

Maximum Torque/Minimum Flux Control of Interior Permanent Magnet Synchronous Motor Based on Magnetic Energy Model

Masashi Takiguchi, Toshiaki Murata, Junji Tamura and *Takeshi Tsuchiya
Kitami Institute of Technology *Hokkaido Institute of Technology
Kitami Institute of Technology, 165 Koen-Cho
Kitami, Hokkaido, 090-8507, Japan
Tel.: +81-157 / (26) – 9274
Fax: +81-157 / (23) – 9450
E-Mail: muratatm@mail.kitami-it.ac.jp

Keywords

«Interior permanent magnet synchronous motor», «Magnetic energy», «Maximum torque», «Optimal regulator», «sliding mode control»

Abstract

This paper presents a field oriented direct torque control system in Interior Permanent Magnet Synchronous Motor (IPMSM) which controls torque to be the maximum and flux to be the minimum simultaneously. In order to control torque directly, a concept of magnetic energy conversion is introduced. A state space model described by magnetic energies which one is stayed in the field and the other is converted to torque are presented. Direct torque control based on the state space model is more robust to disturbance such as load torque change. A PWM inverter-fed IPMSM control system has been driven by field oriented control algorithm without chattering applied on sliding mode control. The validity of the proposed method is confirmed by simulation results.

1. Introduction

Interior permanent magnet synchronous machines are widely used in variable industrial drive applications [1]-[4]. A field oriented control is said to be effective for AC drives with high dynamic performances. In many cases, however, the nature of this control is difficult to hold desired torque for parameter variations, or disturbance for load torque change. In order to control torque directly, which it means to control torque as output, modeling method controlling magnetic energy converted to torque is proposed.

This model is derived from a viewpoint of magnetic energy conversion selected the magnetic energy stored in the field and the one converted to the torque as control quantities. A direct torque controller with speed control and maximum torque under minimum flux is constructed by state space model described based on magnetic energies. Firstly, an optimal regulator with full state feedback is designed by a multi-input and multi-output optimal regulator theory. Next, in accordance with a method of Equivalent Control of variable structure control [8], the direct torque controller based on the *Digital Sliding Mode Theory (DSM)* is designed by selecting a switching surface as feedback gain of the optimal regulator under stable field orientation, for example, the armature current $i_d = 0$ control. Therefore both the switching of the converter and inverter of the IPMSM drive system can be smooth without chattering. The validity of the proposed control method using PWM inverter-fed IPMSM drive with sliding mode algorithm is confirmed by simulation results. The results of this simulation show that the system satisfies high control demands, which are speed follow and disturbance robustness.

2. Definition of magnetic energy

Nomenclature

v_d, v_q :	d- and q-axis stator voltages
i_d, i_q :	d- and q-axis stator currents
i_{cd}, i_{cq} :	d- and q-axis core loss currents
i_{md}, i_{mq} :	d- and q-axis magnetizing currents
λ_d, λ_q :	d- and q-axis stator flux linkages
R_a, R_m :	stator and core loss resistances
L_d, L_q :	d- and q-axis stator inductances
Φ_a :	permanent magnet flux linkage
τ_e, τ_L :	electromagnetic and load torques
J :	moment of inertia
D :	friction coefficient
ω :	electrical speed

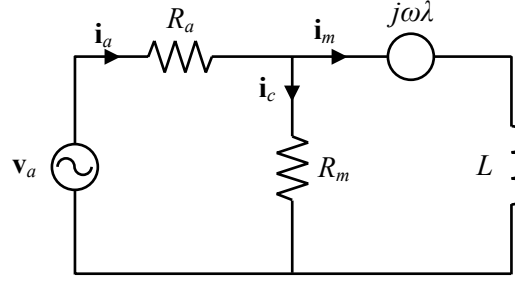


Fig. 1: Equivalent Circuit considering core loss into account

An equivalent circuit taking core loss into account is shown in Fig.1. The voltage and the flux linkage equations of the d-q axes at rotating reference frame fixed rotor are as follows.

