

# Game-theoretic Approach for Complete Vehicle Energy Management

H. Chen, J.T.B.A. Kessels, M.C.F. Donkers and S. Weiland

**Abstract**—This paper describes a game-theoretic approach and the calculation of an online implementable strategy for solving the Complete Vehicle Energy Management (CVEM) problem, which aims at minimizing the global fuel consumption of a hybrid heavy-duty truck including all its auxiliary systems. The approach is based on a two-level single-leader multi-follower game, in which the driver is considered as a leader and each controlled auxiliary is considered as a follower. In the first level, a sequential game between the leader (driver) and each follower (auxiliary) is played and the corresponding Stackelberg strategy is computed offline and stored in a lookup table. In the second level, a simultaneous game is played among all followers and an online iterative process is introduced to find an approximation of a Nash equilibrium for all followers. This approach is tested on a hybrid heavy-duty truck model where a high-voltage battery and an electric refrigerated semi-trailer are considered. The performance in terms of fuel economy is found close to the true optimal solution.

## I. INTRODUCTION

Energy management for Hybrid Electric Vehicles (HEVs) is an important research topic in the automotive industry. Generally speaking, the objective in energy management is to minimize the overall operational cost along a route while satisfying certain boundary conditions. A lot of research has been done on optimally splitting the driving power demand between the combustion engine and the electric motor (see e.g., [3], [10]). Popular techniques for solving this problem include dynamic programming (DP) [2], Pontryagin's Minimum Principle [5], and Equivalent-Consumption Minimization Strategies (ECMS) [11], [12]. A significant number of papers provide comparative studies or surveys on energy management for HEVs, see, e.g., [14], [13], [15], [9].

The problem of Complete Vehicle Energy Management (CVEM) is recently introduced in [6]. In CVEM, all powerflows and auxiliaries with energy buffers in the vehicle are taken into account in the energy management algorithm, so that inefficient operation at (sub)system level is avoided. It is recognized that traditional techniques for vehicle energy management will face limitations in handling future vehicle complexity in terms of development time and cost. As a result, an alternative approach is needed which enables CVEM targeting at optimizing the global vehicle efficiency. At the same time, the energy management algorithm should have a

flexible structure such that plug and play and easy scaling-up can be achieved [6].

The focus of this paper is to present a game-theoretic [1] solution concept for solving the CVEM problem. In [4], a single-leader single-follower framework related to a parallel HEV application is developed where only one auxiliary (the battery) is considered. In the aforementioned paper, a sequential game is formulated between the driver and the powertrain. The driver, considered as the leader, selects the powertrain operating condition  $w(t) \in W$  and announces it. The powertrain, considered as the follower, with the knowledge of  $w(t)$ , then selects the powertrain control variable  $u(t) \in U$  as a response to the leader's action. To make this idea amenable for CVEM, the single-leader single-follower model is extended to a single-leader multi-follower approach where the driver is considered as the leader and all the controlled auxiliaries are considered as followers. In this paper, we show how such a game-theoretic model can be applied to solve the CVEM problem through a two-level game play. The main contributions of this paper are the development of the game-theoretic solution concept for CVEM leading to an online implementable algorithm and applying it on a vehicle simulation model.

The remainder of this paper is structured as follows: Section II presents the modelling of the components in the simulation model. Section III formulates the CVEM problem as an optimization problem. Section IV describes the game-theoretic solution concept and the algorithm for generating online implementable strategies. Section V compares the simulation results of several different strategies with the game-theoretic approach. The conclusions are given in Section VI.

## II. COMPONENT MODELLING

The type of vehicle we consider in this paper is a parallel hybrid heavy-duty truck equipped with a high-voltage battery and an electric refrigerated semi-trailer (Figure 1). The drive train block contains all the drive train components including the gearbox, wheels and the complete vehicle mass. The goal of the energy management strategy is to control the battery power  $P_B$  and the electric power consumed by the

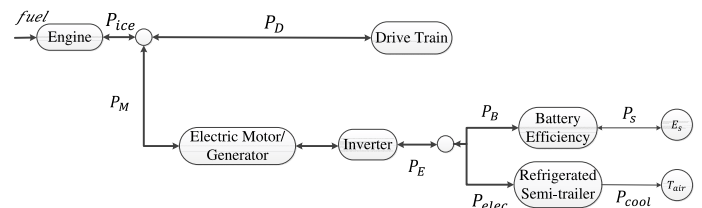


Figure 1. Power flows within the vehicle simulation model

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refrigerated semi-trailer  $P_{elec}$  such that the fuel consumption is minimized. It is assumed that the drivability is not influenced when different energy management strategies are applied. This implies that the vehicle speed, the gear ratio, and drive train torque remain fixed. Before formalizing the CVEM problem, we give the models describing the components of the vehicle under consideration.

