitable in urban areas. For traveling on highways, GreenGPS will perform similarly to a traditional GPS.

There are a few works done so far for the efficient management of plug-in hybrid vehicles. Some works as in [13] [14], were done by targeting the problem of maximizing the profits from vehicle-to-grid (V2G) service by selling stored electricity to the grid or by participating in frequency regulation. However, none of these works considered the problem of driving on highways and scheduling of alternative use of fuels. The authors in [8] addressed the issue of minimizing the EV recharging waiting time through intelligently scheduling recharging activities. The authors theoretically formulated the minimum waiting time for the problem of recharging scheduling. Based on the analysis, they proposed a distributed scheme for the purpose of optimizing the recharging schedule. However, their model is limited only to electric vehicles with a mere objective of minimizing the waiting time in the charging stations only. The model only considers a single highway and cannot find the routing path for a complex highway system. In this paper, we address the navigation management problem for Hybrid Electric Vehicles for long trips, especially on highways. Our model is more comprehensive (considering PHEV properties) and practical (based on user requirements).

7. CONCLUSION

Plug-in hybrid vehicles require switching to gasoline or recharging their batteries for long trips. Recharging batter-

ies takes a longer time compared to the refueling of gasoline. Due to these characteristics, a flexible navigation management scheme is required for them. In this paper, we have presented a SMT-based formal modeling of the PHEV navigation management problem. Our model offers an optimal management plan that includes the route, along with the potential charging points. This satisfies all the constraints on the fuel cost, the traveling time, and the intermediate points of interest. We have also presented a price-based navigation control technique to achieve better load balancing for the system. We implemented both of our models, ran simulation experiments using Z3 SMT solver and evaluated their scalability. We observed that the running time of our navigation management model lies within 60 seconds for a highway system of 500 exits and similar number of charging stations, while that of our control model lies within 120 milliseconds for a problem of 100 vehicles and 250 exits. In the future, we plan to address the challenges like variable speed for fuel economy, explicit road congestion status, vehicle-to-grid (V2G) applications, etc.

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