$$\mathbf{v}_a = R_a \mathbf{i}_a + R_m \mathbf{i}_c \quad (1)$$

$$R_m \mathbf{i}_c = \frac{d\lambda}{dt} + j\omega\lambda \quad (2)$$

$$\mathbf{i}_a = \mathbf{i}_c + \mathbf{i}_m \quad (3)$$

Where, variables such as voltage, flux linkage and current mean vector defined as follows [9].

$$\mathbf{v}_a = v_d + jv_q, \quad \lambda = \lambda_d + j\lambda_q, \quad \mathbf{i}_a = i_d + ji_q \quad (4)$$

A flux linkage:

$$\lambda_d = L_d i_{md} + \Phi_a, \quad \lambda_q = L_q i_{mq} \quad (5)$$

Then, an electromagnetic torque is expressed by the following equation.

$$\tau_e = P_n (\lambda_d i_{mq} - \lambda_q i_{md}) = P_n \{ (L_d - L_q) i_{md} i_{mq} + \Phi_a i_{mq} \} \quad (6)$$

Where, P_n denotes a pole pair number. The torque and rotor speed are also related by,

$$\tau_e = \frac{J}{P_n} \frac{d\omega}{dt} + \frac{D}{P_n} \omega + \tau_L \quad (7)$$

where J is inertia of the rotor, D is damping constant. An angular rotor speed means electrical quantities. By multiplying both side of Eq. (1) and (2) by \mathbf{i}_a^* and \mathbf{i}_m^* (the complex conjugate of \mathbf{i}_a and \mathbf{i}_m), the following relations are obtained.

$$\mathbf{i}_a^* \mathbf{v}_a = \mathbf{i}_a^* R_a \mathbf{i}_a + \mathbf{i}_a^* R_m \mathbf{i}_c \quad (8)$$

$$\mathbf{i}_m^* \mathbf{v}_a = \mathbf{i}_m^* \frac{d\lambda}{dt} + \mathbf{i}_m^* j\omega\lambda \quad (9)$$

Adding Eq. (8) and (9), next energy balance is obtained as follows.

$$\mathbf{i}_a^* \mathbf{v}_a = \mathbf{i}_a^* R_a \mathbf{i}_a + \mathbf{i}_c^* R_m \mathbf{i}_c + \mathbf{i}_m^* \frac{d\lambda}{dt} + \mathbf{i}_m^* j\omega\lambda \quad (10)$$

The right hand side of Eq. (10) is in turn armature copper loss, core loss, magnetic energy and a real part of the last term is output power.

Then we take notice of next magnetic energies.

$$W_\tau = \lambda_d i_{mq} - \lambda_q i_{md} \quad (11)$$

$$W_q = \lambda_d i_{md} + \lambda_q i_{mq} \quad (12)$$

These relations are obtained from Eq. (10) as follows.

$$i_m^* \frac{d\lambda}{dt} = (i_{md} - j i_{mq}) \left(\frac{d\lambda_d}{dt} + j \frac{d\lambda_q}{dt} \right) = i_{md} \frac{d\lambda_d}{dt} + i_{mq} \frac{d\lambda_q}{dt} - j \left\{ i_{mq} \frac{d\lambda_d}{dt} - i_{md} \frac{d\lambda_q}{dt} \right\} \quad (13)$$

It is noticed that the magnetic energy W_τ is converted to torque and the magnetic energy W_q is stored in the field. State equations described by the magnetic energies are obtained as follows.