#### A. Engine

The internal combustion engine is represented by its fuel consumption as function of requested power  $P_{ice}$  and its rotational speed  $\omega$ . It is given by the following quadratic function which is obtained by approximating a mean-value static fuel consumption map. That is, with  $\dot{m}_{fuel}(t)$ ,  $t \in \mathbb{N}$  denoting the discrete samples of the fuel mass rate, we consider

$$\dot{m}_{fuel} = \max(a(\omega)P_{ice}^2 + b(\omega)P_{ice} + c(\omega), 0). \quad (1)$$

In (1),  $a(\omega)$ ,  $b(\omega)$  and  $c(\omega)$  are real-valued coefficients that depend on the engine rotational speed  $\omega$ . From the power balance indicated in Figure 1, we have  $P_{ice} = P_D + P_M$ . Here,  $P_D$  and  $P_M$  represent the requested driving power and the electric motor/generator power on the mechanical side, respectively. For simplicity, we assume efficiency one for the electric motor/generator and the inverter, i.e.,  $P_M = P_E$  where  $P_E$  denotes the aggregated auxiliary power requests (a feasible way to incorporate energy conversion loss in the game-theoretic solution framework will be discussed in *Remark 1* in Section IV-A). Using this assumption and again the power balances in Figure 1, we have

$$P_{ice} = P_D + P_E = P_D + P_B + P_{elec}. \quad (2)$$

#### B. Battery

The dynamics of the battery energy  $E_s$  are modelled as an integration of the net power  $P_s$  stored in or retrieved from the battery denoted as  $P_s$ . For a given sample time  $h$ , the discrete time model for  $E_s$  is given below

$$E_s(t+1) = E_s(t) + h \cdot P_s(t). \quad (3)$$

Using the approach described in [8], the power loss of operating the battery is taken into account

$$P_B \approx \gamma_{bat} P_s^2 + P_s \quad (4)$$

where a positive constant  $\gamma_{bat}$  represents the quadratic loss during charging and discharging.

#### C. Refrigerated Semi-trailer

The air temperature inside the refrigerated semi-trailer is governed by an energy balance where heat exchange takes place in two ways: the heat infiltration  $Q_{leak}$  induced by air exchange between the air inside and outside the semi-trailer and the cooling power  $P_{cool}$  from the compressor (Figure 2). As a result, the air temperature  $T_{air}$  in the semi-trailer can be described by the discrete time model

$$T_{air}(t+1) = \frac{h}{C_{reef}}(Q_{leak}(t) - P_{cool}(t)) + T_{air}(t) \quad (5)$$

where  $C_{reef}$  is the equivalent heat capacity of the semi-trailer. For simplicity, we assume no power loss in converting electric power to cooling power. Thus, we have  $P_{elec} = P_{cool}$ .

### III. PROBLEM FORMULATION

The objective of energy management is to minimize the overall fuel usage within a time interval  $t \in \{0, 1, \dots, t_f - 1\}$  for some  $t_f \in \mathbb{N}$ , while satisfying several constraints. This can be formulated by the following optimization problem in discrete time

$$\min \sum_{t=0}^{t_f-1} \dot{m}_{fuel}(t) \cdot h \quad (6)$$

subject to (2) - (5) and

$$P_{B,min} \leq P_B(t) \leq P_{B,max}, \quad (7)$$

$$P_{elec,min} \leq P_{elec}(t) \leq P_{elec,max}, \quad (8)$$

$$E_{min} \leq E_s(t) \leq E_{max}, \quad (9)$$

$$T_{air,min} \leq T_{air}(t) \leq T_{air,max}, \quad (10)$$

$$E_s(t_f) = E_s(0), \quad (11)$$

$$T_{air}(t_f) = T_{air}(0). \quad (12)$$

In order to have an energy-sustaining system, constraints (11) and (12) force  $E_s$  and  $T_{air}$  at the end of the drive-cycle to go back to their initial values. (7)-(10) impose power limits and limit the operating range of the battery and the semi-trailer. The control variables are  $P_B$  and  $P_{elec}$ . After they are determined,  $P_{ice}$ ,  $P_M$  and  $P_E$  can be found through (2).

### IV. GAME-THEORETIC APPROACH

#### A. Solution Concept

In this section, we present the two-level single-leader multi-follower game-theoretic solution to the CVEM problem given in the previous section. We provide first the solution concept and then the implementation details. It is argued in [7] that feedback Stackelberg equilibria [1] may provide a suitable framework for formulating and solving the game problems relevant to powertrain control applications. In the single-leader single-follower framework related to the HEV application, the driver is considered as the leader and the powertrain is considered as the follower. In this paper, we extend the framework of [4] towards CVEM. To do so, we propose a single-leader multi-follower game model that allows all the controlled auxiliaries to be included. In this framework, the driver is selected as a leader and each controlled auxiliary is selected as a follower. Assuming there are  $N$  auxiliaries to be controlled, we can then define the set of  $N$  followers  $F = \{f_1, \dots, f_N\}$ . The leader can select its action  $w(t)$  from an admissible set  $W$ . Every follower  $f_i$ ,  $i \in \{1, \dots, N\}$  selects its control variable  $u_i(t)$  from the admissible set  $U_i$ . In the remainder of this paper, the leader's action  $w$  represents the requested wheel

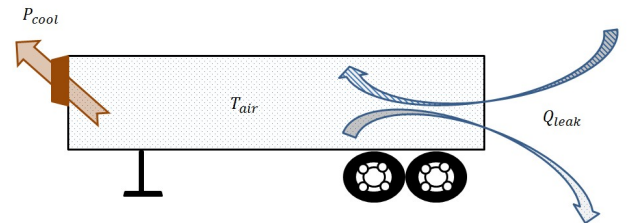


Figure 2. Refrigerated semi-trailer model

torque and rotational speed at the input side of the gearbox and  $W = \{(\tau, \omega) | \tau_{min} \leq \tau \leq \tau_{max}, \omega_{min} \leq \omega \leq \omega_{max}\}$ . The control variable  $u_i(t)$  for follower  $f_i$  is the requested power and  $U_i = \{P_i | P_{i,min} \leq P_i \leq P_{i,max}\}$ . Each follower has a corresponding state  $x_i$  which varies within the state space  $X_i$ . And each follower has a model describing the dynamic behavior of  $x_i$  as given below by a discrete time model

$$x_i(t+1) = g_i(x_i(t), u_i(t)).$$

In the ideal situation where energy conversion takes place without power loss, we can express the engine power as  $P_{ice} = P_D + \sum_i P_i = \tau\omega + \sum_i P_i$ . Hence, the fuel consumption (6) can be expressed by  $u_i(t)$  and  $w(t)$  using (1).

Typically, a driver drives the vehicle according to personal habits without explicitly thinking about fuel economy. If we consider the driver as a player in the game, it would then be logical to assume that this player's action mimics the typical behavior of a real-world driver. For this reason, let us define a cost function for the driver within the prediction horizon  $[0, t_p - 1]$  for some positive integer  $t_p \leq t_f$  as

$$J_w = \sum_{m=0}^{t_p-1} h \cdot G(w_m). \quad (13)$$

The function  $G(w)$ , as introduced by the authors in [4], is like a probability distribution function for a typical drive-cycle (see also Section V). By maximizing the function  $J_w$ , i.e.,  $\max_w J_w$ , the driver selects the maximum likelihood of his/her drive-cycle. Subsequently, the control strategy for the  $i$ th auxiliary reacts on the driver's action by minimizing the fuel consumption subject to certain constraint, i.e.,  $u_i$  minimizes  $J_{aux,i}$  where

$$J_{aux,i} = \sum_{m=0}^{t_p-1} h \cdot \dot{m}_{fuel}(m) + \mu_i(x_{i,t_p} - x_{i,0})^2 \quad (14)$$

where  $x_{i,0}$  and  $x_{i,t_p}$  denote the initial and end value of  $x_i$  within the prediction horizon, respectively and  $\mu_i$  is a positive weighting parameter. Note that the terminal constraints (11) and (12) are converted into soft constraints by the terminal cost  $\mu_i(x_{i,t_p} - x_{i,0})^2$ . Altogether, the cost function  $J_i$  associated to each  $f_i \in F$  emerges where the driver wants to maximize by selecting  $w_m \in W$  and  $f_i$  wants to minimize by selecting  $u_{i,m} \in U_i$ . With the denotation  $u_{-i,m} = \sum_{j \neq i} u_{j,m}$ ,  $J_i$  is given as

$$J_i(w, x_{i,0}, u_{i,\cdot}, u_{-i,\cdot}) = \sum_{m=0}^{t_p-1} L_i(w_m, u_{i,m}, u_{-i,m}) + \mu_i(x_{i,t_p} - x_{i,0})^2 \quad (15)$$

where  $L_i(w_m, u_{i,m}, u_{-i,m}) = h \cdot (\dot{m}_{fuel}(m) + \beta G(w_m))$ . A positive weighting parameter  $\beta$  is pre-multiplied to the  $G(w)$  function. By varying  $\beta$  from infinity to zero, the cost function  $J_i$  varies from (13) to (14) from the driver's perspective but it remains unaffected from the perspective of the auxiliary because the auxiliary cannot decide on the value of  $G(w)$ .