$$\frac{d\omega}{dt} = -\frac{D}{J} \omega + \frac{P_n^2}{J} W_\tau + \frac{P_n}{J} \tau_L \quad (14)$$

$$\frac{dW_q}{dt} = 2\omega W_\tau + 2P_a' \quad (15)$$

$$\frac{dW_\tau}{dt} = 2\omega W_q - 2P_q \quad (16)$$

Where,

$$P_a' = P_a - R_a (i_d^2 + i_q^2) - R_m (i_{cd}^2 + i_{cq}^2) \quad (17)$$

$$P_a = v_d i_d + v_q i_q \quad (18)$$

$$P_q = v_q i_d - v_d i_q \quad (19)$$

Control inputs are active power and reactive power. The magnetic energies W_τ and W_q can be controlled as output satisfying the demand that have been made. It should be noticed that the direct torque control means to torque as output. The direct torque control system can be constructed using the energy model.

3. Maximum torque/minimum flux control

A condition of minimum flux for given torque is a condition of the core loss to be minimum. In order to seek for the condition for maximum torque, the following equations are arranged,

$$\lambda = \sqrt{\lambda_d^2 + \lambda_q^2} \quad (20)$$

$$i_{mq} = \frac{\sqrt{\lambda^2 - (L_d i_{md} + \Phi_a)^2}}{L_q} \quad (21)$$

Then, the electromagnetic torque is obtained as follows.

$$\tau = \frac{P_n}{L_q} (L_d i_{md} + \Phi_a) \sqrt{\lambda^2 - (L_d i_{md} + \Phi_a)^2} - P_n i_{md} \sqrt{\lambda^2 - (L_d i_{md} + \Phi_a)^2} \quad (22)$$

Putting a developed torque expressed by the magnetizing current i_{md} , $\partial\tau/\partial i_{md}$, then, a quadric equation in relation to the d-axis magnetizing current is obtained as follows.

$$2L_d^2 (L_q - L_d) i_{md}^2 + L_d \Phi_a (3L_q - 4L_d) i_{md} + \Phi_a^2 (L_q - 2L_d) - (L_q - L_d) \lambda = 0 \quad (23)$$

Considering the flux-weakening control,

$$i_{md} = \frac{-4\Phi_a (L_q - L_d) + L_q \Phi_a - \sqrt{L_q^2 \Phi_a^2 + 8(L_q - L_d)^2 \lambda^2}}{4L_d (L_q - L_d)} = -\frac{\Phi_a + \Delta\Phi_d}{L_d} \quad (24)$$

Substituting Eq. (24) into Eq. (21),

$$i_{mq} = \frac{\sqrt{\lambda^2 + \Delta\Phi_d^2}}{L_q} \quad (25)$$

a desired magnetizing currents are,

$$i_{md}^R = -\frac{\Phi_a + \Delta\Phi_d}{L_d}, \quad i_{mq}^R = \frac{\sqrt{\lambda^2 + \Delta\Phi_d^2}}{L_q} \quad (26)$$

Where,

$$\Delta\Phi_d = \frac{-L_q \Phi_a + \sqrt{(L_q \Phi_a)^2 + 8(L_q - L_d) \lambda^2}}{4(L_q - L_d)} \quad (27)$$

Substituting from Eq. (11), (12), (26)

$$W_q^R = \frac{(\Phi_a + \Delta\Phi_d)\Delta\Phi_a}{L_d} + \frac{L_d^2 L_q W_\tau}{(L_d - L_q)^2 (\Phi_a + \Delta\Phi_d)^2 - 2\Phi_a L_d (L_d - L_q)(\Phi_a + \Delta\Phi_d) + L_d^2 \Phi_a^2} \quad (28)$$

4. Direct torque control system

4.1 Optimal regulator

A discrete time form of controlled object and output are,

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \quad (29)$$

$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) \quad (30)$$

Where,

$$\mathbf{A} = e^{\mathbf{A}_c t}, \quad \mathbf{B} = \int_0^T e^{\mathbf{A}_c \tau} d\tau \cdot \mathbf{B}_c, \quad \mathbf{E} = \int_0^T e^{\mathbf{A}_c \tau} d\tau \cdot \mathbf{E}_c \quad (31)$$

$$e^{\mathbf{A}_c t} = \mathbf{I} + \mathbf{A}_c t + \frac{1}{2!}(\mathbf{A}_c t)^2 + \dots + \frac{1}{n!}(\mathbf{A}_c t)^n + \dots \quad (32)$$

In this paper, terms are expanded to n=150 terms. The linearized matrixes are as follows.