This observation suggests that the higher we select  $\beta$ , the more confidence we put in the function  $G(w)$ . On the contrary, a choice of zero for  $\beta$  indicates that we have no information about the driving behavior and a worst-case driver is assumed, i.e., a driver who maximizes the fuel consumption.

The solution concept of the single-leader multi-follower game model consists of two levels of games. The first level consists of  $N$  multistage sequential games between the leader and each follower. In the second level, all the followers are

involved in a simultaneous game. The two-level game will be explained in the following paragraphs.

**1) Stackelberg Equilibrium:** Consider a two-player multi-stage sequential game between the leader and the follower  $f_i$  with common cost function  $J_i$ ,  $i \in \{1, \dots, N\}$ . The leader tries to maximize  $J_i$  and the follower tries to minimize it. Assume the power demands from the other auxiliaries, i.e.,  $\{u_{-i,l}\}_{l=0}^{t_p-1}$  to be known over the prediction horizon  $t_p - 1$ . At the first stage, given the information of  $x_{i,0}$  and  $u_{-i,0}$ , the leader selects  $w_0$  first. With the knowledge of  $x_{i,0}$ ,  $w_0$  and  $u_{-i,0}$ , the follower  $f_i$  reacts by selecting  $u_{i,0}$ . The state  $x_i$  then evolves according to  $x_{i,1} = g_i(x_{i,0}, u_{i,0})$  and  $x_{i,1}$  is passed to the next stage of the game. This is repeated until all  $t_p - 1$  stages are completed. By playing this dynamic antagonistic game, we obtain a pair of sequences  $(w_{i,\cdot}^*, u_{i,\cdot}^*)$  which satisfies

$$J_i(w_{i,\cdot}^*, u_{i,\cdot}^*) = \max_{w_0 \in W} \min_{u_{i,0} \in U_i} \dots \max_{w_{t_p-1} \in W} \min_{u_{i,t_p-1} \in U_i} J_i(w, u_{i,\cdot}).$$

Such a pair of sequences  $(w_{i,\cdot}^*, u_{i,\cdot}^*)$  constitutes a feedback Stackelberg equilibrium pair [7]. The multistage dynamic game at stage  $m$  can be defined as the optimal cost-to-go function  $V_{i,m}$  in a recursive way,

$$V_{i,t_p}(x_{i,t_p}, \emptyset) = \mu_i(x_{i,t_p} - x_{i,0})^2, \quad (16)$$

$$V_{i,m}(x_i, \{u_{-i,l}\}_{l=m}^{t_p-1}) = \max_{w \in W} \min_{u_i \in U_i} \{L_i(w, u_i, u_{-i,m}) + V_{i,m+1}(g_i(x_i, u_i), \{u_{-i,l}\}_{l=m+1}^{t_p-1})\} \quad (17)$$

for  $m = t_p - 1, t_p - 2, \dots, 0$ . Once  $x_{i,0}$  and  $\{u_{-i,l}\}_{l=0}^{t_p-1}$  are given, (16)-(17) provide the feedback Stackelberg equilibrium pair  $(w_{i,\cdot}^*, u_{i,\cdot}^*)$  where  $w_{i,\cdot}^*$  and  $u_{i,\cdot}^*$  are the maximizers and minimizers of (17), respectively. The Stackelberg strategy is interpreted as the follower  $f_i$  tries to optimize its future cost under the assumption that the Stackelberg equilibrium pair will be played by the leader and follower in the future. It should be noted that the Stackelberg strategy does not need the exact information of decisions from other followers but the summation of them. This observation is crucial for the plug and play feature because no variables need to be added or removed to the strategy if an auxiliary is added to or removed from the vehicle.

**2) Nash Equilibrium:** Now we have seen how to derive the Stackelberg strategy for any follower  $f_i$  given any profile of  $\{u_{-i,l}\}_{l=0}^{t_p-1}$ . Next we assume all the followers play their Stackelberg strategies. At stage  $m$ , because of the interdependency in their Stackelberg strategies, all the followers need to negotiate over their decisions and try to reach a mutual agreement. This negotiation gives rise to the second-level simultaneous game among all the followers leading to an  $N$ -tuple equilibrium strategies  $\{u_{1,m}^e, u_{2,m}^e, \dots, u_{N,m}^e\}$ . When every follower is playing its equilibrium strategy, (17) becomes

$$V_{i,m} = \max_{w \in W} \min_{u_i \in U_i} \{L_i(w, u_i, u_{-i,m}^e) + V_{i,m+1}(g_i(x_i, u_i), \{u_{-i,l}^e\}_{l=m+1}^{t_p-1})\}. \quad (18)$$

These equilibrium strategies satisfy

$$u_{i,m}^e = \arg \min_{u_i \in U_i} \{L_i(w, u_i, u_{-i,m}^e) + V_{i,m+1}\} \quad (19)$$

for all  $i \in \{1, \dots, N\}$ . The objective now is to efficiently solve (19). We will propose an approximate solution in Section IV-B that uses an additional assumption. It can be shown that the set  $\{u_{1,m}^e, u_{2,m}^e, \dots, u_{N,m}^e\}$  are in fact a Nash equilibrium that satisfies

$$J_i(x_{i,0}, u_{i,\cdot}^e, u_{-i,\cdot}^e, w_{i,\cdot}^e) \leq J_i(x_{i,0}, u_{i,\cdot}, u_{-i,\cdot}^e, w_{i,\cdot}^e) \quad (20)$$

for all  $i \in \{1, \dots, N\}$  and  $u_{i,\cdot} \in U_i$  where  $w_{i,\cdot}^e$  are the maximizers of (18).

*Remark 1:* In the discussion until now, we have assumed no power loss during energy conversion. It is very well possible to incorporate power loss in the current framework. Consider the HEV topology shown in Figure 3. Assume there exists an efficiency map  $P_{aux} = \gamma(\sum_{i=1}^N u_i)$  to convert the aggregated power from all auxiliaries to the power requested at the engine side. This map is also supposed to be known to each auxiliary. With the knowledge of  $u_{-i}$ , follower  $f_i$  can express the engine power  $P_{ice}$  as

$$P_{ice} = \gamma(u_i + u_{-i}) + P_D. \quad (21)$$

Hence, (15) can still be formulated with  $x_{i,0}$ ,  $u_{i,m}$ ,  $w_m$  and  $u_{-i,m}$ .

### B. Online Implementable Strategies

Considering the limitations of online computational power in vehicles, we cannot implement the entire two-level game in real time at each time instance. Hence, to obtain online implementable strategies, we make the assumption for each follower that the equilibrium strategies from other followers remain constant within the prediction horizon, i.e.,  $u_{-i,0}^e = u_{-i,1}^e = \dots = u_{-i,t_p-1}^e$ . As a result, to compute the Stackelberg strategy of follower  $f_i$ , we only need the first element of  $\{u_{-i,l}^e\}_{l=0}^{t_p-1}$  rather than the entire sequence of it. As a result, (18) simplifies to

$$\hat{V}_{i,t}(x_i, u_{-i,0}^e) = \max_{w \in W} \min_{u_i \in U_i} \{L_i(w, u_i, u_{-i,0}^e) + \hat{V}_{i,t+1}(g_i(x_i, u_i), u_{-i,0}^e)\}. \quad (22)$$

The equilibrium strategies  $\{u_{1,0}^e, u_{2,0}^e, \dots, u_{N,0}^e\}$  can be found by the following algorithm, if convergence is achieved.

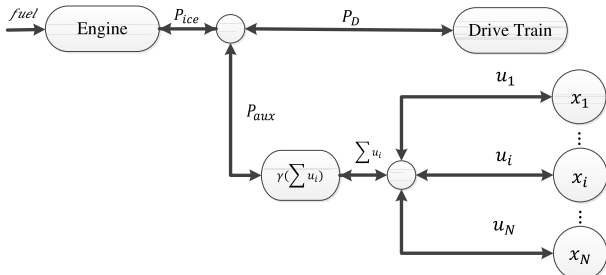


Figure 3. HEV topology with  $N$  auxiliaries

Set  $k = 0$ , (23)

Let  $\hat{u}_{-i}^k = \hat{u}_{-i}^0$ , for  $i = 1, \dots, N$

Repeat

$k = k + 1$ ,

for  $i = 1, \dots, N$

$$\hat{u}_i^k = \arg \min_{u_i \in U_i} \{L_i(w, u_i, \hat{u}_{-i}^{k-1}) + \hat{V}_{i,1}\}, \quad (24)$$

end

Until  $\hat{u}_i^k \approx \hat{u}_i^{k-1}$  for all  $i \in \{1, \dots, N\}$

Let  $u_{i,0}^e = \hat{u}_i^k$ , for  $i = 1, \dots, N$ . (25)