Where, the superscript d denotes a steady state operating point.

$$\mathbf{A}_c = \begin{bmatrix} -\frac{D}{J} & 0 & \frac{P_n^2}{J} \\ -2W_\tau^d & 0 & -2\omega^d \\ 2W_q^d & 2\omega^d & 0 \end{bmatrix}, \quad \mathbf{B}_c = \begin{bmatrix} 0 & 0 \\ 2 & 0 \\ 0 & -2 \end{bmatrix}, \quad \mathbf{E}_c = \begin{bmatrix} \frac{P_n}{J} \\ 0 \\ 0 \end{bmatrix} \quad (33)$$

State variables and control input and output for speed control and field oriented control are,

$$\mathbf{x}(k) = \begin{bmatrix} \omega(k) \\ W_q(k) \\ W_\tau(k) \end{bmatrix}, \quad \mathbf{u}(k) = \begin{bmatrix} P_a'(k) \\ P_q(k) \end{bmatrix}, \quad \mathbf{y}(k) = \begin{bmatrix} \omega(k) \\ W_q(k) \end{bmatrix} \quad (34)$$

Errors for speed control and the torque maximum control for the desired magnetic energy are defined as follows.

$$\mathbf{e}(k) = \mathbf{R}(k) - \mathbf{y}(k) \quad (35)$$

$$\mathbf{R}(k) = \begin{bmatrix} \omega^R(k) \\ W_q^R(k) \end{bmatrix} \quad \mathbf{R}: \text{reference} \quad (36)$$

An augmented system that includes the error $\mathbf{e}(k)$ and first difference $\Delta\mathbf{x}(k)=\mathbf{x}(k)-\mathbf{x}(k-1)$ and the incremental input $\Delta\mathbf{u}(k)=\mathbf{u}(k)-\mathbf{u}(k-1)$ can be defined as follows.

$$\begin{bmatrix} \mathbf{e}(k+1) \\ \Delta\mathbf{x}(k) \end{bmatrix} = \begin{bmatrix} \mathbf{I} & -\mathbf{CA} \\ \mathbf{0} & \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{e}(k) \\ \Delta\mathbf{x}(k) \end{bmatrix} + \begin{bmatrix} -\mathbf{CB} \\ \mathbf{B} \end{bmatrix} \Delta\mathbf{u}(k) = \Phi\mathbf{X}(k) + \mathbf{G}\Delta\mathbf{u}(k) \quad (37)$$

In order to obtain the optimal control input, let define a next performance index \mathbf{PI} ,

$$\mathbf{PI} = \sum_{k=1}^{\infty} [\mathbf{X}_0^T(k) \mathbf{Q} \mathbf{X}_0(k) + \Delta\mathbf{u}^T(k) \mathbf{H} \Delta\mathbf{u}(k)] \quad (38)$$

The optimal control input minimizing the performance index \mathbf{PI} is, for example, setting $\partial\mathbf{PI}/\partial\Delta\mathbf{u}=0$,

$$\Delta\mathbf{u}(k) = -[\mathbf{H} + \mathbf{G}^T \mathbf{P} \mathbf{G}]^{-1} \mathbf{G}^T \mathbf{P} \Phi\mathbf{X}(k) = \mathbf{F}_B \mathbf{X}(k) \quad (39)$$

The matrix \mathbf{P} in Eq. (39) can be obtained by the next steady state Riccati equation as follows.