$\hat{u}_i^k$  is the estimate for  $u_{i,0}^e$  at the  $k$ th iteration and  $\hat{u}_{-i}^0$  are initial guesses for  $u_{-i,0}^e$ . To further reduce the online computational burden, we split the two-level game into an offline and an online part. The first level sequential game is played offline where we grid  $w$ ,  $x_i$  and estimate for  $u_{-i,0}^e$  over the admissible space. On each grid point, (22) computes the Stackelberg strategy for follower  $f_i$  at the first step of the prediction horizon, i.e.,

$$\hat{u}_{i,0} = \arg \min_{u_i \in U_i} \{L_i + \hat{V}_{i,1}\}. \quad (26)$$

$\hat{u}_{i,0}$  is then stored in a lookup table. Once every follower gets its lookup table, the second level simultaneous game takes place online where a receding horizon principle is adopted. At each sample time, (23) - (25) are carried out. Because (26) is already stored in a lookup table for every auxiliary, (24) involves finding values in the lookup tables corresponding to the given  $w$ ,  $x_i$  and the estimate for  $u_{-i,0}^e$  at  $k-1$  iteration, i.e.,  $\hat{u}_{-i}^{k-1}$ . Figure 4 illustrates the online iterative process using lookup tables and a central unit called the Energy Management System Operator (EMSO).

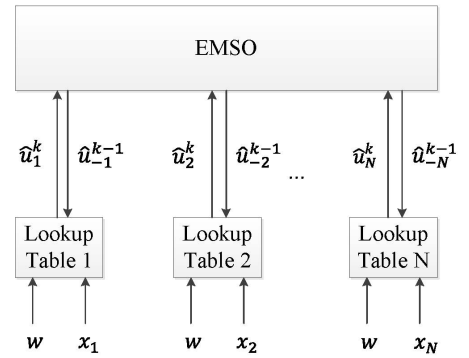


Figure 4. Online simultaneous game

In Figure 4, lookup table  $i$  contains the Stackelberg strategy  $\hat{u}_{i,0}$  obtained offline for follower  $f_i$ . At each time instant, every  $f_i$  is informed of the current value of  $w$  and  $x_i$ . With this information, it proposes its strategy  $\hat{u}_i^1$  by making an initial guess of  $\hat{u}_{-i}^0$ . This information is gathered by the EMSO. After collecting decisions from all the followers, the EMSO sends  $\hat{u}_{-i}^1$  back to each  $f_i$ . Based on the updated information of  $\hat{u}_{-i}^1$ , each  $f_i$  then updates its decision  $\hat{u}_i^2$ . If it's possible, this



procedure is repeated until an equilibrium is reached such that  $\hat{u}_i^k \approx \hat{u}_i^{k-1}$  for all  $i \in \{1, \dots, N\}$ . The equilibrium strategies are implemented for all the followers at that time instant. And at the next sample time, this iterative process is repeated again. Due to the fact that the assumption  $u_{-i,0}^e = u_{-i,1}^e = \dots = u_{-i,t_p-1}^e$  may not always be valid, the equilibrium strategies implemented at each sample time are just approximations of the Nash equilibrium described in Section IV-A2.

## V. CASE STUDY

### A. Simulation Conditions

To solve the problem formulated in Section III, the game-theoretic approach is implemented on the HEV model described in Section II and evaluated for a typical long-haul drive-cycle. The drive-cycle is also used to construct  $G(w)$  shown in Figure 5. The x and y axis correspond to the requested wheel rotational speed and torque at the input side of the gearbox, respectively. The value of  $G(w)$  at a certain operating condition  $w$  indicates the static probability of such condition to occur in the long-haul drive-cycle. In the game-theoretic controller design, a large value is assigned to  $\beta$  in (15) to express confidence that the driver will act according to the statistics in  $G(w)$ .

For the driver's decision space, we assume the vehicle's inertia to be infinitely large. Therefore,  $\omega$  does not change within the prediction horizon. As a result, the decision space for the driver reduces to one dimension as  $W = \{(\tau, \omega) | \tau_{min} \leq \tau \leq \tau_{max}, \omega = \omega_0\}$ .

### B. Strategies Evaluation

With the above given conditions, the following strategies are compared:

1) *Baseline*: The baseline strategy assumes no battery usage and the refrigerated semi-trailer is fixed at  $6^\circ C$ .

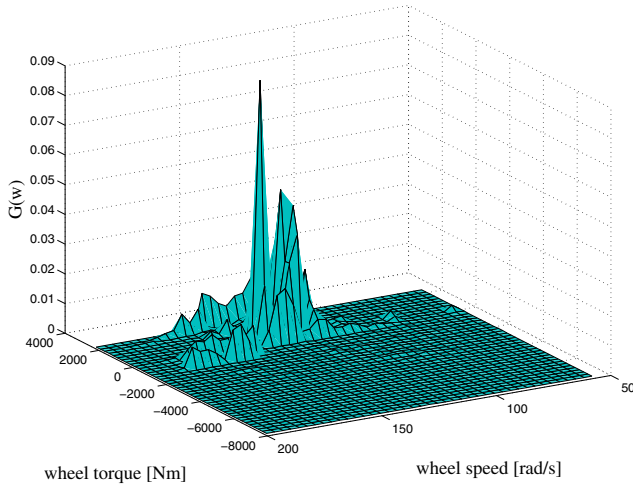


Figure 5. Function  $G(w)$  in (15)

2) *Equivalent Consumption Minimization Strategy (ECMS)*: In this case, the approach denoted as QP1 in [8] is used to compute the optimal  $P_s$ . Similar to [8], the equivalent consumption factor  $\lambda$  is adjusted with a PI controller. The semi-trailer cooling power  $P_{elec}$  is fixed similar to baseline strategy.