$$\mathbf{P} = \mathbf{Q} + \Phi^T \mathbf{P} \mathbf{G} - [\mathbf{H} + \mathbf{G}^T \mathbf{P} \mathbf{G}]^{-1} \mathbf{G}^T \mathbf{P} \Phi \quad (40)$$

Where, $\mathbf{Q}=\text{diag}[Q_1 \ Q_2 \ 0 \ 0 \ 0]$ is positive semidefinite symmetric matrix. The term diag indicates the diagonal matrix.

$$Q_1 = 10.0, \quad Q_2 = 10.0, \quad H_1 = 0.01, \quad H_2 = 0.01 \quad (41)$$

4.2 Sliding mode control

A linear function of state variable $\delta(k)$ is defined as follows.

$$\delta(k) = \mathbf{S} \mathbf{X}(k) = \mathbf{F}_B \mathbf{X}(k) \quad (42)$$

The matrix \mathbf{S} is designed so that the system of Eq. (37) can be stable. A more straightforward technique easily applicable to multi-input system is the method of equivalent control as proposed by Utkin [8]. The method of equivalent control is a means for determining the system motion restricted the switch surface $\sigma(\mathbf{x})=0$. An Equivalent control input is as follows [8].

$$\mathbf{u}_{eq}(k) = -(\mathbf{S} \mathbf{G})^{-1} \mathbf{S}(\Phi - \mathbf{I}) \mathbf{X}(k) \quad (43)$$

In order to avoid chattering, the control input is accomplished by assuming it is composed by the sum of two components, equivalent control input $\mathbf{u}_{eq}(k)$ whose state is kept on hyperplane at next sampling-time, and nonlinear control input $\mathbf{u}_{nl}(k)$ whose state moves to neighborhood of the sliding plane without chattering.

$$\mathbf{u}(k) = \mathbf{u}_{eq}(k) + \mathbf{u}_{nl}(k) \quad (44)$$

Next Liapunov function $\mathbf{V}(k)$ is defined to solve the control input $\mathbf{u}_{nl}(k)$.

$$\mathbf{V}(k) = \frac{1}{2} \boldsymbol{\sigma}^2(k) \quad (45)$$

From the definition of the equivalent control of sliding mode and the definition of sliding plane,

$$\boldsymbol{\sigma}(k) = \mathbf{S}\mathbf{X}_0(k) = \mathbf{0}, \quad \boldsymbol{\sigma}(k) = \boldsymbol{\sigma}(k+1) = \mathbf{0} \quad (46)$$

$$\Delta \boldsymbol{\sigma}(k+1) = \boldsymbol{\sigma}(k+1) - \boldsymbol{\sigma}(k) \quad (47)$$

Squaring the both side of Eq. (47),

$$\boldsymbol{\sigma}^2(k+1) = \boldsymbol{\sigma}^2(k) + 2\boldsymbol{\sigma}\Delta \boldsymbol{\sigma}(k+1) + \Delta \boldsymbol{\sigma}^2(k+1) \quad (48)$$

The control input can be satisfied the next relation.

$$2\boldsymbol{\sigma}(k)\Delta \boldsymbol{\sigma}(k+1) + \Delta \boldsymbol{\sigma}^2(k+1) < 0 \quad (49)$$

Then the following relation can be

$$\mathbf{V}(k+1) < \mathbf{V}(k) \quad (50)$$

From Eq. (40), (42) and (43), the next relation is obtained.

$$\boldsymbol{\sigma}(k+1) = \boldsymbol{\sigma}(k) + \mathbf{S}\mathbf{G}\Delta \mathbf{u}_{nl}(k) \quad (51)$$

The following inequality is obtained to restrain state in the switching plane. $\Delta \mathbf{u}_{nl}(k)$ being satisfactory (51) becomes,

$$\Delta \mathbf{u}_{nl}(k) = -\eta(\mathbf{S}\mathbf{G})^{-1} \mathbf{S}\mathbf{X}_0(k) \quad (52)$$

Where, the boundary of a factor η is within $0 \leq \eta \leq 2.0$.

Table I: Parameters of the IPMSM

Rated output [W]	475	Core loss resistance $R_m[\Omega]$	300
Rated speed [r/min]	1800	d-axis inductance $L_d[\text{mH}]$	9.0
Rated torque [N-m]	2.52	q-axis inductance $L_q[\text{mH}]$	22.5
Number of pole pair P_n	2	Magnetic flux linkage $\Phi_a[\text{Wb}]$	0.1
Stator resistance $R_a[\Omega]$	0.5	Inertia $J[\text{Kg} \cdot \text{m}^2]$	2.55×10^{-3}

5. Simulation Result

In order to verify the usefulness of the proposed magnetic energy model, the simulation study of the PWM inverter-fed IPMSM drive system has been made using the model of the overall system, which works as shown in Fig.2 is developed. Tables I shows the parameters of the IPMSM. A PWM based on a state space vector method is considered by control inputs from Eq. (43) and (52) without chattering. The primary desired angular frequency ω and the stator voltages v_d and v_q in a synchronously rotating reference frame are converted to the α - β axes at rest in Appendix. Fig.3 shows responses for desired speed under given load torque. The rotor speed is tracking to the desired speed for suitable control input P_a . The core loss can be controlled to be zero, therefore the maximum torque/minimum flux control can be achieved well. It is noticed that the maximum torque/minimum flux control does not mean maximum efficiency, but it is worthy to minimize the core loss under the maximum developed torque.

The direct torque controller is full state feedback with torque. In order to verify the effect of the torque feedback, Fig.4 shows a step response for abrupt load torque as disturbance. As mentioned early, the reactive power has been selected as the control input. The developed torque is controlled by injecting the immediate reactive control input shown in Fig4. The control algorithm of the proposed method can be implemented like a look-up table method [4].

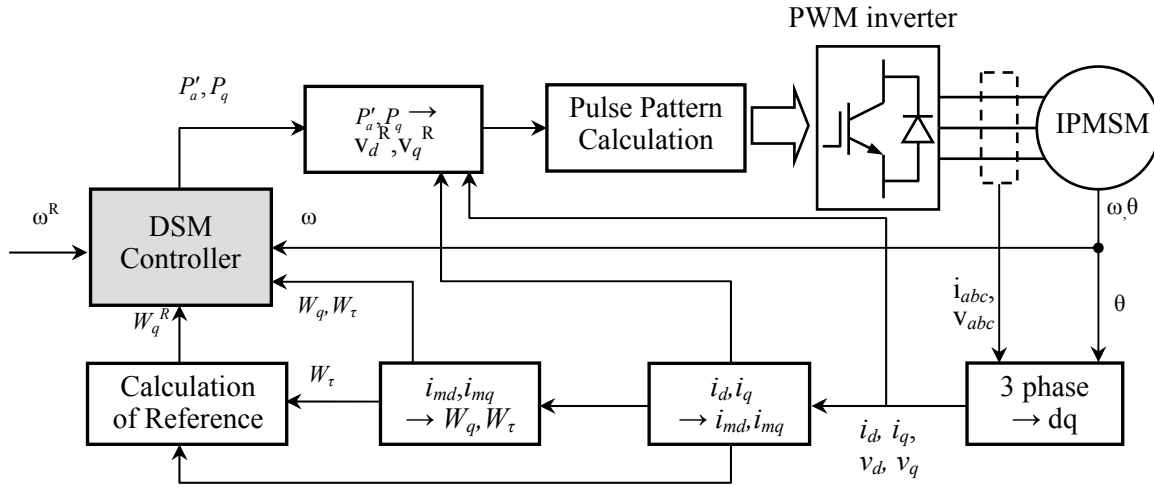


Fig. 2: Control system structure

6. Conclusion

In this paper, the direct torque control which control torque as output was presented from a view point of magnetic energy conversion adding to the magnetic energy converted to torque, until now this magnetic energy was not considered. The validity of the proposed model was confirmed from simulation results for maximum torque/minimum flux control method. The most important matter can be an estimation of a core loss resistance. A real time identification of the core loss resistance may be a very useful with laboratory experiment in near future.