3) *Smart Battery Only (SBO)*: In this case, the optimal battery power  $P_B$  is determined using the online implementable game-theoretic approach (Section IV-B). The prediction horizon  $t_p$  is chosen as 8 for the smart battery. Next, the semi-trailer cooling power  $P_{elec}$  is fixed similar to baseline strategy and ECMS.

4) *Smart Battery+Smart Trailer (SBST)*: In this case, the optimal battery power  $P_B$  and the semi-trailer cooling power  $P_{elec}$  are determined both using the online implementable game-theoretic approach (Section IV-B). The prediction horizon for the smart trailer is 4 rather than 8 (since the semi-trailer has a smaller buffer capacity compared to the battery).

5) *Dynamic Programming (DP)*: Given the exact drive-cycle information, dynamic programming is carried out to determine the optimal battery power and the semi-trailer cooling power corresponding to the following cost function:

$$J_{DP} = \sum_{t=0}^{t_f-1} h \cdot \dot{m}_{fuel}(t) + \mu_{DP,1}(E_s(t_f) - E_s(0))^2 + \mu_{DP,2}(T_{air}(t_f) - T_{air}(0))^2 \quad (27)$$

where  $t_f$  denotes the complete length of the drive-cycle;  $\mu_{DP,1}$  and  $\mu_{DP,2}$  are tuning parameters such that  $E_s$  and  $T_{air}$  go back to their initial values at the end of the drive-cycle. Table I summarizes the parameters used in the simulations. In all simulations, the end values of  $E_s$  and  $T_{air}$  get close enough to their initial values such that no correction is needed for the fuel consumption result.

Table I. SIMULATION PARAMETERS

Name	Value	Name	Value
$P_{B,min}$ [kW]	-110	$P_{B,max}$ [kW]	110
$P_{elec,min}$ [kW]	0	$P_{elec,max}$ [kW]	5.8
$E_{min}/E_{bat}^*$	0.3	$E_{max}/E_{bat}^*$	0.7
$T_{air,min}$ [ $^\circ C$ ]	4	$T_{air,max}$ [ $^\circ C$ ]	6
$\gamma_{bat}$ [-]	$1.12 \times 10^{-6}$	$C_{reef}$ [J/K]	35000

\* $E_{bat}$  denotes the maximum energy that can be stored in the battery.

### C. Simulation Results

The fuel consumption results are summarized in Table II. It can be seen that SBST results in a close to optimal fuel consumption compared to the solution derived from DP. Another observation is that SBST brings 0.08% more fuel savings compared to SBO. This is because SBST utilizes both the buffer capacities of the battery and the refrigerated semi-trailer. One scenario is illustrated in the Figure 6. In this figure, it is shown that when braking occurs, the free regenerative energy is stored in the battery and the semi-trailer proportionally. It can be observed that the battery takes the regenerative energy when the braking power is large and the semi-trailer starts to take over when the braking power drops.

In this application, the semi-trailer shows a relatively small potential for fuel savings because a tight temperature bound is imposed and the heat capacity of the air inside the semi-trailer is small. Besides the fuel benefits, another advantage of the game-theoretic approach is that the intelligence of the semi-trailer is added without re-calibration of the intelligence of the battery which shows the plug and play feature.

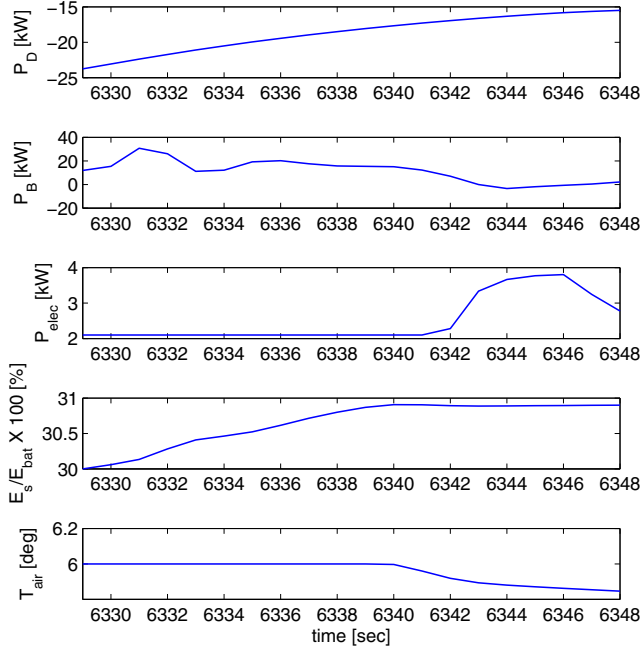


Figure 6. Regenerative energy stored in both the semi-trailer and the battery (lower plot)