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Appendix

Space vector

A space vector is defined as follows [9]. A vector of α - β axes at rest is,

$$\mathbf{v}_a^{\alpha-\beta} = \sqrt{\frac{2}{3}} E (S_a + aS_b + a^2 S_c) = v_\alpha + jv_\beta$$

$$S_{a,b,c} = \begin{cases} 0 & \text{upper on} \\ 1 & \text{lower on} \end{cases}$$

$$a = e^{-j\frac{2}{3}\pi} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}, \quad a^2 = e^{+j\frac{2}{3}\pi} = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$$

In this paper, The SVM principle is based on the switching between two adjacent active vectors and a zero vectors during one switching period [3].

A current and a flux linkage vector are defined in a similar way.

$$\mathbf{i}_a^{\alpha-\beta} = i_\alpha + ji_\beta, \quad \boldsymbol{\lambda}^{\alpha-\beta} = \lambda_\alpha + j\lambda_\beta$$

A transformation to the d-q axes synchronously rotating reference frame with rotor is,

$$\mathbf{f}^{d-q} = \mathbf{f}^{\alpha-\beta} e^{-j\omega t}$$

The d-q axes equations of the IPMSM are obtained from Eq.(1) to (3).

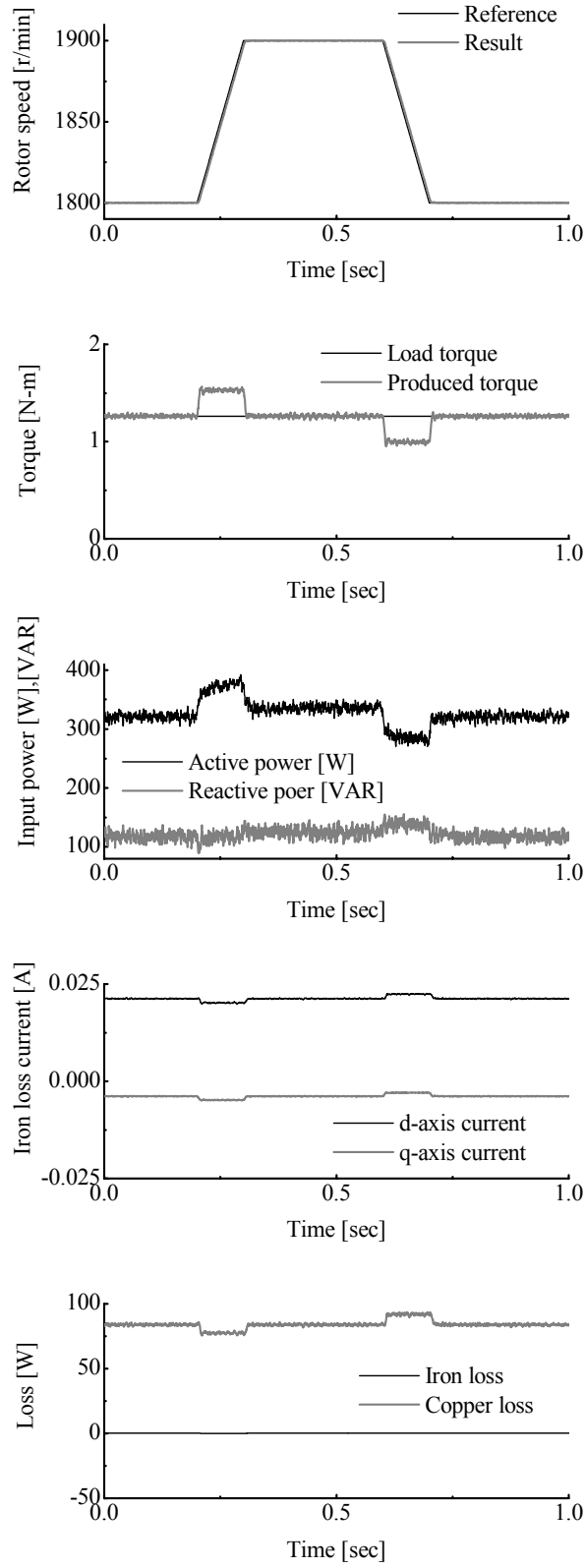


Fig. 3: Responses for desired speed

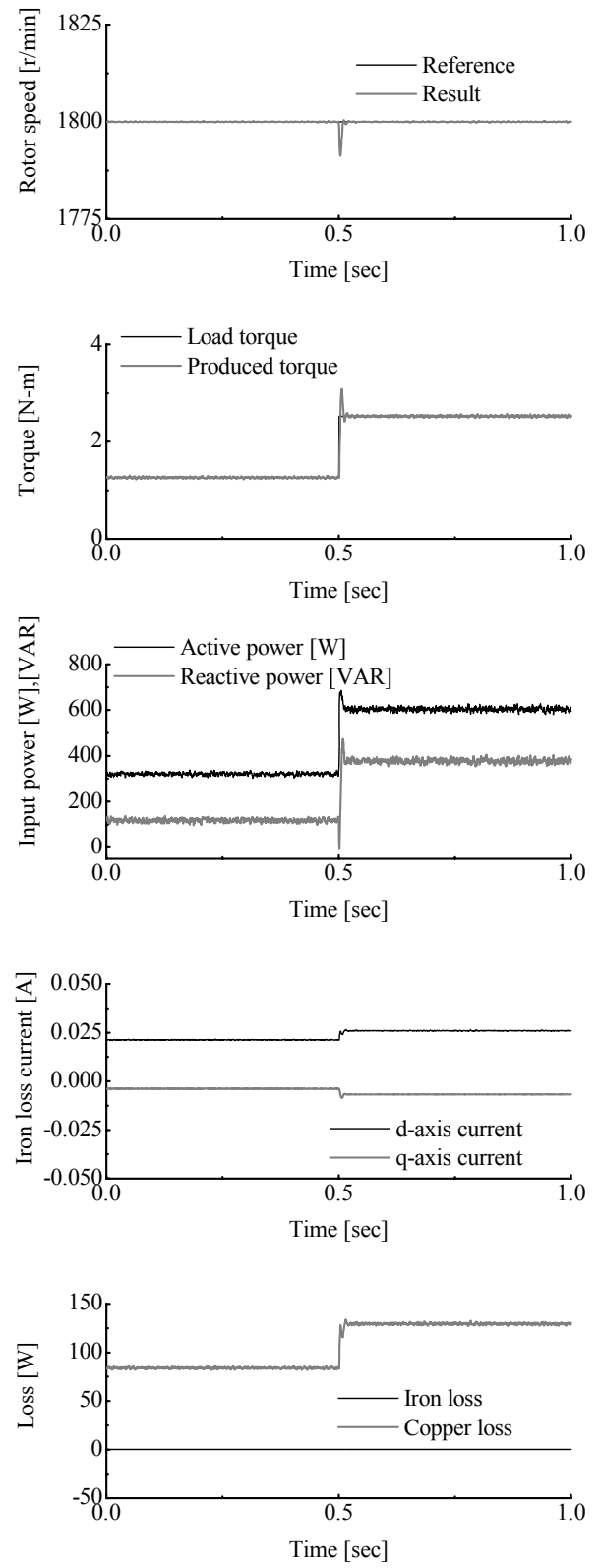


Fig. 4: Responses for desired torque