Table II. FUEL CONSUMPTION OVERVIEW

	Baseline	ECMS	SBO	SBST	DP
Fuel consumption [kg]	60.08	57.43	57.16	57.11	56.91
Percentage [%]	100	95.59	95.14	95.06	94.72

## VI. CONCLUSION

This paper describes a game-theoretic solution concept for the CVEM problem. The solution concept is based on a single-leader multi-follower game model. The CVEM problem is solved through a two-level game resulting in online implementable strategies using lookup tables. This approach shows a plug and play potential as it provides a modular solution for each auxiliary and limits the information sharing amongst auxiliaries. A case study is carried out for a parallel HEV. The game-theoretic approach shows a close to optimal performance in the simulation.

## REFERENCES

- [1] T. Başar and G. J. Olsder. *Dynamic Noncooperative Game Theory*. SIAM, Philadelphia, PA, USA, 1982.
- [2] D. P. Bertsekas. *Dynamic Programming and Optimal Control*. Athena Scientific, Belmont, MA, 1995.

- [3] S. Delprat, J. Lauber, T.-M. Guerra, and J. Rimaux. Control of a parallel hybrid powertrain: optimal control. *Vehicular Technology, IEEE Transactions on*, 53(3):872–881, May 2004.
- [4] C. Dextreit and I.V. Kolmanovsky. Game theory controller for hybrid electric vehicles. *Control Systems Technology, IEEE Transactions on*, 22(2):652–663, March 2014.
- [5] H. P. Geering. *Optimal Control with Engineering Applications*. Springer, Berlin, 2007.
- [6] J.T.B.A. Kessels, J.H.M. Martens, P.P.J. van den Bosch, and W.H.A. Hendrix. Smart vehicle powernet enabling complete vehicle energy management. In *Vehicle Power and Propulsion Conference (VPPC), 2012 IEEE*, pages 938–943, Seoul, Korea, Oct 2012.
- [7] I. Kolmanovsky and I. Siverguina. Feasibility assessment and operating policy optimization of automotive powertrains with uncertainties using game theory. In *ASME International Mechanical Engineering Congress and Exposition, Proceedings*, volume 2, pages 1189–1196, 2001.
- [8] M. Koot, J.T.B.A. Kessels, B. de Jager, W.P.M.H. Heemels, P.P.J. Van den Bosch, and M. Steinbuch. Energy management strategies for vehicular electric power systems. *Vehicular Technology, IEEE Transactions on*, 54(3):771–782, May 2005.
- [9] A.A. Malikopoulos. Supervisory power management control algorithms for hybrid electric vehicles: A survey. *Intelligent Transportation Systems, IEEE Transactions on*, accepted for publication 2014.
- [10] C. Musardo, G. Rizzoni, and B. Staccia. A-ECMS: An adaptive algorithm for hybrid electric vehicle energy management. In *Decision and Control, 2005 and 2005 European Control Conference. CDC-ECC '05. 44th IEEE Conference on*, pages 1816–1823, Dec 2005.
- [11] G. Paganelli, T. M. Guerra, S. Delprat, J.-J. Santin, M. Delhom, and E. Combes. Simulation and assessment of power control strategies for a parallel hybrid car. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 214(7):705–717, 2000.
- [12] Gino Paganelli, Gabriele Ercole, Avra Brahma, Yann Guezennec, and Giorgio Rizzoni. General supervisory control policy for the energy optimization of charge-sustaining hybrid electric vehicles. *{SAE} Review*, 22(4):511 – 518, 2001.
- [13] P. Pisu and G. Rizzoni. A comparative study of supervisory control strategies for hybrid electric vehicles. *Control Systems Technology, IEEE Transactions on*, 15(3):506–518, May 2007.
- [14] A. Sciarretta and L. Guzzella. Control of hybrid electric vehicles. *Control Systems, IEEE*, 27(2):60–70, April 2007.
- [15] L. Serrao. *A Comparative Analysis of Energy Management Strategies for Hybrid Electric Vehicles*. PhD thesis, Department of Mechanical Engineering, The Ohio State University, Columbus, 2009.